Map Projections for Global and Continental Data Sets and an Analysis of Pixel Distortion Caused by Reprojection

Daniel R. Steinwand, John A. Hutchinson, and John P. Snyder

Abstract

With growing emphasis on global monitoring, research using remotely sensed data and geographic information systems is increasingly focused on large regions studied at small scales. These global change studies require the integration of data sets from several sources that are reprojected to a common map base. In small-area, large-scale studies the choice of a map projection has little effect on data quality. In global change studies the effects of map projection properties on data quality are more apparent, and the choice of projection is more significant. To aid compilers of global and continental data sets, six equal-area projections were chosen: the interrupted Goode Homolosine, the interrupted Mollweide, the Wagner IV, and the Wagner VII for global maps; the Lambert Azimuthal Equal-Area for hemisphere maps; and the Oblated Equal-Area and the Lambert Azimuthal Equal-Area for continental maps. Distortions in small-scale maps caused by reprojection, and the additional distortions incurred when reprojecting raster images, were quantified and graphically depicted. For raster images, the errors caused by the usual resampling methods (pixel brightness level interpolation) were responsible for much of the additional error where the local resolution and scale change were the greatest.

Introduction

Data transformation and a suitable map projection are necessary when registering remotely sensed data to a map base. For studies of small areas at large scale, raster data are often registered to a topographic map base using, for example, the Universal Transverse Mercator or Lambert Conformal Conic projections. Errors caused by reprojection are usually not significant because projection properties have less effect on data quality than other factors for study areas that extend only over a topographic quadrangle.

For large study areas, problems caused by map projection characteristics may arise that are not significant for smaller study areas. When several small data sets must be merged or large data sets reregistered to a common map base, the distortion due to a map projection change must be considered. For example, data may be in a projection that represents the North Pole as a line and are reprojected to a map projection where the North Pole is a point. In such a case, feature compression and data loss will occur at and near the pole. Data degradation caused by changing projections is not always severe, but the larger the study area, the more significant the distortions can be.

The goals of this study were to select map projections for use with data sets of global, hemispheric, or continental extent and to identify distortions introduced (1) during the transformation of data to and from different projections, and (2) during the reprojection of raster data.

Map Projection Properties and Classes

The distortion characteristics of a map projection depend on its properties. There are several schemes for classifying map projections based on their properties, but in this study, projections were classified as equidistant, conformal, or equal-area.

All maps distort distances, because it is impossible to perfectly portray the round Earth on a flat map. Equidistant projections, such as the Azimuthal Equidistant, show distances correctly through one or two points, but most other distances are distorted (Figure 1).

Conformal projections manipulate distance distortion to preserve local angles or shapes, but not areas. On a conformal map projection, a very small circle on the globe will project to a circle on the map, but not of the same size. The three most commonly used conformal projections form a

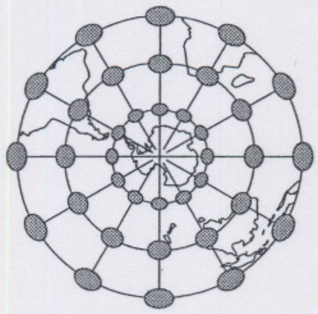

Figure 1. Azimuthal Equidistant, centered on the South Pole. Distances are shown correctly through the center of the projection.

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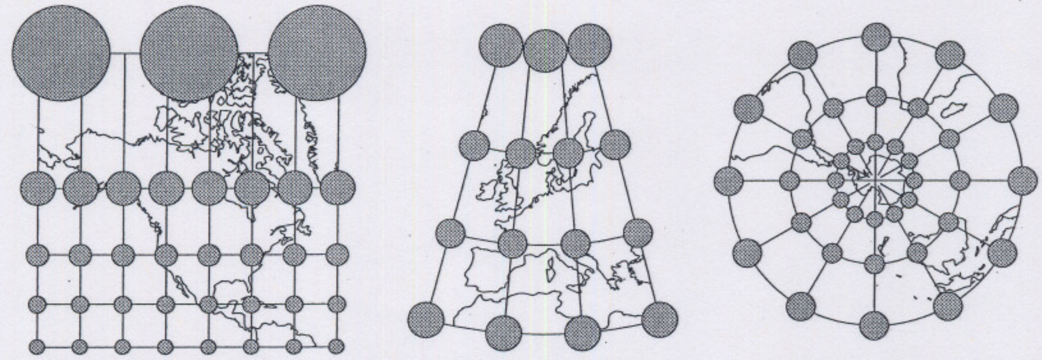

Figure 2. Conformal projections: a small circle on the globe projects to a circle on the map, but of different size. The Mercator, Lambert Conformal Conic, and Stereographic (l to r) form a mathematically related family of conformal projections.

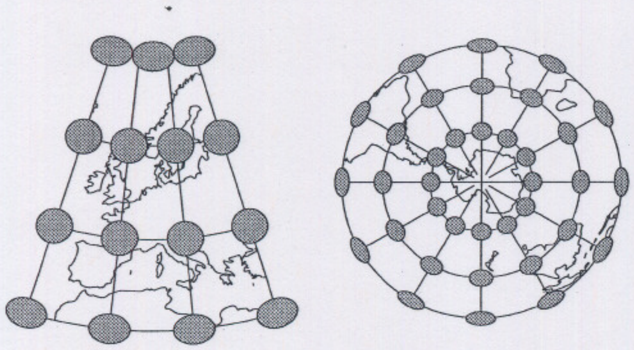

Figure 3. Equal area projections: a small circle on the globe projects to an ellipse on the map, but not of the same size. The Albers Equal-Area Conic (l) and Lambert Azimuthal Equal-Area (r) are two equal-area projections among the many in common use.

mathematically related family: Mercator, Lambert Conformal Conic, and Stereographic (Figure 2).

Equal-area projections preserve areas and sizes, but not angles or shapes. A very small circle on the globe will generally project to an ellipse on an equal-area projection, but the ellipse will have the same area as the circle. Just as a circle can be formed into many different-shaped (but equal-in-area) ellipses, so there are many equal-area projections, including the Albers Equal-Area Conic and the Lambert Azimuthal Equal-Area (Figure 3).

No projection can show distances, angles, and areas all correctly; this is only possible on a globe. Some projections, however, are neither equidistant, conformal, nor equal-area. For example, the Robinson projection does not preserve angles or areas but achieves a better look. It avoids the shearing near the poles characteristic of many equal-area projections, without the excessive area distortion of the conformal Mercator (Figure 4).

