A Quadtree For Global Information Storage †

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QUADTREES

The hierarchical data structure known as a quadtree has advantages fur geographic data storage. In this structure a two-dimensional geometric region is recursively decomposed into quadrants. Each of the four quadrants becomes a node in the quadtree. A larger quadrant is a node at a higher hierarchical level of the quadtree, and smaller quadrants appear at lower levels. The advantage of this structure is that the regular decomposition provides for simple and efficient data storage, retrieval, and processing. The simplicity stems front the geometric regularity of the decomposition into squares, and the efficiency is obtained try storing only those nodes containing data of interest. A comprehensive review of quadtrees can be found In Samet (1984).

Most of the applications of quadtrees to date have been to binary images (e.g., Klinger and Dyer 1976), but there have also been recent algorithmic developments that yield results similar to those used in geographic data processing. These include computation of geometric properties such as area calculation, centroid determination, comparison of images (Shneier 1981), connected component labeling, neighbor detection (Samet 1981a, b), distance transforms (Samet 1982), image segmentation, data smoothing (Ranade and Shneier 1981), and edge enhancement (Ranade 1981). Because of these advances, several investigators (Samet 1984, Peuquet 1984, Mark and Lauzon 1985) have proposed the use of quadtrees for geographic information storage. Additional work useful for this purpose has included the development of procedures for the conversion of data from raster to quadtree format (Dyer, Rosenfeld, and Samet 1980). The storage efficiency of quadtrees has been calculated (Dyer 1982) and enhanced by using linear coding techniques (Gargantini 1982). The possibility of using artificial intelligence to improve a very large quadtree-based geographical information system has even been considered (Chen 1984, 1985; Smith and Pazner 1984). Clearly this is an area of active research and promise.

LARGE GEOGRAPHIC INFORMATION SYSTEMS

Several large inventories of geographical Information have been assembled in computer readable form in recent decades. One of the largest of these is the Canada Land Information System. There it was recognized that a hierarchical form of information storage would be useful. Since the system covers a large two-dimensional region (Canada), the methods devised were similar to those now used to represent images. Specifically, an index quadtree was built up to access map sheets. The Morton number was proposed to manage these map frames (Morton 1966), so that sequential recordings on magnetic tape always stored neighboring maps near each other. The data for each map, however, are encoded as vectors, not quadtrees. Thus, the Canada Land Inventory provides a vector environment but uses a quadtree-like indexing scheme (Calkins and Marble 1980).

Geographic Information systems are now proliferating at the same time as interest in global ecology is growing. Thus, it is natural to attempt to devise a means to index data for the entire world. One such proposal (Spies and Paulson 1981) includes a hierarchical structure, in which each parent node has a different number of descendants. The entire sphere is first divided into 36 *zones,* each of which is subdivided into 3,060 *regions*, and these are then broken into 15 *districts.* The districts are subdivided into 64 *blocks;* within each block an array of 151 X 151 *points* is defined. There is an abrupt change at 50 degrees latitude, north and south, to compensate for the convergence of the meridians. The irregularity of the hierarchy makes calculation complex; thus, the system cannot benefit from the theoretical and practical advantages that have been demonstrated for regular hierarchical decompositions such as quadtrees. The finest resolution of this system, as proposed, is fixed at three arc-seconds of latitude-longitude, or about 93 meters.

The difficulty, of course, is that the topology and geometry of the earth differ from that of a square image of pixels. We are aware of three existing proposals to deal with this difficulty. The first two make use of map projections, while a third attacks the spherical problem directly. After discussing these, we examine alternative possibilities and provide detailed calculations for one of these.

FURTHER PROPOSALS

One proposal (Bcaudet, Chan, and Goldshlak 1973, Chair and O'Neill 1975, O'Neill and Laubscher 1976) is to project the surface of the earth onto a cube, and their to let each of the six sides of this cube be covered by a set of rectangular coordinates. Each square face can then be decomposed hierarchically into a set of nested squares or pixels. The method of transferring the ellipsoidal model of the earth onto the cube uses a new, mathematically complex, map projection. This is a variant of the polyhedral projections used for some topographic surveys in the eighteenth and nineteenth centuries. Exhaustive treatment of alternative projections onto a cube can be found in Fisher and Miller (1944) and Steers (1965). A difficulty consists in the positioning of the seams of the box relative to the earth. In our opinion, this approach seems too complex and too arbitrary, and, thus, unsatisfactory.

