Dürer Transforms

A Research Challenge

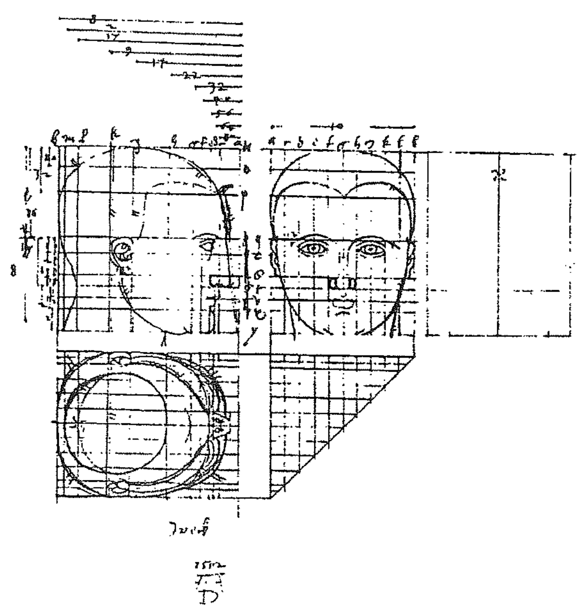
by

W. R. Tobler

MCMLXXXIV

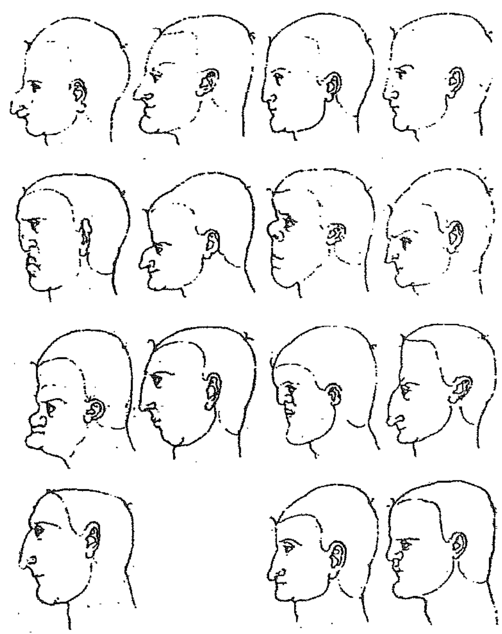
Albrecht Dürer's <u>Vier Bücher von Menschlicher Proportionen,</u> posthumously published in 1528, is today catalogued with manuals on "How to Draw the Human Figure" and is discussed in art history books almost only by title. Dürer's studies in this work are also often seen as a somewhat misguided attempt to define human beauty in terms of ideal proportions, and are thus connected with concepts going back to Vitruvius, Leonardo da Vinci, and others. The forward connection to Lautensack, Putti, Topffer, Cozens, Daumier, etc., can best be followed in Gombrich (1977). The significance of Dürer's book seems to me to be much greater. What is often not recognized is the connection of his proportionate studies to the subject currently known in biology as allometry (Gould 1966). Several other connections of contemporary relevance can also be made.

In the work cited Dürer introduces illustrations of the human head in the three orthographic views familiar to us today from elementary mechanical drawing. Superimposed on these illustrations are a series of perpendicular lines, drawn through identifiable features forming, as it were, a rectangular coordinate system. Recall that this is considerably before the time of Rene Descartes (1596-1660) and the spread of "Cartesian" coordinates or analytic geometry. Rectilinear grids were of course in prior use as is evidenced, inter alia, by the archaeologic discovery of ancient towns with orthogonal street patterns and construction grids in Egyptian drawings.