Identification of Global and Continental Map Projections

This study primarily involved raster data sets. Because the analysis of raster data is based on the areas of image pixels, not necessarily their shape or distance, equal-area projections were chosen over conformal or equidistant. The best equal-area map projection for a global or continental data set is the projection with the least distortion for the area and with the optimum parameters. The latter includes central latitude, longitude, and standard parallels or other constants affecting the specific distortion pattern.

There are strong indications that the optimum equal-area map projection of a given region will have a line of constant distortion following the limits of the region, a principle proven in the 19th century for conformal maps. When attempting to select a projection that most satisfactorily approaches this ideal for a given region, conflicting situations soon become apparent:

- For a world map, the criteria for selection is subjective because the relative importance of land versus water portions, of polar versus equatorial regions, and of straight parallels versus curved parallels affect the decision, as well as the overall appearance;
- For continental regions, irregular lines of constant distortion that follow coastlines require complicated formulas and more uncommon projections; and
- The choice of a map projection is determined by whether the map of the region will be used independently or whether it should fit maps of adjacent regions and, therefore, be on the same projection as that of the larger region;

Equal-area world map projections have been the subject of numerous papers; therefore, projection selection was based on these papers and on an evaluation of distortion. If land and ocean data are not needed on the same map, an interrupted projection can be used to reduce distortion. In this case, the interrupted Goode Homolosine or interrupted Mollweide are recommended, interrupted for land or water in standard formats (see Cover Image). For uninterrupted world maps, recommended projections are Wagner IV (same as Putnins P2') or Wagner VII (same as Wagner-Hammer) (Plate 1).

Equal-area hemispheric map projections need little debate. If all parts of a hemisphere are to be given equal importance, the Lambert Azimuthal Equal-Area projection, centered on the center of the hemisphere desired, is ideal because its circular lines of constant distortion include a line following the limit of the hemisphere (Plate 2).

For maps of continents or oceans, the method of least squares can be applied to determine a minimum-error projec-

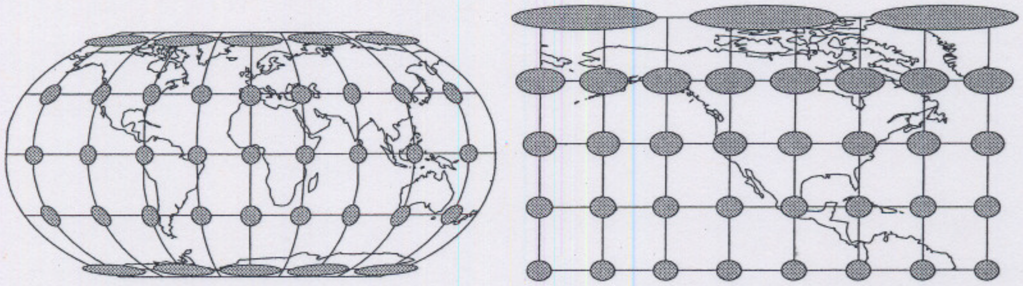

Figure 4. Projections neither conformal nor equal area: a small circle on the globe projects to an ellipse of a different shape and size on the map. The Robinson (l), Plate Carrée (r), and the Azimuthal Equidistant (figure 1) have special properties that preclude correct depiction of shapes and sizes.

tion (within a given category) for the region. Snyder (1985) used this method for certain conformal map projections, and the same principle has now been applied to the oblique Lambert Azimuthal Equal-Area projection (standard) and its more general case, the Oblated Equal-Area projection (recently developed) (Snyder, 1988).

Regions benefiting most from these projections are circular (for the Lambert) and symmetrically oval or rectangular (for the Oblated). In principle, the continents tend to fall into the second category. Small sections of continents, about 5 degrees square, except for Antarctica, were used in the present study for minimum-error calculations. These sections are numbered 162 for North America, 107 for South America, 182 for Africa and the Mid-East, 103 for Europe and Asian Turkey, 213 for Asia, except for the Mid-East, and 161 for Australia.

When the principle of minimum error using least squares was applied to the oblique Lambert Azimuthal Equal-Area projection, two parameters, the latitude and longitude of the center, were optimized. For the Oblated Equal-Area projection, there were five parameters to optimize. These are the latitude and longitude of the center, two shape constants, and the rotation from due north of the major axis of the ovals or rectangles of constant distortion. The attempts to optimize can lead to unresolved iteration problems in some cases.

After optimization, it was concluded that, although the Oblated Equal-Area was a slight improvement over the oblique Lambert for all the continents, the 1 to 10 percent improvement in root-mean-square error (RMSE) does not justify the complication of using the new projection. An exception is North America, where the Oblated projection shows almost a 30-percent improvement in RMSE over the Lambert and, therefore, is recommended (Figure 5). For Antarctica, no attempt was made to move the center of the projection away from the South Pole. Plate 3 shows examples of continental maps.

The parameters, aside from scale, for the selections for world map projections involve the central meridian of the two Wagner projections and the central meridians and longitude limits of the lobes of the two interrupted projections. These are readily determined from existing maps or may be changed to suit the perceptual needs.

For continental maps, the recommended parameters, rounded off because of the subjective nature of the 5° elements, are as follows:

North America:	Oblated Equal-Area with center 48°N, 95°W, shape constants m = 1.33, n = 2.27, rotation = 13.95° -or- Lambert with center 50°N, 100°W
South America:	Lambert with center 15°S, 60°W
Europe:	Lambert with center 55°N, 20°E
Africa:	Lambert with center 5°N, 20°E
Asia:	Lambert with center 45°N, 100°E
Australasia:	Lambert with center 15°S, 135°E
Antarctica:	Polar Lambert centered on South Pole

Distortion Introduced by Changing Map Projection

The recommended equal-area map projections nominally result in the least possible distortion of shapes and distances for global and continental data sets. Such distortions are, however, significant. To show them, the ellipse of distortion — an established way of showing projection characteristics — was first applied, and methods were developed to show distortion introduced during the reprojection of data.

The ellipse of distortion, or indicatrix, was developed by the 19th-century French mathematician Tissot as a way to graphically depict distortion on map projections by showing what happens to an infinitesimally small circle when it is projected from the spherical Earth to a flat map. The ellipse of distortion aids in the selection of a projection to use for a particular area by showing the distortions that result from projection characteristics. For example, the Albers Equal-Area Conic projection fits the conterminous U.S. with little shape distortion, assuming standard parallels at 29.5°N and 45.5°N, but creates foreshortening toward the pole if applied to all of North America. The Lambert Azimuthal Equal-Area, centered at 50°N and 100°W, does not fit the United States as well as the Albers, but its circular pattern of distortion gives a better fit to North America as a whole (Figure 6).