More promising is the recent proposal to base a global scheme on the existing Universal Transverse Mercator (UTM) map zones. These are widely used by the military agencies of the western world. The specific proposal (Mark and Lauzon 1985) is to cut each of the 60 UTM zones into subzones, and then into square patches which are subsequently ordered by their Morton numbers. The patches are subdivided into 256 x 256 pixels of 30 meters on a side. The particular choice of numbers here is based on detailed algorithmic considerations involving 10-bit VAX word lengths and Morton number indexing. The proposal has the advantage of meshing nicely with existing practice at mapping agencies that produce local (not global) data. A major disadvantage is that the boundaries between zones introduce unconformities, making it difficult to use this system for the whole world.

In order to define a regular hierarchical data structure for the entire earth, it would be expected that all nodes of the tree should have equal numbers of descendants, and simple and similar shapes. On a plane there are several patterns - triangles, squares, and hexagons - that satisfy these criteria. These perfect hierarchical recursions do not exist on a sphere, and the beginning point for most related studies has been the platonic solids, known since the fourth century B. C. There are five, and only five, such solids: tetrahedron, hexahedron (cube), octahedron, dodecahedron, and icosahedron (Holden 1971, Fisher and Miller 1944). Each facet on these solids is a regular polygon with equal-length sides and equal interior angles. If one allows such a solid to be inscribed within a sphere then the vertices touch the sphere. The great circles that connect these vertices will bound a curved patch of spherical surface. There are some complications if one attempts to adapt this scheme to an ellipsoid, but these are not severe iii the present context.

One can build a recursively hierarchical quadtree on the square facets of a cube, but the boundaries of most of these facet edges are curves on the sphere which are not coincident with the latitude-longitude lines, and the sub-facet edges are not even great circles unless the gnomonic projection is used. The decompositions of the other platonic solids involve similar difficulties. It is, of course, possible to consider non-regular solids, such as the combinations used in Buckminster Fuller's geodesic domes (Popko 1968), but this has not been proposed for computerized storage of geographic information. We do not consider it a useful direction.

The most detailed proposal is for a regular, four-fold decomposition of spherical triangles (Dutton 1984). The sphere can be covered by twenty such triangles, based on the vertices of an icosahedron, or, using fewer triangles, a tetrahedron or an octahedron. Such a recursion is shown in Figure 1. Table 1 shows what happens to the triangles by recursive decomposition into four descendants for a tetrahedron and for an octahedron (both yield the same result). Table 2 shows the variation for the icosahedron. As one reads down the rows of the tables, each spherical triangle is cut into four smaller spherical triangles by connecting the midpoints of each edge. The innermost triangle keeps its equilateral shape during the recursion but the other interior angles change from their original values (120 degrees for the tetrahedron, 90 degrees for the octahedron, and 72 degrees for the icosahedron) to the 60 degrees of the limiting plane triangle, while the outermost triangles retain only one angle at the original value of 120, 90, or 72 degrees. It is clear that a regular hierarchical decomposition is achieved in this manner, since each step of the hierarchy has the same number of descendants. But the shapes and areas change, and the edges are very irregularly arranged on the sphere. The same would he true For a three-fold decomposition, joining vertices to the centroid, instead of the four-fold one.

DESIGNING A GLOBAL COORDINATE SYSTEM

One feature of a global information system is the potentially large amount of information: there are nearly $1.5 \times l0^{15}$ square meters of surface area. The variety of potential data types is also large and extremely diverse - some are numerical, others are categorical, many are vector-valued. In some cases, the data consist of only points or lines; in other eases, complete area coverage is required. Recalling these facts, we attempt to satisfy the following principles:

1. A hierarchical data structure is required. This will allow data storage at different levels of resolution.
2. A regular hierarchy is more efficacious than an irregular hierarchy. A regular hierarchy has the same number of descendants at each level. The two-dimensional quadtree is a good example of such a regular hierarchy. Consequently, all nodes can be manipulated by the same programs and algorithms.
3. It should be easy to convert from existing structures used for geographic information systems, and back again. We include geographical maps within the set of existing structures. This leads to the next principle.
4.The system should have a clear and simple relation to the currently well-known system of latitude and longitude coordinates. In particular, we believe that, for use at all scales - global, international, and local - the system should directly refer to the meridians and parallels.