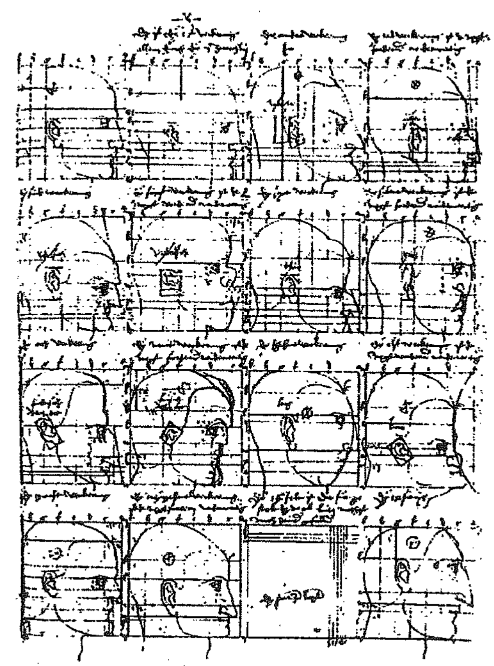


Head by Dürer, 1512, in orthographic views with rectilinear lines defining position

Dürer's mathematical bent is well known to art historians but almost all of the studies have concentrated on his use of projective geometry and properly treat this within the early tradition of Alberti and other innovators of the Renaissance. A very limited number of short papers describe and analyze Dürer's mathematical talents as represented by his "measurement" book Underweysung der Messung... (1525). Especially emphasized is his work on spirals and polygons. In these studies only minimal reference to, and no deep treatments of his Bücher von Menschlicher Proportionen can be found. It is my contention that this work is even more important in the subtlety of its mathematical ideas than is the "measurements" book. What Dürer does is to introduce a method of modifying figures, heads, and faces to obtain deformed views of the originals. Some of these, when pushed to extremes, could become grotesque - Diirer warns against this. And this was not his intent. He was studying human physionomy.



OBVERSE



REVERSE

Coordinate transformations by Albrecht Dürer, circa 1524 (Strauss 1972). Notice the rotated ears in line three.

From a modern point of view what Dürer did, especially in "book" three, was to introduce the method of coordinate transformations, very similar to that used in D'Arcy Thompson's famous biological classic On Growth and Form (1917) and still the subject of active mathematical and scientific research (Bookstein 1977, Fain 1975, Tobler 1977). This method allows us to depart from the notion of idealized proportions and to study the variations of form which occur in nature. Dürer's coordinate

Scorpaena (Fig. 152) are easily derived by substituting a system of triangular, or radial, co-ordinates for the rectangular ones in which we had inscribed *Polyprion*. The very curious fish *Antigonia capros*, an oceanic relative of our own boar-fish, conforms closely to the peculiar deformation represented in Fig. 153.

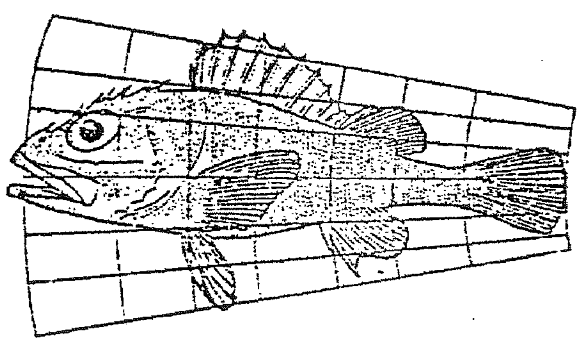


Fig. 152. *Scorpaena* sp.

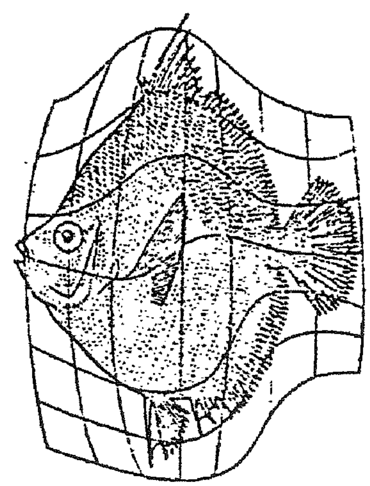


Fig. 153. *Antigonia capros*.