In global change studies, the job is often not one of compiling a map from original source materials, but rather of using data sets already compiled in a particular projection. For example, a data set compiled in the optimal projection for the United States (Albers), may need to be reprojected to Lambert Azimuthal Equal-Area as part of a data set for all of North America. The converse may also occur when a data set for a particular country must be extracted and reprojected from data compiled for a large area.

In either case, data are transformed not from sphere to

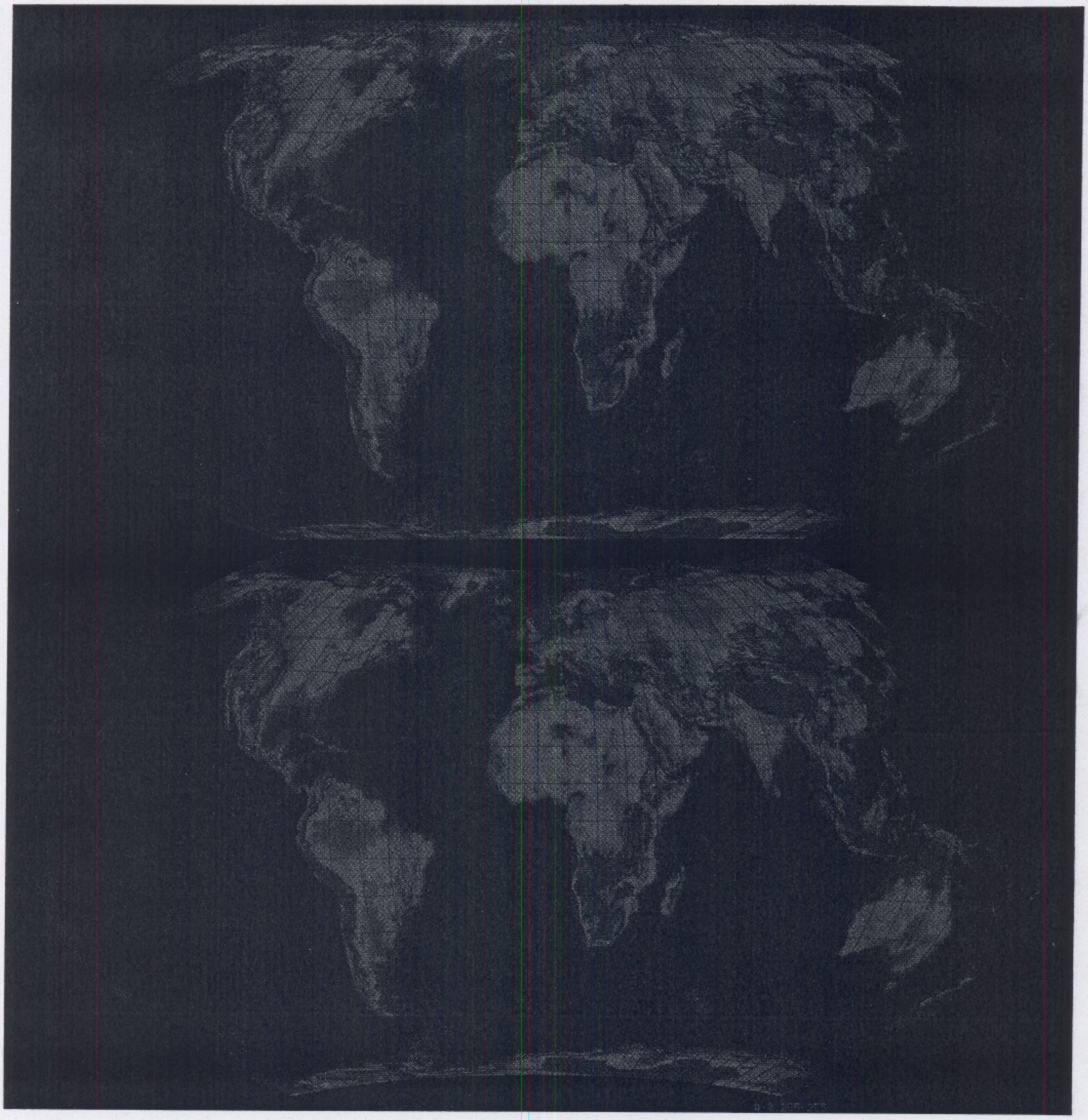

Plate 1. The Wagner IV (top) and the Wagner VII (bottom)

projection, but from projection to projection. Calculations for the ellipse of distortion are able to show only what happens to data during transformation from sphere to Albers, or from sphere to Lambert, but the distortion incurred in transforming a data set from Albers to Lambert (or vice versa) combines the distortion inherent in both projections. A way to show the effects of distortion in reprojection was needed.

To show this distortion, a set of grid squares was created in the Lambert Azimuthal Equal-Area projection and reprojected to the Albers Equal-Area projection (Figure 7). (Note

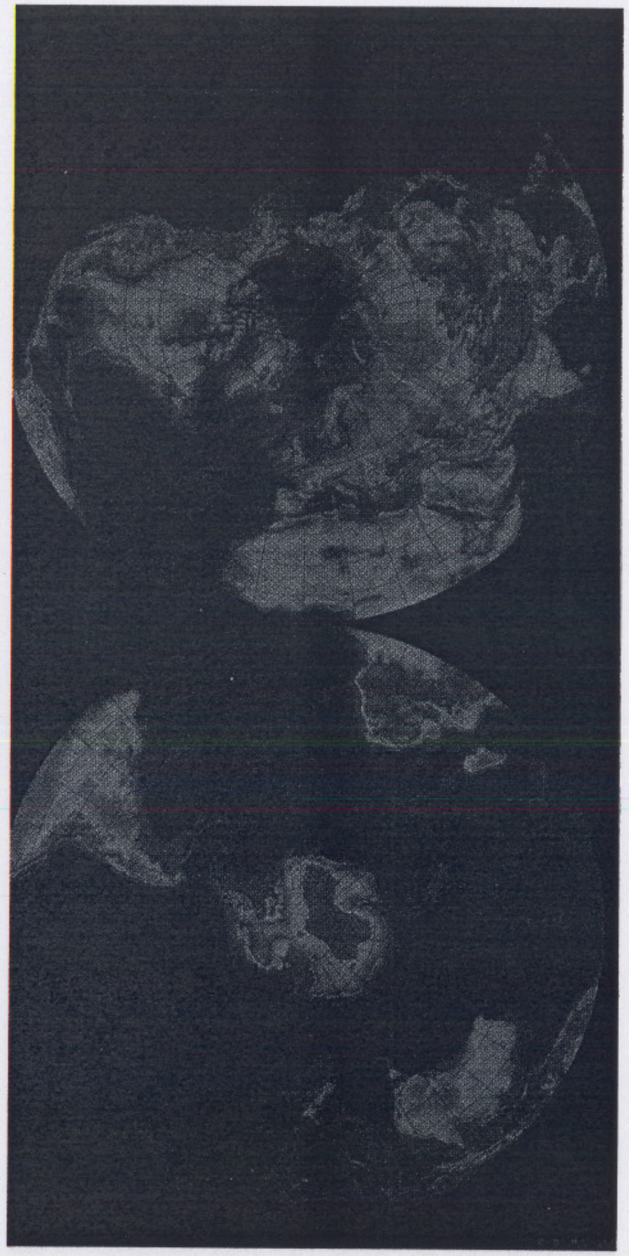

Plate 2. Lambert Azimuthal Equal-Area centered on the North Pole (top) and centered on the South Pole (bottom).

