Fractal Enhancement of Cartographic Line Detail

Geoffrey H. Dutton

ABSTRACT. **In** plane geometry curves have a dimension of exactly 1 and no width. In nature, all curvilinear features have width, and most have dimension greater than 1, but less than 2. Many phenomena, such as coastlines, have the same "look," even when viewed at greatly varying scales. The former property is called "fractional dimensionality," and the latter is called "self similarity." Curves digitized from maps may be analyzed to obtain measures of these properties, and knowledge of them can be used to manipulate the shape of cartographic objects. An algorithm is described which enhances the detail of digitized curves by altering their dimensionality in parametrically controlled, self-similar fashion. Illustrations show boundaries processed by the algorithm.

Measuring and Modelling Irregularity in Nature

Only in the mind and works of man do straight lines exist. Rarely does Nature rule with a straightedge, and even these lines are rough, seldom extending *very* far. But surrounded by rectilinear artifacts, it is understandable why humans try to measure and model the world with Euclidean precepts. Frustration in making certain measurements and in modelling many natural forms can be attributed to this view of space itself, in which distance between two given points is assumed to be Pythagorean.

Suppose one is surveying a section of coastline and wants to calculate its length accurately and map *it. A series of* closely-spaced sightings must be made at the high-water mark. The cumulative distance along these points can then be accurately computed, and it is invariably greater than the crow's-flight distance spanning the stretch of coast. Fig. IA represents the profile of a fictitious coastline. Its surveyed approximation is plotted in Fig. IB, and the crow's-flight version of it is shown in Fig. 1C. A greater number of sightings yields a closer approximation to actual length and shape, even though the rate of increase of length slows.

This lesson in approximation has several morals. One is that surveyors run into a real law of diminishing returns when trying for centimeter accuracies in the lengths of complicated boundaries. Another is that the difficulty, *hence* the probable error, in measuring coastlines and the like varies from place to place. In Fig. 1A, it is obvious that there is much more irregularity in the lower part of the coast than in the upper part. This may be due to the *former* being composed of rock outcroppings and the latter being a sandy beach. But in trying to express this qualitative difference quantitatively, one finds scientific vocabulary confusing and inadequate. Literature in geography and image processing abounds with indices that characterize the shapes of point sets and linear and areal features (Stoddard, 1965; Boyce and Clark, 1964; Bunge, 1961; Boots, 1972). However, to paraphrase Pavlidis (1978), these indices are destructive of information and provide neither a general linguistic model nor a measure suitable to allow manipulation of cartographic detail. One wishes for a measure of geometric complexity and irregularity that is as general as that of

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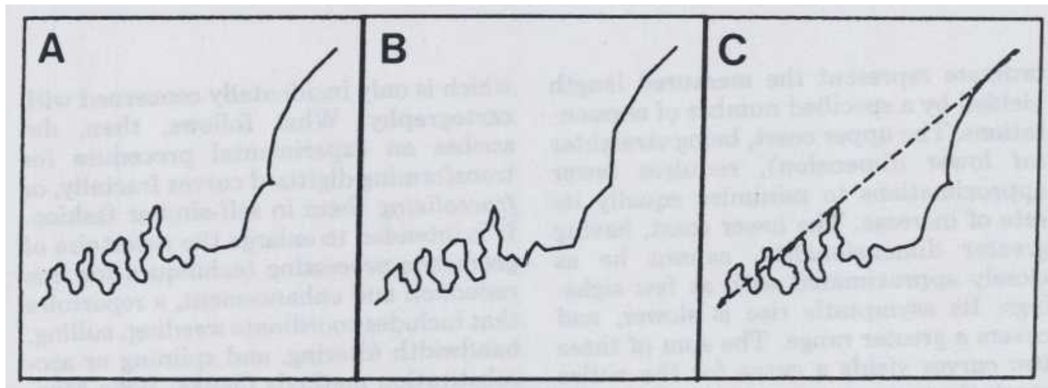

Fig. 1. A coastline and its approximations: (A) the original coastline, (B) segmented approximation. (C) original coastline with superimposed trend line.

entropy in thermodynamics. Fortunately, foundations for such a vocabulary and for such measures have *been developed.*

Irregularity as Fractional
Dimensionality and Self Similarity

A suitably general approach to quantifying the complexity of irregular forms, and one that directly confronts the dilemmas of Euclidean measurement, is that of Mandelbrot (1977). The phenomena that he addresses-natural forms arising from forces such as turbulence, curdling, Brownian motion, and erosion-have at all scales two related properties, *self* similarity and *fractional dimensionality. Self* similarity means that a portion of an object when isolated and enlarged exhibits *the same* characteristic complexity as the object as a whole. The shapes revealed may be highly irregular, and none may be exactly alike, but they will have the same kind of irregularity over a wide range of scales. Fractional dimensionality means that the Euclidean dimension that normally characterizes *a* form (1 for lines, 2 for areas, 3 for volumes) represents only the integer part of the true dimension of the form, which is a fraction.'