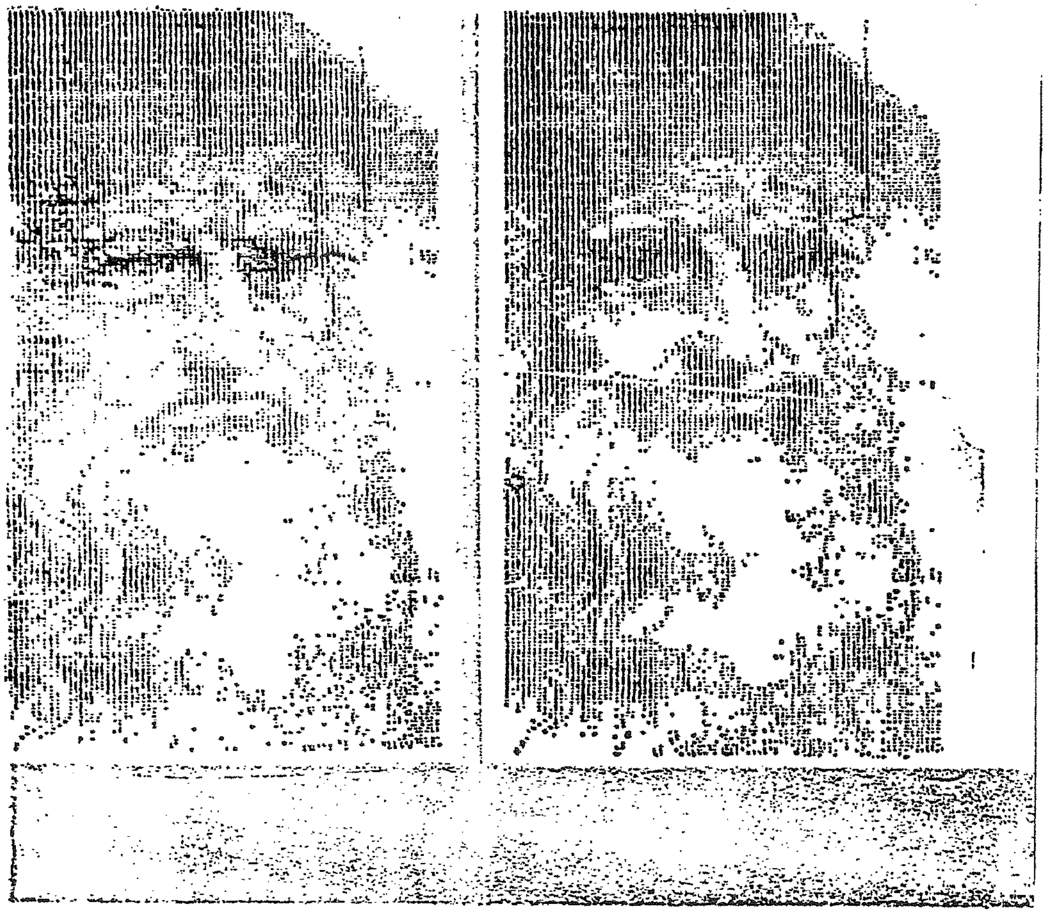
The method of coordinate transformations in Biology (Thompson 1917).

transformations are all essentially two dimensional and, according to the literature, all fall into the class of affine or projective transformations. The literature is in this instance clearly wrong and several of the transformations are non-linear, quite subtle, and intriguing. One group consists of what appear to be separable or piecewise linear transformations and others appear not to have been discussed in any mathematical literature at all. The name "Dürer Transforms" is proposed for these. In general they are all continuous transformations and therefore can be studied by analytic methods that are now well understood. Dürer of course did not describe his transformations by mathematical equations as would be done today.

The more interesting of Dürer's proportionate studies deal with the human face. This allows yet another connection to be made to modern research. There is currently interest on the part of psychologists on how people recognize, distinguish, analyze, and remember faces (Daives et al 1981). Psychology as a field of course did not exist in the sixteenth century, but 'Dürer transforms', with only a small extension of the actual methods used by Dürer, allow one to devise experiments to test recognition, and to measure the amount by which faces differ from each other and thus to obtain an indication of the similarity of two faces, or of one face at two different times. Francis Galton with his great interest in familial resemblances would have been delighted, but he seems not to have known Dürer's work in this field. There are implications beyond psychology too, in genetics and in criminology, where comparisons of faces occur.

