

# Symbolization of Map Projection Distortion: A Review

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**ABSTRACT:** One of the most fundamental steps in map creation is the transformation of information from the surface of a globe onto a flat map. Mapmakers have developed and used hundreds of different map projections over the past 2,000 years, yet there is no perfect choice because every map projection uniquely alters some aspect of space during the transformation process. Detailed information about the type, amount, and distribution of distortion is essential for choosing the best projection for a particular map or data set. The distortion inherent in projections can be measured and symbolized much like any other map variable. Methods for symbolizing map projection distortion are reviewed, with each method described and illustrated in graphical form. The symbolization methods are collected under ten separate headings organized from simple to more complex in terms of interpretation. Most of these methods are highly effective at communicating distortion, yet they are rarely used beyond textbooks and technical documentation. Map projections and the distortions they carry need to be better understood by spatial data developers, distributors, and users. Map distortion should be carried along with map data as confidence layers, and the easily accessible distortion displays should be available to help in the selection of map projections. There is a suitably wide array of symbolization methods to match any need from basic education to research.

**KEYWORDS:** Map projection distortion, cartographic symbolization, visualization

## Introduction

Cartographers have been portraying our globe on flat surfaces for approximately 2,000 years (Snyder 1993). The results have been far from perfect. It is impossible to depict rounded objects, such as the globe, on flat surfaces with complete fidelity. Thus, all maps are flawed in some way. The somewhat flawed and flattened geospatial products that result take forms such as maps printed on paper, virtual maps on computer screens, or virtual maps in computer databases. These flat map products cannot be true representations of our globe. Alternatives to flattening the globe such as Dutton's global hierarchical coordinate system (Dutton 2000), or methods to perform spatial analysis on the surface of the globe (NCGIA 1997), are being developed but, in the meantime, map projections must be used.

As the globe is flattened, areas may expand or contract, distances may change, and angles may be bent out of shape. The amount and types of distortion introduced may be controlled through the selection of appropriate map projections. Choosing the best projection for a map or map product is a task that has haunted cartographers for ages. The

secret lies in choosing an appropriate projection that will allow the final product, in whatever form, to retain the most important properties for a particular use. Cartographers have measured, categorized, and organized various distortion characteristics resulting from the transformation of the globe to the flattened map. The most common types of distortion that are measured include changes in scale, distance, area, and maximum angular deformation.

In addition to the inevitable distortions listed above, the data structure used to store map coordinate and attribute information adds another dimension to the problem of map projection distortion. Steinwand et al. (1995) present an argument for specifically addressing the projection of raster data sets as opposed to vector data sets. When projecting raster data sets, pixel attribute values may change or pixels may multiply to fill space or may be squeezed out entirely. Two visualization methods were devised by Steinwand et al. (1995) to convey the special problems of transforming raster data.

Maps that show projection distortion have been included as backdrops to a base map, as supplemental insets, or as parallel maps. For example, Snyder and Voxland (1989) created parallel distortion maps for most maps in their *Album of Map Projections*. As another example, Canters and Declair (1989) provided a display of isolines of angular and area distortion on their maps in *The World in Perspective: A Directory of World Map Projections*. Cartographers have shown remarkable

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versatility in the display of distortion using familiar shapes, direct comparisons of isolines of distortion or graticules, and interactive visualization tools.

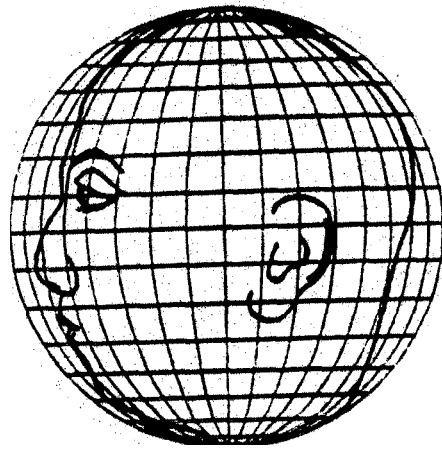
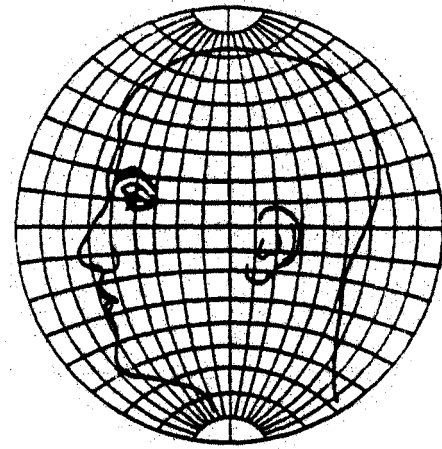
Map users and researchers have a wide range of possibilities for evaluating and symbolizing map projection distortion. In this paper, these symbolization methods are presented under 10 separate headings. From familiar shapes to checkerboards, these cartographic symbolization methods are arranged in approximate order of interpretive complexity. We have chosen to provide a variety of methods because this review is intended as a resource for a broad community of users with a wide range of needs and cartographic background.

## Ten Distortion Methods

### Familiar Shapes

The use of a familiar shape to visualize distortion is the simplest and often the most engaging of methods for symbolizing distortion. A face or regular shape is created on a base map and then transformed (Figure 1). The human head or face is a popular theme. A number of examples were listed by Tobler (1964), who included work by Reeves (1910), Gedymin (1946), and others. Reeves (1910) employed a single profile of a face that stretched nearly from pole to pole in drawings for an educational lecture. An illustration of part of his work is included in Figure 1. His objective was to "give an idea at a glance of the exaggerations and distortions of a few of the best-known projections." He mentioned having created these displays specifically "for a lecture to boys" but noted that there are "those of us who can more readily appreciate what the shape of the average man's head is than they can the general form of the land masses of the earth." Robinson et al. (1995) used the profile in the same fashion as Reeves, with a single head covering both the northern and southern hemispheres.

Gedymin's (1946) use of the human face to symbolize map projection distortion differed from the approaches of Reeves (1910) and Robinson et al. (1995). Gedymin inscribed the face twice, once in the southern hemisphere and once in the northern. The effect on a Mercator projection is a classic alien head with small facial features and an enormous cranium in the Northern Hemisphere and a normal cranium and ballooning neck in the Southern Hemisphere. This two-headed version



**Figure 1.** Profile of the human head is an example of the use of familiar shapes for depicting distortion. The profile is shown over a graticule in the globular projection at top and is shown in the orthographic projection in the bottom view (after Reeves 1910).

effectively conveys the concept of increased distortion with increased distance from the equator north and south on the Mercator.

A rather different example is found in the *Hammond International Atlas of the World* (Hammond, Inc. 1994) that uses a cartoon face created from circles, a triangle, and a line. The reader is instructed to think of the facial features as continents. The face—a simple plan of circle eyes, triangle nose, and a straight-line mouth—is transformed into four projections. The cartoon face method seems more friendly, when compared to the sometimes grotesquely distorted human heads, yet it is hard to say which method is most effective. The more cheerful cartoon face may be more inviting to examination while the grotesquely distorted head may provide a greater initial visual impact.