Mandelbrot treats dimension as a continuum, in which the integer Euclidean dimensions merely represent limiting *cases of topological genera,* unlikely to occur in nature. Thus the coastline in Fig. 1A might have an approximate overall dimensionality of 1.2, but its two dissimilar subsections have different structure and dimensionality. The more irregular lower portion may have a dimension of nearly 1.3, while the smoother upper part may be of a lower dimension, less than 1.1. There is only one version of the coast that has a dimensrionality of *exactly I (its* Euclidean dimensionality), and that is the trendline shown in Fig. 1C.

This difference in dimensionality is quantified in Fig. 2. On this graph the abscissa symbolizes the number of sightings (or line segments) used to approximate the entire coast (top curve) and its lower (middle curve) and upper (bottom curve) portions. *The values* read from the

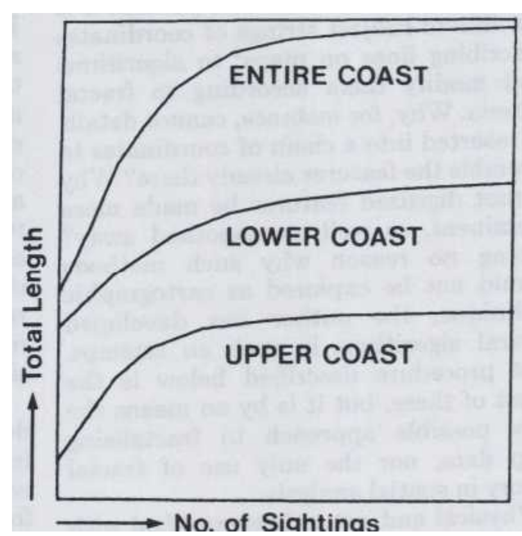

Fig. 2. Dependence of length on fractal limensionality and scale of measurement.

ordinate represent the measured length yielded by a specified number of segmentations. The upper coast, being straighter (of lower dimension), requires fewer approximations to minimize equally its rate of increase. The lower coast, having greater dimensionality, cannot be as closely approximated with as few sightings. Its asymptotic rise is slower, and covers a greater range. The sum *of these* two curves yields a curve for the entire coast, the top one.

Fractal Forms in Cartography

Digitized map data resemble fractals much more than they resemble (continuous) functions which mathematicians normally study. Although certain cartographic objects, including both boundaries and terrain, can be approximated using real functions, e.g., trigonometric series, the difficulty remains of representing nonperiodic map features as well as ones that are not single-valued, i.e., places where curves reverse direction. Furthermore, fractal curves, such as coastlines, can have islands, whose number and size distribution varies with fractal dimensionality (Mandelbrot, 1977, p. 45). Given these similarities, perhaps it is possible to subject strings of coordinates describing lines on maps' to algorithms that modify them according to fractal criteria. Why, for instance, cannot details be inserted into a chain of coordinates to resemble the features already there? Why cannot digitized features be made more prominent, as well as smoothed away? Seeing no reason why such methods should not be explored as cartographic technique, the author has developed several algorithms in such an attempt. The procedure described below is the latest of these, but it is by no means the only possible approach to fractalizing map data, nor the only use of fractal theory in spatial analysis.

Physical and natural science deal with many fractal phenomena, and the application of fractal concepts to geography has only just begun. Examples of such analyses abound in Mandelbrot's essay, which is only incidentally concerned with cartography. What follows, then, describes an experimental procedure for transforming digitized curves fractally, or fractalizing them in self-similar fashion. It is intended to enlarge the repertoire of geometric processing techniques for line reduction and enhancement, a repertoire *that includes* coordinate weeding, culling, bandwidth filtering, and splining or arc-substitution methods (Jenks, 1980; Morrison, 1975; Douglas and Peucker, 1973; Rhind, 1973; Ramer, 1972).

