

MEDIEVAL DISTORTIONS: THE PROJECTIONS OF ANCIENT MAPS[†]

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ABSTRACT. Estimates of the map projection employed for an ancient map is a prerequisite for a variety of other studies. The preliminary evaluation presented here has yielded empirical equations for the Hereford map and illustrated the agreement of a portolan chart with an oblique Mercator projection

The study of ancient maps provides one of the fascinating aspects of historical geography. Such maps can be analyzed for many purposes and from several points of view. The following comments refer only to the estimation of the map projection implied by the ancient mappae mundi and portolan charts. Evaluation of the map projection of these old maps is of assistance in the determination of the accuracy of the maps, and may provide insight into their method of construction. Modern theories regarding the ancients' perception of the world also may require consideration of the map projection employed for maps.

The maps in the two classes under investigation do not contain any indication of the terrestrial graticule of latitude and longitude. This has led some students to conclude that the maps are not based on any map projection.¹ This point requires clarification. Certainly the lack of the graticule does not imply the absence of a projection. Even modern maps are occasionally published without this grid.² More telling is the high probability the sphericity of the earth was unknown to, or was not considered relevant by, the individuals who constructed the maps. If this is the case the maps would be constructed as though the earth were flat. Inconsistencies between the plotting and the observational information then might arise; these inconsistencies could be attributed to the (unavoidable) errors in one or the other, or both. For a small area the errors and inconsistencies might be quite small and could go unnoticed. Inconsistencies are not necessary or inevitable, however. No set of observational information specifying the location of any terrestrial position by not more than two independent measures will lead to inconsistencies when plotted. This is true whether the earth is considered round or flat. In either of the above events it is correct to say that the map is not based on a map projection *only* in the sense that the cartographer involved was not consciously employing a map projection.³ But, as one learns from any elementary work on map making, every map requires a map projection. The ancient maps therefore are implicitly referred to some map projection.

The next difficulty, it seems, occurs if it is assumed that this implicit projection is one of the now-known projections. For example, the portolan charts have been compared with charts drawn on Mercator's projection and on the square projection.⁴ Suppose that the match is sufficiently poor to conclude that the chart is not drawn on either of these two projections. This does not prove that the chart is not based on a map projection; such a conclusion can in fact never be drawn if one accepts the notion of an implicit map projection. The search, then, must continue for a map projection, that may be any of the several hundred now known, or may be one that is completely unknown today. The problem can be attacked from several directions. Most helpful is an examination of the method of construction of the maps, if this is known. Typically this turns out to be inconclusive, and is in fact one of the questions that many have attempted to answer. But there are some hints that allow the range of possible projections to be narrowed down to a relatively limited few. A second obvious approach is to attempt to sketch the lines of latitude and

longitude on the map, as estimated by identification of locations shown thereon. Examination of the graticule, its curvature and so on, should provide hints as to a reasonable family of projections.

A crucial point has now been reached. On the basis of some study it is postulated that "this" projection forms the basis for the map. How is this hypothesis to be tested? The test generally consists of superimposing a map drawn on the postulated projection over the original map, with a scale adjustment and shifting until the best average coincidence is obtained. Since one knows that the agreement of the two maps will not be perfect, the question is now one of deciding how much agreement is necessary before the hypothesis is to be accepted. The procedure outlined below does not answer this question, but it does allow one to say, with relative precision, how great the agreement is, and thus permits one to rank, from greatest to least agreement, all projections for which one cares to carry through the necessary operations. The method requires extensive observations and computations but is perfectly general and may be applied whenever it is desired to ascertain the agreement of a questionable map with a particular map projection. The necessary calculations may seem formidable but entail less than five minutes on a modern high-speed digital computer.

To begin it is necessary to identify a large number of points on the map. By identify is meant to record the modern latitudes and longitudes of these points. The next step is to record the map coordinates of these same points. Any system of coordinates will serve but rectangular (x , y) coordinates are the most convenient. It does not matter in the least what units are used for the coordinates, and it is not necessary to determine the scale of the chart or any distances thereon.⁵ Nor does the orientation of the grid system matter. The postulated map projection will be defined by a mathematical relation of the form:

$$X = f(\phi, \lambda), \quad Y = g(\phi, \lambda),$$

where ϕ is the latitude and λ is the longitude, and X and Y are the corresponding map projection coordinates. Using this relation calculate the map projection coordinates of all of the points identified by latitude and longitude. This is where the points should be if the projection were to give a perfect fit. These calculated coordinates (X , Y) are now to be compared to the observed map coordinates (x , y). Since the coordinates employed for the recording of the observations were quite arbitrary, a different result would be obtained for each possible set of recording coordinates. It is therefore necessary to apply the mathematical equivalent of rotation and change of scale, as might be obtained by use of an optical reducing-enlarging instrument, to give the best possible overall average fit. This is given by a least squares Euclidean transformation, which brings the numbers given the recorded observations into the same units as the calculated map projection coordinates (see Appendix). The comparison of the map projection with the map is now made by calculating the difference between the observed and calculated locations for corresponding points. The correlation coefficient between the two sets of coordinates gives the amount of agreement, and areas of greatest disagreement may provide hints as to a more suitable map projection. With the same observational information the entire procedure can be repeated a second time for a new map projection, and so on.

As one already has recorded, for the foregoing operations, the latitudes and longitudes of a large number of places along with the map locations (in some arbitrary coordinate system) of these same identified points, one can continue by obtaining an empirical estimate of the equations defining the map projection. The problem can be phrased thus: find the equations which, when entered with any specific latitude and longitude value, result in the observed x and y coordinate values. The mathematical theory indicates that this can always be achieved with a

sufficiently complicated equation. In practice it is difficult to obtain a perfect fit and one is generally content with an equation which reproduces the observations with a high degree of reliability. This is reasonable since it is not desired to reproduce minor errors, such as might be caused by the shrinkage which has occurred on a 500-year-old map. The mathematical fitting procedure generally employed is the method of least squares polynomial or Fourier series curve fitting.⁶ The equations obtained in this manner enable one to calculate, and draw, the latitude and longitude graticule at any desired interval. Equally importantly, they allow calculation of Tissot's measures of map projection distortion.⁷ Formulation of the problem in terms of the distortion is independent of any hypothesis regarding the specific map projection, but can be employed to infer whether or not the projection has specific properties. This in turn may be of assistance in determining the projection. A word of caution is necessary here since the necessary differentiation of the empirical equations may be subject to large errors.⁸ The same comment holds true if a graphic determination of Tissot's measures is applied to visually sketched lines of latitude and longitude.⁹

THE HEREFORD MAP

The Hereford map¹⁰ is one of the most famous of the surviving medieval mappae mundi (Fig. 1). Certainly it is one of the largest (53 x 65 inches). It is a product of the later Middle Ages, *circa* 1283, and is still in the cathedral at Hereford, England. In many respects the map represents a culmination of 1,000 years of mapping efforts, having precedents from Roman times. The map has been the subject of at least one book, and several monographs and articles.¹¹ At least three large reproductions of the map have been published. A detailed description of the map, therefore, is not necessary. The form of the map follows the typical circular T-in-O style, with Jerusalem in the center and Asia (paradise) at the top. In many respects the map is more a representation of religious reality than geographical reality. On the other hand, the map preserves most of the topological properties¹² of a map projection; if this were not the case the drawing would not be recognizable as a map.