The cartoon face used in the *Hammond Atlas* combines simple geometric figures into a face, but

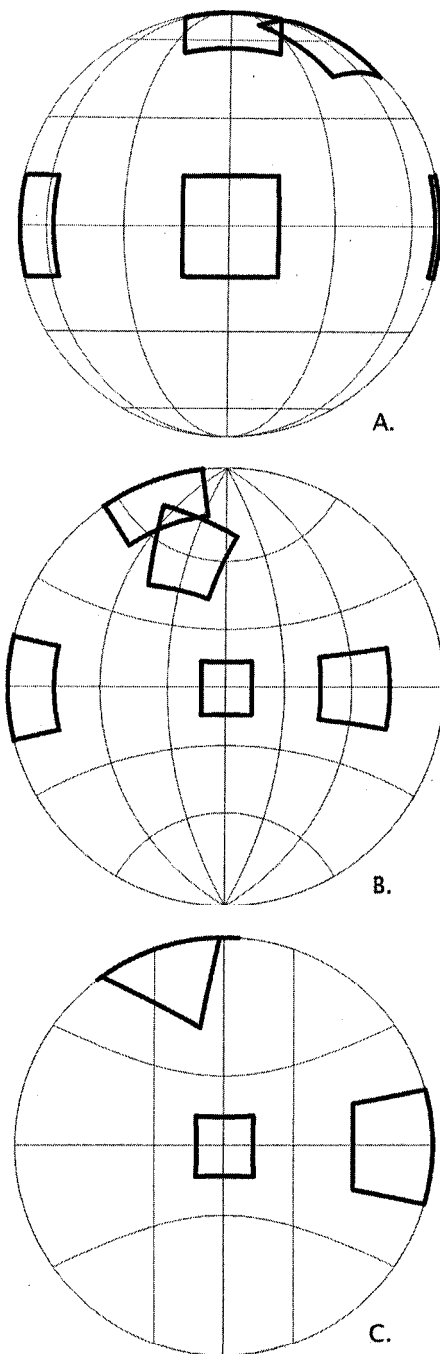


Figure 2. Spherical "squares" is another example of the use of familiar shapes for depicting distortion (after Chamberlin 1947). The projections illustrated are: A. Orthographic; B. Stereographic; and C. Gnomonic.

simple shapes are also used individually or in sets. For example, Figure 2 shows a set of three projections, each with five "spherical" squares created after Chamberlin's (1947) model. A more continuous view of distortion is seen in Figure 3, an illustration created after Fisher and Miller (1944) who employed an unbroken net of equilateral spherical

triangles and compared these to a set of equilateral planar triangles.

The use of familiar shapes is the most accessible of all of the visualization methods. Familiar shapes are limited, however, in that the amount and characteristics of distortion are not communicated explicitly when using this method. Furthermore, the use of a familiar shape is implemented as a learning device rather than as an analytical tool. On the positive side, while a student may or may not grasp the notion of transforming between the surface of the globe and a flat map, the essential idea is that something has changed. Features now appear in different shapes or sizes in an immediately recognizable way. Familiar shapes are highly recommended as an introduction to the concept of map projection distortion.

#### Interactive Display of Distortion

The use of a simple geometric shape is the basis for displaying map projection distortion in an interactive software package created by Brainerd and Pang (1998). It is intended as a tool for map users in a wide range of applications and disciplines. In the software, a ring floats freely upon the surface of the globe like a contact lens and is referred to as the "Floating Ring" by the developers. The ring can be scaled as well as moved over the surface of the globe. It can be shown as a simple outline or a continuous raster fill that depicts either area or angular distortion. The user is able to interactively manipulate the ring on a globe-like view on one screen and see the resultant distortions on a second screen. The strength of this approach is the interactivity of the software combined with multiple visual cues.

Figure 4 shows the contents of the two map viewing sections of the software screens for the Floating Ring. On the left is the globe-like view with a cone centered over the North Pole. The cone explicitly refers to the classification system for map projections that relates each projection type to a simple planar geometric shape, cone, or cylinder. A ring is shown floating on the surface of the globe in Southern Africa. On the right side of the graphic is the map that results from the application of a conic projection. The floating ring over Southern Africa now appears distorted in both shape and area.

The software requires further development, and there are theoretical issues that influence its design and reduce its possible effectiveness. These issues are understandable given that this contribution to cartographic visualization comes from computer science. During future development

of the floating ring software, the inclusion of a cartographer in the development team might assist in making this an even more effective exploratory learning tool for map projections.

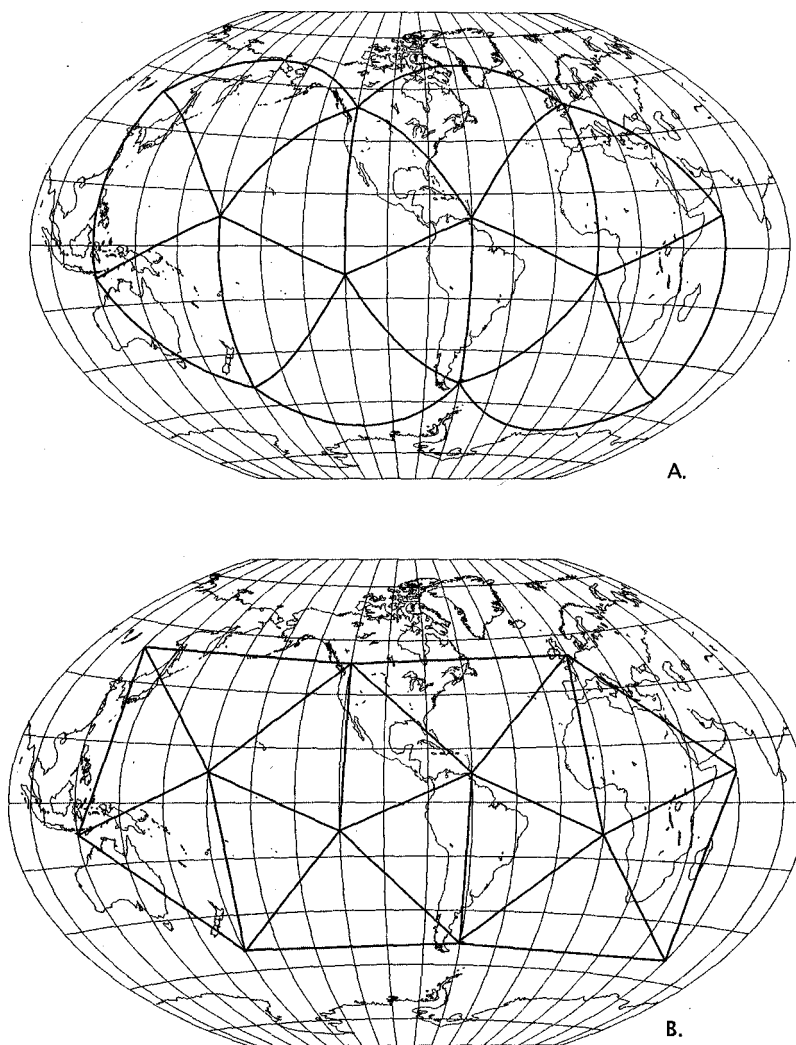
### Comparisons

The mental act of comparison is inherent in nearly all of the symbolization methods described in this paper. For example, the familiar shapes method previously described functions by allowing the viewer to compare the original shape to the transformed shapes. Of particular interest, however, is the comparison of sets of isolines, shorelines, and graticules. Comparisons, like familiar shapes, are frequently used in educational contexts, although they also have been employed in analytical studies.