Parametric Fractalization of *Digitized Curves*

Like splining, in which smooth, mathematically-defined arcs are inserted in place of one or more segments along a chain, fractalizing can increase a chain's total length. Unlike it and other methods for coordinate reduction and chain smoothing, fractalizing permits features to be *exaggerated* and smaller-scale features to be *introduced into* digitized curves, as well as allowing features to be *eliminated.* The exaggerations and additions are not arbitrary forms introduced to the chain but are caricatures and recursions of forms already found there. Because the procedure can be applied recursively, there are *geometric* similarities between smaller features introduced and larger features already existing in cartographic lines. Such enhancements of chains may not have as regular an appearance as a chain smoothing via arc-substitution, but they may preserve more concretely the qualities of the original chain. The following describes the most recent approach to fractalization, which reconfigures chains to desired dimensionality and detail.

As currently implemented, the procedure is given a list of coordinates for an input chain and returns a fractalized version of it in a separate array, transformed according to four parameters:

(1) a Sinuosity Dimension (SD) (
2) a Uniformity Coefficient (UC) (
3) a Straightness Tolerance (ST) (
4) a Smoothness Tolerance (SM)

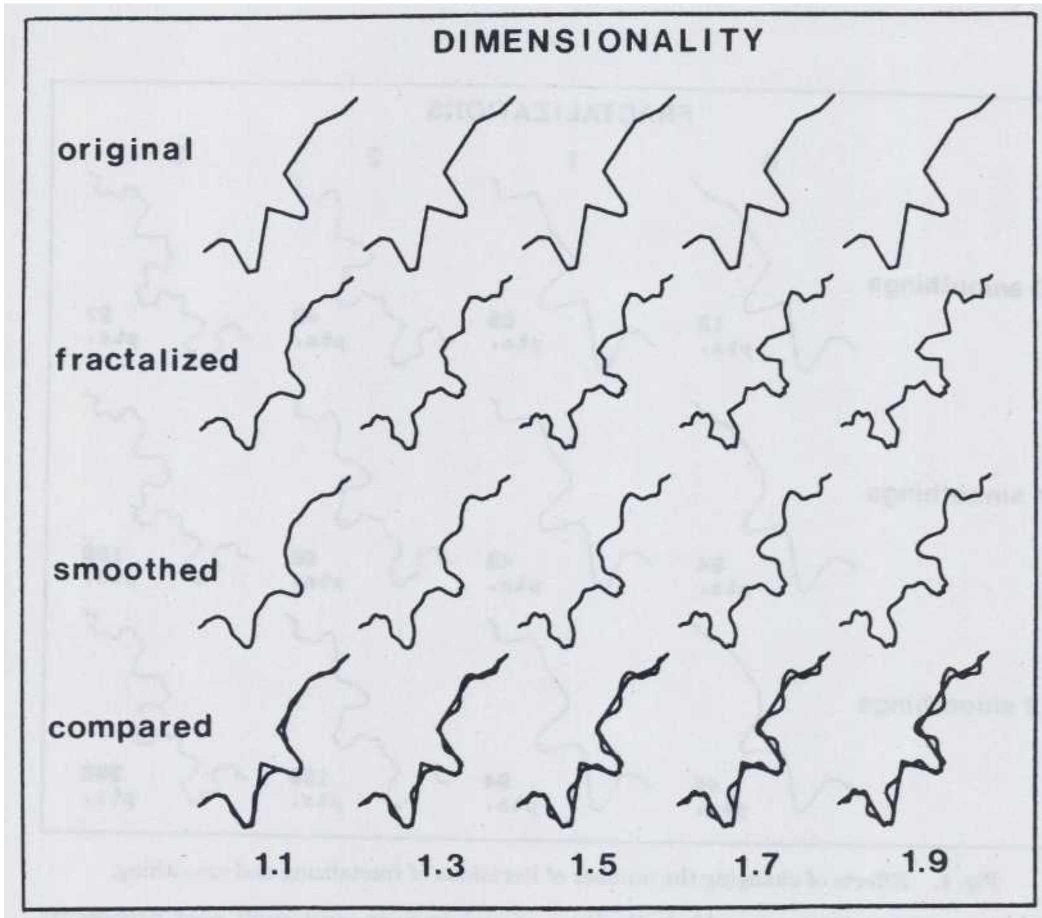

Fig. 3. Enhancement of a 13-point chain, showing the effects of varying SD.

The Sinuosity Dimension

The Sinuosity Dimension, SD, prescribes the amount of waviness that chains should possess after fractalization, but is tempered by the Uniformity Coefficient, discussed below. SD is a real number between 1 (minimum sinuosity) and 2 (maximum sinuosity), and specifies a fractal dimension to characterize processed chains.