5. The purposes of a geographic information system are primarily for planning, analysis, and inventorying of geographic phenomena. We argue that coverage must be uniform and that every element of area must have an equal probability of entering the system. This suggests that the world should be partitioned into chunks of equal size.

The question is whether or not all of these properties can be combined into one system. An obvious choice would be to use the latitude-longitude quadrilaterals directly, as in Figure 2. A great many data are already indexed by latitude and longitude (see, for example, Thomas and Henderson-Sellers 1985; Matthews 1985). There is further precedent for such a system in the indexing convention used for the International Millionth Map of the World (United Nations 1951). The disadvantage of this international indexing system is that it does not envision further subdivision of the map sheets. Of course, a well-known difficulty with the direct use of the latitude-longitude system is that the meridians converge toward the poles, with the result that quadrilaterals defined by meridians and parallels decrease in area as the poles are approached, and are not square.

Isothermal Coordinates

One cartographic method that overcomes some of the foregoing difficulties is the introduction of isothermal coordinates on the sphere or ellipsoid (Adams 1949, Thomas 1952, Hubeny 1953). Only the spherical case is treated here, since (the extension to the ellipsoid is well-known, though computationally somewhat more involved). Isothermal coordinates introduce a system of infinitesimal squares on a surface by requiring that the first fundamental form (Gauss 1825) simplifies to $ds^2 = E(u, v)\ (du^2 + dv^2)$. For a sphere this requires an integration resulting in what is known as isometric latitude. This new latitude, φ', is related to the geographical latitude, φ, by $\varphi' = \mathrm{Ln}\ \tan(\pi/4 + \varphi/2)$ and is equivalent to the use of Mercator's projection (Figure 3). The difficulty with this is that data in the polar regions are impossible to represent, and that the areas of the squares on the surface are not constant. The advantage is that any conformal map projection can be used to induce such a system of coordinates on the sphere; the Mercator projection, of course, is the only one that preserves the meridians as one of the sets of coordinates.

Authalic Coordinates

Another cartographic coordinate system is used for equal area mappings (Adams 1949), usually of an ellipsoid onto a sphere, and this provides a partitioning of the sphere into quadrilaterals of equal area (Figure 4). The simplest of these equal area coordinates modifies only the spacing of the parallels by introducing an authalic latitude, $\varphi' = \sin\varphi$. This is equivalent to the use of Lambert's cylindrical equal area projection (Figure 5a) on which a system of equal-sized squares can be drawn (Figure 5b). The difference between Figures 5a and 5b is the location of the latitude lines. In one case they are a uniform distance apart on (the sphere) and in the other case they are spaced to obtain quadrilaterals of equal area. These quadrilaterals become the square nodes of our quadtree.

One way to derive the authalic spacing is to consider the subdivision of a spherical quadrilateral into four equal-sized pieces (Figure 6). The longitude bounds of the quadrilateral are λ_1 and λ_2 the other two edges are given by φ_1 and φ_2. The area, A, of the quadrilateral is

$$A = R2 \int_{\lambda_1}^{\lambda_2} d\lambda \int_{\varphi_1}^{\varphi_2} \cos\varphi \, d\varphi$$

$$= (\lambda_2 - \lambda_1)(\sin\varphi_2 - \sin\varphi_1).$$

This equation demonstrates the separability of the coordinates. A convenient way to divide the area into four sub-areas is to first split along a longitude line, λ_0, which satisfies

$$\lambda_0 - \lambda_1 = \lambda_2 - \lambda_0 = 0.5\,(\lambda_2 - \lambda_1) \quad \text{or} \quad \lambda_9 = 0.5\,(\lambda_2 + \lambda_1).$$

Splitting along the parallel requires that

$$\sin\varphi_o - \sin\varphi_1 = \sin\varphi_2 - \sin\varphi_o = 0.5\,(\sin\varphi_2 - \sin\varphi_1)$$

or $\sin\varphi_o = 0.5\,(\sin\varphi_2 + \sin\varphi_1)$.