In Figure 5, the Lambert azimuthal equal-area projection is superimposed on the stereographic projection at the same scale and with the same origin at 0 degrees latitude and 72.5 degrees west longitude. Features overlap perfectly near the origin of the map. This is the corresponding area of low distortion on both projections. A comparison of the graticules and the shorelines indicates an increasing offset with increased distance from the origin.

Although the scale, origin, and extent of these two maps are the same, the two projections differ greatly from one another. Due to an increasing scale distortion that increases radially from the projection center, only the centers of the projections remain true to scale. The stereographic projection has the property of conformality where local angles are retained in the output map. To maintain conformality, scale, and hence, area enlargement increases dramatically with distance from the origin of the projection. The Lambert azimuthal equal-area projection is less dramatic in exposing the angular distortion that results from maintaining the equal-area property. Still, land masses and countries towards the outer edge of the maps, such as

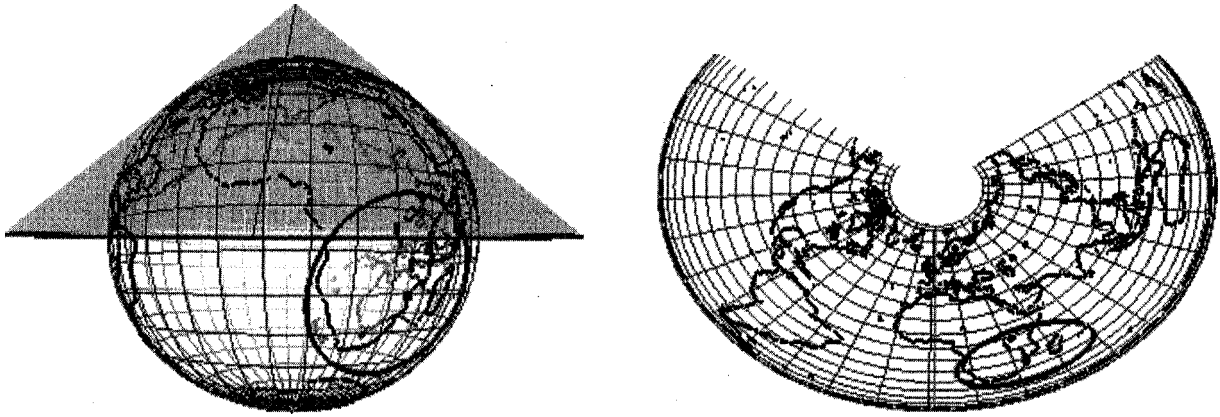
Antarctica



**Figure 3.** Spherical and plane triangles on the Winkle Tripel projection is a visualization similar to Fisher and Miller (1944). Familiar shapes such as the triangles can also provide distortion measurements over large areas.

and Greenland, have a highly altered appearance. Another example is found in the online materials of Peter Dana (1995) which effectively compare the Lambert conformal conic to the Albers equal-area conic for North and Central America. The comparison approach is excellent as an educational device because it goes a step beyond the use of familiar shapes by overlaying one map directly upon the other. The close juxtaposition of the map features encourages detailed examination of the differences due to the projections.

Although comparisons of graticules and shorelines are often used for educational purposes, they have analytical applications as well. Maling (1992) provided an example of comparing isolines of dis



**Figure 4.** Contents from map views in the Floating Ring interactive software. The globe-like view on the left indicates the orientation and type of projection that is shown on the right: namely, a conic projection centered over the North Pole. The left map view also shows the size and orientation of the floating ring on the globe. The right map view shows the resultant conic projection and floating ring.

tortion for two projections over an approximate outline of Europe. His goal was to select the projection that would have minimal distortion characteristics for a GIS termed: Coordinated Information on the European Environment (CORINE). Maling (1992) compared the relative merit of the azimuthal equal-area projection and Bonne's projection by overlaying isolines of maximum angular distortion on an approximate coastline of the North American continent, and he compared isolines of particular scales on the normal aspect of the Mercator, transverse Mercator, and the stereographic projection for a conformal map of Latin America.

Overall, comparisons are one of the most flexible and accessible methods for serving both educational and analytical purposes. Having the graticules, shorelines or isolines overlapping on the same map makes feature comparison very easy for map users. Comparisons are beneficial from a mapmaker's perspective because they are very simple to create. The mapmaker can employ existing software packages to layer vector maps for an effective cartographic visualization. For example, the ArcView 3.2 software package was used to create Figure 5.

#### Isolines of Projection Distortion

Although isolines are mentioned in the previous section on comparisons, they are an important symbolization method in their own right. The use of isolines for the display of map projection distortion is effective for depicting the magnitude and the distribution of any distortion measurement. Isolines are lines that represent equality with respect to a given variable such as contour lines of equal elevation and isobars of atmospheric

pressure. Isolines can be used to map any variable that is assumed continuous. When applied to distortion, isolines are used (often with shading between the lines) to identify the distribution and magnitude of area exaggeration or angular deformation. They can be used to depict any distortion measure, however. Isolines were used extensively by Maling (1992), by Bugayevskiy and Snyder (1995), and almost exclusively by Canters and Declair (1989) in their directory of world map projections. The isoline approach also is a very common distortion method in reference books on projections.

The two world maps in Figure 6 depict map projection distortion on the Winkle Tripel projection. This projection was recently adopted by the National Geographic Society after using the very similar Robinson projection from 1988 to 1998 because the "Winkel Tripel improves on the Robinson by distorting the polar areas less" (Miller 1999). The upper map depicts degree measurements of angular distortion and the lower indicates percentage of scale distortion. This projection is neither equivalent nor conformal, but instead seeks to portray the world in a balanced fashion suitable for world reference maps.

Isoline mapping is a quantitative symbolization method and provides absolute values of distortion. In addition to its effectiveness at portraying the amount and distribution of distortion, the ability to determine absolute values is its greatest strength as a symbolization method.