The Hereford map is believed to have been prepared by copying some other ancient map, with supplementary information obtained from itineraries. This does not provide much assistance in an initial guess at a map projection. The many authors who have discussed the Hereford map have ignored, or rejected, the question of a map projection and no assistance can be gained here, though these authors are extremely valuable in providing identification of names. The form of the map, however, suggests an azimuthal projection, perhaps with a larger scale at the center of the map. The orthographic projection is one (of many) which has these properties. The empirical observations suggest that this is not a poor guess.

Konrad Miller's 3/7th size edition of the Hereford map was employed to identify 155 locations. This is a tedious operation and subject to error. Each identification consisted of map coordinates, and modern latitude and longitude.¹³ The dispersion of the observations is rather uneven since it proved impossible to identify anything in some parts of the map, particularly the margins. Misidentifications may have occurred, in which event the consequent analysis will be slightly distorted.

Direct correlations between the latitude and the y coordinate, and between the longitude and the x coordinate are high (Fig. 2). This is to be expected since the Hereford map preserves most of the topological properties of any map projection. The results of further computations indicate that the square projection provides a seventy-three percent match to the Hereford map, and that

an oblique orthographic projection centered at Jerusalem provides an eighty-four percent match. Using a polynomial approximation the locus of lines of latitude and longitude for the Hereford map can be reproduced with a fidelity of ninety-five percent. A computer print of these calculated lines in fact agrees quite well with a manually interpolated graticule. The details of these results are given in the Appendix.

Comparison of the Hereford map with only two known map projections cannot be said to provide an exhaustive study. Nor can it be said that the map is drawn on the orthographic projection, though this provides a better fit than does the square projection. Examination of the interpolated lines of latitude and longitude does, however, reinforce the hypothesis of European antecedents for the map, on the grounds that the larger scale will be in the vicinity of areas with which the cartographer is most familiar. The only apparent exception is the Jerusalem region.

PORTOLAN CHARTS

One of the most interesting classes of maps in the venerable history of cartography are the early sailing charts depicting the vicinity of the Mediterranean Sea. The oldest existent map is estimated to have been drawn in the latter portion of the thirteenth century. The fame of these representations rests in part on their accuracy relative to other European maps of the same period. When contrasted with the contemporaneous T-in-O maps, for example, the charts appear outstandingly more correct. The earliest of these sailing charts do not contain any indication of the terrestrial grid but carry an extensive set of symmetrical, criss-crossing lines. A voluminous though somewhat controversial, literature is available concerning the antecedents, method of compilation, construction, and employment of these charts.¹⁴

Several authors have come to the conclusion that the portolan charts are not based on a map projection¹⁵. This view has already been rejected here on *a priori* grounds, in accordance with the modern interpretation of map projections¹⁶. In contradistinction to the T-in-O maps, where the suggestion is usually rejected out of hand, the very great accuracy of the portolan charts has prompted questions concerning the map projection of these charts. The method of construction of the portolan charts, though not definitely known, does provide some hints. Perhaps the compass was involved and the observational information available for compilation consisted of loxodromic directions. This suggests Mercator's projection. The magnetic meridians¹⁷ are not coincident with the geographical meridians, however, so that a magnetic error should exist on the maps. A difference between "rose north" and true north can in fact be observed on the charts. Breusing then reasoned that the compass, if it contributed to the development of these charts, would yield readings resulting in a projection with curved parallels. This he took to require a conic projection. Fiorini came to the conclusion that the projection should be an oblique azimuthal equidistant, since both directional and distance information seem to have been available, and since the central rose provides a convenient point of departure for the plotting of the map. Another projection which has been proposed is the square projection. This choice is apparently derived from the subsequent use of this projection for sailing charts.

It should be possible to perform the operations previously outlined and applied in the case of the Hereford map to all of the foregoing projections relative to a portolan chart. The necessary identification is difficult since the maps themselves are rare (even facsimiles are rare), much of the script is difficult to read and translate, and determination of the modern equivalent locations is often impossible, or at best tedious. Unfortunately some previous scholars published only their conclusions, not the detailed observations employed in the analysis. This is especially true of Wagner's otherwise excellent study¹⁸ Wagner sketched lines of latitude and longitude on detailed

tracings of several portolan charts and demonstrated that the length of mile employed differs along the west coast of Europe from that employed in the interior of the Mediterranean Sea.¹⁹ He also illustrated an abrupt jog in the path of the parallels through Greece on the map and attributed this to a method of construction in which the map is made by piecing together separate, local charts. He concluded that the portolan charts are not based on a map projection, thus rejecting the notion of an implicit map projection. Wagner presented a careful analysis of all of these points. Steger proceeded in a less detailed but similar manner and concluded, on the basis of map interpolations, that the meridians and parallels are straight lines when sketched in on the charts. He thus refuted Breusing's suggestion. Steger's demonstration, however, can be disputed on two grounds. His study did not include the Black Sea, which is where the curvature might be most noticeable, and his meridians and parallels are virtually all determined by only two points. A more recent study of the portolan charts by Clos-Arceduc compared them to a Mercator chart at approximately the same scale. His reasoning was that Mercator's projection provides a better fit than does the square projection. He further illustrated that the eastern portion of the charts is too far north, only, it must be noted in relation to Mercator's projection.

Computation by the method employed for the Hereford map would allow estimation of the degree of association between the charts and each of the projections. Calculation of Tissot's measures of distortion from an empirical equation should also be of assistance. Thus, if Fiorini's postulate of an oblique azimuthal equidistant projection is correct, one should find that the one value of the linear distortion in some direction is always unity at every point on the map. Similarly if the charts are loxodromic they should be conformal, or very nearly so. A result demonstrating very minor and randomly distributed angular error would provide additional evidence for the use of the compass in the construction of the charts. This procedure, via Tissot's theorem, has the distinct advantage that it is independent of any particular *a priori* hypothesis regarding the nature of the map projection.

Two additional projections can be proposed for the portolan charts. The first of these is an oblique Mercator projection. The difference between rose north and true north can be taken as a guide to the obliquity. One such projection²⁰ is illustrated and compared with an outline from an early portolan chart in the figure (Fig. 4). The discrepancy at the eastern extremity of the map is less than for the previously proposed projections. The medians converge slightly as required by Breusing. An oblique or square projection would yield similar results, however. Comparison of several small tracings of photographs of early portolan charts further indicates that there exists a considerable variation between individual portolan charts. Suppose one were to take all the known portolan charts that do not contain any indication of latitude and longitude and compare them with some standard chart, perhaps the *Carte Pisane*. In each case the correlation will be somewhat less than perfect. A postulated map projection should not be expected to perform better. Perhaps an appropriate strategy would be to assume that the map projection of the portolan charts is determined if the variance between the postulated map projection is less than or equal to the variance between individual portolan charts.