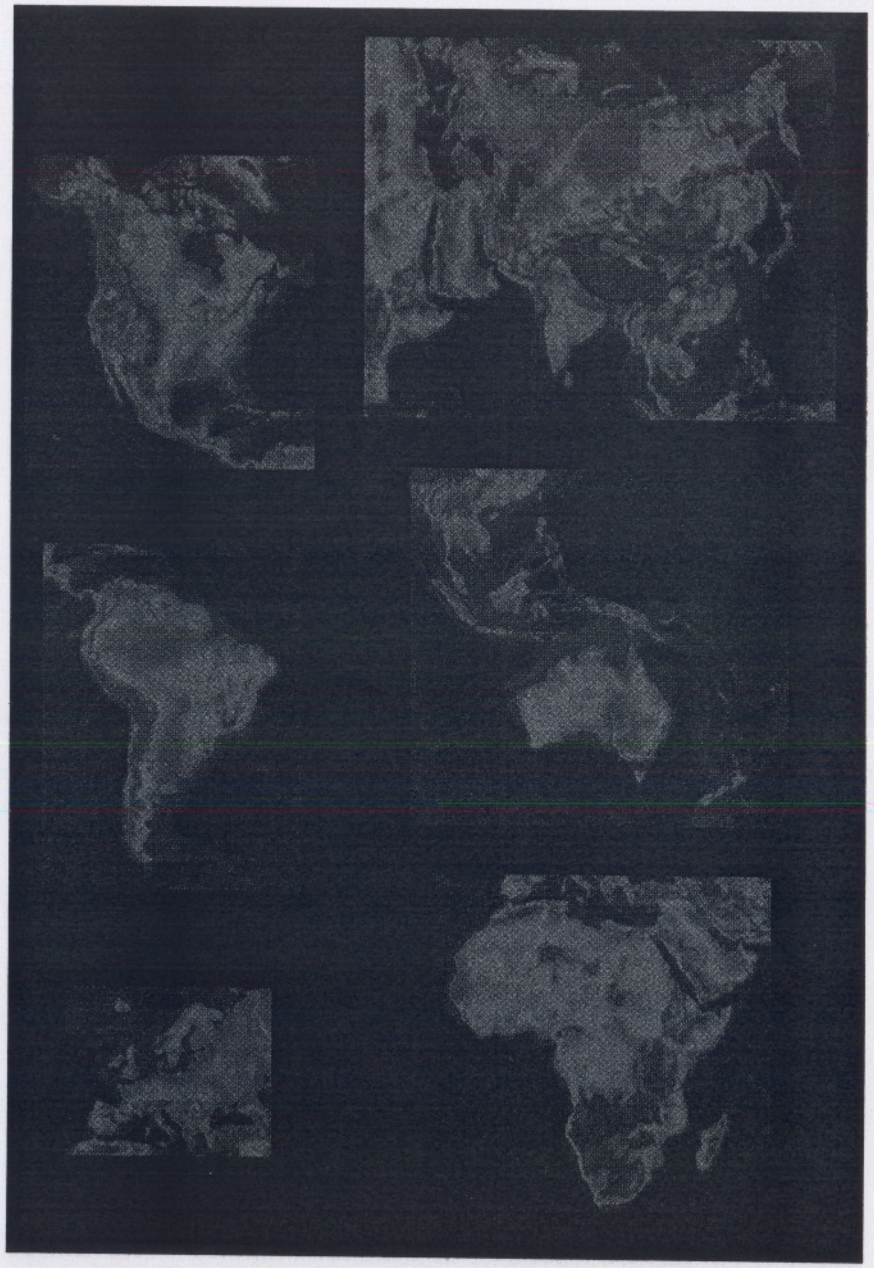

Plate 3. Examples of the recommended continental projections.

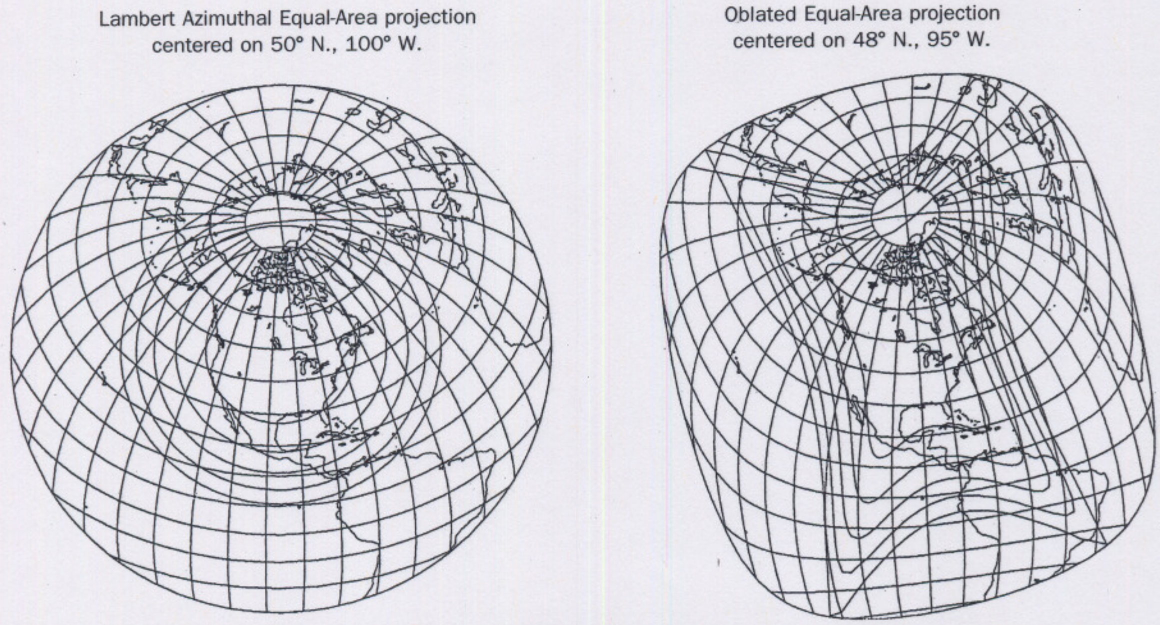

Figure 5. Projections for North America, with isocols of linear scale distortion from 1.02 to 1.10 in increments of 0.02. Oblated Equal-Area has 30 percent less overall scale distortion than the Lambert Azimuthal Equal-Area.