The Proposed Studies

At this point what is needed is a careful examination of Dürer's proportionate drawings in order to rewrite them in equation form. Preliminary study has indicated that this is feasible - and fascinating - but not simple. Some of the transformations turn out to be mathematically very delicate. Dürer in one instance slightly rotates an ear. Rotation as such is not very complicated but this is a local rotation and nearly everything, except the ear, stays fixed. The connecting tissue between the ear and the fixed portions of the face must be allowed to change too but in a continuous manner that decreases in amount as one moves away from the ear, and without tearing. The only comparable transformation given in mathematical form of which I am aware is that of converting the dour DaVinci image into a smiling one by moving the edges of the mouth upward. The Soviet engineer Fain needs to use the mathematical theory devised by S. Lie (1842-1899) to obtain this result. The movement applied by Fain is a translation and is simpler than the to-be-analyzed rotation used by Dürer.



В.С. ФАЙН

A local coordinate transformation (Fain 1975).

Once the mathematical analysis is complete then the next step is to put all of the results into an organized monograph - basically along the lines of this text but in greater scholarly detail, flushing out the arguments and connections, with emphasis on relations to other activities in the 450 years since Dürer's time. To this end a modest amount of additional bibliographic work is required.

The transformation equations, once specified, will allow this aspect of Dürer's work to be mimicked quite easily on a graphics display device attached to a computer. Indeed, such a facility is a prerequisite for the work to be undertaken. But the foregoing "quite easily" can be understood at several distinct levels. At one level it would be easy for a mathematician/computer programmer to replicate specific effects achieved by Dürer, once the analysis is available. At another level we can imagine an artist with a computer controlled "fun-house mirror" used to introduce either subtle or gross distortions into an image, or attempting to warp one face so that it appears more similar to another. A computer facility which would allow an artist to do this at a console, at a useful level of realism, and in a manner considered to be "quite easy" would in fact be difficult to achieve today. The difficulty is rather deep, like trying to automate cartooning or the drawing of caricatures. Dürer's work is relevant to such a task, but does not reach that level of analysis. However, a computer console and display implemented at an intermediate level might be achieved without too great an investment and should make for an attractive hands-on museum exhibit to be manipulated by children (or adults). Pulling in color images of individuals and then manipulating these could be quite enchanting. How difficult this would be depends in part on the number of free parameters in the transformations. An additional benefit of the research would be to popularize the method of coordinate transformations and thus bring it to the attention of potential users. A subsidiary effect would be to reinforce the relevance of art and art history to contemporary life. Equally important the research would demonstrate the value of mathematical study applied to objects not normally considered mathematical, and might help reduce the alleged gulf between scientifically and artistically focused individuals, often considered a problem in our society.

BIBLIOGRAPHY

N. Badler, S. Platt, 1981, "Animating Facial Expressions", Computer Graphics (siggraph) 15,3:245-252.

F. Bookstein, 1977, "The Study of Shape Transformation after D'Arcy Thompson", Mathematical Bioscience, 34:177-219.

F. Bookstein, 1977, "The Measurement of Biological Shape and Shape Change", Ph.D. Thesis, Ann Arbor, University of Michigan.

H. Chernoff, 1973, "The Use of Faces to Represent Points in k-dimensional space graphically", J. Am. Stat. Assn., 68:361-368.

W. Conway, 1958, The Writings of Albrecht Durer, London, P. Owen.

G. Davies, H. Ellis, J. Shepherd, eds., 1981, Perceiving and Remembering Faces, New York, Academic Press.

A. Dürer, 1528, Vier Bücher Von Menschlicher Proportionen, facsimile edition, Zürich, J. Stocker-Schmid, 1969.

V. Fain, 1975, Algorithmicheskoye Modelirovanie Formoobravaniya, Moskva, Izdatelslvo Nauka.

J. Giesen, 1929, Dürer's Proportions Studien im Rahmen der allgemeinen Proportionsentwicklung, Bonn, K. Schroeder.

E. Gombrich, 1977, Art and Illusion, 5th ed., Oxford, Phaidon.

E. Gombrich, E. Kris, 1940, Caricature, London, Penguin

S. Gould, 1966, "Allometry and Size in Ontogeny and Phylogeny", Biological Review, 41:587-607.

R. Heath, 1901, Albrecht Durer, London, Low Marston

J. Hoffmann, 1971 "Dürers Verhältnis zur Mathematik", pp. 132-151 of Albrecht Dürers Umwelt, Nuremberg, Verein für Geschichte der Stadt.