A final postulated projection might be referred to as an "oblique magnetic Mercator projection." This can be conceived of as a Mercator projection based on magnetic meridians and parallels.²¹ On such a map the geographical grid would appear distorted in the vicinity of local magnetic anomalies. This is a very appealing hypothesis, but it is apparently not possible to determine the locus of the magnetic meridians in the years 1200 to 1300 A. D. so that the empirical curve fitting procedure appears appropriate. Such a magnetic Mercator should be approximately conformal and investigation of the angular distortion is suggested.

Without performing the actual analysis, it is not possible to draw further conclusions concerning the portolan charts. Any such analysis would need to be quite carefully done, since the charts will probably fit several map projections very closely.

CONCLUSIONS

It has been possible to answer only a few of the substantive questions concerning the possible methods of preparation of medieval maps. The strategy developed in this preliminary analysis, however, seems to offer some improvement over procedures employed in previous studies. For the student of the history of geography the techniques also allow estimates of the rate of cartographic progress. Imhof, for example, has recently published small illustrations of two old maps of the Canton of Zürich.²² On the basis of these rather small illustrations, the simple calculations imply an improvement of twenty-eight percent in the mapping of Zürich between 1566 and 1667, and an improvement of only two percent between 1667 and 1965. This result in turn suggests the hypothesis of an S-shaped growth curve for the history of positional accuracy on maps. The same approach as outlined here can be applied, with obvious modifications, to maps which do contain the latitude and longitude graticule, either to determine the perhaps unknown map projection, or to make estimates of the accuracy of the maps if the projection is known.

Appendix

The procedure employed in estimating the amount of correlation between the observed map locations and a specific map projection is given here. Let the complex number $W_j = x_j + iy_j$, ($i^2 = -1$) be the map location of the j^{th} point of identified latitude and longitude. Let $Z_j = X_j + iY_j$ be the location of this same point on some map projection. Transform the arbitrary map recording coordinates to a new system, $W' = x'_j + iy'_j$, by applying a translation, rotation, and change of scale. This is no more than an assignment of new numbers to the recording coordinates and in no way changes the interrelations of the points. The transformation is of the form $W' = A + B W$, where A and B are complex numbers determined by application of the least squares criterion in a manner such that the residual

$$\sum_{j=1}^N |W'_j - Z_j|^2$$

is a minimum.

The amount of correlation between the transformed observational locations and the locations on the postulated map projection (calculated from the estimated latitudes and longitudes) is then given as the ratio of the regression variance to the total variance, as in ordinary correlation methods. The square of this complex correlation times 100 provides a measure of the percentage agreement between the two maps. This entire procedure is repeated for each postulated map projection. In the case of the Hereford map there were three comparisons; the square projection, the oblique orthographic projection, and the polynomial approximation. The map projection computations were all performed for a sphere of unit radius.

The coefficients of the unweighted polynomial approximation ($R^2 = 0.945$) to the Hereford map are given in Tables 1 and 2. It is cautioned that terms cannot be dropped from this equation without changing the coefficients. The latitude (ϕ) and longitude (λ) values are to be entered in

radian units. The computations were performed with the assistance of the University of Michigan computing center.

Notes:

¹ A. E. Nordenskiold refers to these maps as paratropical; *Periplus, An Essay on the Early history of Charts and sailing Directions* (Stockholm: Bather translations, 1897).

² R. E. Dahlberg, "Maps without Projections", *The Journal of Geography*, Vol. 60 (1961), pp 213-18.

³Similar comments apply to an engineering survey of a small area.

⁴ H. Wagner, "Das Ratsel der Kompasskarten im Lichte der Gesamtentwickelung der Seekarten," *Verhandlungen*, XI Deutsches Geographentages, Bremen, 1895, pp. 65-87;

E. Steger, "Untersuchung über italienische Seekarten des Mittelalters auf Grund der kartometrischen Methode" Dissertation, Gottingen, 1896;

M. Fiorini, *Le projezioni delle carte geografiche*, Bologna, 1881;

A. Breusing, "Zur Geschichte der Kartographie," *Zeitschrift fur Wissenschaftliche Geographie*, 11(1881), p. 168 ff;

M. A. Clos-Arceduc, "L'Enigme des Portulans: Etude sur la Projection et le mode de construction des cartes à rumbz du XIV^e et du XV^e Siecle," *Bulletin*, Comité des Travaux Historiques et Scientifiques, Section de Géographie, LXIX (1956), pp. 215-31.

The square projection is also known by the names plate carrée, simple cylindrical, and cylindrical equal-spaced projection.

⁵ Readers of the earlier treatises (especially Wagner, *op. cit*, footnote 4) will recognize this as a distinct advantage.

⁶ For a discussion of curve fitting see any elementary work on numerical analysis or an intermediate work on statistical methods. If there are some locations that can be identified with a greater degree of reliability than others, a weighting procedure may be employed.

⁷ M. A. Tissot, Mémoire sur la Représentaion des Surfaces (Paris; Gautier-Villars, 1881)

⁸Cf., B. Arden, *An Introduction to Digital Computing* (New York: Addison-Wesley, 1963).

⁹ G. A. Ginzburg, "A Practical Method of Determining Distortion on Maps," *Geodezist* (Moscow), 10 (1935), pp. 49-57.

¹⁰ The materials in this section are summarized from: S. Jones, "The Hereford Map Projection," M.A. thesis, Department of Geography, University of Michigan, Ann Arbor, 1964, 39 pp., and are used with the kind permission of Ms. Jones.

¹¹ W. L. Bevan and H. W. Phillott, *Medieval Geography; An Essay in Illustration of the Hereford Mappa Mundi* (London: 1873);

K. Miller, *Die Herefordkarte* (Stuttgart, 1903);

G. R. Crone, *The World Map by Richard of Haldingham* (London: Royal Geographical Society, 1954) (with an extensive set of references).

¹² R. H. Bing, "Elementary Point Set Topology", *American Mathematical Monthly*, 67, 7 part II (Aug./Sept., 1960)

¹³ See Table 2.

¹⁴ K. Kretschmer, *Die Italienischen Portolane des Mittelalters*, Heft 13 of the Veröffentlichungen des Institutes für Meereskunde und des Geographischen Institutes an der Universität Berlin, 1909, 687 pp;

A. Cortesao, *Cartografia e Cartografos portugueses dos séculos XV e XVI* (Lisboa: 1935);

Y. Kamal, *Hallucinations scientifiques* (Leiden: E. J. Brill, 1937), 95 pp; for additional references see: W. W. Ristow, and C. E. LeGear, *A Guide to Historical Cartography*, 2nd ed. (Washington: Library of Congress, 1960).

¹⁵ Among these are Nordenskiold, Wagner, Bagarow, and M. Eckert.

¹⁶ Cf Clos-Arceduc, *op. cit.*, footnote 4.

¹⁷ Cf A. N. Strahler, *The Earth Sciences* (New York: Harper & Row, 1963), Figure 9.15, p. 152.

¹⁸ Wagner, *op. cit.*, footnote 4; and also see his "The Origin of the Medieval Italian Nautical Charts" (London: Report of the Sixth International Geographical Congress, 1895).

¹⁹ For a plausible explanation of the phenomena that differs from that given by Wagner, see Clos-Arceduc, *op. cit.*, footnote 4, p. 226.