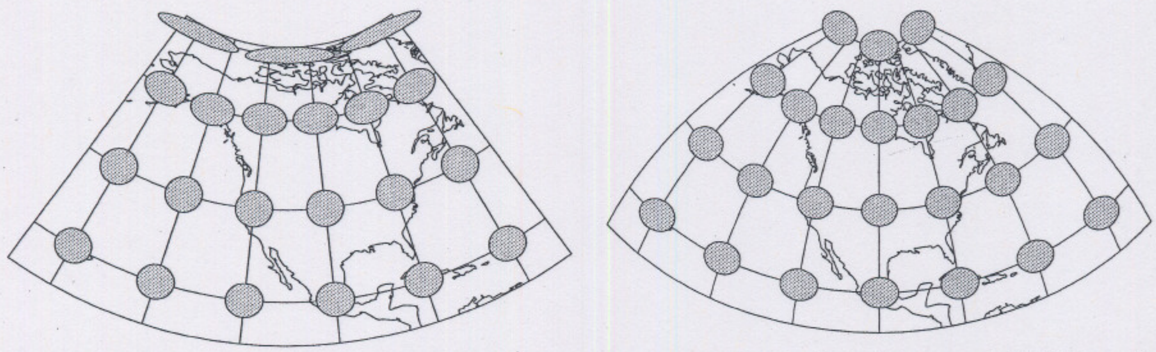

Figure 6. Albers Equal-Area Conic (l) with optimum standard parallels for the United States, at 29.5° N. and 45.5° N., creates foreshortening if extended to all of North America. Lambert Azimuthal Equal-Area (r), centered at 50° N., 100° W., gives a better fit to the continent as a whole because of its circular pattern of distortion.

that all projection 1 to projection 2 transformations are actually projection 1 to geographic to projection 2 transformations). The resulting grid squares show low distortion near the map center and along the standard parallels, but a high degree of skewing near the North Pole. The resulting grid squares maintain the correct size, because both Lambert and Albers maintain equal areas, but the change in shape of the grids is caused by the differing patterns of angular distortion between the two projections.

The underlying assumption in many geographic information systems (GIS) and image processing packages is that the geographic areas involved are small and the distortion incurred is not significant. For studies of global data sets, however, data may have to be stretched or compressed considerably during reprojection based on the properties of the two projections, where the projections are centered, and the extent of area covered.

A product that indicates problems caused by reprojection and shows them graphically is a plot of the Global Vegetation Index (GVI) data set geometry in the Robinson projection. The GVI is a raster data set archived by the National Oceanic and Atmospheric Administration in the Mer-

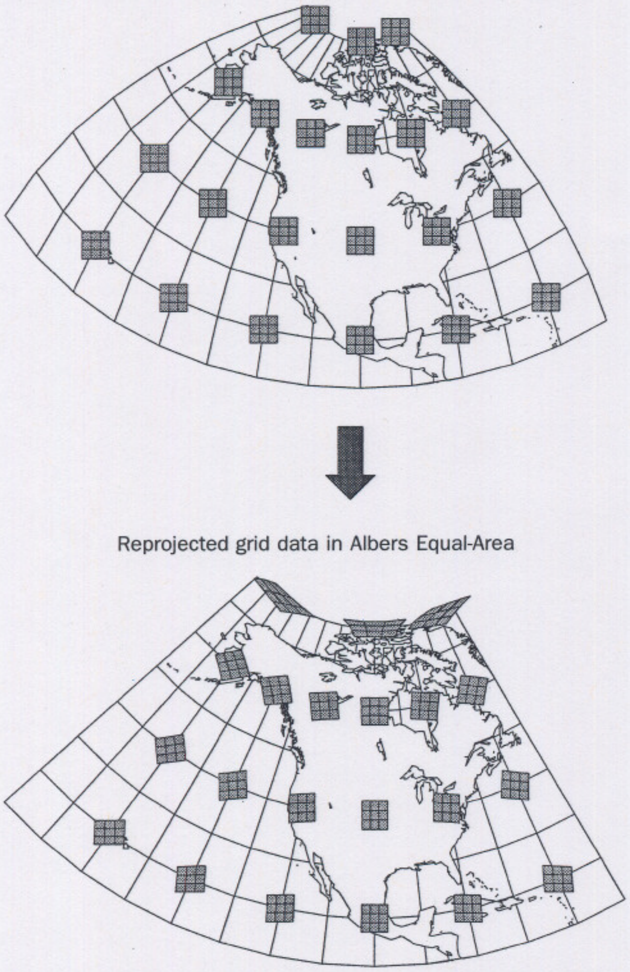

Figure 7. Reprojection from Lambert Azimuthal Equal-Area to Albers Equal-Area. Grid squares on Albers show low distortion near map center, but skewing near North Pole.

