Global Spatial Analysis †

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We now have available several books on spatial analysis {Anselin 1988; Arbia 1989; Cliff & Ord 1981;Cressie 1991; Haining 1990; Gaile & Willmott 1981; Griffith 1988, 1990; Ripley 1981, 1990; Unwin 1981, Upton & Fingleton 1985). This is most encouraging, but is it not strange that almost none of these works considers that the earth is homeomorphic to a sphere? The term sphere does not even occur in the index in most of these books. This is also true of older books like Berry & Marble (1968) and Bennet (1979). A similar criticism applies to all the books published in the last two decades under titles similar to "Statistics for Geographers", or "Statistical Geography", etc. This seems a most curious omission. The same comment seems to apply to the (literally) hundreds of "Geographical Information Systems" (GIS's). The one exception, explicitly designed to consider the spheroidal earth, is the "Hipparchus" system developed by Hrvoje Lukatela of Calgary, Alberta (Lukatela 1987). One other system, GIS-Plus, uses latitude and longitude for its coordinate system. Most GIS's can convert from latitude and longitude to map projections but do not treat the spherical units as their primary referencing system. And, although there has been a continued plea for more analysis capability, most of the current GIS's do not claim to be "Geographical Analysis systems" (GAS's). Facilitation of analysis is the thrust of a current (1992) National Center for Geographic Information and Analysis (NCGIA) study initiative and I have argued that this should be extended to include analysis of global distributions (see Appendix).

The inference that I make from my observation is that the greatest demand is for parochial, local studies and not for global analysis. This is inconsistent with the increased interest in global problems, especially as they relate to the global environment, terrestrial warming, ozone depletion, and so on, and even to global economic relations. How can this "flat earth" syndrome be overcome and round earth thinking be brought into the textbooks, mainstream research papers and monographs, and GIS's or GAS's? Much blame must be put on the teaching of Euclidean geometry in the elementary schools instead of the more natural earth oriented Riemann (elliptical) geometry. It is probably hopeless to attempt to change this. Should everyone know that the circumference of a circle on a sphere increases as two pi times the sine of the radius, measured in radians as an arc on the surface, (which means it eventually goes to zero), and that the area of a circle increases in proportion to the square of the sine of one half of the circle's radius? Or that the circumference of a circle on the surface of an ellipsoid depends not only on the circle's radius but also on where one puts the center {Blaschke 1949)? No one seems to teach analytic geometry on the sphere; the most recent book which I have found is nearly 100 years old (Heger 1908). Knowing these sorts of things would make it much easier to understand the complexities of "geographical circles", the set of all places attainable within a given number of hours (or dollars). These circles have a circumference bounded by isochrones (isotims), the radii are time geodesics, and the circles' shape depends on where, and when, one begins to travel. These circles often have disjoint pieces, or holes, on the surface of the earth due to air travel. Thus transportation systems induce a geometry even more complicated than that of a sphere or ellipsoid.

The only fields in which global analysis routinely appears are geodesy, meteorology, and oceanography, collectively sometimes known as geophysics (Moritz 1980). Geologists today increasingly invoke spherical ideas, especially since the development of the theory of continental drift, and a statistician occasionally wanders into this domain {Mardia 1972, Watson 1983). Thus I can cite recent books such as that of Fisher, Lewis, & Embleton, (1987), Washington & Parkinson (1986), and Daley (1991), and older ones (e.g., Chapman & Bartels 1940) that consider the geometric nature of the earth. Generally these works are subject specific and specialized. Still they provide a starting place.