Now each of the subpartitions has area

$$[0.5\,(\lambda_2 - \lambda_1)]\,[0.5\,(\sin\varphi_2 - \sin\varphi_1)]$$

$$= 0.25\,(\lambda_2 - \lambda_1)(\sin\varphi_2 - \sin\varphi_1)$$

$$= 0.25\,A,$$

as was desired. Each of the sub-areas can be divided recursively in this same manner. This result is equivalent to using Lambert's cylindrical equal area map projection, shown in Figure 5. Adaptation to an ellipsoid is straightforward.

It is not necessary to use the idea of a map projection at all in a computerized geographic information system. The traditional map functions both as a data storage device and as a geographic display. These two uses can now be separated. The concept of terrestrial coordinates, however, is needed, and these are always interpretable as some map projection. Thus, we have equated isothermal coordinates with Mercator's projection, conventional latitude and longitude with the square projection (Figure 2), and authalic coordinates with one of Lambert's projections. Lambert's map can even be squashed in an equal area manner by the transformation

$$\begin{bmatrix} x \\ y \end{bmatrix}' = \begin{bmatrix} \frac{1}{\sqrt{\pi}} & 0 \\ 0 & \sqrt{\pi} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

to put the entire world within a perfect square (Figure 7). A quadtree based on this grid could recursively subdivide the earth into equal-sized plane squares whose sides lie along the images of the meridians and parallels, and which cover identical-sized areas on the earth. Alternate equal area mappings of the earth into a square have been given by Close (1949, p. 88) and by

Gringorten (1972), but these will not serve as well for geographic data storage.

CONCLUSION: A GLOBAL QUADTREE

Reverting now to the simpler authalic latitude, the entire earth can he inserted into a quadtree as follows:

1. The root (level 0) of the global quadtree is the entire earth. It has four descendants at level 1, two of these corresponding to the western and eastern hemispheres, respectively, the other two being null nodes.
2. Each hemisphere has four descendants, and these are recursively divided into four equal-sized areas until sufficient resolution (see Table 3) has been obtained.
3. Each node is a spherical quadrilateral with four bounding lines made up of meridians and parallels intersecting at right angles. The only exception is when one of the quadrilateral boundaries coincides with one of the poles.
4. Each node has the same terrestrial area as all other nudes at the same hierarchical level. Each node has four times the area of its subordinant (child) nodes.
5. The number of levels in the hierarchy depends on the resolution required to store the data for the problem at hand.

Table 3, column two, lists the resolution at each level nf such a global quadtree, assuming a globe of 6,380,000 meter radius, with an equatorial length of circa 40,000,000 meters. The number of leaves at any level, L, is, for $L > 0$, equal to $4^L/2$. The spherical area of each leaf node is $8\pi R^2/4^L$, and the spatial resolution is the square root of this quantity. Working backwards, from the resolution to the hierarchical level, we use

$$L = A - B \, \mathrm{Ln}\,(r),$$

Where r is the desired spatial resolution in meters,

$$A = 24.9308731 = [\mathrm{Ln}\,(8) + \mathrm{Ln}\,(\pi) + 2\,\mathrm{Ln}\,(R) / \mathrm{Ln}\,(4),$$
$$B = 1.44269504 = 2 / \mathrm{Ln}\,(4),$$