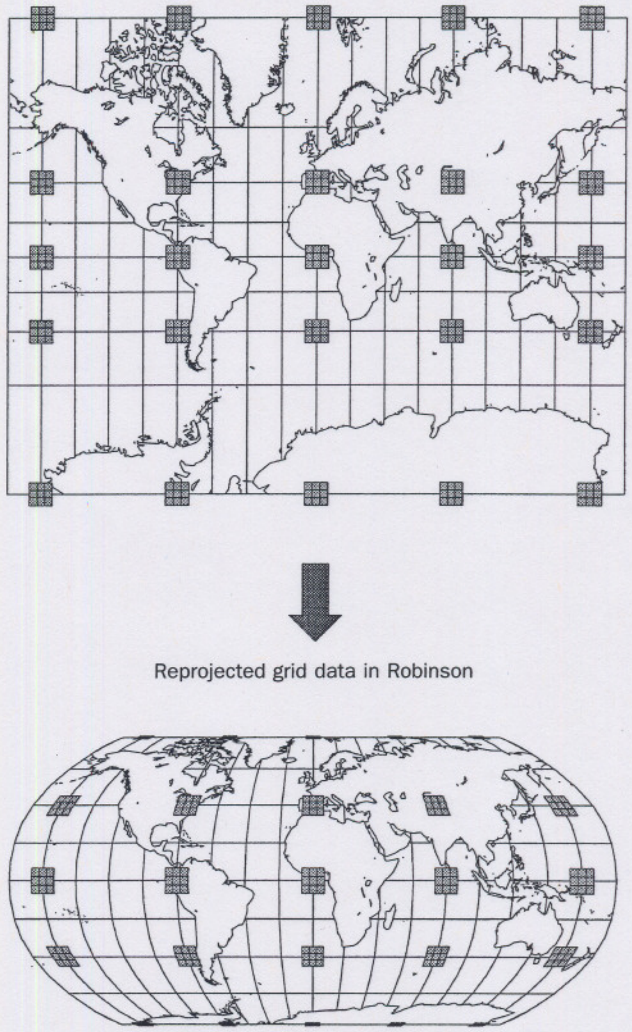

Figure 8. Reprojection from Mercator to Robinson. Grid squares on Robinson are skewed toward east/west margins of map and foreshortened near poles.

cator projection. To show possible distortion problems, a set of grid squares was created in Mercator and reprojected to Robinson (Figure 8). Grid squares were skewed near the equator and extremely foreshortened toward both poles, an expected result from the combination of Mercator scale expansion and Robinson scale compression toward the North and South poles.

Data stored in geographic space (in effect, a Plate Carrée projection) has no advantage over data stored in any other projection and introduces extreme distortions of size and shape (see Figure 4). Reprojection from the Plate Carrée can introduce moderate skewing if the product is in Robinson, or severe skewing and compression if the product is Lambert Azimuthal Equal-Area (Figure 9).

Distortions Introduced During Reprojection of Raster Data

The grid square maps presented in the previous section can be used to conceptualize what happens to raster data when reprojected. Reprojection of raster data sets involves calculation of a geometric transformation model, which is used to warp images in the original projection space to the selected projection space. The amount of data distortion incurred when reprojecting raster data depends primarily on the relative geometry of the two projections and on the local scale and resolution changes made to the images.

Image pixels are arranged in a rectangular grid, each pixel having the same size as the other pixels in the image. This grid aligns nicely with the concept of projection coordinates, and there is a one-to-one correspondence between image coordinates and projection coordinates in georegistered, map-projected images. Projection change disrupts this regularity by warping the grid of one projection to fit another. If raster data were continuous, they would stretch, compress, and bend to conform to the new geometry.

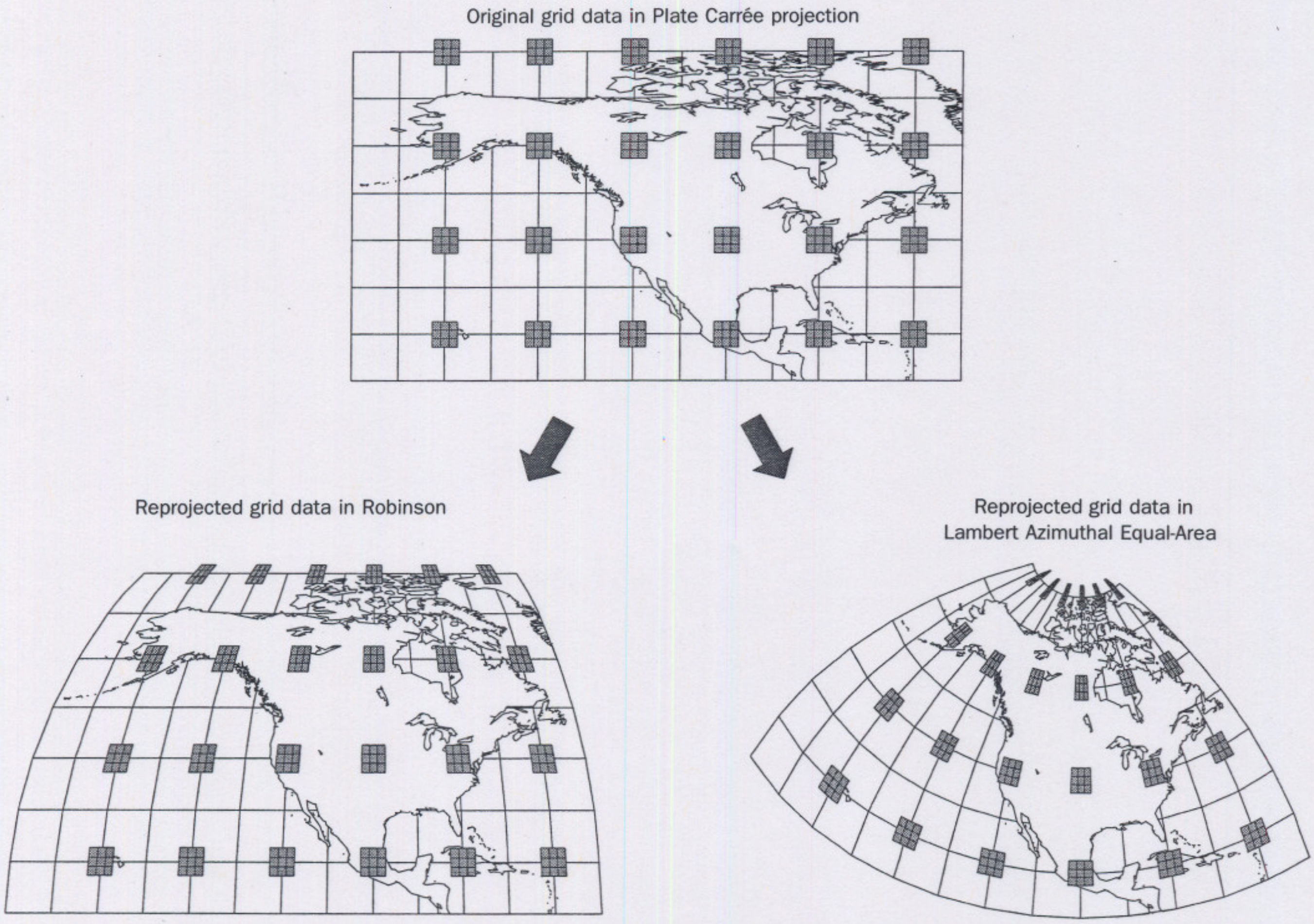

Figure 9. Reprojection from Plate Carrée to Robinson, with moderate shearing of grid squares, and from Plate Carrée to Lambert Azimuthal Equal-Area, with severe shearing of grid squares.

