Supplemental Notes
Exponential growth and doubling time

I. Demographic accounting equation

\[ P(t) = P(0) + B(0,t) - D(0,t) + I(0,t) - E(0,t) \]

- \( P(t) \) = Population at time, \( t \)
- \( P(0) \) = Population at time, 0
- \( B(0,t) \) = Births over the interval (0,\( t \))
- \( D(0,t) \) = Deaths over the interval (0,\( t \))
- \( I(0,t) \) = Immigration over the interval (0,\( t \))
- \( E(0,t) \) = Emigration over the interval (0,\( t \))

Rates (‘crude rates’ usually expressed per 1,000)
- Birth rate: \( b = \frac{B(0,t)}{P(0)} \), Crude Birth Rate (CBR) = \( b \times 1,000 \)
- Death rate: \( d = \frac{D(0,t)}{P(0)} \), Crude Death Rate (CDR) = \( d \times 1,000 \)
- Growth rate: \( r = b - d + (i - e) \)

II. Exponential growth - four variable model

Provides a simple means of population projection assuming a fixed growth rate, \( r \).

Projection: \( P(t) = P(0)e^{rt} \)

Algebraic restatements —

1. Find \( t \) given \( r \), \( P(0) \), and \( P(t) \):
   \[ t = \frac{\ln\left(\frac{P(t)}{P(0)}\right)}{r} \]

2. Find \( r \) given \( t \), \( P(0) \), and \( P(t) \):
   \[ r = \frac{\ln\left(\frac{P(t)}{P(0)}\right)}{t} \]

3. Find \( P(0) \) given \( r \), \( t \), and \( P(t) \):
   \[ \frac{P(t)}{e^{rt}} = P(0) \]

Doubling time is a special case of (1) when \( P(t)/P(0) = 2 \).
III. Examples and problems

Q: Given an initial population of $P(0)$ growing at fixed rate, $r$, what will be the population in $t$ periods?

Examples:
Syria: $P(1990)= 500$, $b=0.033$, $d=0.006$, what is $P(2000)$?

Ans. $r = 0.033-0.006=0.027$
$P(2000) = 500*\exp(10*0.027) = 655$


Ans. $r = (42-17)/1000 = 0.025$
$P(2010) = 400 * \exp(20*0.025) = 659$

Group problem:
If the population of Haiti was 6,075,000 in 1990 and the $CBR=35$ and $CDR=14$ remain constant over the next forty years, what will be the population after forty years? Assume Haiti experience no international migration.

Q: How soon will the population double if it continues to grow at rate, $r$?

Examples:
Syria? Ans. $\ln(2)/0.027 = 25.7$ years

Afghanistan? Ans. $\ln(2)/0.025 = 27.7$ years

Group problems
What is the population doubling time for India if $CBR=25$, $CDR=9$?

What is the population tripling time for Mozambique if $CBR=41$, $CDR=28$?