If one were to develop a course of studies introducing analysis on the earth, treated spherical, what should the class of problems to be treated cover? One approach is to take the standard statistics book and redo the problems on a sphere. The surface of the earth is topologically spherical, even with bumps like the Matterhorn; it is a two dimensional manifold lacking an edge, but with some discontinuities and unsmooth derivatives, and is perhaps even fractal-like if we don't look too closely. The boundaries above and below are of no concern so that one is immediately into two dimensional spatial statistics [which, as Bunge (1966) would remind us, neglects the danger from intercontinental missiles]. First, of course, come the two dimensional descriptive statistics, frequency histograms, density functions, means, variances, binomials, Gaussian normals, transformations, etc., all on the spherical surface. About the only idea from this set that one routinely encounters is the population center of the United States and how it moves from census to census. Even the few papers on centrography generally only consider planar values. Then it perhaps gets more interesting. Scatter diagrams and correlation between observations given by latitude and longitude. We need to know how to rotate to principal axes, a pair of orthogonal great circles; Factor analysis on a sphere would need this. Or fit a "straight line", which now becomes an arc of a great circle through a set of point locations; that is, find the pole of this great circle on the sphere. Or does a spherical quadratic (small circle, parabola, or hyperbola), cubic, quartic, or quintic fit better? Or a loxodrome, or some other transcendental curve? Or a nice spline, or a weighted nonparametric smooth curve? Here we have some types of questions addressed in Fisher, Lewis, & Embleton, (1987). Is this interesting? Do we need spherical markerboards for the classroom demonstrations? Now move on to the analysis of spherical point patterns in the style of planar analysis {Diggle 1983, Getis & Boots 1978). A fleeting reference to Voronoi polygons on the sphere is found in Okabe, Boots, & Sugihara, (1992). Quadrat analysis must obviously be modified to become quadrilateral analysis. Or one can map the surface of the earth into an equal area planar square (Tobler & Chen 1986) thereby distorting angles and distances, or a conformal square (Peirce 1879) to distort areas and distances. In geology the equal area Schmidt net (known to geographers as Lambert's (1772) azimuthal equal area map projection) and the stereographic projection are often used. Spatial sampling takes a different form on spheres {Giacaglia & Lundquist 1972; White, Kimerling & Overton 1992). If we have some variable discovered or measured at point (i.e., latitude and longitude) locations we are led, inter alia, to interpolation questions long studied in meteorology under the name of "objective analysis" (Gandin 1963; Bengtsson, Ghil, & Kallen, 1981; Thiebaux & Pedder 1987), or "spatial analysis" (Daley 1991, p. 30), or as "collocation" in geodesy (Moritz & Suenkel 1978), and recently popularized in geology and ecology as "Kriging". Most of this work is Euclidean and planar but interpolation and contouring on a sphere has not been completely neglected (Lawson 1984a, 1984b; Renka 1984; Diggle & Fisher 1985; Willmott, Rowe, & Philpot, 1985) and the problem has also been approached using spherical splines {Wahba 1981; Dierckx 1984; Hobbs 1985). To simplify a global distribution we can do a form of trend analysis or spatial filtering on the sphere, using spherical harmonics as the basis functions (Neumann 1838; Prey 1922; Haurwitz & Craig 1952; Jones & Gallet 1962; Heiskanen & Moritz 1967; Balmino, Lambeck, & Kaula, 1973; Barraclough 1978; Colombo 1981; Tobler 1992). Then it is also natural to ask whether the data are auto-correlated, or cross-correlated with another variable on the earth. For analysis purposes this leads to the computation of a spectrum, and the spectra of two spherical arrangements can be used to measure their degree of relatedness through the cross spectrum (Kaula 1967; Rayner 1971; Schwartrauber 1979).

The usual distinctions can still be made. Some observations are measured on a nominal scale, some are ordinal, some interval and some are ratio variables. The analyses can be parametric or nonparametric, confirmatory or exploratory (leading to EGAS's ?). The bulk of the geophysical literature treats all phenomena as fields. This means that they exist everywhere, in space and time. Usually they consist of ratio variables that are at least continuous and differentiable. A few odd things occur such as the piecewise continuous Ocean Function; it takes on the value one where the surface of the earth is ocean, zero elsewhere {Munk & MacDonald 1960; Lambeck 1988). In most of the literature the prototypical variable is a scalar; a single number (having magnitude only) at each location. The two dimensional topographic surface, measured as a departure from a datum, or a barometric pressure surface, are simple examples. Beyond this are the time-varying vector fields (with magnitude and direction) such as wind, or components of terrestrial magnetism. Occasionally a field variable is a tensor; for example a matrix of strain coefficients at each point. Another example is an anisotropic velocity surface in which a propagation velocity at a point can differ depending on whether movement is to the north or to the south across the point, or to the east or to the west or to the northwest, etc. In other words the traversing speed at every point depends on the direction moved across that point. A simple instance is a freeway in which the speed along the path differs drastically from that in the perpendicular direction (across the road). In the world of Geographic Information Systems the field variables seem to appear as rasters, with each component of a multicoinponent system considered a sample on a regular tesselation, and then stored as a "layer". This often works well but I imagine it causes difficulty when one wants to convert to a different naming convention, such as converting from rectangular to polar coordinates, or (on a sphere), converting to an oblique pole. In such a case a vector field does not change at all, but all of the components take on new numerical values. This means changing the values in all of the appropriate layers.

From a different point of view one can analyze point patterns on a sphere, or line patterns, or networks, or area patterns, etc. These distinctions are made in Geographic Information Systems, but geographic analysts face problems that are different from those that occur in geophysics. For example the Gross National Product (GNP) is a variable defined as a spatial aggregate having meaning only for nation states. Should it be assigned as a density to all the locations to which it applies, or should it be assigned to a point location? The first option is probably preferable since it can then be treated as a piecewise continuous scalar. Or is it a set variable requiring a new way of thinking? Rates (of disease incidence for example) involve comparable problems. Similarly, foreign trade data fit into a country by country table applicable to the entire world, and as such are not representable as a simple scalar. More complex analyses are required for such data than are necessary for the simpler scalar, vector or tensor representable phenomena. And then there is the ill-defined problem of modifiable areal units, and the problem of data conversion between such units (Tobler 1990). For this problem it is relatively simple to rewrite the pycnophylactic reallocation algorithm (Tobler 1979, 1996) in a spherical form. Many classical geographical models relating to interaction, diffusion (Hägerstrand 1967), migration (Dorigo & Tobler 1983), and location-allocation also ought to be rewritten in this spherical manner. If we look beyond statistics to include modeling then our pedagogic work ought to include the two dimensional spherical variants of classical analysis: analytic geometry, trigonometry, vector analysis, calculus {Osserman 1977), partial differential (especially the diffusion and the wave) equations, finite differences or elements, networks, transition matrices, etc. all on the surface of a sphere (or ellipsoid). As discussed elsewhere (Tobler 1991) we really wish to use the computer systems (GIS's) as backdrops upon which to run simulations or analytical models.

The usual flat earth treatment assumes only a small portion of the sphere. Small enough that earth curvature can be neglected; about 300 km on a side. Then one uses flat maps on a projection chosen so that the error is not appreciable. One can teach people how to compensate for map projection errors (Maling 1988), but it seems easier just to forget about working on maps. Instead, use map projections in the Transform - Solve - Invert paradigm (Kao 1967; Eves 1980). For example, in problems dealing with great circles it makes sense to convert from latitude and longitude to gnomonic projection coordinates, solve the problem in these most - natural - for - the - problem coordinates, and then go back to latitude and longitude or to a different projection for another part of the problem or, e.g., for display purposes to the orthographic projection. The map on the gnomonic projection never needs to be seen. Mercator's projection can be used similarly when dealing with angles in a small area, and the stereographic projection is used in meteorology because it leaves the Laplacian invariant. The associated Legendre functions used as the orthonormal basis for the latitudinal variation in a spherical harmonic expansion make implicit use of Lambert's equal area cylindrical map projection in this same way (Lambert 1772; Hobson 1931). A greater facility with spherical geometry might confine some explicit uses of projections to these sorts of analytical transformations. Certainly the computation facilities for this are now available. Visual displays are of course extremely important to all analytical investigations, especially exploratory ones, and map projections are still useful for this purpose. Clearly there is a need to extend GIS's to deal with and display spherical data, i.e., we must have an Exploratory Global Analysis and Display System (EGADS). For the output of geographical studies the systems must be capable of producing maps of the whole world. But one computer atlas now eschews all conventional map projections and instead uses an image of a globe that can be rotated to be positioned for a clear view at any desired location - it uses a perspective view of the earth for this. In this sense map projections, my specialty, are now obsolete. Reporting information in latitude and longitude suffices since computations in global coordinates is relatively simple, and should always be considered.

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ABSTRACT
It would be expected that geographers doing spatial analysis would take into account the curved nature of the earth. But they generally do not. This essay explores some directions that such analyses might take.

KEYWORDS: Spherical earth, geographic study

APPENDIX
Anonymous reviewers have cited relevant papers by Bartholdi & Goldsman (2001), Dutton (1999), Miller(2000), Raskin (1994), Raskin, Funk, & Webber (1997), Tobler (2001), and by Yang, Snyder, & Tobler (2001) that have been published since the 1992 presentation. These have been added to the references.

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