Raster data, however, are discrete values and do not conform to these new geometries. Bent lines become stair-stepped arcs. Area compression results in the reduction of image resolution (that is, fewer pixels represent a given area). Area enlargement results in the replication of image pixels (the local scale changes, but resolution does not — more pixels represent a given area, but detail is not better) (Figure 10). If the grid squares introduced above were to represent image pixels, then the warped grid squares represent the skewing and compression necessary for raster data to fully depict the transformation. Data loss results from dropping or changing image pixels during the reprojection process. When the resolution of discrete data is reduced, there is no way to recover the original values from the reduced data — the information has been lost. It is, therefore, important to be aware of the potential information loss that may occur when reprojecting large-area raster data sets.

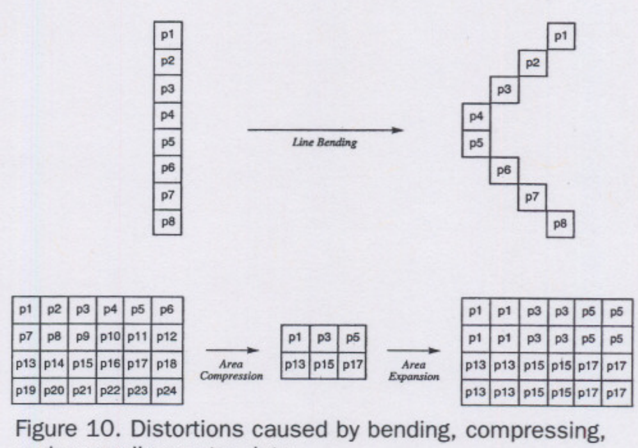

Figure 10. Distortions caused by bending, compressing, and expanding raster data.

The next stage of the study was to move from a graphic depiction to a more detailed examination and quantification of data degradation during projection change. Distortions in the reprojection of raster data are due to the distortion inher-

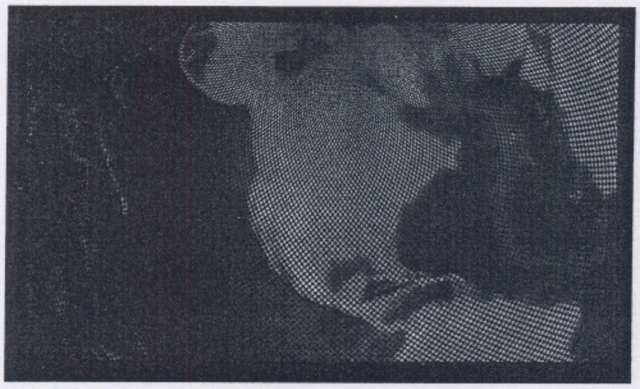

Figure 11. Distortion image of North America, with data reprojected from Plate Carrée to the Lambert Azimuthal Equal-Area.

ent in projection change and to the distortion created by resampling discrete pixel values. Image processing functions were developed to assist in the quantification and visualization of these errors. In this first example, an elevation image in the Plate Carrée projection of the North American continent was reprojected to the Lambert Azimuthal Equal-Area projection. As expected, reprojecting from Plate Carrée to the Lambert results in severe skewing and compression of the data (see Figure 9). To better show distortion effects on raster data, a checkerboard image with alternating 10 by 10 black-and-white squares was reprojected to the same geometry, giving a similar visual representation of the geometric error as the grid plots of the previous sections. These two data sets were combined to show projection change distortions over the area of interest (Figure 11).

The grid distortion plot (see Figure 9) of the area was used to calculate the distortion due to the projection change alone using a GIS. The checkerboard image blocks were then counted to calculate the distortion due to the projection and the image warping. Table 1 shows these results. Note that the percent of original area used for projection change and discrete image pixels can only approach the percent of original area used for projection change alone. Also note that the last table entry was not countable because of the extreme data reduction in that part of the image. This condition will be addressed in the next example.

The second example again consists of the reprojection of images in the Plate Carrée to the Lambert Azimuthal Equal-Area projection. In this example the original image is again a checkerboard image, alternating 10 by 10 black-and-white squares in the Plate Carrée projection representing the Northern Hemisphere. The resulting image is in the Lambert Azimuthal Equal-Area projection centered on the North Pole. This is an extreme example because the first line of 4,320 pixels in the original image, which represents 90°N, is reprojected to a point (one pixel) in the center of the resulting image, and the line of 4,320 pixels in the original image representing the Equator are reprojected to a circle (Figure 12). Note that, on the Equator of the resulting map, the checkerboard can be seen, although skewed. As the North Pole is approached, the identification of the breaks in the checkerboards is impossible to determine due to the reduction in image resolution that took place during the projection transformation. Additional software was, therefore, developed to identify which pixels in the original image were used to create the final image (Figure 13). In this example, only 50 percent of the data in the Plate Carrée image were used in the polar image. While experimenting with the pixel size, it was found that reducing the pixel size in the final projection only moved the distortions to a higher latitude and increased final image size.

These two examples used nearest-neighbor resampling. During the image warping process, pixel values in the final image are determined by taking the original image pixel coordinate determined by the mapping transformation and rounding it to the nearest line/sample integer location; no interpolation of neighboring pixels is performed. Nearest-neighbor resampling is often used by scientists who work with class data because it does not create new classes in the image warping process. However, this method of determining pixel values used in areas of high geometric distortion can result in a data sampling that is not representative of the area and often results in blocky looking data. An interpolating resampler such as bilinear interpolation (Colwell, 1983),

Table 1. Distortions Due to Projection Change and Image Warping

Latitude/longitude		Percent of original area used: Projection change alone	Percent of original area used: Projection change and discrete image pixels
20°N	160°W	94	81
40°N	160°W	77	66
60°N	100°W	50	41
60°N	60°W	50	41
80°N	60°W	17	Not countable

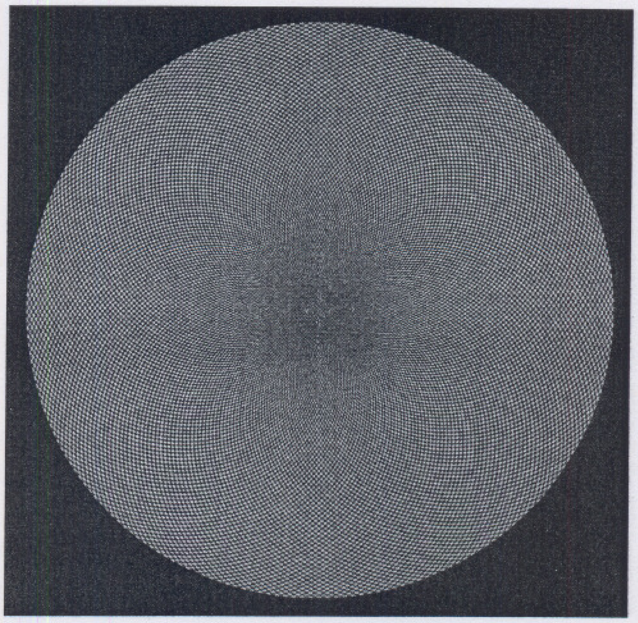

Figure 12. Distortion image of the Northern Hemisphere, centered on the North Pole. Reprojection is from Plate Carrée to Lambert Azimuthal Equal-Area.