#### Perspective Surface

Although not frequently used for symbolization of projection distortion, a perspective surface may

be employed because it provides a very intuitive cartographic display. The surface can represent a wide variety of different distortion measurements, such as the magnitude of angular or area distortion. Clarke and Teague (1998) used this method to create a series of small multiple displays with distortion mapped separately for east-west, north-south, and the overall magnitude of positional error.

Figure 7 portrays results from evaluating the pixel changes that occur during the projection of a raster data set to a Mollweide projection. A raster data structure is essentially a matrix, with each element being described as a pixel. Each time a raster data set is transformed, scale change may cause change in pixel attribute values or the number of pixels. Pixels might be deleted in areas of scale reduction, while pixels may be duplicated in areas of scale expansion. Pixel duplication is evaluated at  $10^0$  increments over the map. The map is presented here as a perspective surface with dots indicating the location of data points. The Mollweide projection shows minimum distortion at approximately 40 degrees north and south of the equator, where the projection is true to scale. In Figure 7, these locations appear as "valleys" along the parallels at approximately 40 degrees north and south. Beyond 40 degrees north and south, distortion increases rapidly towards the poles.

There are minor difficulties associated with creating and viewing world map projection distortion as a continuous surface. A general principle is that map projection distortion is at a minimum at particular parallels or central points and increases towards a maximum at the edges of the map. A world map is generally located such that minimal distortion is at the center of the map. In Figure 7, distortion is viewed as a saddle-like object where the extremes of distortion occur at the edge. Finding a suitable viewpoint for the surface so that the distortion surface is easily seen can be challenging.

The perspective surface is highly effective at presenting a continuous depiction of the distribution and relative magnitude of a variety of distortion measurements. A surface can be used alone

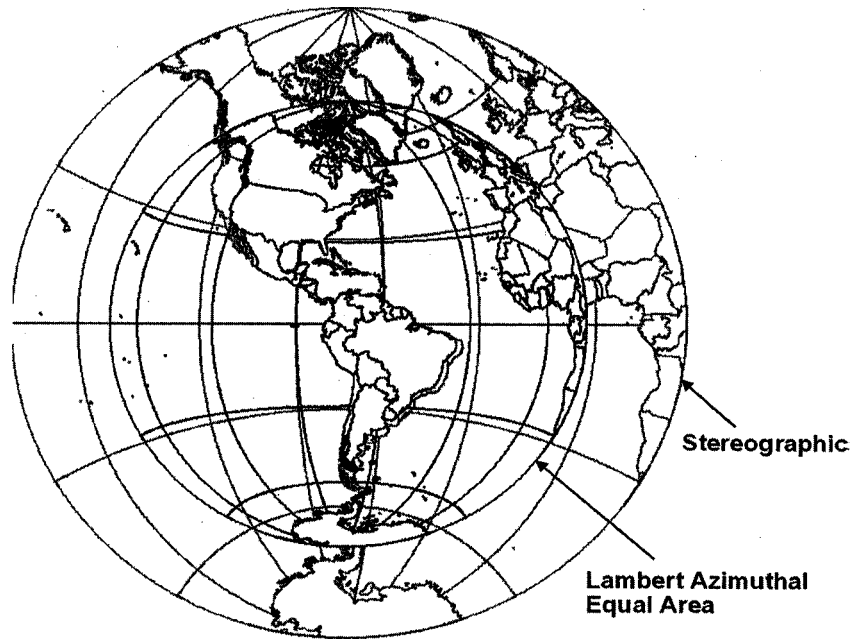


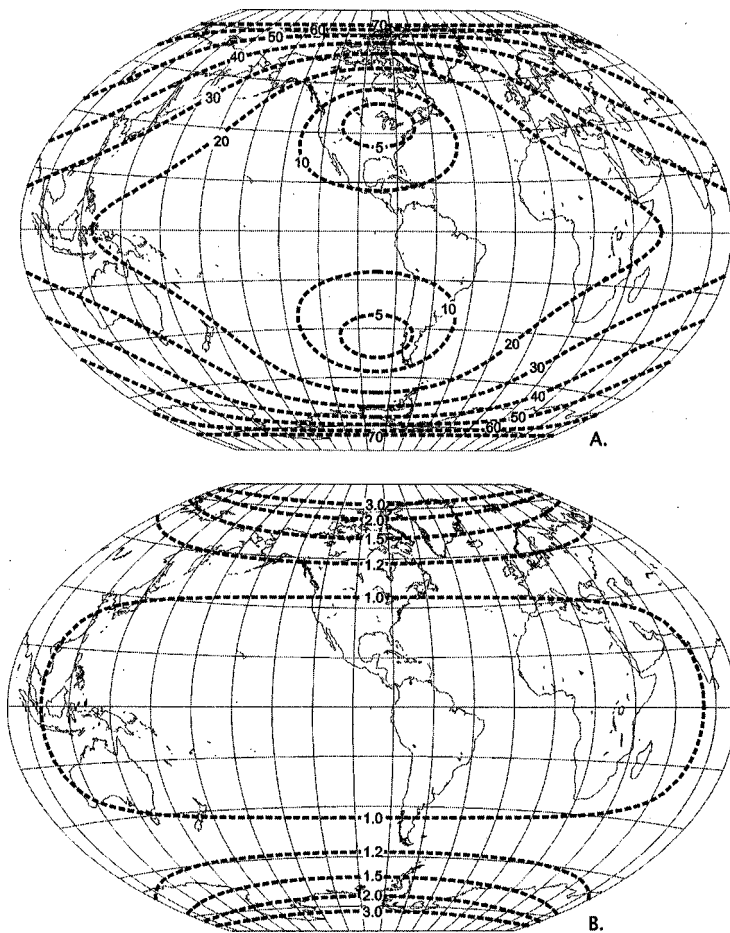
Figure 5. Comparison of the Lambert azimuthal equal-area and stereographic projections using graticules, shorelines, and country boundaries. Point of origin is  $0^{\circ}\text{N}$ ,  $72.5^{\circ}\text{W}$ .

to depict the magnitude and spatial distribution of distortion, but this symbolization method may have the greatest impact when used along with a primary map or dataset to illustrate the consequences of projection selection. For example, small inset maps of a perspective surface might be provided to data users as an integral part of the data documentation.

#### Color Methods

Similar to the perspective surface, color methods can provide a continuous display of distortion across a map. Distortion is symbolized by assigning a numerical measure to each pixel of a raster dataset. To illustrate, Figure 8 shows the same data shown as a perspective surface. Compare the distribution and relative magnitude of distortion in Figures 7 and 8. A continuous color distortion display may consist of a single variable or multiple variables. The color method can be nearly as intuitive as the perspective surface when a single variable is being mapped, and it has the advantage of allowing the entire distortion distribution to be viewed at one time.