and R is the spherical radius in meters. At level 15 the resolution is close to that of the meteorological satellites; at level 19 it is close to the resolution of multispectral scanner images from the LANDSAT system; at level 21 Thematic Mapper images from LANDSAT D can be represented without loss of detail, and at level 22 SPOT images can be well represented. Conventional large-scale topographic maps are also approximately at this level. At levels 25 through 27 the resolution suffices for most aerial photography. Finally, even geodetic ground control points, with an accuracy of centimeters, can be represented at level 30. The cumulative number of nodes needed to reach this level could be overwhelmingly large but for the fact that only those nodes containing data are stored in linear quadtrees. The point is that virtually any resolution can be accommodated by this global quadtree. For comparison, the last two columns of Table 3 illustrate the difficulty of using latitude and longitude directly. The hierarchical partitioning results, in equal increments in both latitude and longitude coordinates but very unequal areas of the leaf nodes, depending on the latitude. Those nodes representing places nearest to the equator are larger than the most polarwardly located nodes. Taking these two extremes, the table shows the variation in the mean resolution (the square root of the spherical area). The judicious use of authalic coordinates avoids this problem and provides a simple, yet convenient and effectual scheme for the storage of geographic information on a global basis.

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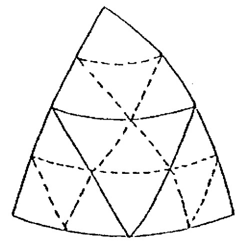

Fig. 1. Recursion of a Spherical Triangle with Out-degree Four

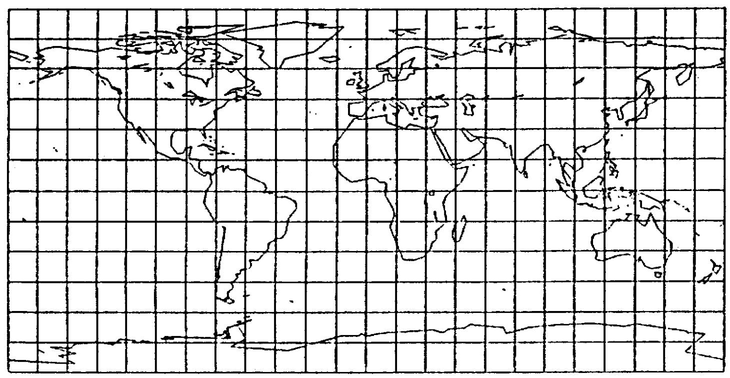

Fig. 2. A World Map with Latitude-Longitude Graticule. Square Projection

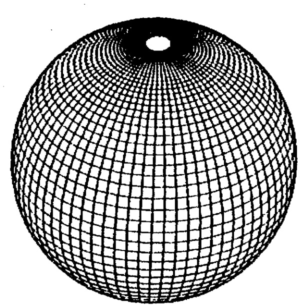

Fig. 3. Isothermal Coordinates on a Sphere (Orthographic View), Showing Subdivision of Sphere into "Squares"

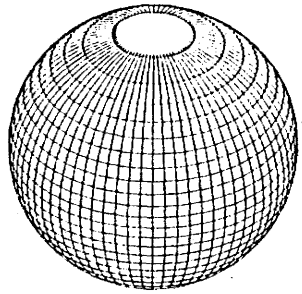

FIG. 4. Authalic Coordinates on a Sphere (Orthographic View), Showing Subdivision of Sphere into Quadrilaterals of Equal Area

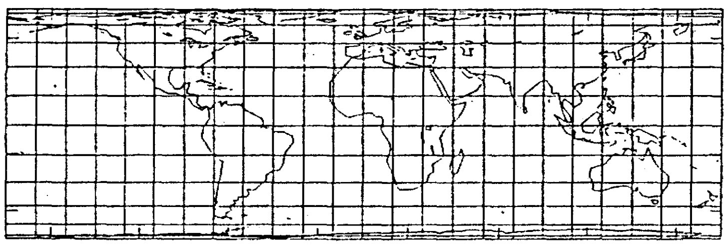

FIG. 5a. Lambert's Cylindrical Equal Area Map Projection with 15-Degree Intervals of Latitude and Longitude