Fig. 3 demonstrates the *effect of varying* SD from 1.1 to 1.9 in fractalizing a 13-point chain. Three iterations of fractalization were performed (second row), followed by one of smoothing (third row). The enhancements are overlaid with the original chain in the fourth row. UC, discussed below, is fixed at 1.0 in each of the examples in Fig. 3.

Ignoring for the moment the effect of UC, the algorithm works by moving along a chain, relocating each vertex in the general - direction of its angle bisector. *The movement forms a* new angle which *will be* the same at all vertices, as determined by SD. Since all vertices are potentially subject to displacement, it is geometrically necessary to introduce additional vertices, which are located at the midpoint of each segment along the chain. These serve as fixed points, and successive ones are connected to form triangle *bases, where* the apex is the vertex between these midpoints. To relocate such an apex, it is necessary and sufficient to know (a) the desired angle of the apex, which SD implies, and (b) the current proportions of its two legs. In

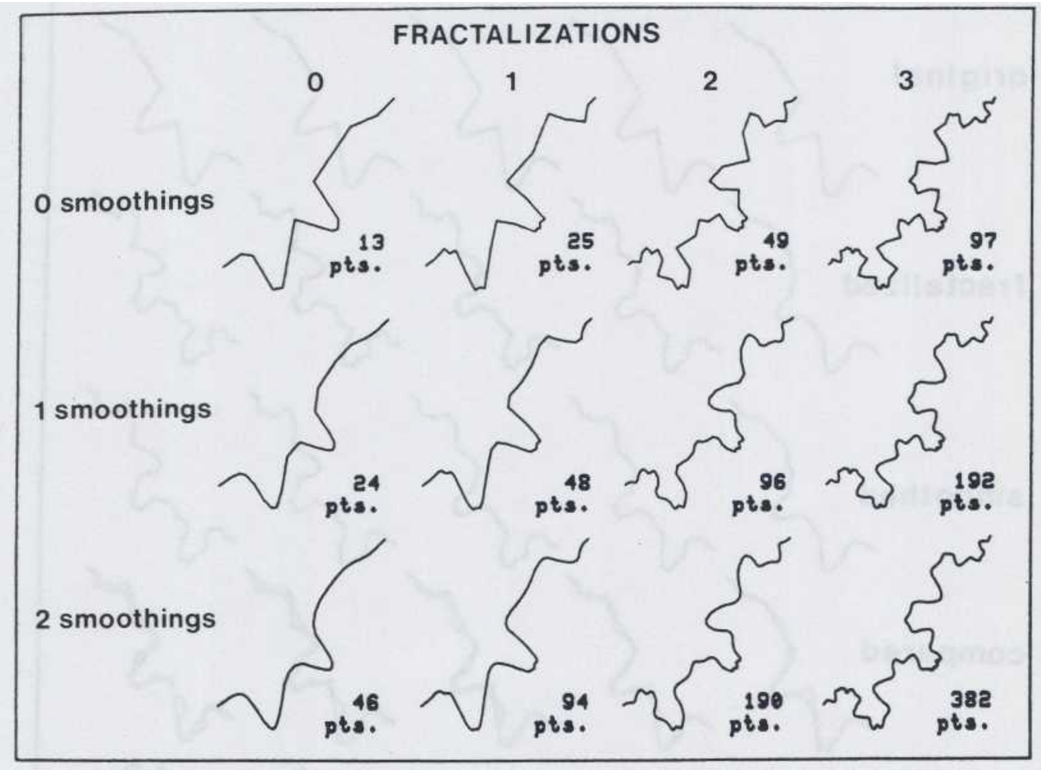

Fig. 4. Effects of changing the number of iterations of fractalizing and smoothing.

order to maintain local self-similarity, this proportion is constrained to be the same after fractalizing as before. The actual computations require solving trigonometric *identities* involving the law of sines, law of cosines, and sums and differences of sines and cosines. The result is to standardize all junctions at the angle determined by SD and to introduce intermediate vertices (at segment midpoints) having unstandardized included angles. This provides the *degree of freedom* needed to standardize the original junctions. Successive iterations of the procedure will then standardize the angles at the former midpoints, once again halving all segments.

The effect of this, as the second row of Fig. 3 shows, is to create rather mechanical-looking chains. Such figures are not very "cartographic," although they may have certain stylistic uses in thematic mapping. Their zigzags may be smoothed, as in the third row of Fig. 3, to soften visual impact, and such post-processing should be applied to chains thus fractalized. Usually, however, the goal is not to standardize the geometry of a chain but to influence it in a self-similar fashion. Therein lies the utility of the second parameter, UC.