²⁰ The oblique Mercator projection has been chosen so that the great circle passing through 37.77° N, 1° W and 37° N, 30° E becomes the projection equator. The axis of the projection is the great circle arc connecting the oblique pole at 55.55° N, 14.58° E. the angle between grid north and the geographical meridian is 1°50' E at the center of the map. An earlier attempt with an 8.25° obliquity (as suggested by Clos-Arceduc) provided a poorer fit.

²¹ This postulate can also be found in Clos-Arceduc, *op. cit.*, footnote 4, p. 222.

²² E. Imhof, "Beiträge zur Geschichte der topogrphischen Kartographie", *International Yearbook of Cartography*, 4 (1964), pp. 129-53.

[†] *Annals, Association of American Geographers*, 56, 2 (June 1966), pp. 351-360

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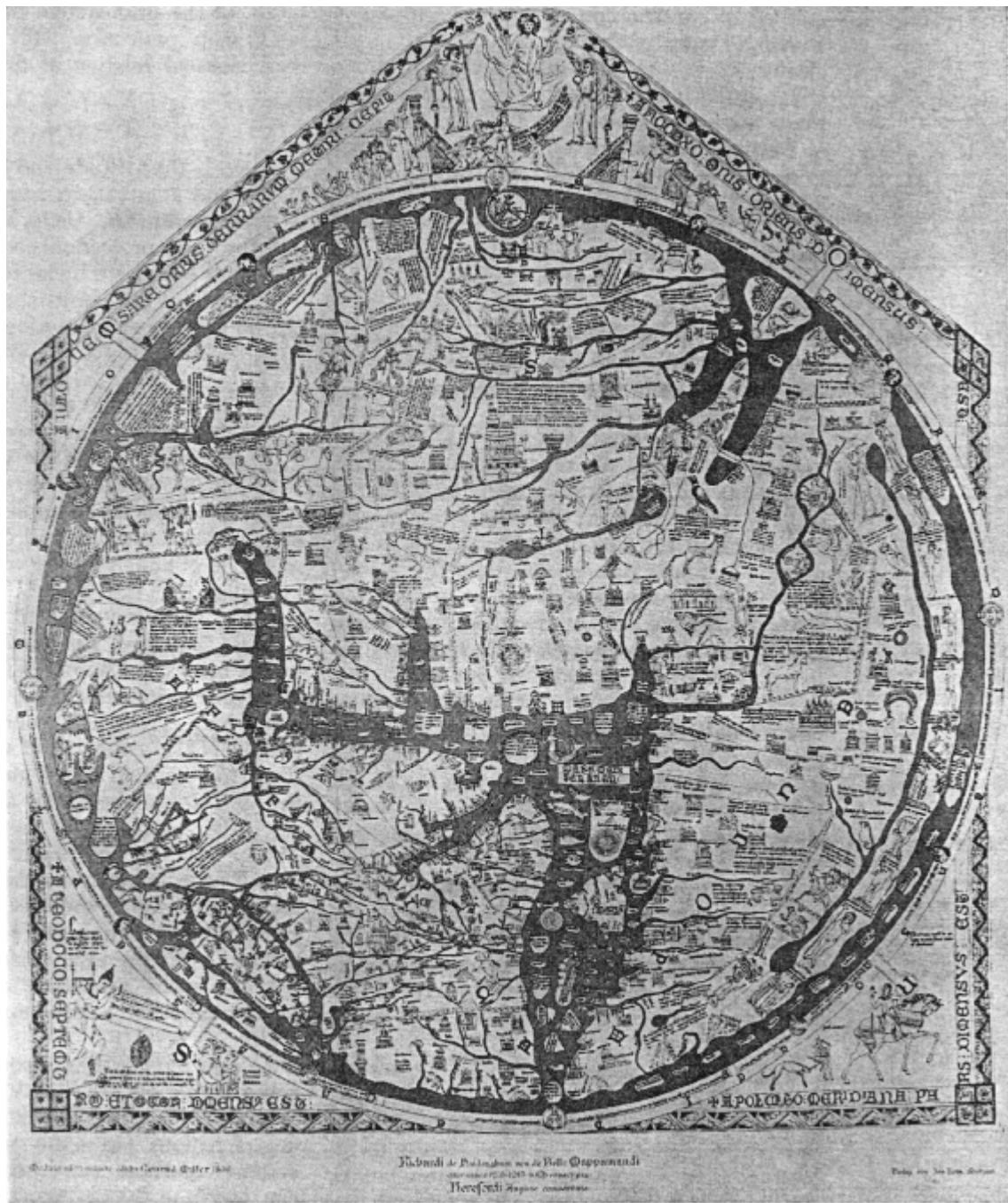


FIG. 1. Konrad Miller's edition of the Hereford Map.

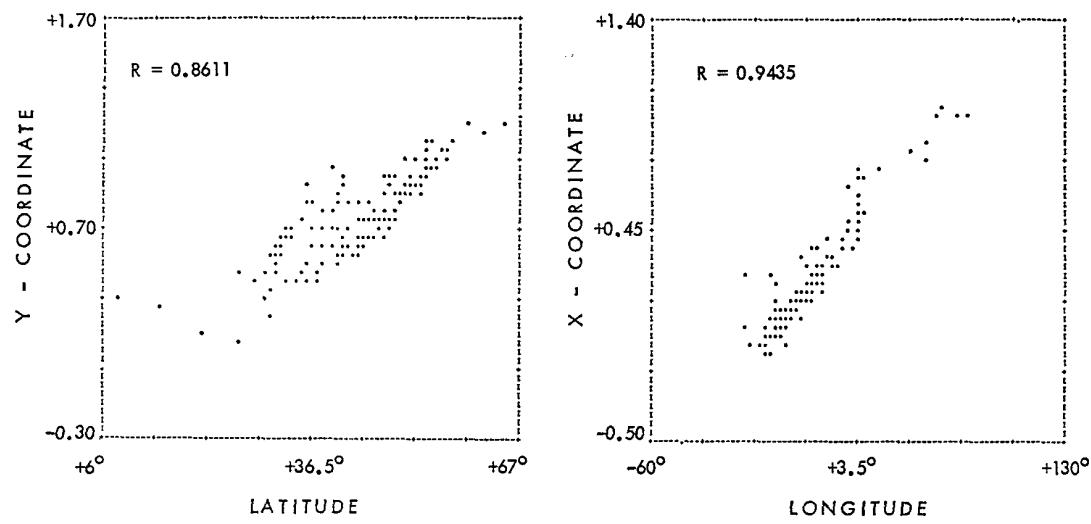


FIG. 2. Scatter diagrams illustrating the correlation between latitude and longitude and rectangular coordinates (after rotation and scaling) on the Hereford Map.



FIG. 3. Sketch of the latitude and longitude graticule as interpolated for the Hereford Map.