Color methods by Clarke and Mulcahy (1995) present a multivariate display using hues. The east-west scale distortion along the parallels was mapped to red, the north-south scale distortion along the meridians to green, and the convergence angle to blue. This same approach was used



**Figure 6. Distortion expressed as isolines on the Winkle Tripel projection. A. Degree of angular deformation; B. Scale change.**

by Clarke and Teague (1998) to depict locational distortion for a comparison of two vector maps. They presented several versions employing the red-green-blue (RGB) and the hue-saturation-intensity (HSI) color models. Other examples of color methods by Shuey and Tessar (1999) and Shuey and Murphy (2000) depict distortion in area, angle and shape. Each distortion variable is assigned to a raster data layer (or a band) and referred to as "distortion channel." Each distortion channel is then symbolized using an additive RGB color model.

Color methods provide a visually continuous representation of the magnitude of distortion. Both the spatial distribution and degree of distortion are effectively portrayed. Color methods also have the advantage of being able to display more than one variable at the same time. These methods may be particularly useful to researchers accustomed to viewing global data sets in continuous color displays. As with perspective surfaces, color methods can be effectively used as supplements to illustrate the consequences of projection selection.

### Tissot Indicatrix

Tissot's century-old method is an elegant and effective means for depicting map projection distortion. Tissot used a display method called the indicatrix, which is visually based on the transformation of small circles. Unlike the circle used as a familiar shape, the indicatrix is based upon Tissot's (1881) theorem of map projection distortion. Imagine an infinitely small circle on the globe. This circle is then transformed by a map projection and, in the process, it changes in size and/or shape due to scale change. The circles may remain circular but more often become ellipses with the semimajor and semi-minor axis reflecting the amount, type, and direction of the projection distortion. Tissot's indicatrix has been used extensively to

enhance understanding of projections in specialized cartographic literature (Snyder and Voxland 1989). The indicatrix is also used as an educational tool in explaining map projection distortion in many contemporary textbooks in cartography (e.g., Dent 1999; Jones 1997; Kraak and Ormeling 1996; and Robinson et. al. 1995).

To illustrate Tissot's indicatrix, ellipses are placed at the intersections of 30° graticules in Figure 9. The graticule in Figure 9A shows the changes in shape that relate to angular and area distortion. For example, the sinusoidal projection has the geometric property of equal area, and so any area on the map accurately reflects the same relative area on the globe. Using Tissot's indicatrix, the ellipses indicate the equal area property by retaining the same area throughout the map. The amount and direction of maximum angular change are also indicated, as ellipses are increasingly distorted with increased distance from the center of the projection.

In the second example in Figure 9B, the ellipses have retained a circular shape. This time the projection is the Mercator that has the property of conformality, meaning all local angles remain true. In this case, ellipses near the equator are the only ones exhibiting near-true equal area properties. With increased distance from the equator, north or south, the amount of area distortion increases dramatically.

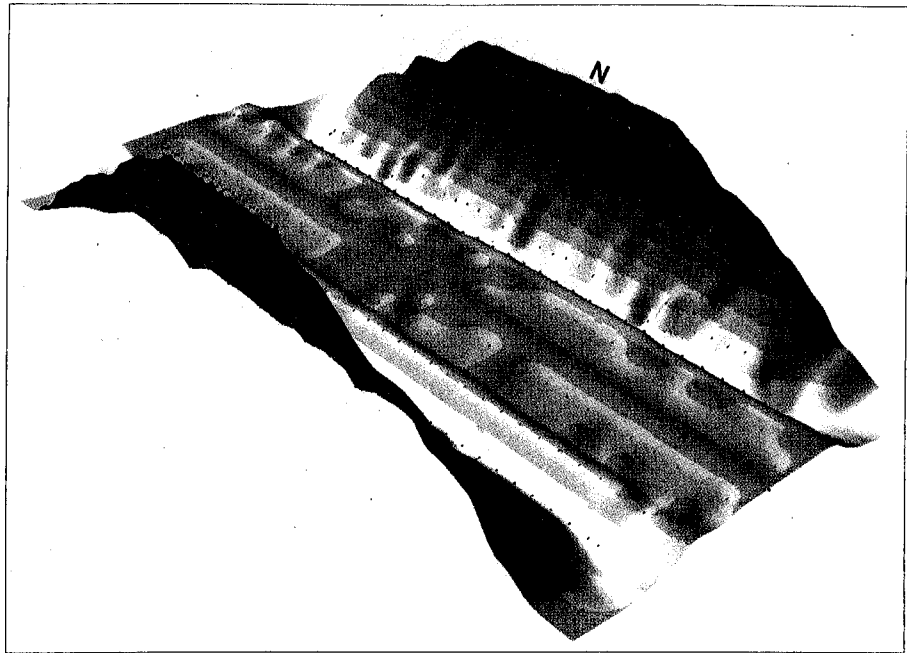
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Although Tissot's method is effective and widely accepted there have been criticisms of it. One problem is that the indicatrix is frequently interpreted to be limited to the forward transformation from the globe to a map projection only. In computer applications there is more frequently a need to move data between various projections. A second problem is that the distortion distribution shown by Tissot's indicatrix only describes the infinitesimal areas near the center of the ellipses, which is not the same as the area of the ellipses indicated visually. This may lead to misinterpretations of the location of distortion. A third problem, raised by Brainerd and Pang (1998), is that the use of the indicatrix obscures underlying data. We feel that ellipses may be superimposed on a thematic map without obscuring the primary map data by symbolizing the ellipses by an outline only and not filling the symbols.

#### The Map Graticule

Many projections are evaluated or classified based upon the appearance of the latitude and longitude grid-known as the graticule-on a map. Robinson et al. (1995) argued that the evaluation of many projections could be accomplished effectively by a visual analysis whereby the characteristics of the graticule on a map are compared to the graticule on a reference globe (Figure 10). They listed nine comparative visual characteristics of the graticule on a globe.

- Parallels are parallel.
- Parallels, when shown at a constant interval, are spaced equally on meridians.
- Meridians and great circles on a globe appear as straight lines when viewed orthogonally (looking straight down), which is the way we look at a flat map.
- Meridians converge toward the poles and diverge toward the equator.
- Meridians, when shown at a constant interval, are equally spaced on the parallels, but their



**Figure 7.** Surface representation of distortion where the height of the surface indicates the amount of pixel loss for a raster data set that has been transformed to the Mollweide projection. Bands of increasing color intensity reinforce the use of height to illustrate increasing distortion.