The results of varying *the* number of iterations of both fractalizing and smoothing is demonstrated in Fig. 4. In it the same chain used in Fig. 3 is subjected to combinations of zero to three fractali*zations and zero to two* smoothings. In each case, SD - 1.7, UC = 1.0, SM = 0.0, and ST = infinity. The effect of additional iterations generating smaller-scale details is evident for fractalization, while for smoothing, additional *iterations* soften detail without really eliminating it.

The Uniformity Coefficient

Should extremely regular chains not be desired, UC can he used to prevent their

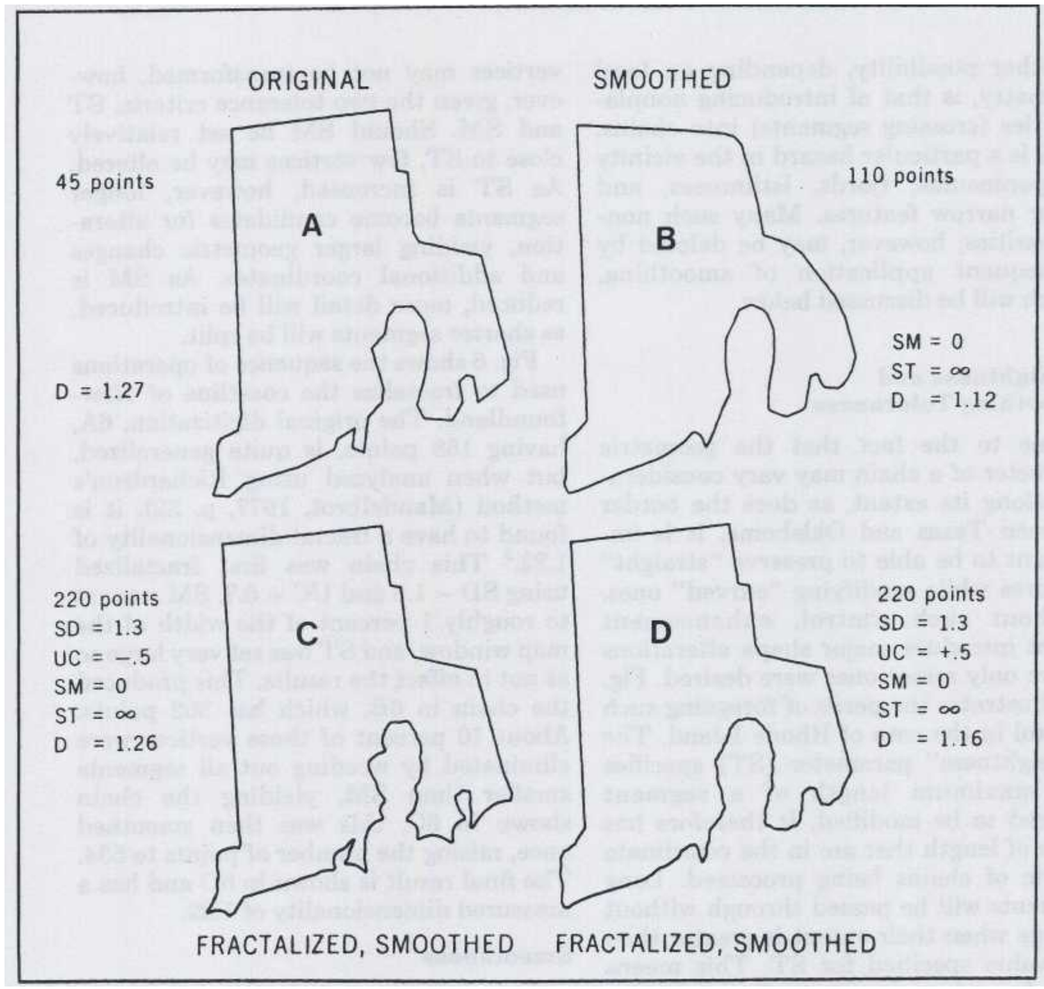

Fig. 5. Effects of uniformity (UC) and straightness (ST) on enhancement.

formation. This parameter specifies the degree to which junctions may vary from whatever angle SD specifies. When UC is 1 (its maximum), the dimensionality of a fractalized chain is held constant at SD throughout its length. When UC is zero, the appearance of chains will be unaffected by fractalizing them, although they will gain extra coordinates at segment midpoints. Given any value of UC intermediate between 0 and 1, the algorithm will displace vertices toward their "standardized" positions through a distance proportional to UC. That is, if UC equals 0.5, a vertex will not be displaced to the location giving it a local dimensionality of SD but will be moved only halfway towards that point. This allows SD to influence the dimension of chains without fully standardizing them.