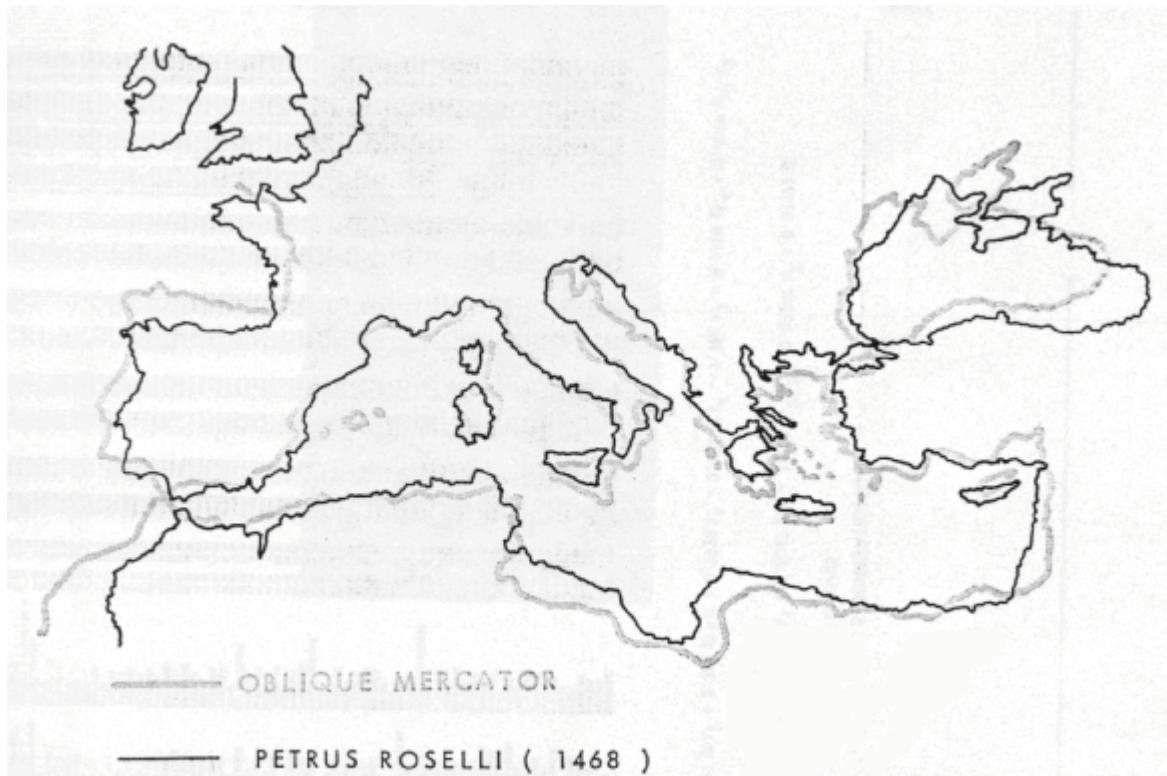


FIG. 4. Comparison of the outline from the Petrus Roselli portolan chart of 1468 with an outline on an oblique Mercator projection.

Table 1: Polynomial Approximation to the Hereford Map based on 155 observations
Standard error of the estimate, in radians; $S_x = 05176$; $S_y = 0.05785$

$$X = 0.2174 + 0.4752 \lambda - 0.3383 \varphi^5 \lambda + 0.01529 \lambda^6 + 0.4530 \varphi^6 - 0.5129 \varphi^2 - 0.5587 \varphi \lambda^5 \\ - 1.258 \varphi^4 \lambda^2 + 1.657 \varphi^2 \lambda^4 + 0.6047 \varphi \lambda + - 1.179 \varphi^3 \lambda^3 + 0.09247 \lambda^2 + 0.01191 \varphi.$$

$$Y = 0.4399 + 2.956 \varphi^2 - 2.023 \varphi \lambda - 3.324 \varphi^4 \lambda^2 + 0.004231 \varphi^5 \lambda - 1.712 \varphi + 2.445 \varphi^2 \lambda^4 \\ + 0.0591 \varphi \lambda^2 + 2.305 \varphi^2 \lambda + 0.3928 \lambda^6 - 3.934 \varphi \lambda^4 - 1.186 \lambda^2 - 0.5789 \varphi^4 + 0.4399 \varphi^3 \lambda^3.$$