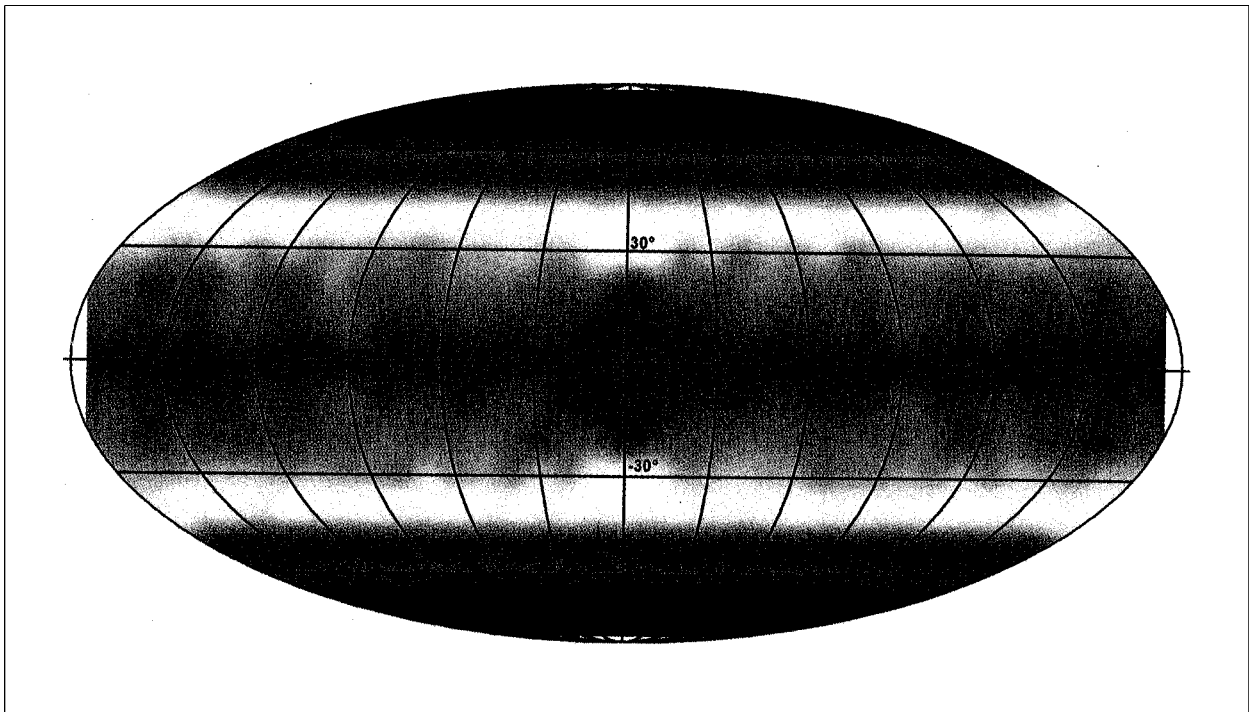
spacing decreases from the equator to the pole.

- When both are shown with the same intervals, meridians and parallels are equally spaced at or near the equator.
- When both are shown with the same intervals, meridians at 60° latitude are half as far apart as parallels.
- Parallels and meridians always intersect at right angles.
- The surface area bounded by any two parallels and two meridians (a given distance apart) is the same anywhere between the same parallels.

The globe-like map appearing in Figure 10 is an orthographic projection. The orthographic projection is the preferred choice when a globe-like map view is required. Robinson (1986) makes extensive use of the orthographic as a reference globe specifically for assessing world map projections. Each of the previously listed characteristics of the graticule on a globe can be illustrated using this globe-like projection. To evaluate another projection, comparisons can be made between the globe-like graticule and the cylindrical projection above or the pseudocylindrical projection below.

The significance of the appearance of the graticule was highlighted by Canters and Declair (1989). They classified projections into a system' based upon Maling (1973) and Tobler (1962). Canters





**Figure 8.** Color method showing the same data as the surface in Figure 6, where the continuously increasing color intensity indicates the amount of pixel loss for a raster data set that has been transformed to the Mollweide projection.

and Declair arranged 68 world projections into a three-by-four matrix. Along both axes, various characteristics of the graticule were described. On one axis, the various classes of projections are grouped as:

- Polyconic, where both parallels and meridians are curved;
- Pseudocylindrical, which have "curved meridians and straight parallels;"
- Pseudoconic, which "represents meridians as concurrent curves and parallels as concentric circular arcs;" and
- Cylindrical, which "comprises projections where both parallels and meridians are represented by parallel straight lines."

The other axis consists of the description of the parallels as being equally spaced, having decreasing parallel spacing, or having increasing parallel spacing (Canters and Declair 1989).

The appearance of the graticule is useful in the construction and use of these classification systems described above and for the identification of unknown projections. The appearance of the graticule may be used to help during map projection selection. A study of the characteristics of the graticule is also useful in an educational context to illustrate some of the basic map projection distortion concepts. The use of the graticule in this instance is somewhat limited because the graticule does not directly symbolize scale, area, or angular change

as clearly as Tissot's indicatrix, isolines, color methods, or a perspective surface.

### Grid Squares

The last two methods presented here—grid squares and checkerboard—were developed during a search for the best projection for a global raster database to be distributed by the USGS. Steinwand et al. (1995) were the first to address, and publish on, the problem of selecting the "best" projection for this type of data. Both grid squares and checkerboard methods were developed primarily to explore the difference between projecting raster and vector data, but both methods are useful for depicting projection distortion in general.

The grid squares method is visually similar to Tissot's indicatrix, with squares replacing circles. The grid squares method consists of the creation of sets of three-by-three vector-structured grids located at the intersection of selected parallels and meridians. The original grid squares shown in Figure 11A were formed on a Plate Carree projection and then transformed to the Lambert azimuthal equal area projection as seen in Figure 11B.

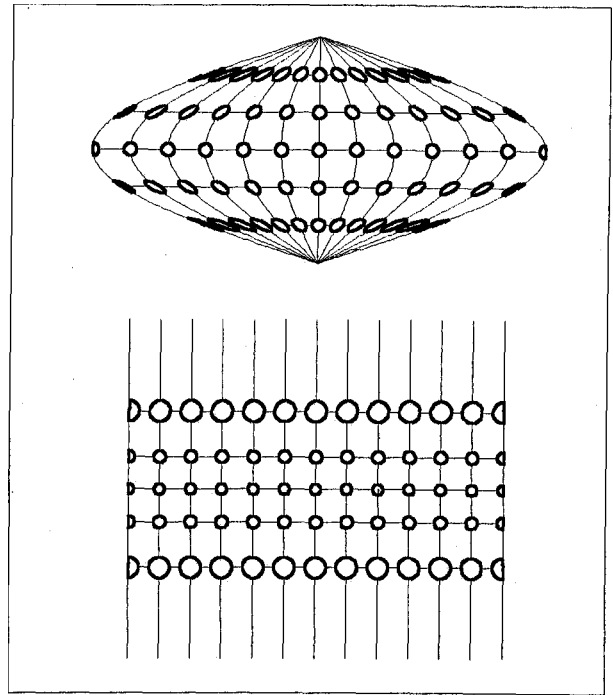
Although this method can be thought of as a raster-styled Tissot's indicatrix, there is no explicit formula for determining angular distortion. Area is measured over a discrete region rather than

within an infinitesimally small area. Nevertheless, the type, amount, direction, and overall spatial distribution of map projection distortion throughout the map are communicated effectively by this method. Grid squares also have the advantage of identification, with the raster data structure highlighting the data changes that occur when selecting different projections. The primary advantage of using the grid squares over Tissot's indicatrix is that this method is not limited to the globe-to-map case and can be used for examining changes in distortion between different projections.