UC also may be set to less than zero, but not less than -1. When negative values are used, their effect is to exaggerate junctions; rather than being displaced toward a specific angle, they are displaced in the opposite direction. The different effects of the sign of UC are illustrated in Figs. 5C and 5D. With negative values of UC, the farther from the standardized location a vertex initially is, the farther it will be displaced away from it. Negative displacements can cause a junction to reverse direction, should its dimension initially be less than SD.

Another possibility, depending on local geometry, is that of introducing nonplanarities (crossing segments) into chains. This is a particular hazard in the vicinity of peninsulas, fjords, isthmuses, and other narrow features. Many such nonplanarities, however, may be deleted by subsequent application of smoothing, which will be discussed below.

Straightness and Smoothing Tolerances

Due to the fact that the geometric character of a chain may vary considerably along its extent, as does the border between Texas and Oklahoma, it is important to be able to preserve "straight" features while modifying "curved" ones. Without such control, enhancement might introduce major shape alterations where only minor ones were *desired. Fig.* 5B illustrates the perils of foregoing such control in the case of Rhode Island. The " straightness" *parameter* (ST) specifies the maximum length of a segment allowed to be modified. It therefore has units of length that are in the coordinate metric of chains being processed. Long segments will be passed through without change when their extent is greater than the value specified for ST. This means that, for enhancement to affect a vertex, *both* segments that meet *there must* be less than ST units in length.

In a similar but inverse fashion, the " Smoothness Tolerance" specifies the smallest segment allowed to be modified. SM determines the fineness of detail produced by fractalizing anywhere along a chain. Detail may also be limited by stopping the procedure after a fixed number of iterations. Unlike ST, however, SM can and should be altered depending on the scale at which chains are to be displayed. Together, then, ST and SM define the upper and lower limits for the size of features subject to enhancement and thus constrain the overall amount of added detail.

One application of the procedure will nominally double the number of segments in each chain processed. Certain vertices may not be transformed, however, given the two tolerance criteria, ST and SM. Should SM be set relatively close to ST, few vertices may be altered. As ST is increased, however, longer segments become candidates for alteration, yielding larger geometric changes and additional coordinates. As SM is reduced, more detail will be introduced, as shorter segments will be split.

Fig. 6 shows *the* sequence of operations used to fractalize the coastline of Newfoundland. *The original* digitization, 6A, having 158 points, is quite generalized, but when analyzed using Richardson's method (Mandelbrot, 1977, p. 32), it is found to have a fractal dimensionality of 1.22.[3] This chain was first fractalized using SD = 1.3 and UC = 0.7. SM was set to roughly 1 percent of the width of the map window, and ST was set very large so as not to affect the results. This produced the chain in 6B, which has 302 points. About 10 percent of these vertices were eliminated by weeding out all segments smaller than SM, yielding the chain shown in 6C; this was then smoothed once, raising the number of points to 534. The final result is shown in 6D and has a measured dimensionality of 1.22.

Smoothness

As stated earlier, the visual quality of chains that have been fractalized may be rather jagged, the degree of which depends on SD and SM. Before plotting chains, it is thus usually necessary to smooth them, which does not appreciably alter their dimensionality. The method used is a form of splining, similar to *the* technique developed by Chaikin (1974, with comments by Riesenfeld, 1975). Both are fast methods, but the author's is a bit slower due to its capability to spline to different curvatures.

The smoothing procedure is illustrated in Fig. 7. During smoothing, each vertex is literally "snipped off" the chain somewhere between the vertex and the midpoints of the two segments that join at the vertex. In Fig. 7, either one, two, or four iterations of smoothing were used, each with Roundness set at .25, .50, and