TABLE 2

OBSERVED AND CALCULATED VALUES FOR THE HEREFORD MAP

NUMBER	MAP NAME	MODERN NAME	GEOGRAPHICAL COORDINATES	MAP COORDINATES ¹	SQUARE PROJECTION COORDINATES ²	ORTHOGRAPHIC PROJECTION COORDINATES ³	POLYNOMIAL COORDINATES ⁴	VALUES FROM TISSOT'S INDICATRIX ⁵									
								LAT.	LONG.	X	Y	X'	Y'	A	B	S	2 W
1	ORCADES	ORKNEYS	59 00 -3 00	.21142 1.18544	-.05236 1.02974	.08039 1.02353	.18555 1.1451	3.15594	.56482	1.78255	88.2768						
2	YSLAND	ICELAND	65 00 -18 00	.24464 1.18423	-.31416 1.13446	.03088 1.14561	.25106 1.1637	4.70039	1.23402	5.80779	71.4054						
3	FAURE I	FAURES	62 00 -6 30	.25216 1.16867	-.11345 1.08210	.07493 1.07755	.26736 1.2077	3.90939	.13095	.51195	138.5137						
4	FL WISARA	WISARA I.	53 30 8 25	.14696 1.16671	.14690 0.93375	.15938 0.91888	.17807 1.0364	2.64599	.36401	.96317	98.5998						
5	FL ALBANA	ELBE R.	53 50 9 20	.16565 1.11339	.16290 0.93957	.16966 0.92428	.19032 1.0597	2.68443	.26637	.71559	110.0856						
6	FL FISTULA	VISTULA R.	54 00 19 00	.02616 1.11006	.033161 0.94830	.27108 0.9354	.23635 1.2272	2.71272	.16376	.44423	124.7843						
7	ABERTEENE	ABERDEEN	57 09 -2 09	.11168 1.06624	-.03752 0.97446	.07607 .99131	.13570 1.0919	2.80404	.78659	2.20563	68.3699						
8	FL EMISA	EMS R.	53 20 15 15	.12581 0.96495	.12654 0.93084	.14690 0.91638	.16594 1.0177	2.60278	.46007	1.19747	88.7866						
9	BRENA	BREMEN	53 05 8 49	.15832 0.98497	.15388 0.97260	.16230 0.91132	.17611 1.0253	2.63129	.38444	1.01684	96.1273						
10	EDENBURGH	EDINBURGH	55 55 -3 10	.08316 0.94165	.09793 0.97979	.08654 0.9522	.235505 0.03124	2.63487	.502886								
11	JOH	PERTH	56 25 -3 25	.10483 1.07244	.09563 0.96666	.08980 0.9999	.23401 0.9672	2.63401	.98886	2.59940	54.1177						
12	FARDIN	VERDEN	52 57 9 15	.16444 0.98018	.16144 0.97415	.16637 0.9884	.17813 1.0253	2.63553	.37568	.99010	97.2643						
13	S. ANDR.	ST ANDREWS	56 20 -2 45	.11006 1.05148	.04800 0.98320	.06598 0.97878	.18036 1.0654	2.66126	.94512	2.462637	56.4493						
14	OLDELINBURGH	OLDENBURG	53 07 8 12	.17407 0.94834	.14312 0.97206	.15996 0.92193	.17103 1.0192	2.61648	.0107	1.11832	92.5590						
15	BENCUR	BANGOR	53 12 -4 05	.00748 0.92617	.07127 0.92852	.03567 0.92678	.01547 0.9704	2.07511	1.38856	2.91208	20.7678						
16	DIVELIN	DUBLIN	53 20 -6 15	.03528 0.10386	.10908 0.93084	.01662 0.92928	.01732 0.9839	1.99932	1.58490	3.09575	14.6044						
17	CARLUA	CARLISLE	54 55 -3 00	.06475 0.10476	.05236 0.95848	.05545 0.95462	.04552 0.10215	2.07518	1.39998	1.13412	2.72073	41.9550					
18	ALSTAD	HALBERSTADT	51 54 11 04	.19214 1.02879	.19315 0.90583	.18315 0.88996	.18249 0.10083	2.43478	.39710	1.04706	95.1607						
19	CASTELLO NOVO	NEWCASTLE ON TYNE	54 55 -1 37	.10713 1.02115	.02822 0.95848	.06799 0.95253	.08612 0.10223	2.45709	1.01807	2.50149	48.9237						
20	BRAGA	PRAGUE	50 05 14 26	.25957 1.02689	.25191 0.87412	.21753 0.85774	.19871 0.9876	2.68058	.46822	1.25510	89.2728						
21	SNAWEDON	MT. SNOWDON	53 05 -3 55	.04351 0.99289	.06836 0.92648	.03643 0.92443	.01616 0.9666	2.09120	1.38420	2.89443	23.4752						
22	REDE	DRUM	54 45 -1 40	.07602 0.99710	.02909 0.95557	.06555 0.94968	.08018 0.10169	2.43089	.03999	2.52810	47.2480						
23	AMPHIMATA	ARMAGH	54 21 -6 42	-.00456 0.98115	.11694 0.94899	.01915 0.95132	-.00575 0.10146	2.14107	1.47444	3.15689	21.2500						
24	CESTRIA	CHESTER	53 12 -2 55	.06187 0.97874	.05091 0.92852	.04642 0.92483	.03364 0.9679	2.16016	1.29081	2.78835	29.1818						
25	M. CLEOE	CLEE HILL	52 24 -2 38	.02674 0.95725	.04596 0.91455	.04846 0.91045	.02554 0.9422	2.07979	1.32577	2.75731	25.5836						
26	SNOTINGHAM	NOTTINGHAM	53 55 -1 10	.06468 0.95706	.02036 0.92357	.06161 0.91732	.05535 0.9577	2.21711	1.16425	2.58128	36.2838						
27	CEDARA	KILDARE	53 23 -6 55	.07409 0.97300	.03747 0.92648	.00906 0.93000	-.03259 0.9810	1.92704	1.61972	3.12127	9.9416						
28	SCOBESBRI	SHREWSBURY	52 43 -2 30	.01661 0.94533	.04945 0.91979	.04459 0.91958	.02695 0.9521	2.10323	1.32192	2.78045	26.3868						