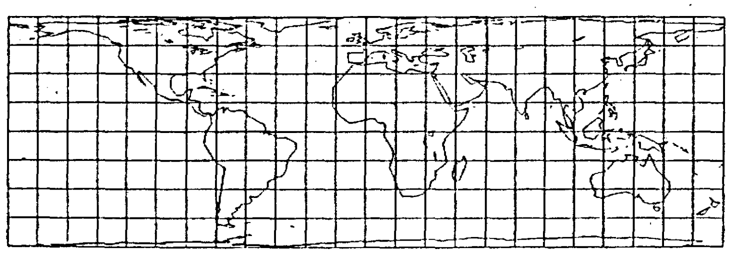

FIG. 5b. Lambert's Cylindrical Equal Area Map Projection with Authalic Grid

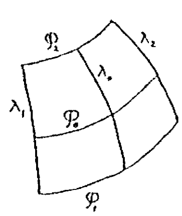

FIG. 6. Decomposition of a Quadrilateral into Four Equal Pieces

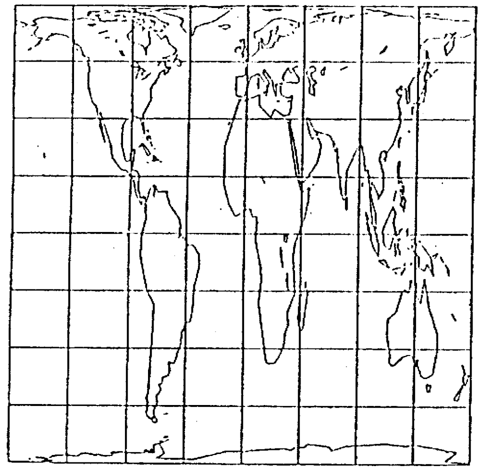

Fig. 7. Equal Area Map of the World in a Square

TABLE 1
Regular Decomposition Parameters of Spherical Triangles (Tetrahedron and Octahedron)

Edge Length/Radius of Central Triangle	Inner Angles of Central Triangle	Triangle Area at Center /Triangle Area at Corner
1.91063324	120.00000	2.99999993
1.57079631	90.00000	5.21483803
1.04719754	70.52870	6.18001553
0.58568553	62.96431	6.47154247
0.30266527	60.76636	6.54840632
0.15264673	60.19326	6.56788856
0.07649056	60.04842	6.57277607
0.03826627	60.01211	6.57399905
0.01913576	60.00303	6.57430502
0.00956821	60.00076	6.57438215
0.00478415	60.00019	6.57440396
0.00239208	60.00005	6.57441956

TABLE 2
Regular Decomposition Parameters of Spherical Triangles (Icosahedron)

Edge Length/Radius of Central Triangle	Inner Angles of Central Triangle	Triangle area at Center Triangle Area at Corner
1.10714872	72.00000	1.20312725
0.62831853	63.43495	1.27451087
0.32636622	60.89277	1.29396202
0.16483370	60.22546	1.29893173
0.08262747	60.05651	1.30018095
0.04134020	60.01414	1.30049368
0.02067341	60.00353	1.30057189
0.01033712	60.00088	1.30059147
0.00516861	60.00022	1.30059645
0.00258431	60.00006	1.30059804

TABLE 3

Resolution for 30 Levels of Global Quadtree

Level (L)	Authalic Latitude	Geographic Latitude: Equatorial	Geographic Latitude: Polar case
0	22,616,511	22,616,511	22,616,511
1	15,992,288	15,992,288	15,992,288
2	7,996,144	7,996,144	7,996,144
3	3,998,072	4,751,536	3,059,992
4	1,999,036	2,473,264	1,103,066
5	999,518	1,248,687	391,880
6	499,759	625,852	138,717
7	249,880	313,115	49,059
8	124,940	156,581	17,346
9	62,470	78,293	6,133
10	31,235	39,147	2,168
11	15,618	19,574	767
12	7,809	9,787	271
13	3,904	4,893	96
14	1,952	2,447	34
15	976.09	1,223.35	12.06
16	488.05	611.67	4.26
17	244.02	305.84	1.65
18	122.01	152.92	0.67
19	61.01	76.46	0.00
20	30.50	38.23	—
21	15.25	19.11	—
22	7.63	9.56	—
23	3.81	4.78	—
24	1.91	2.39	—
25	0.95	1.19	—
26	0.48	0.60	—
27	0.24	0.30	—
28	0.12	0.15	—
29	0.06	0.07	—
30	0.03	0.01	—

Mean resolution given in meters.

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