### Checkerboard Method

The checkerboard method was the second method developed by Steinwand et al. (1995) as a raster-based analytical tool. A regular checkerboard pattern mimics the state of the discrete raster structure, which does not respond to projection transformations like vector data. Vector data consist of locations or points that may stand alone to represent a feature or may be combined to form lines and polygons. These vector data are transformed point-by-point in a manner very similar to the way cartographers have manually transformed map features for centuries. The transformation of a raster data structure is a different matter. Raster data consist of a matrix of data values usually referred to as pixels. The transformation of a raster data set occurs in two stages. First, a new geometric space for the projected matrix is created. Then, for each pixel in the new geometric space, the original matrix is searched for the appropriate attribute value, and this attribute value is then transferred to the new projected matrix. The result is often a dramatic change in map features. Lines may become stair-stepped and pixels duplicated or dropped. Since raster data are not as flexible concerning geometric transformations as vector data, the checkerboard assists in communicating the differences between the two structures.

In the checkerboard method, a data set is developed with alternating colored squares and is then transformed by map projection. The pattern formed by the alternating hues or values exposes the spatial pattern and distribution of distortion and shape alterations, which provides insight into the limitations of the raster structure in terms of geometrical alterations. The checkerboard projected in Figure 12 is a fragment from a global vector-based grid data set by Shuey and Murphy (2000). It was originally created on a curved sphere and transformed to a Plate Carree projection. It is used here to show the extent of distortion that occurs to the individual pixels.

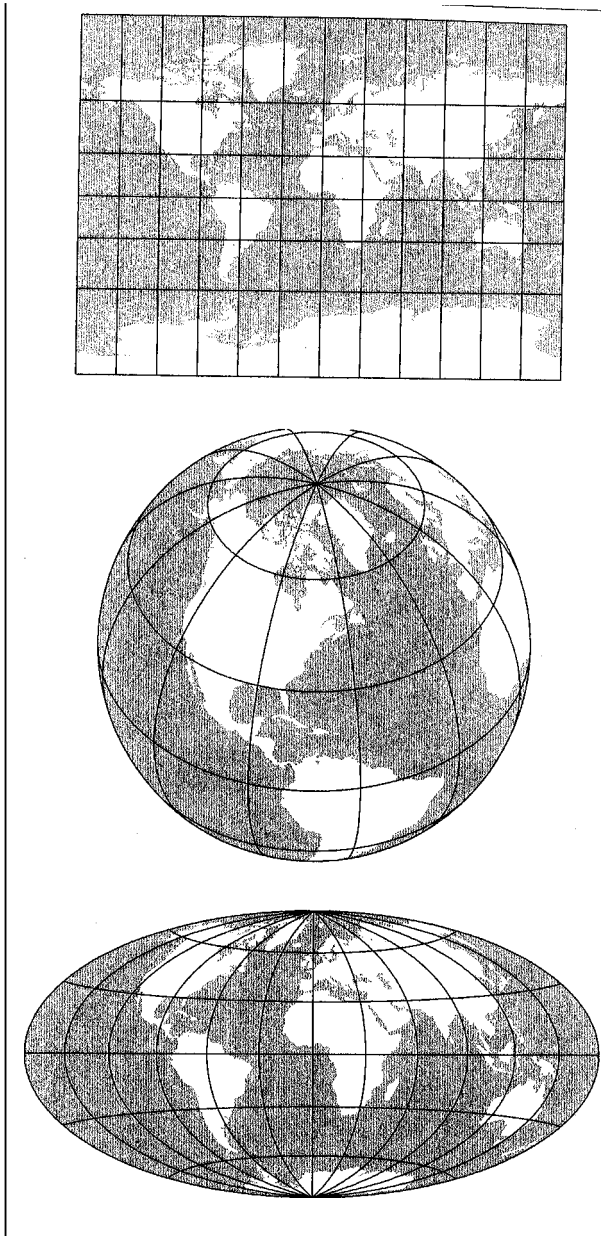


**Figure 9.** Tissot's indicatrix shown at 300 increments. Upper map shows the equal-area sinusoidal projection and the lower map, the conformal Mercator projection.

The checkerboard method is useful for educational purposes, resembling the use of familiar shapes, but it has an important advantage. In addition to communicating the basic concept of geometric changes occurring during projection transformation, the checkerboard illustrates the changes that occur within a raster data structure. Users of spatial information systems such as image processing or geographic information systems are unaware that there is any difference between the transformations of vector versus raster data. The checkerboard effectively addresses this issue.

### Discussion

This review has outlined the principal methods of cartographic display of map projection distortion. Clearly, many methods have been designed and implemented, and most of them are highly effective at communicating the spatial pattern of map projection distortion. Nevertheless, the use of distortion display is negligible beyond cartographic and map projection textbooks, manuals, and technical documentation. This is unfortunate, given that map and GIS users often have an unrealistic level of faith in their base map and little knowledge of projections. Nyerges and Jankowski (1989) went so far as to say, "few people, even few cartographers, commonly know which projection is good



**Figure 10.** Graticule patterns for several projections including the orthographic projection as reference globe at the center, the Mercator above, and the Hammer below.

for what purpose and the trade-offs involved." For the GIS user working at continental and global scales, an understanding of projections is critical during the data capture, management (registration, merging, overlay), and analysis stages.

Cartographic education has tended to treat map projections as peripheral, too technical, simplistically, or not at all. This trend has continued throughout the more general area of GIScience. Democratization of the access and use of spatial data has brought the projection issue to the forefront because important decisions are often based

upon spatial information. Raster GIS and image processing, with its massive amount of hidden image resampling, can lead to greater errors than provided by the vector systems, yet neither is problem free from the projection standpoint.

Compounding the problem of deficiencies in the educational process, the support for map projections in many software packages is limited. Often only a few projections are provided to the user. In comparing the implementation of equal area world map projections in commercial software, Mulcahy (1999) found that ESRI's ArcInfo 7.04 software had the greatest number at seven, while the ERDAS Imagine 8.1 and Clark Lab's Idrisi software packages had only one each. Furthermore, there is no access to distortion measurement or the display of projection distortion in popular GIS and remote sensing software.

Distortion displays should be closely linked with their respective data sets and even with map products. Map projection distortion, and, indeed, any other spatial error, could be considered a confidence, or uncertainty overlay, that can be carried along with the map data during analysis and applied as a reality check or accuracy filter when necessary. This might be called truth in labeling. If distortion displays were provided, the GIS user would be encouraged to pay considerably more attention to projection issues throughout the entire mapping process. The inherent distortion could be included as a parallel or inset map or as an overlay to the primary map.

Shuey and Tessar (1999) hope to implement a Web based on-demand service that would allow a user to submit a data set to a geographic server. The server would then calculate distortion levels and return an image depicting the combined distortion in area, angles and shapes. Providing information in the form of distortion displays on demand is an important step leading towards the optimal use of map projections.

## Conclusion

Much creative energy has been expended at symbolizing map projection distortion. Some distortion symbolization methods seem logical expressions of the analytical treatment of distortion such as using the graticule, the Tissot indicatrix, the color methods, the grid squares and the checkerboard. Other symbolization methods are borrowed from the standard cartographic symbolization toolbox such as the isoline method or perspective surface. Finally, a number of methods seem inspired simply by the need to communicate map projection distort

tion to inexperienced users, such as familiar shapes, interactive display of distortion, or comparisons. There is a wide array of methods to choose from to suit any need for depicting map projection distortion.

The arrangement of symbolization methods and many of the comments by the authors on the effectiveness of the methods are subjective. Future studies may include the testing of the authors' opinions with various user groups, from children being introduced to map projections, to researchers studying various aspects of global systems. These methods can be presented in many settings, from textbooks and popular GIS and remote sensing software to on-demand Web-based servers. The authors hope that increased awareness of these methods of projection distortion symbolization will enhance understanding of the principles and consequences of projection distortion.

#### ACKNOWLEDGMENTS

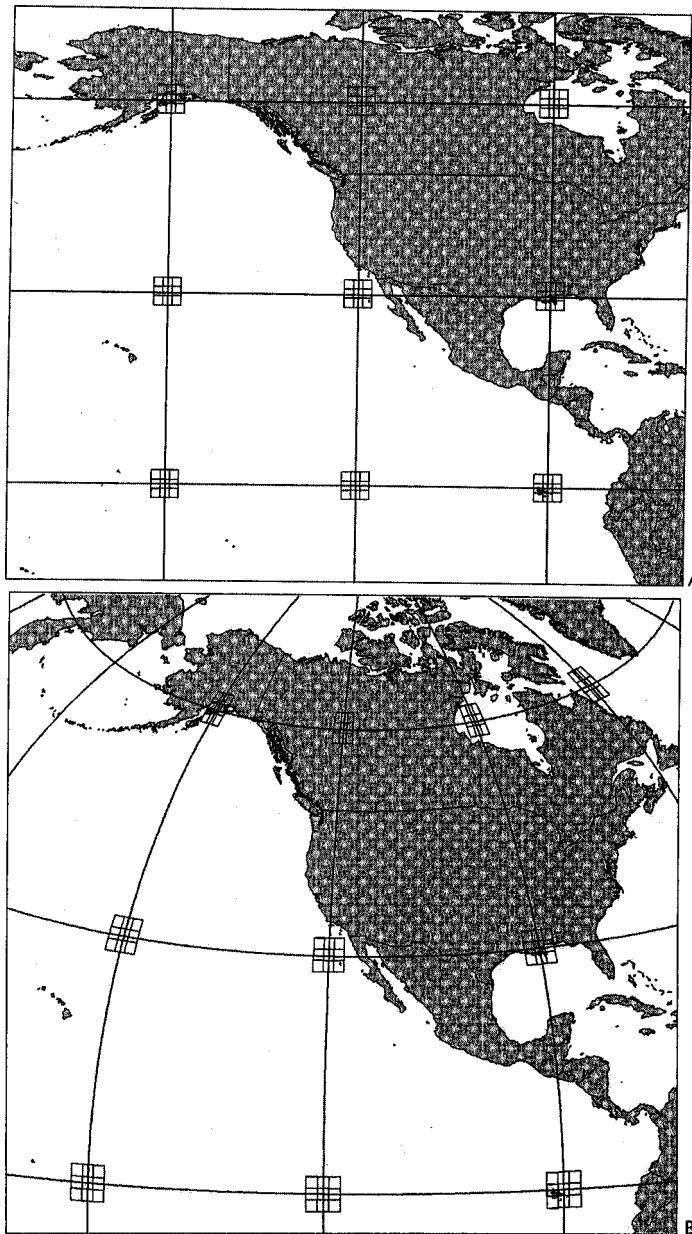
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#### NOTE

Numerous terms specific to the map projection literature are described in a glossary prepared for this work. It is located at <<http://www.geog.ucsb.edu/~kclarke/Papers/Distortion.html>>.

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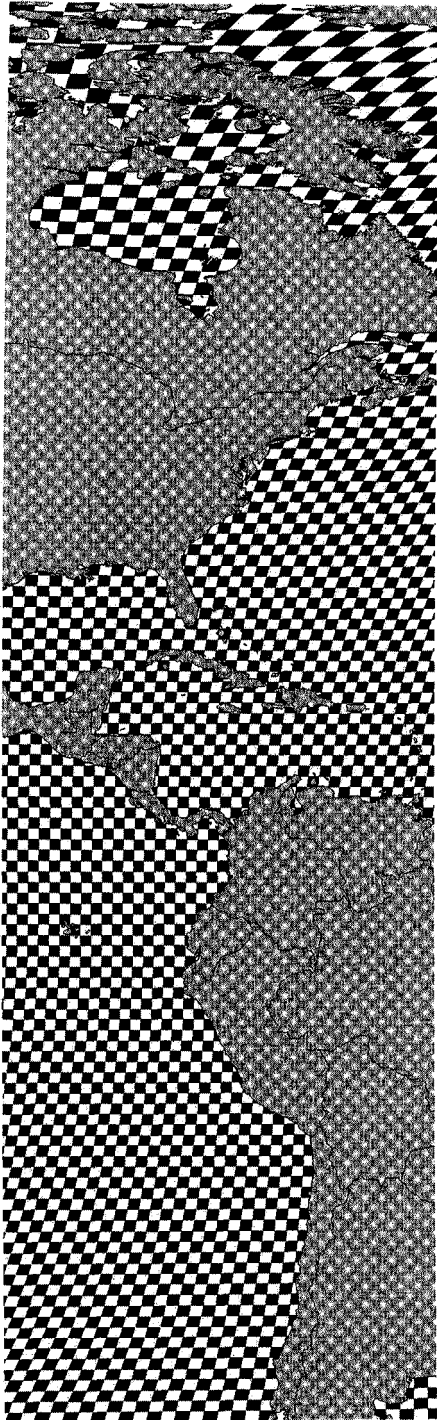
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**Figure 11.** Grid squares portrayed before and after transformation by a projection. A. Set of grid squares located at graticule intersections on the Plate Carree projection. B. The same set of grid squares after a transformation to the Lambert azimuthal equal-area projection (after Steinward et al. 1995).

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**Figure 12.** The checkboard image was created from a data set developed by Shuey and Murphy (2000), which was originally created on a curved sphere and then transformed to a Plate Carree projection. The variations in the size and shape of the individual squares indicate the degree of data change.

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