29	BONONIA	BOLOGNE	50 45 1 40	.10401 0.97113	.00797 0.88775	.07847 0.87544	.07542 0.98864	2.15638	1.07504	2.31959	39.0680						
30	WRCESTR.	WORCESTER	52 10 -2 10	.02116 0.93006	.03782 0.90408	.06051 0.90571	.02934 0.9339	2.08031	1.30143	2.70738	26.6319						
31	LINCOLN	LINCOLN	53 15 -3 33	.07762 0.94271	.09010 0.92939	.07979 0.92078	.02881 0.9344	2.33014	1.39417	2.31856	47.7324						
32	FATNA	VIENNA	48 14 16 20	.02216 0.96677	.08576 0.92807	.08183 0.91471	.02185 0.95119	2.35919	1.29819	2.31856	77.1111						
33	PAZAEI	BUDAPEST	47 30 19 05	.27813 0.96817	.33307 0.92903	.27102 0.87777	.27009 0.93837	1.74852	0.74757								
34	NORMANTON	NORTHAMPTON	52 15 -0 55	.04581 0.91600	.01600 0.91933	.08020 0.92808	.02556 0.9422	2.21549	1.00950	2.34043	43.2183						
35	OXFORD	OXFORD	51 45 -1 15	.03637 0.92034	.02182 0.90321	.05494 0.88695	.03754 0.9195	2.08858	1.24994	2.61040	29.0971						
36	ELY	ELY	52 25 6 10	.06347 0.92384	.00582 0.91484	.07370 0.90567	.06957 0.94235	2.06946	1.9426	2.23900	1.07540	2.40784	41.1059				
37	CIMERACUM	CAMBRAI	50 10 3 11	.09056 0.92735	.05556 0.87557	.09345 0.87554	.06347 0.92811	2.19077	.99948	2.18943	43.8528						
38	AGRIPPINA COLONIA	COLOGNE	50 57 6 57	.15017 0.9307	.12130 0.88925	.13635 0.87441	.13501 0.9263	2.42937	1.7033	1.71000	66.8301						
39	BATHE	BATH	51 25 -2 25	-.00016 0.90459	.04218 0.87939	.04170 0.89296	.01584 0.9105	1.99227	1.36249	2.71445	21.6402						
40	COLESSTRIA	COLCHESTER	51 52 0 25	.05203 0.91600	.01513 0.90524	.07661 0.87661	.06940 0.9250	2.21205	1.07015	2.36723	40.7187						
41	LAESBURGH	REGENSBURG	49 02 12 04	.21660 0.93265	.21060 0.85579	.18900 0.83855	.17579 0.9121	2.55381	1.62494	1.59597	74.7172						
42	LONDONA	LONDON	50 39 3 07	.03016 0.90306	.00204 0.90338	.00950 0.88634	.05434 0.9114	2.13877	1.15328	2.46661	34.8377						
43	LEODUM	LEGE	50 39 5 35	.11976 0.91640	.07474 0.90745	.10209 0.86998	.11691 0.9042	2.34390	1.81070	1.90183	58.2011						
44	BOSFORUS TRACIUS	S. OF CONSTANTINOPLE	50 22 7 35	.14284 0.94748	.10615 0.94748	.10615 0.94748	.13734 0.9104	2.42121	1.71350	1.72754	66.0183						
45	FLUENTIA	COLENTE	52 30 -6 45	.04244 0.94964	.07475 0.90749	.01968 0.92879	1.89541 0.9902	2.92624	1.65334	3.13399	7.8184						
46	FL SCENE	SHANNON R.	50 45 -3 30	.02216 0.97328	.08777 0.90709	.08297 0.98431	.08297 0.98431	1.88211 0.9831	1.88211 0.9831	1.46655	2.75861	14.2247					
47	EXCESTRIA	EXETER	50 45 -3 30	.02216 0.97328	.08777 0.90709	.08297 0.98431	1.88211 0.9831	1.88211 0.9831	1.46655	2.75861	14.2247						
48	CANTURIA	CANTERBURY	51 15 1 00	.01419 0.87889	.01745 0.87448	.08048 0.75717	.07517 0.98448	2.16316 0.98448	2.42937	1.7033	1.71000	66.8301					
49	SALZBURGH	SALZBURG	47 48 13 01	.23437 0.97664	.22718 0.84247	.19842 0.84247	.16355 0.92756	2.42937	1.7033	1.71000	66.8301						
50	CONSTANTINOPOLIS	CONSTANTINOPLE	41 02 29 00	.35559 0.92110	.50615 0.76167	.41055 0.76167	.41055 0.50590	2.28327 0.98529	2.28327 0.98529	1.89674	2.12196	2.96035	57.1545				
51	WINTONA	WINCHESTER	51 05 -1 20	.01017 0.86735	.02327 0.89157	.05090 0.88559	.08850 0.85381	2.02310 0.98533	2.02310 0.98533	2.61058	2.61058	2.61058	15.5508				
52	MAGONTIA	MAINZ	50 00 8 13	.14073 0.88419	.14341 0.87266	.14748 0.85676	.14169 0.9029	2.42870	1.70794	1.71938	64.5419						
53	METIS	METZ	49 09 6 10	.15498 0.84766	.10763 0.85783	.12251 0.84266	.12371 0.86069	2.27632 0.87307	2.27632 0.87307	52.9187	52.9187	52.9187					
54	AUGUSTA	AUGSBURGH	49 23 10 53	.18428 0.88439	.18995 0.84445	.17411 0.82644	.16251 0.87820	2.47017 0.71143	2.47017 0.71143	1.75734	1.75734	1.75734	67.1166				
55	SAMARCAN	SAMARKAND	39 40 67 00	.08167 0.97210	.16937 0.92311	.10231 0.86354	.08249 0.8523	2.16251 0.85383	2.16251 0.85383	1.85383	1.85383	1.85383					
56	NAMETIS	NANTES	47 13 -1 33	.01158 0.85652	.02705 0.80749	.00873 0.81106	.00873 0.81106	2.16251 0.85383	2.16251 0.85383	1.85383	1.85383	1.85383					
57	WORMIA	WORMS	49 37 8 21	.13692 0.86724	.14573 0.85957	.14806 0.84988	.										