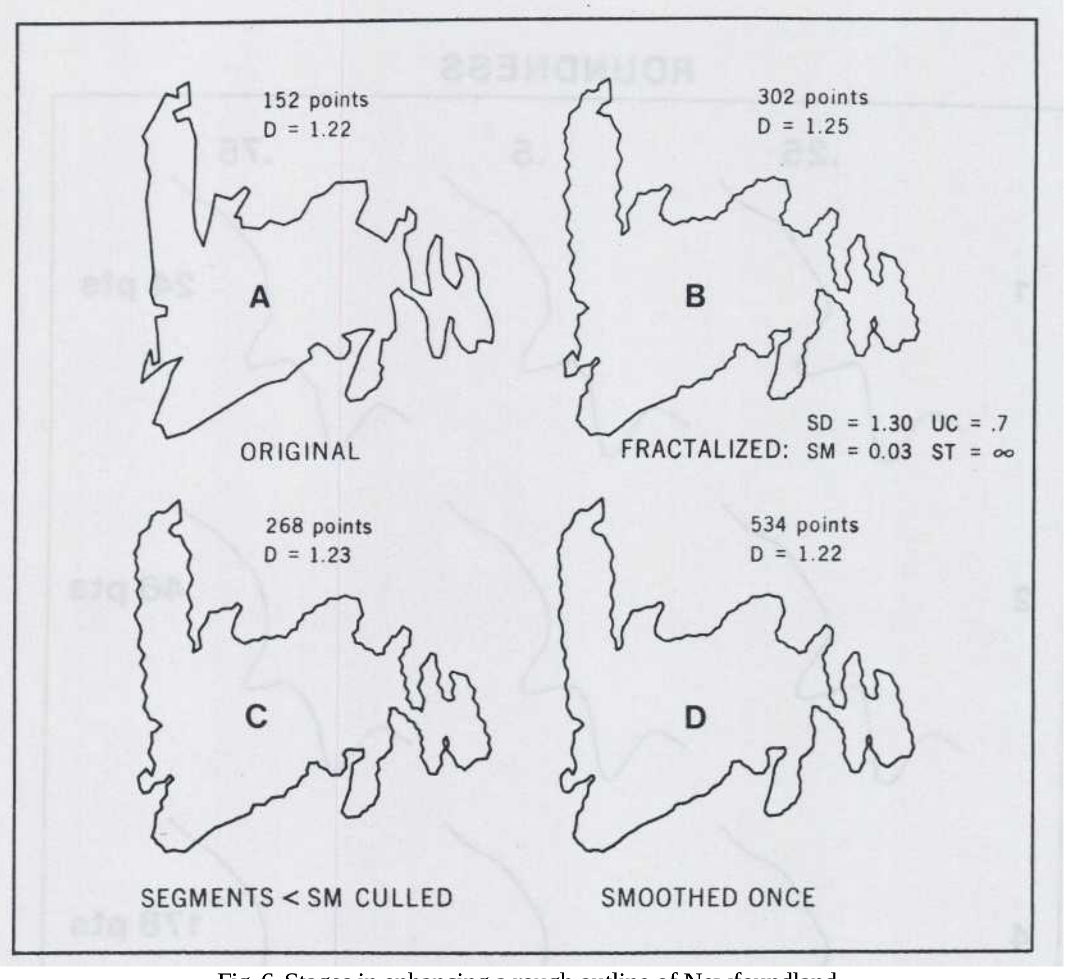

Fig. 6. Stages in enhancing a rough outline of Newfoundland.

.75. SM and ST were set so as not to affect the results. Chaikin's algorithm was developed to rasterize smooth versions of digitized lines; the author's *version similarly can produce a series of* adjacent raster coordinates if SM is set to a suitably small value and the procedure is called recursively.

Illustrated Examples

To demonstrate fractalization in a familiar context, a file of the contiguous U. S. state boundaries and *its* enhancements are *presented in Figs. 8* through 18. These figures are reproduced at four graphic scales, roughly varying from 1:30,000,000 to 1:3,000,000, with views zooming from the entire file to its north *west corner,* the Puget Sound region.

Source data for these figures consist of state boundaries extracted from the County DIME File, distributed by the U.S. Bureau of the Census. This extract contains 11,541 points, but its resolution varies due to inconsistencies in the source maps originally used in digitizing. To standardize its resolution and to generate a suitably sparse representation with which to test enhancement, the coordinates were converted to miles on an *Alber's* equal-area projection, and these converted coordinates were then filtered, using the Douglas-Peucker Bandwidth-Tolerance Algorithm (Douglas and Peucker, 1973), at a bandwidth of 2 mi. This eliminated over 90 percent of the