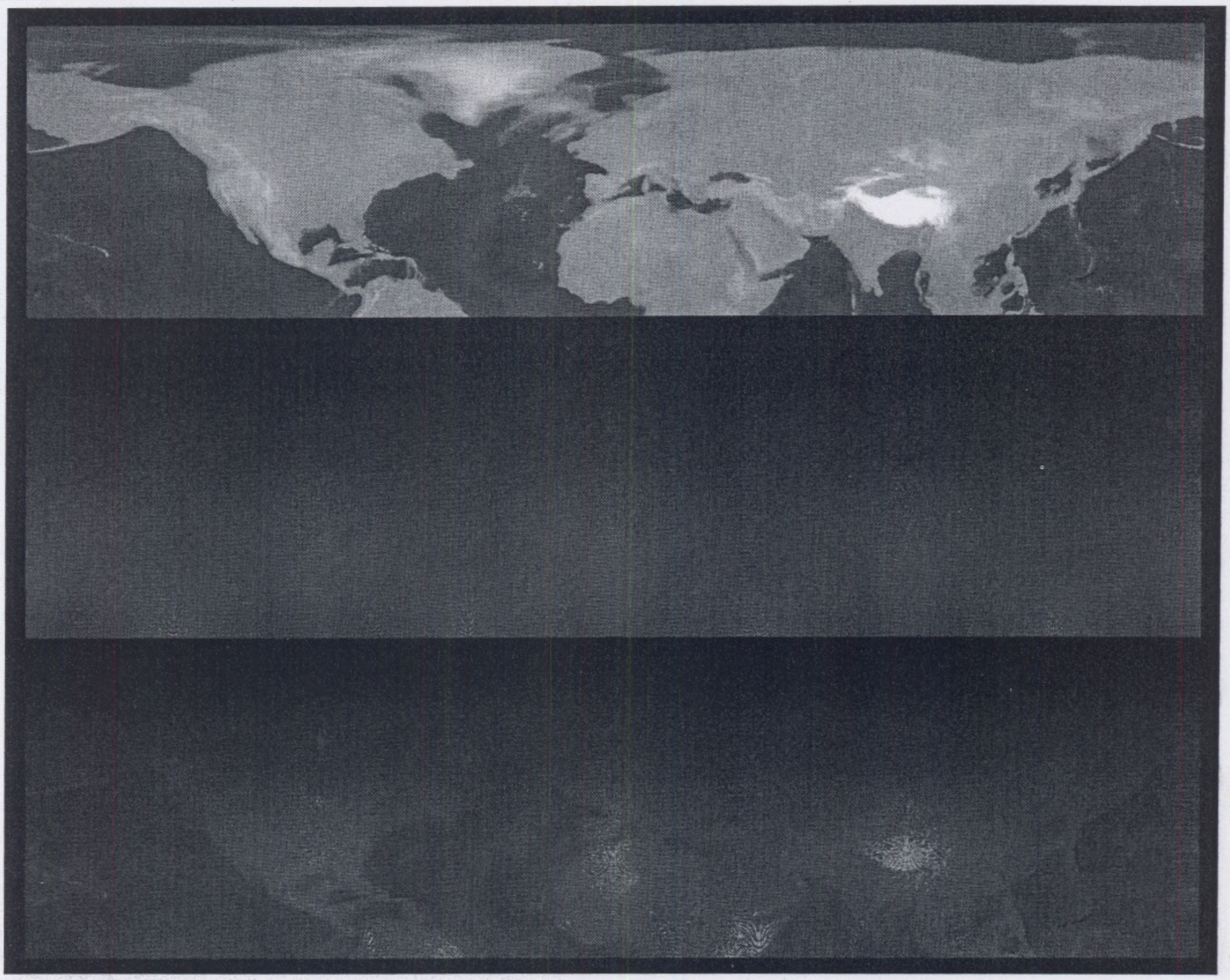

Figure 13. Data use images. The Plate Carrée original data (top), map of pixel usage (middle), and combination of the two (bottom).

which uses the four neighboring pixels, or cubic convolution (Park and Schowengerdt, 1983), which uses the 16 neighboring pixels, produces a more geometrically accurate (but radiometrically smoother) result in large-scale studies (that is, Landsat images on 7.5-minute quadrangles). However, in large-area, small-scale studies, the use of these small neighborhood interpolators does little to improve the type of errors seen in these examples. New interpolation techniques that take into account the appropriate resolution and scale changes for a given data point are needed. This should be determined for each pixel in the final image because the amount of resolution and scale distortion is often not constant throughout the warped image space.

Conclusions

This study has identified equal-area map projections for data sets that cover the globe, a hemisphere, or a continent. These are the interrupted Goode Homolosine, the interrupted Mollweide, the Wagner IV, and the Wagner VII for global maps; the Lambert Azimuthal Equal-Area for hemispheric maps; and the oblated Equal-Area and the Lambert Azimuthal Equal-Area for continental maps.

In addition, this study identified and developed software tools to visualize and quantify distortions due to both the projection change and the problems associated with the reprojection of raster image data of global or continental extent. Users and processors of large-area data sets must be aware of these distortions and take steps to reduce them to ensure better information quality in data products.

One of the main problems encountered when reprojecting raster data is the determination of final image pixel values. Further investigation into resampling methods that take into account the amount of resolution reduction and scale change is needed. These resampling methods must adapt to resolution changes that are not constant throughout the final image space.

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References

Colwell, R.N., 1983. *Manual of Remote Sensing, Second Edition,* Volume 1, Chapter 17, Data Processing and Reprocessing: American Society of Photogrammetry, Falls Church, Virginia, pp. 734–737,

Park, S.K., and R.A. Schowengerdt, 1983. Image Reconstruction By Parametric Cubic Convolution, *Computer Vision, Graphics, and Image Processing 23,* Academic Press Inc., pp. 258–275.

Snyder, J.P., 1985. *Computer-Assisted Map Projection Research,* U.S. Geological Survey Bulletin 1629.

———, 1988. New Equal-Area Map Projections For Noncircular Regions, *The American Cartographer,* 15(4):341–355.

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