H. Huntley, 1970, The Divine Proportion, New York, Dover.

T. Kanade, 1977, Computer Recognition of Human Faces, Basel, Birkhauser.

D. Lenorovitz, 1975, "Development of a Prototype Computer Assisted System to be used as an Aid in Facial Recognition Tasks", Ph.D. thesis, Buffalo, State University of New York.

K. May, 1967, "Mathematics and Art", The Mathematics Teacher, LX, 6 (Oct); 568-572.

S. Monneret, 1970, Durer, Wildt translation, New York, Excalibur.

E. Panowsky, 1955, The Life and Art of Albrecht Durer, 4th ed., Princeton, University Press.

F. Parke, 1974, "A Parametric Model for Human Faces", Ph.D. Thesis, Salt Lake City, University of Utah.

K. Pearson, 1424, Life, Letters and Labours of Frances Galton, 3 vols, Cambridge, University Press.

F. Russell, 1976, The World of Durer, New York, Time

E. Schroder, 1980, Dürer: Kunst and Geometrie, Basel, Birkhauser.

M. Steck, 1969, Albrecht Dürer als Kunsthistoriker, Zurich, J. Stocker

W. Strauss, 1972, The Human Figure by Albrecht Dürer (Dresden Sketchbook), New York, Dover.

W. Strauss, 1973, The Complete Engravings, Etchings, and Drypoints of Albrecht Dürer, 2nd ed., New York, Dover.

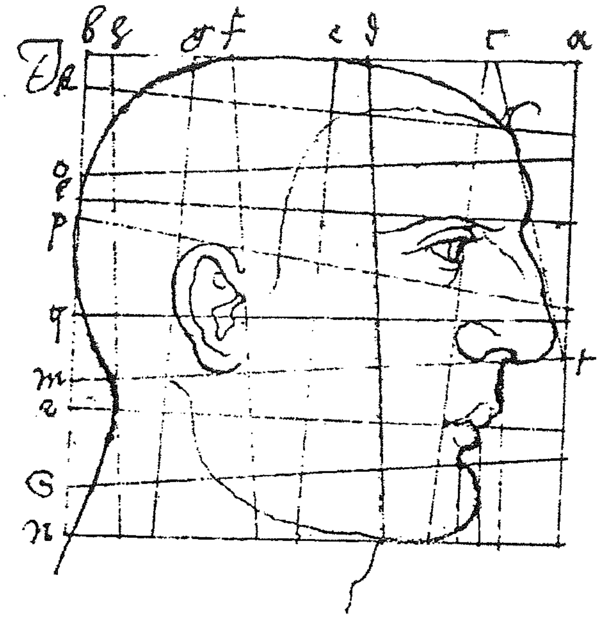
P. Strieder, 1976, The Hidden Dürer, Arnoldo Translation, Chicago, Rand McNally.

D. Thompson, 1917, On Growth and Form, Cambridge, University Press.

W. Tobler, 1977, Bidimensional Regression, Santa Barbara, Department of Geography, UCSB.

J. Trost, 1859, Die Proportions lehre Dürers nach ihren wesentlichen Bestimmungen in übersichtlicher Darstellung, Wien.

R. Zaunick, H. Salie, 1970, "Diirer als Mathematicker", pp. 171-172 of J. Poggendorff's Biographisch-Literarisches Handwörterbuch der exakten Natur Wissenschaften, Berlin, Vol. VII, a, supplement.



Problem:

Find $x' = f(x,y) \quad y' = g(x,y)$

Such that* $x'(0) = 0 \quad x'(1) = 1 \quad y'(0) = 0 \quad y'(1) = 1$

$$\frac{\partial x'}{\partial x} > 0 \quad \frac{\partial y'}{\partial y} > 0 \qquad J = \frac{\partial x'}{\partial x}\frac{\partial y'}{\partial y} - \frac{\partial x'}{\partial y}\frac{\partial y'}{\partial x} > 0$$

Solution (preliminary):

One approximate solution is

$$x' = x + a_1 \, (2y-1) \sin (k_1 \pi x)$$

$$y' = y + a_2 \, (2x-1) \sin (k_2 \pi y)$$

Subject to $|a_i| < 1/k_i \pi$. For k_i odd this is symmetric.

*The line tangent to the forehead and nose does not satisfy these conditions; it is not a coordinate line.