Table 2 continued,

74 TOLOSA	TOULOUSE	43 35	1 28	.01879	75827	.02540	74047	.05127	74634	.03900	4504	1.11074	1.13344	2.04143	26.8453
75 VERONA	VERONA	45 27	11 00	.19948	77451	.19194	79325	.17044	7739	.16214	7439	2.32956	2.07012	2.02499	54.2741
76 PICTAVIS	POITIERS	46 25	0 20	.03103	74884	.00582	81363	.04872	.80257	.02385	.7483	1.84448	1.23024	2.29647	23.7181
77 CARNOTUM	CHARTRES	48 25	1 30	.05271	75164	.02618	84503	.04866	83404	.04782	.8087	1.98657	1.17502	2.33464	29.8026
78 LUGDUNUM	LYON	45 45	4 50	.10148	75796	.08434	79849	.09740	78226	.08562	.7261	2.01208	1.04555	2.14998	35.8232
79 LARIS	LARISA	39 36	22 25	.34534	79953	.39124	.69115	.32185	.67332	.32059	.7166	2.61128	.98002	2.59911	54.0303
80 IAC BENACUS	I. GARD	45 45	10 45	.14552	75815	.18762	79849	.16818	77912	.15903	.7721	2.33146	.84347	2.01313	54.7044
81 BACTRUM	BALKH	36 44	64 58	.78478	84095	1.1682	.64112	.93887	.81678	.88060	.8769	3.70442	.27322	1.01211	19.2259
82 CORINTHUS	CORINTH	37 55	22 55	.28783	76555	.39997	.66177	.33166	.64200	.34091	.6594	2.53848	1.00214	2.54982	51.4348
83 FL MIMBUS	R. MINHO	41 52	-8 50	-.10164	.70942	-.15417	.73071	-.07290	.73451	-.11337	.6944	2.21927	.67994	1.50897	64.1393
84 TURONIS	TOURS	47 20	0 40	.05009	.72226	.01164	.82612	.05354	.81560	.03101	.7724	1.90102	1.22704	2.33266	24.8867
85 AUTISODORUM	AUXERRE	47 49	3 35	.07719	.73277	.06254	.83456	.08940	.82090	.07376	.7927	2.05929	1.07386	2.21144	36.4623
86 FL PADUA	R. PO	45 05	12 30	.18015	.74611	.21817	.78340	.18874	.76326	.17977	.7654	2.38033	.85611	2.04466	54.3427
87 AURELIANUM	ORLEANS	47 53	1 57	.05692	.71913	.03403	.83630	.07169	.82455	.05171	.7923	1.98319	1.19592	2.29954	30.3679
88 MEDIOCILANUM	MILAN	45 27	9 10	.13890	.72242	.15999	.79725	.14699	.77400	.14064	.7448	2.22810	.92110	2.05231	49.0418
89 RAVENNA	RAVENNA	44 45	10 13	.19307	.73155	.21262	.76322	.19497	.75469	.17795	.7402	2.34475	.98528	2.02991	53.4806
90 CIPRO	CYPRIUS	35 00	33 30	.37074	.75169	.39149	.61947	.45542	.61424	.5017	.7117	2.63007	.07399	2.08495	53.4699
91 LUCANIA	LUCANIE	46 23	6 40	.10167	.73990	.11436	.79114	.12105	.78435	.11629	.74335	2.15000	.80267	2.08479	54.1162
92 VENICIA	VENICE	45 26	12 20	.21005	.72794	.21526	.79296	.18717	.72713	.17710	.77474	2.40113	.21072	.57.8811	
93 FL DORIAS	R. DOURO	41 05	-8 40	-.10757	.67028	-.15126	.71704	-.07483	.72182	-.10742	.6712	2.20581	.44100	1.41392	64.4893
94 TOLETUM	TOLEDO	39 53	-4 02	.02628	.68061	.07030	.69610	.02649	.68544	.03129	.5762	1.93422	.76290	1.51586	50.1940
95 GARGANUS	TESTA DEL GARGANO	41 46	16 13	.21828	.70691	.26303	.72955	.23514	.70740	.23444	.69683	2.42218	.92473	2.23984	53.1558
96 ATHENE	ATHENS	37 54	22 52	.33897	.71156	.41655	.66148	.34333	.64423	.32425	.6749	2.58203	.24059	1.99938	
97 AUGUSTUDUNUM	AUTUN	46 59	4 19	.07946	.67426	.07534	.82001	.05905	.80498	.08083	.7666	2.05055	1.06045	2.17451	37.1152
98 TIRUS	TYRE	33 17	35 11	.07943	.67456	.11404	.58090	.15279	.58843	.14812	.6554	2.82029	1.05363	2.97243	54.2594
99 VIENNA	BIENNE	45 31	4 52	.09103	.66845	.08494	.79442	.09717	.77793	.08991	.7183	2.00197	1.06032	2.13674	35.4071
100 PLACENTIA	PIACENZA	45 02	9 41	.14527	.67546	.16901	.78398	.15909	.76613	.14719	.7348	2.23441	.92132	2.05861	49.1767
101 BONONIA	BOLOGNA	44 30	11 21	.16689	.67827	.19805	.77667	.17381	.75617	.16775	.7331	2.29893	.89775	2.06488	51.8791
102 TARENTUM	TARANTO	40 26	17 12	.23263	.68127	.30200	.70268	.24868	.68370	.25911	.64336	2.39945	.95219	2.28473	51.1446
103 AUGUSTA	AOSTA	45 44	7 19	.10864	.65971	.12770	.79202	.12674	.78003	.11786	.7397	2.14153	.97264	2.08285	44.0936
104 FLORENTIA	FLORENCE	43 47	11 15	.17372	.66813	.19635	.76416	.17165	.74297	.16834	.7059	2.25469	.92617	2.08827	49.3728
105 RODOS I	RHODES	36 30	28 00	.37492	.68688	.48869	.63705	.40883	.62673	.41399	.6891	2.68299	1.04375	2.00337	52.1899
106 PISA	PISA	43 42	10 24	.16539	.65763	.18151	.76300	.16091	.74196	.15628	.6947	2.20003	.9459	2.08994	47.0569
107 FL SOETIS	GUDAOLQUIVER R.	34 48	-6 23	-.10386	.59913	-.11141	.64272	-.06373	.63771	-.04773	.5265	1.99723	.32774	1.05411	71.1759
108 SALERNA	NAMONNE	45 11	1 10	.09707	.65310	.10564	.75690	.10564	.75707	.10564	.6581	1.83031	1.00113	2.04178	26.2357
109 TUBUS BABEL	BABYL'ON	32 45	44 30	.05070	.66877	.77647	.75740	.67042	.68749	.75678	.7159	3.22033	.57.32243	3.36446	
110 NOLA	NOLA	40 55	14 33	.22339	.64795	.25955	.71493	.21270	.69056	.18179	.65658	2.27087	.54.95922	2.17824	53.2811
111 JERUSALEM	JERUSALEM	31 46	35 14	.47808	.64795	.61494	.55443	.53441	.56124	.55679	.589	2.79117	.03019	2.02186	53.9225
112 ARELAS	ARLES	43 39	4 41	.07847	.59519	.08174	.76184	.08977	.74396	.08435	.6549	1.90404	.08874	2.07523	31.6736
113 MASSILLA	MARSEILLE	43 20	5 20	.10015	.59200	.09308	.75631	.10714	.743747	.09374	.6446	1.91885	1.07445	2.06149	32.7702
114 GENUA	GENOA	44 24	8 55	.12183	.60080	.15563	.77493	.14351	.75489	.13668	.7051	2.15731	.94033	2.07649	45.0993
115 MAR GALILEE	SEA OF GALILEE	32 50	35 25	.61496	.66466	.62105	.58507	.53538	.53819	.55172	.58172	2.81622	1.0457	2.95017	54.4842
116 ROMA	ROME	41 45	12 15	.19298	.60451	.21380	.72867	.18257	.70543	.18777	.6444	2.19692	.96254	2.11461	45.9953
117 SONFENTA	COSENZA	39 15	16 16	.29192	.60330	.28919	.68504	.23249	.66050	.25057	.6062	2.27295	.97462	2.21527	47.1295
118 CORDUBA	CORDOBA	37 54	-4 46	-.08671	.55176	-.08319	.66148	-.04242	.65385	-.03119	.5325	1.93895	.47708	1.25422	50.8790
119 FL HIBERUS	E BRO R.	40 44	0 50	-.05962	.55527	.01454	.71093	.03484	.69505	.03739	.5524	1.76954	1.01262	1.92225	31.5637
120 CORSICA I	CORSICA	42 30	9 00	.04877	.56930	.15700	.74716	.14137	.71987	.14424	.6397	2.05748	1.02645	2.04293	40.3273
121 DAMASCUS	DAMASCUS	33 35	36 28	.67196	.65000	.63446	.58614	.54993	.59640	.55610	.6853	2.89303	1.05943	3.06495	55.2801
122 ZAGINTHUS	ZANTE I.	37 45	20 40	.33126	.60037	.36070	.65886	.29957	.63670	.31500	.6145	2.41529	.99265	2.39755	49.3470
123 BETHLEHEM	BETHLEHEM	31 41	35 11	.44576	.60969	.61404	.55996	.53418	.55682	.55682	.5815	2.73159	1.03877	2.81018	53.8512
124 NEAPOLIS	NAPLES	40 51	14 26	.22588	.57149	.25191	.71297	.21103	.68919	.21874	.6389	2.26024	.95812	2.17326	47.9168
125 SALERNA	SALERNO	40 41	14 43	.24666	.62899	.25688	.71006	.21499	.68467	.22208	.6384	2.27436	.95924	2.18161	47.9944
126 EBRON	EBRON	31 32	35 05	.55013	.61913	.61232	.55634	.53323	.55628	.55673	.5734	2.71727	1.02419	2.78643	53.7109
127 SARDINIA	SARDINIA	32 51	25 29	.55277	.61942	.55277	.61794	.54760	.61360	.54548	.5853	1.91015	1.02387	2.66473	35.7448
128 ERICO	JERICHO	31 51	35 29	.59003	.59402	.48100	.55399	.53847	.55353	.55955	.5874	2.26487	1.02173	2.60335	54.7448
129 PALERNA	PALERMO	38 07	12 23	.14389	.51550	.22348	.64423	.19417	.64423	.2242	.5227	2.03047	.99012	2.02097	49.9036
130 CUZA	GAZA	31 29	15 20	.45820	.55620	.60127	.54949	.52465	.55211	.54838	.5644	2.68120	1.03473	2.74247	53.1005
131 BERSEBEE	BEERSHEBA	31 15	34 45	.54540	.58201	.40650	.54542	.52629	.55001	.55450	.5572	2.48197	1.03115	2.73070	53.2940
132 PETRA	PETRA	30 26	35 30	.72995	.52745	.52942	.54477	.54477	.53253	.52525	.5207	2.03594	.99729	2.03044	40.0506
133 UNNAMED	SICILY	37 34	14 10	.17303	.50275	.24725	.64549	.20718	.64549	.24292	.2531	2.03594	.99729	2.03044	
134 OPPIDUM	CEUTA	35 58	-5 23	-.03745	.49997	-.03936	.52127	-.02521	.4992	-.02370	.4926	1.91876	.47644	1.02464	48.4462
135 FL MALUVA	MULUVI R.	35 20	2 30	.03290	.47909	.04363	.61668	.04391	.59274	.09105	.4123	1.58550	.84163	1.33440	35.6947
136 CARPATHIAS I	SCARPANTO	35 40	27 10	.40618	.52741	.47415	.62520	.39670	.6052	.41359	.6409	2.59739	1.03570	2.69013	50.9162
137 CRETA I	CRETE	35 10	25 00	.27208	.50455	.43633	.61377	.36761	.59578	.36868	.64607	2.12036	.99150	2.10638	42.4020
138 CARTAGO MAGNA	LATER CARTHAGE	37 00	10 20	.19095	.49975	.18035	.64727	.15230	.61742	.18807	.4715	1.79432	1.01299	1.81970	32.3864
139 FORTUNATE IS	CANARY AND MADEIRA														

Here' is a better view of the original:

