

# Error Analysis on Grid-Based Slope and Aspect Algorithms

Qiming Zhou and Xuejun Liu

## Abstract

*Slope and aspect are the most frequently used surface geomorphic parameters in terrain analysis. While derived from grid DEM, the parameters often display noticeable errors due to errors (a) in data, (b) inherent in data structure, and (c) created by algorithms. It has been observed that some controversial results were reported in evaluating the results by various slope and aspect algorithms, largely because of the variety in assessment methodology and the difficulties in separating errors in data and those generated by the algorithms. This paper reports the study that assesses and compares the results from numerous grid-based slope and aspect algorithms using an analytical approach. Tests were made based on artificial polynomial surfaces which can be defined by mathematical formulae, with controllable "added" data errors. By this approach, different algorithms were quantitatively tested and their error components were analyzed. Thus, their suitability and tolerance related to DEM data characteristics can be described.*

## Introduction

Slope and aspect have been regarded as two of the most important geomorphic parameters, as they not only efficiently describe the relief and structure of the land surface, but are also widely applied as vital parameters in hydrological models (Band, 1986; Moore *et al.*, 1988; Quinn *et al.*, 1991; 1995; Jenson, 1994), landslide monitoring and analysis (Duan and Grant, 2000), mass movement and soil erosion studies (Dietrich *et al.*, 1993; Desmet and Govers, 1996a; Mitasova *et al.*, 1996; Biesemans *et al.*, 2000) and landuse planning (Desmet and Govers, 1996b; Stephen and Irvin, 2000).

In most applications today, slope and aspect are typically derived from DEM that is based on raster data structure. To date, there have been numerous mathematical models and algorithms that calculate slope and aspect from elevation data (e.g., Sharpnack and Akin, 1969; Fleming and Hoffer, 1979; Horn, 1981; Unwin, 1981; O'Callaghan and Mark, 1984; Zevenbergen and Thorn, 1987; Wood, 1996). Although the mathematical definition of slope and aspect is quite clear, its implementation based on grid-based DEM may vary, since some assumptions must be made on how the continuous surface is approximated by discrete sample points (i.e., grid cells). The variation in such implementation would only present minor problems in applications such as surface visualization and classification, but its impact on terrain analysis that is based on quantitative models could be very significant (Zhou *et al.*, 1998; Tang, 2000; Zhou and Liu, 2002). It was pointed out that selection of algorithms could be a critical

factor that might create a great impact on the analytical results (Moore, 1996; Burrough and McDonnell, 1998).

Studies have been conducted to analyze the errors created by the slope and aspect algorithms with a variety of approaches and methodologies. One approach emphasizes errors in DEM itself while paying little attention on models. For example, Skidmore (1989) and Florinsky (1998) analyzed slope and aspect errors using the actual survey data. Another approach focused on mathematical models while ignoring DEM errors (e.g., Hodgson, 1995; Jones, 1998). The conclusions from these studies have been quite different, sometimes controversial. For example, Skidmore (1989) concluded that third-order finite difference method (Horn 1981) derived better results than the second-order finite difference method. His finding was also confirmed by Florinsky (1998). On the other hand, Hodgson (1995) and Jones (1998) stated that the latter performed better than the former in computing slope and aspect values.

To make a *fair* comparison between different slope and aspect algorithms, three pre-conditions must be satisfied:

1. The source of error should be identified;
2. The kinds of errors must be independent, identifiable and controllable in the test so that their impact can be quantified; and
3. A reference (or *true value*) must be established to make the results comparable.

Zhou and Liu (2002) reported a method that is based on mathematical surfaces to evaluate the errors generated by flow routing algorithms in digital terrain analysis. Using this method, errors created by an algorithm can be isolated from DEM errors so that the quantitative measures can be made to compare a derived, specific catchment area with the *true value* defined by the polynomial surfaces.

Using an analytical approach, Florinsky (1998) developed test for the precision of four methods for computing partial derivatives of elevations, and produced formulae for Root Mean Square Error (RMSE) of four local topographic variables including slope and aspect as focused by this paper. Florinsky also used a real-world DEM to map the error distribution for visual analysis, but the contribution from the DEM data error largely remained unknown.

This paper reports the study that attempts to extend the above methods into the test of slope and aspect calculation. In this study, we extended Florinsky's method to general terms and designed tests on artificial polynomial surfaces with controllable data errors. Two representative surfaces, namely, the inverse ellipsoid and Gauss synthetic surface, were used to test six commonly-employed slope and aspect algorithms

Q. Zhou is with the Department of Geography, Hong Kong Baptist University, Kowloon Tong, Kowloon, Hong Kong, China (qiming@hkbu.edu.hk).

X. Liu is with the School of Geographical Science, Nanjing Normal University, Nanjing 210097, China.

Photogrammetric Engineering & Remote Sensing  
Vol. 70, No. 8, August 2004, pp. 957-962.

0099-1112/04/7008-0957/\$3.00/0  
© 2004 American Society for Photogrammetry  
and Remote Sensing



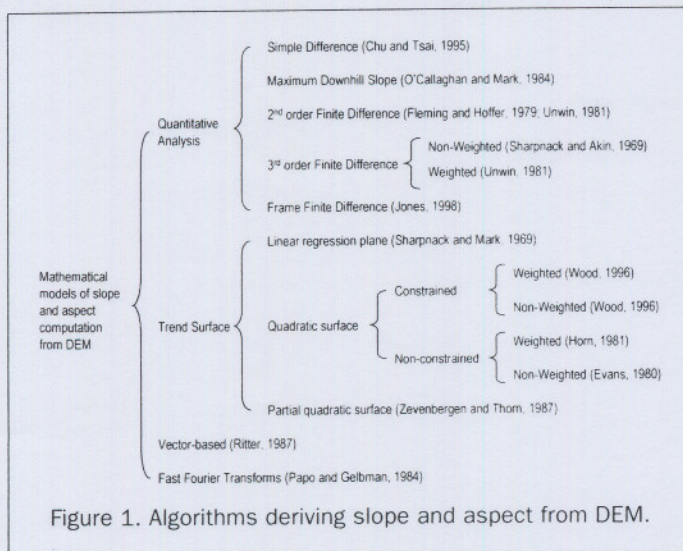


Figure 1. Algorithms deriving slope and aspect from DEM.

found in literature and GIS software. The derived slope and aspect values were then compared with the *true value* derived by mathematical inference. Quantitative analysis can then be conducted for the comparison of the algorithm accuracies.

### Slope and Aspect Algorithms

At a given point on a surface  $z = f(x, y)$ , the slope ( $S$ ) and aspect ( $A$ ) is defined as a function of gradients at  $X$  and  $Y$  (i.e., W-E and N-S) directions, i.e.:

$$S = \arctan \sqrt{f_x^2 + f_y^2} \quad (2.1)$$

$$A = 270^\circ + \arctan\left(\frac{f_y}{f_x}\right) - 90^\circ \frac{f_x}{|f_x|} \quad (2.2)$$

where  $f_x$  and  $f_y$  are the gradients at W-E and N-S directions, respectively.

From the above equations, it is clear that the key for slope and aspect computation is the calculation of  $f_x$  and  $f_y$ . Using a grid-based DEM, the common approach is to use a moving  $3 \times 3$  window to derive finite differential or local surface fitting polynomial for the calculation. Figure 1 shows various methods found in the literature and GIS software.

TABLE 1. MATHEMATICAL MODELS OF SLOPE AND ASPECT COMPUTATION FROM DEM

|  | 7 | 8 | 9 |  |
|--|---|---|---|--|
|  |   |   |   | $S = \arctan \sqrt{f_x^2 + f_y^2}$   |
|  |   |   |   | $A = 270^\circ + \arctan\left(\frac{f_y}{f_x}\right) - 90^\circ \frac{f_x}{ f_x }$   |
|  |   |   |   | $g = \text{spatial resolution (i.e., grid cell size)}$   |
| $f_x = (z_6 - z_4)/2g, f_y = (z_8 - z_2)/2g$                           |   |   |   | 2 <sup>nd</sup> order finite difference (2FD), Vector-based algorithm  |
| $f_x = (z_3 - z_1 + z_6 - z_4 + z_9 - z_7)/6g$                         |   |   |   | Partial quadratic equation   |
| $f_y = (z_7 - z_1 + z_8 - z_2 + z_9 - z_3)/6g$                         |   |   |   | 3 <sup>rd</sup> order finite difference (3FD), Linear regression plane, Unconstrained quadratic surface, constrained quadratic surface   |
| $f_x = (z_3 - z_1 + 2(z_6 - z_4) + z_9 - z_7)/8g$                      |   |   |   | 3 <sup>rd</sup> order finite difference weighted by reciprocal of squared distance (3FDWRSD), Weighted constrained quadratic surface, Weighted unconstrained quadratic surface |
| $f_y = (z_7 - z_1 + 2(z_8 - z_2) + z_9 - z_3)/8g$                      |   |   |   | 3 <sup>rd</sup> order finite difference weighted by reciprocal of distance (3FDWRD)  |
| $f_x = (z_3 - z_1 + \sqrt{2}(z_6 - z_4) + z_9 - z_7)/(4 + 2\sqrt{2})g$ |   |   |   |  |
| $f_y = (z_7 - z_1 + \sqrt{2}(z_8 - z_2) + z_9 - z_3)/(4 + 2\sqrt{2})g$ |   |   |   |  |
| $f_x = (z_3 - z_1 + z_9 - z_7)/4g$                                     |   |   |   | Frame finite difference (FFD)  |
| $f_y = (z_7 - z_1 + z_9 - z_3)/4g$                                     |   |   |   |  |
| $f_x = (z_5 - z_4)/g, f_y = (z_5 - z_2)/g$                             |   |   |   | Simple difference (Simple-D)   |

Considering the popularity and the use of different algorithms, we have selected six commonly-employed algorithms for test (Table 1):

- Second-order Finite Difference (2FD) (Fleming and Hoffer, 1979; Zevenbergen and Thorne, 1987; Ritter, 1987).
- Third-order Finite Difference (3FD) (Sharpnack and Akin, 1969; Horn, 1981; Wood, 1996).
- Third-order Finite Difference Weighted by Reciprocal of Squared Distance (3FDWRSD) (Horn, 1981).
- Third-order Finite Difference Weighted by Reciprocal of Distance (3FDWRD) (Unwin, 1981).
- Frame Finite Difference (FFD) (Chu and Tsai 1995).
- Simple Difference (SIMPLE-D) (Jones 1998).

### Error Components

The accuracy of slope and aspect computation is directly related to the partial derivatives at  $X$  and  $Y$  direction,  $f_x$  and  $f_y$ , which are estimated by numerous proposed methods. Taking the second-order finite difference as an example, let  $(x, y)$  denote the coordinates of the center cell in a  $3 \times 3$  window and  $g$  denote the DEM spatial resolution (i.e., grid cell size), the partial differential can be expressed as (King, 2000):

$$df_x = \frac{g^2}{6} \left( \frac{f_x'''(\xi_x, y) + f_x'''(\gamma_x, y)}{2} \right) + \frac{dz_8 - dz_2}{2g} \quad (3.1)$$

$$df_y = \frac{g^2}{6} \left( \frac{f_y'''(x, \xi_y) + f_y'''(x, \gamma_y)}{2} \right) + \frac{dz_6 - dz_4}{2g}$$

The first term represents the errors caused by the uncertainty of mathematical model implementation and  $\xi_x, \xi_y, \gamma_x$  and  $\gamma_y$  are dependences of  $f'''$ ,  $\xi_x \in (x, x + g)$ ,  $\xi_y \in (y, y + g)$ ,  $\gamma_x \in (x - g, x)$  and  $\gamma_y \in (y - g, y)$ . Since the relationships between these variables and  $x$  and  $y$  are usually not clear, it is difficult to define their values in application. Thus it is common to set the upper limits for  $f'''$  instead. Let  $M_x$  and  $M_y$  as the upper limits of  $f'''$  in terms of  $x$  and  $y$ , respectively; Equation 3.1 can be altered as:

$$df_x = \frac{g^2}{6} M_x + \frac{dz_8 - dz_2}{2g} \quad (3.2)$$

$$df_y = \frac{g^2}{6} M_y + \frac{dz_6 - dz_4}{2g}$$

The second term of the equation represents DEM data error (including data precision).



In Equation 3.2,  $M_x$  and  $M_y$  are estimated based on the 'worst scenario', usually represent much larger estimates than the actual case, with a given distribution of probability. Let the RMSE of  $M_x$  and  $M_y$  equal to  $M$ , and  $m$  denote the RMSE of the DEM, the result is:

$$m_{f_x}^2 = m_{f_y}^2 = \left(\frac{g^2}{6}\right)^2 M^2 + \frac{m^2}{2g^2} \quad (3.3)$$

Deriving the differentials of slope and aspect equations (Equations 2.1 and 2.2), and considering  $S = \arctan\sqrt{f_x^2 + f_y^2}$  and  $\tan^2 S = f_x^2 + f_y^2$ , the result is:

$$dS = \frac{f_x df_x + f_y df_y}{(1 + \tan^2 S)\tan S} \quad \text{and} \quad dA = \frac{f_y df_x + f_x df_y}{\tan^2 S} \quad (3.4)$$

Referring to Equation 3.3, the RMSE of slope ( $m_S$ ) and aspect ( $m_A$ ) can therefore be expressed as:

$$m_S^2 = \left[ \left(\frac{g^2}{6}\right)^2 M^2 + \frac{m^2}{2g^2} \right] \cos^4 S \quad (3.5)$$

$$m_A^2 = \frac{1}{\tan^2 S} \left[ \left(\frac{g^2}{6}\right)^2 M^2 + \frac{m^2}{2g^2} \right]$$

Note that Equation 3.5 is for the second-order finite difference method. Let  $a = \frac{1}{6}g^2$  and  $b = \frac{1}{\sqrt{2}}g^{-1}$ ; Equation 3.5 can then be expressed in a general form:

$$m_S = \sqrt{a^2 M^2 + b^2 m^2} \cos^2 S \quad (3.6)$$

$$m_A = \frac{1}{\tan S} \sqrt{a^2 M^2 + b^2 m^2}$$

Similar to the above procedure and referring to Table 1, we can derive the RMSE for each selected algorithm as shown in Table 2.

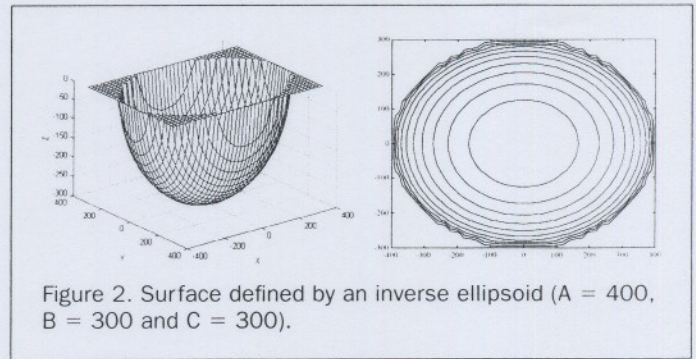
Equation 3.6 and Table 2 show that the overall errors of derived slope and aspect come from three sources:

1. *Algorithm errors*: caused by approximation and sampling of continuous surfaces (Variable  $M$  in Equation 3.6),
2. *DEM data errors*: caused by DEM data capture and generation (Variable  $m$  in Equation 3.6), and
3. *DEM spatial resolution* (i.e., the grid cell size—Variables  $a$  and  $b$  in Equation 3.6, defined in Table 2).

(Note that the coefficient  $b$  of  $m$  for 2FD and 3FD confirms the results reported by Florinsky (1998), which represent special cases for the selected algorithms.)

TABLE 2. THE RMSE FOR SELECTED SLOPE AND ASPECT ALGORITHMS

| Algorithm  | Coefficient $a$ of $M$                    | Coefficient $b$ of $m$        |
|--|---|-------------------------------|
| RMSE of slope  | $m_S = \sqrt{a^2 M^2 + b^2 m^2} \cos^2 S$ |                               |
| RMSE of aspect   | $m_A = \sqrt{a^2 M^2 + b^2 m^2} / \tan S$ |                               |
| 2 <sup>nd</sup> order Finite Difference  | $\frac{1}{6}g^2$                          | $\frac{1}{\sqrt{2}}g^{-1}$    |
| 3 <sup>rd</sup> order Finite Difference  | $\frac{1}{6}g^2$                          | $\frac{1}{\sqrt{6}}g^{-1}$    |
| 3 <sup>rd</sup> order Finite Difference Weighted by Reciprocal of Squared Distance | $\frac{1}{6}g^2$                          | $\frac{1}{\sqrt{5.33}}g^{-1}$ |
| 3 <sup>rd</sup> order Finite Difference Weighted by Reciprocal Distance            | $\frac{1}{6}g^2$                          | $\frac{1}{\sqrt{5.83}}g^{-1}$ |
| Frame Finite Difference  | $\frac{1}{6}g^2$                          | $\frac{1}{2}g^{-1}$           |
| Simple Difference  | $\frac{1}{2}g^2$                          | $2g^{-1}$                     |



### Test Design

By examining Equations 3.5 and 3.6, we understand that for a given DEM spatial resolution ( $g$ ), the accuracy of derived slope and aspect is related to the errors by the algorithm that derives  $f_x$  and  $f_y$  ( $M$ ), and DEM error ( $m$ ). The overall accuracy of slope and aspect is dependent upon which of the  $M$  and  $m$  dominates the analysis. While analyzing the algorithms using the real-world DEM (e.g., Skidmore, 1989; Chang and Tsai, 1991; Florinsky, 1998; Bolstad and Stowe, 1994), the DEM error ( $m$ ) would cause much greater error than that by the algorithm, so that the method would be more appropriate for analyzing the impact of data error on derived slope and aspect values. On the other hand, using DEM defined by mathematical surfaces (Hodgson, 1995; Jones, 1998; Carter, 1992) would eliminate data error, thus the observed errors would only be caused by algorithms. Since the error sources could not be defined, the results of the studies appeared inconclusive.

Our approach is to employ mathematical surfaces, in a similar way to Hodgson (1995) and Jones (1998), but with a complexity that represents a closer approximation to the actual land surface. For this purpose, we have selected inverse ellipsoid (Equation 4.1) and Gauss synthetic surface (Equation 4.2) to define the surfaces and generated DEM for a given resolution (Figure 2 and Figure 3).

$$\frac{x^2}{A^2} + \frac{y^2}{B^2} + \frac{z^2}{C^2} = 1 \quad (z < 0) \quad (4.1)$$

$$z = A \left[ 1 - \left(\frac{x}{m}\right)^2 \right] e^{-\left(\frac{x}{m}\right)^2 - \left(\frac{y}{n}\right)^2} - B \left[ 0.2 \left(\frac{x}{m}\right) - \left(\frac{x}{m}\right)^3 - \left(\frac{y}{n}\right)^5 \right] e^{-\left(\frac{x}{m}\right)^2 - \left(\frac{y}{n}\right)^2} - C e^{-\left(\left(\frac{x}{m}\right)+1\right)^2 - \left(\frac{y}{n}\right)^2} \quad (4.2)$$

where  $A$ ,  $B$  and  $C$  are parameters determining surface relief, and  $m$ ,  $n$  in Equation 4.2 are the parameters controlling the

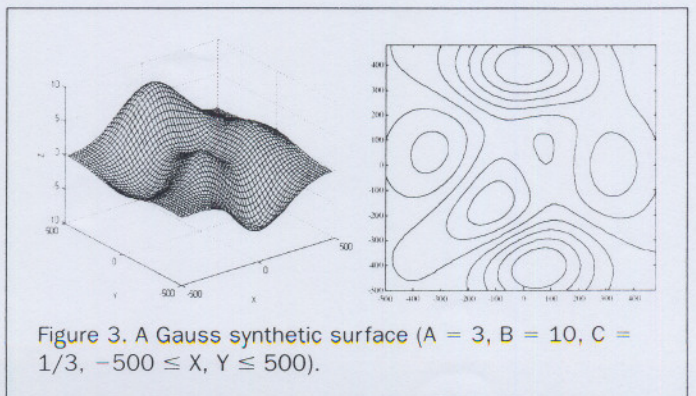




TABLE 3. STATISTICS OF DERIVED SLOPE WITH NO DATA ERROR (UNITS: DEGREES)

| Algorithm | RMSE      |                         | Standard Error |                         | Mean Error |                         | Frequency of Positive and Negative Errors (%) |                         |
|-----------|-----------|-------------------------|----------------|-------------------------|------------|-------------------------|---|-------------------------|
|           | Ellipsoid | Gauss Synthetic Surface | Ellipsoid      | Gauss Synthetic Surface | Ellipsoid  | Gauss Synthetic Surface | Ellipsoid                                     | Gauss Synthetic Surface |
| 2FD       | 0.216     | 0.003                   | 0.189          | 0.002                   | 0.104      | -0.002                  | 100/0   | 15/85                   |
| 3FD       | 0.409     | 0.004                   | 0.368          | 0.002                   | 0.178      | -0.003                  | 100/0   | 9/91                    |
| 3FDWRSD   | 0.358     | 0.003                   | 0.320          | 0.002                   | 0.160      | -0.003                  | 100/0   | 8/92                    |
| 3FDWRD    | 0.384     | 0.004                   | 0.345          | 0.002                   | 0.170      | -0.003                  | 100/0   | 9/91                    |
| FFD       | 0.507     | 0.004                   | 0.459          | 0.003                   | 0.214      | -0.003                  | 100/0   | 10/90                   |
| SIMPLED   | 1.295     | 0.046                   | 1.292          | 0.046                   | 0.078      | -0.002                  | 51/49   | 49/51                   |

TABLE 4. STATISTICS OF DERIVED ASPECT ON WITH NO DATA ERRORS (UNITS: DEGREES)

| Algorithm | RMSE      |                         | Standard Error |                         | Mean Error |                         | Frequency of Positive and Negative Errors (%) |                         |
|-----------|-----------|-------------------------|----------------|-------------------------|------------|-------------------------|---|-------------------------|
|           | Ellipsoid | Gauss Synthetic Surface | Ellipsoid      | Gauss Synthetic Surface | Ellipsoid  | Gauss Synthetic Surface | Ellipsoid                                     | Gauss Synthetic Surface |
| 2FD       | 0.133     | 0.117                   | 0.133          | 0.117                   | 0.000      | -0.008                  | 52/48   | 45/55                   |
| 3FD       | 0.197     | 0.130                   | 0.197          | 0.130                   | -0.001     | 0.000                   | 52/48   | 51/49                   |
| 3FDWRSD   | 0.122     | 0.118                   | 0.122          | 0.118                   | -0.001     | -0.002                  | 52/48   | 48/52                   |
| 3FDWRD    | 0.160     | 0.124                   | 0.160          | 0.124                   | -0.001     | -0.001                  | 52/48   | 50/50                   |
| FFD       | 0.342     | 0.167                   | 0.342          | 0.167                   | -0.002     | 0.004                   | 52/48   | 52/48                   |
| SIMPLED   | 20.76     | 15.13                   | 20.63          | 15.128                  | 2.347      | -0.340                  | 50/50   | 52/48                   |

spatial extent of the surface. The true values of slope and aspect can be computed using the above equations and Equations 2.1 and 2.2.

On the DEM generated from the above surfaces, the selected algorithms have been applied to compute slope and aspect values and statistics were then generated to compare the RMSE between results derived by different algorithms. In order to analyze the impact of DEM data error, we added some noise (i.e., random errors) into the generated DEM to simulate DEM data error.

**Results and Discussion**

Tables 3 and 4 summarize the RMSE comparison results between the selected six algorithms on two surfaces (inverse ellipsoid and Gauss synthetic surface) with no added error (i.e., DEM data error = 0). Figures 4 and 5 shows the RMSE of derived slope and aspect on DEM with various added data errors.

By analyzing Tables 3 and 4, it is shown that on a DEM with high accuracy, the error of derived slope and aspect is sourced from the estimates of partial derivatives  $f_x$  and  $f_y$  and sampling errors. In this case the second-order finite difference method provides a better result than the third-order finite difference methods. For the tested six algorithms, from the best to the worst the order is Second-order Finite Difference (2FD), Third-order Finite Difference Weighted by Reciprocal of Squared Distance (3FDWRSD), Third-order Finite Difference Weighted by Reciprocal of Distance (3FDWRD), Third-order Finite Difference (3FD), Frame Finite Difference (FFD), and Simple Difference (SIMPLE D).

In Equation 3.6, if we ignore the algorithm error (i.e., the first term), we then derive the Florinsky's (1998) RMSE formulae in general terms:

$$m_s = bmc\cos^2S \quad \text{and} \quad m_A = \frac{bm}{\tan S} \quad (5.1)$$

Equation 5.1 suggests that the influence of DEM data error ( $m$ ) relates to slope ( $S$ ) and coefficient  $b$ , which is determined by

grid cell size  $g$ . Referring to Table 2, the third-order finite difference methods appear to be less sensitive to data errors for given  $g$  and  $S$ .

Algorithm error and DEM data error differently influence the accuracy of derived slope and aspect. In general, all the third-order finite difference methods have applied some smoothing effect on the local data window, in order to avoid local relief extremes (Burrough and McDonnell, 1998) for better computation results of surface parameters. On the other hand, the second-order finite difference and simple difference

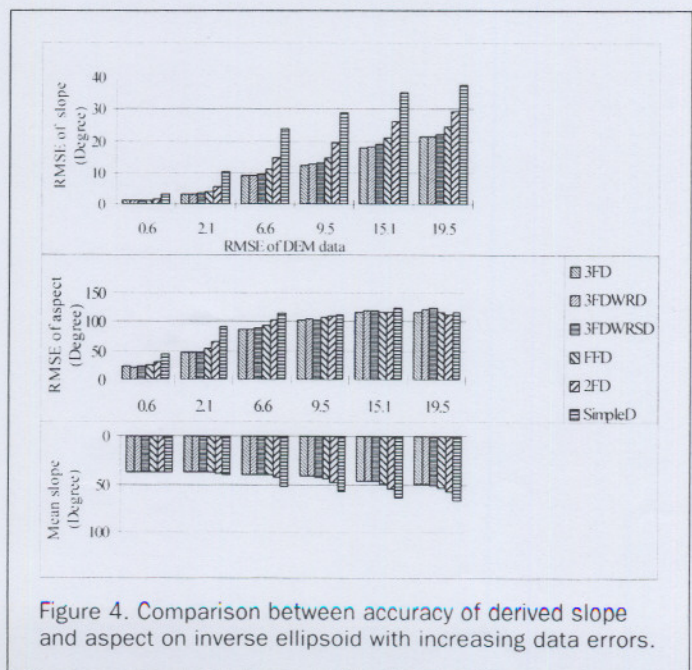


Figure 4. Comparison between accuracy of derived slope and aspect on inverse ellipsoid with increasing data errors.



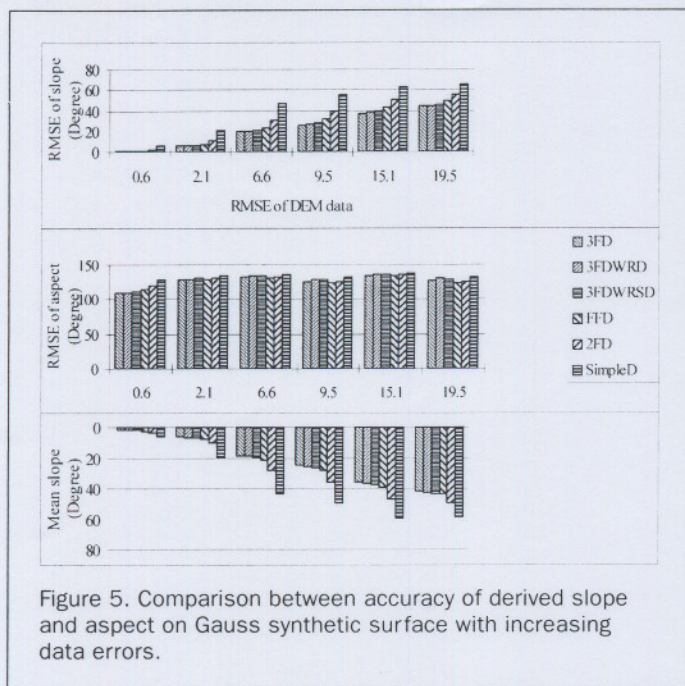


Figure 5. Comparison between accuracy of derived slope and aspect on Gauss synthetic surface with increasing data errors.

methods only utilize a part of samples in the local window, so that they are more sensitive to the data error. This fact is also confirmed by Figures 4 and 5.

The results also confirm the test results reported by Jones (1998). While using an error-free synthetic, trigonometrically defined surface (Morrison's surface III), the 2FD method gave the best results for both gradient and aspect. Using a real-world DEM, where data errors were unavoidable, the 3FD performed best.

## Conclusion

Slope and aspect are two most frequently utilized variables in GIS-based terrain analysis and geographical modeling. There have been numerous analyses on the accuracy of these variables derived from grid-based DEM. The reported findings, however, did not always agree, and sometimes they are controversial. This study attempts to evaluate the issues and establish a fair quantitative measure for assessing various slope and aspect algorithms. With the findings of this study, we can draw the following conclusions:

1. It is important to identify the sources and nature of errors of derived slope and aspect for evaluating algorithms and mathematical models.
2. Evaluation of algorithms and models must be based on an objective, data-independent methodology. Our approach is based on DEM defined by mathematical surfaces, thus the DEM data error can be controlled to make the fair comparison between selected algorithms.
3. On a DEM with high accuracy, the error of derived slope and aspect is sourced from the estimates of partial derivatives  $f_x$  and  $f_y$ , and sampling errors. In this case the second-order finite difference method provides a better result than the third-order finite difference methods.
4. In reality, the influence of DEM data error is in general much larger than the algorithm errors, thus it suggests that the third-order finite difference method would be more appropriate for applications since it is least sensitive to the DEM data error.

Further study will be focused on the impact of grid data structure and other related parameters such as resolution, precision, orientation, and surface complexity on the results of digital terrain analysis. Mathematical models other than slope

and aspect will also be analyzed for their algorithm accuracy and sensitivity to data errors. The real-world tests will be needed to compare with the findings by the theoretical analysis. Based on these analysis, the ultimate goal is to set a conclusive guideline for deriving geomorphic parameters from DEM for a given application project.

## Acknowledgments

This study is supported by Hong Kong Baptist University Faculty Research Grant FRG/98-99/II-35, "3-Dimensional Hydrological Modeling." The constructive comments and suggestions for improving the quality of this paper from anonymous referees are also acknowledged.

## References

- Band, L.E., 1986. Topographic partition of watershed with digital elevation models, *Water Resources Research*, 22(1):15-24.
- Biesemans, J., M.V. Meirvenne, and D. Gabriels, 2000. Extending the RUSLE with the Monte Carlo error propagation technique to predict long-term average off-site sediment accumulation, *Journal of Soil and Water Conservation*, 55(1):35-42.
- Bolstad, P.V., and T.J. Stowe, 1994. An evaluation of DEM accuracy: elevation, slope and aspect, *Photogrammetric Engineering & Remote Sensing*, 60:1327-1332.
- Burrough, P.A., and R.A. McDonnell, 1998. *Principles of Geographical Information System*, Oxford University Press, Oxford, UK, 333 p.
- Carter, J., 1992. The effect of data precision on the calculation of slope and aspect using gridded DEMs, *Cartographica*, 29(1):22-34.
- Chang, K., and B. Tsai, 1991. The effect of DEM resolution on slope and aspect mapping, *Cartography and Geographic Information Systems*, 18:69-77.
- Chu, T.H., and T.H. Tsai, 1995. Comparison of accuracy and algorithms of slope and aspect measures from DEM, in *Proceedings of GIS AM/FM ASIA'95*, 21-24 August, Bangkok:1-11.
- Desmet, P.J.J., and G. Govers, 1996a. Comparison of routing algorithms for digital elevation models and their implication for predicting ephemeral gullies, *International Journal of Geographical Information science*, 10(10):311-331.
- Desmet, P.J.J., and G. Govers, 1996b. A GIS procedure for automatically calculating the USLE LS factor on topographically complex landscape units, *Journal of Soil and Water Conservation*, 51(5): 427-433.
- Dietrich, W.E., C.J. Wilson, D.R. Montgomery, and J. McKean, 1993. Analysis of erosion thresholds, channel networks, and landscape morphology using a digital terrain model, *The Journal of Geology*, 10:259-278.
- Duan, J., and G.E. Grant, 2000. Shallow landslide delineation for steep forest watersheds based on topographic attributes and probability analysis, *Terrain Analysis: Principles and Application* (J.P. Wilson and J.C. Gallant, editors), John Wiley & Sons Press, New York, 311-330.
- Evans, I.S., 1980. An integrated system of terrain analysis and slope mapping, *Zeitschrift für Geomorphologie*, Supplement Band 36: 274-95.
- Fleming, M.D., and R.M. Hoffer, 1979. Machine processing of Landsat MSS data and LARS Technical Report 062879, Laboratory for Applications of Remote Sensing, Purdue University, West Lafayette, Indiana.
- Florinsky, I.V., 1998. Accuracy of local topographic variables derived from digital elevation models, *International Journal of Geographical Information Science*, 12(1):47-61.
- Hodgson, M.E., 1995. What cell size does the computed slope/aspect angle represent? *Photogrammetric Engineering & Remote Sensing*, 61:513-517.
- Horn, B.K.P., 1981. Hill shading and the reflectance map, *Proceedings of IEEE*, 69(1):14-47.
- Jenson, S.K., 1994. Application of hydrologic information automatically extracted from digital elevation models, *Terrain Analysis and Distributed Modelling in Hydrology* (K.J. Beven and I.D. Moore, editors), John Wiley & Sons, Chichester, UK, 35-48.



Jones, K.H., 1998. A comparison of algorithms used to compute hill slope as a property of the DEM, *Computer and Geosciences*, 24(4):315-323.

King, J.T., 2000. *An Introduction to Numerical Calculation* (Translated by Lin C., et al.), Nanjing University Press, Nanjing (In Chinese).

Mitasova, H., J. Hofierka, M. Zlocha, and L.R. Iverson, 1996. Modelling topographic potential for erosion deosition using GIS, *International Journal of Geographical Information Systems*, 10(5):629-641.

Moore, I.D., 1996. Hydrologic Modeling and GIS, *GIS and Environmental Modeling: Progress and Research Issue* (M.F. Gooldchild, L.T. Steyaert, B.O. Parks, M.P. Crane, D.R. Maidment and S. Glendinning editors), GIS World Books, Fort Collins, CO, 143-148.

Moore, I.D., E.M. O'Loughlin, and G.J. Burch, 1988. A contour-based topographic model for hydrological and ecological applications, *Earth Surface Processes and Landforms*, 13:305-320.

O'Callaghan, J.F., and D.M. Mark, 1984. The extraction of drainage networks from digital elevation data, *Computer Vision, Graphics, and Image Processing*, 28:323-344.

Papo, H.B., and E. Gelbman, 1984. Digital terrain models for slope and curvatures, *Photogrammetric Engineering & Remote Sensing*, 50(6):695-701.

Quinn, P.F., K.J. Beven, and R. Lamb, 1995. The  $\ln(a/\tan\beta)$  index: how to calculate it and how to use it within the TOPMODEL framework, *Hydrological Processes*, 9:161-182.

Quinn, P.F., K.J. Beven, P. Chevallier, and O. Planchon, 1991. The prediction of hillslope flow paths for distributed hydrological modelling using digital terrain models, *Terrain analysis and Distributed Modelling in Hydrology* (K.J. Beven and I.D. Moore, editors), John Wiley & Sons, Chichester, UK, 63-83.

Ritter, D., 1987. A vector-based slope and aspect generation algorithm, *Photogrammetric Engineering & Remote Sensing*, 53(8):1109-1111.

Sharpnack, D.A., and G. Akin, 1969. An Algorithm for computing slope and aspect from elevations, *Photogrammetric Survey*, 35: 247-248.

Skidmore, A.K., 1989. A comparison of techniques for the calculation of gradient and aspect from a gridded digital elevation model, *International Journal of Geographical Information Systems*, 3: 323-334.

Stephen, J.V., and B.J. Irvin, 2000. Automated landform classification methods for soil-landscape studies, *Terrain Analysis: Principles and Application* (J.P. Wilson and J.C. Gallant editors), John Wiley & Sons Press, New York, 267-294.

Tang, G., 2000. *A Research on the Accuracy of Digital Elevation Models*, Science Press, Beijing, 165 p.

Unwin, D., 1981. *Introductory Spatial Analysis*. Methuen, London and New York.

Wood, J.D., 1996. *The Geomorphological Characterisation of Digital Elevation Model*, PhD Thesis, University of Leicester.

Zevenbergen, L.W., and C.R. Thorne, 1987. Quantitative analysis of land surface topography, *Earth Surface processes and Landforms*, 12:47-56.

Zhou, Q., P. Wang, and P. Pilesjö, 1998. Accuracy assessment of hydrological modeling algorithms using grid-based digital elevation models, *Proceedings of the International Conference on Modelling Geographical and Environmental Systems with GIS*, 22-25 June 1998, Hong Kong, pp. 257-265.

Zhou, Q., and X. Liu, 2002. Error assessment of grid-based flow routing algorithms used in hydrological models, *International Journal of Geographical Information Science*, 16(8):819-842.

## Certification Seals & Stamps

- Now that you are certified as a remote sensor, photogrammetrist or GIS/LIS mapping scientist and you have that certificate on the wall, make sure everyone knows!
- An embossing seal or rubber stamp adds a certified finishing touch to your professional product.
- You can't carry around your certificate, but your seal or stamp fits in your pocket or briefcase.
- To place your order, fill out the necessary mailing and certification information. Cost is just \$35 for a stamp and \$45 for a seal; these prices include domestic US shipping. International shipping will be billed at cost. *Please allow 3-4 weeks for delivery.*

### SEND COMPLETED FORM WITH YOUR PAYMENT TO:

ASPRS Certification Seals & Stamps, 5410 Grosvenor Lane, Suite 210, Bethesda, MD 20814-2160

NAME \_\_\_\_\_ PHONE \_\_\_\_\_

CERTIFICATION # \_\_\_\_\_ EXPIRATION DATE \_\_\_\_\_

ADDRESS \_\_\_\_\_

CITY \_\_\_\_\_ STATE \_\_\_\_\_ POSTAL CODE \_\_\_\_\_ COUNTRY \_\_\_\_\_

PLEASE SEND ME:  Embossing Seal ..... \$45  Rubber Stamp ..... \$35

METHOD OF PAYMENT:  Check  Visa  MasterCard  American Express

CREDIT CARD ACCOUNT NUMBER \_\_\_\_\_ EXPIRES \_\_\_\_\_

SIGNATURE \_\_\_\_\_ DATE \_\_\_\_\_