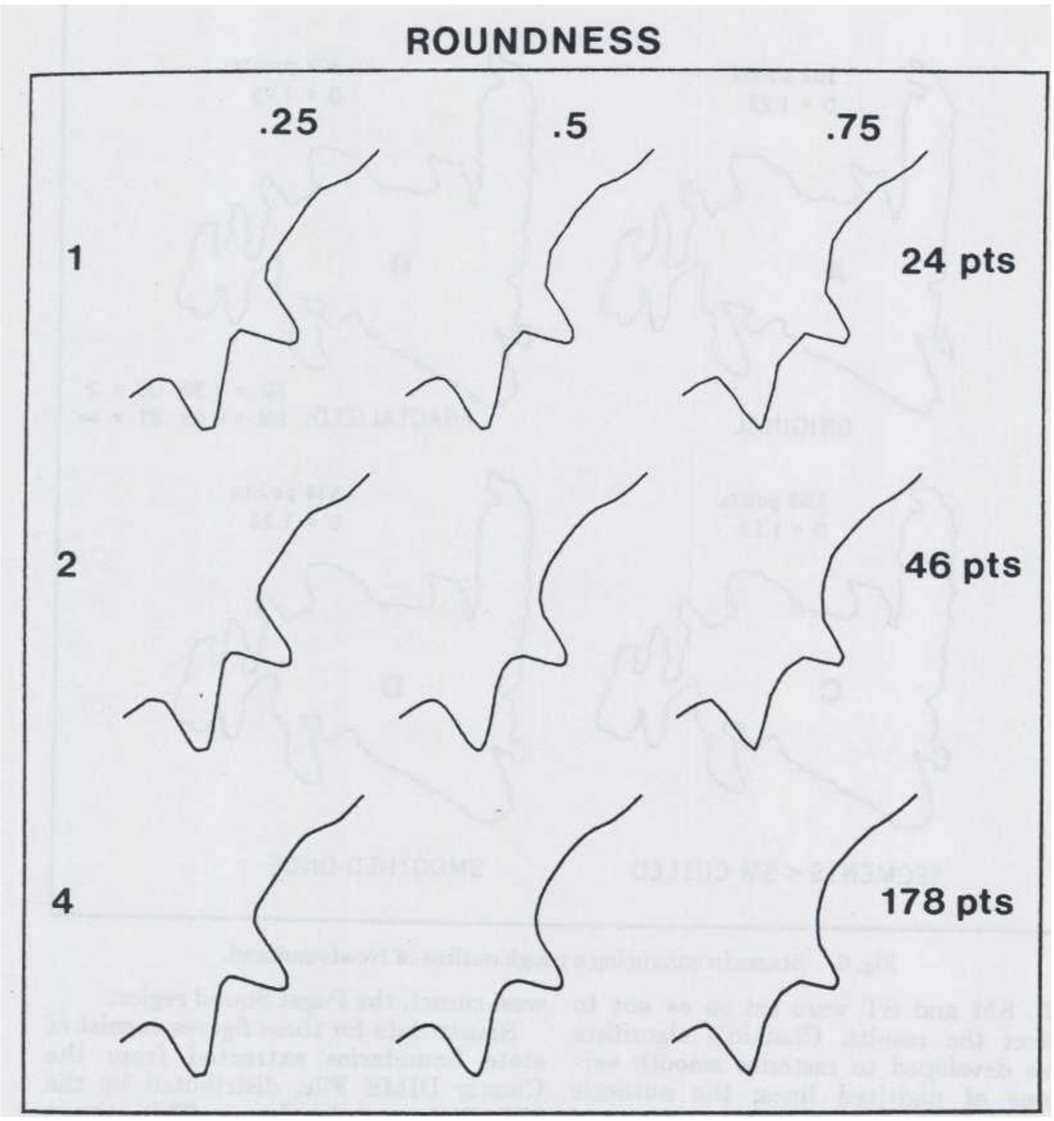

Fig. 7. Smoothing to different curvatures: the effect of varying *Roundness for 1, 2, and 4* smoothings.

original detail, resulting in a U.S. state boundary file having 1,055 points. As drastic as this reduction is, when the filtered file (Fig. 9) is compared to the original (Fig. 8), only a few boundaries appear noticeably degraded.

Under magnification, however, the effects of filtering are quite apparent, as Figs. 12, 14, and 16 reveal, especially in comparison with Fig. 18, a 10x magnification of the unfiltered boundaries. Islands disappear, peninsulas become triangles, and sinuous channels and coasts simply straighten. It is these faint suggestions of shape that fractal enhancement then attempts to rejuvenate in the remaining figures.

The enhancements are displayed in Figs. 10, 11, 13, 15, and 17, with increasing scale. Figs. 10 and 13 display one iteration of fractalization; Figs. 11 and 15, two iterations; and Fig. 17, three itera

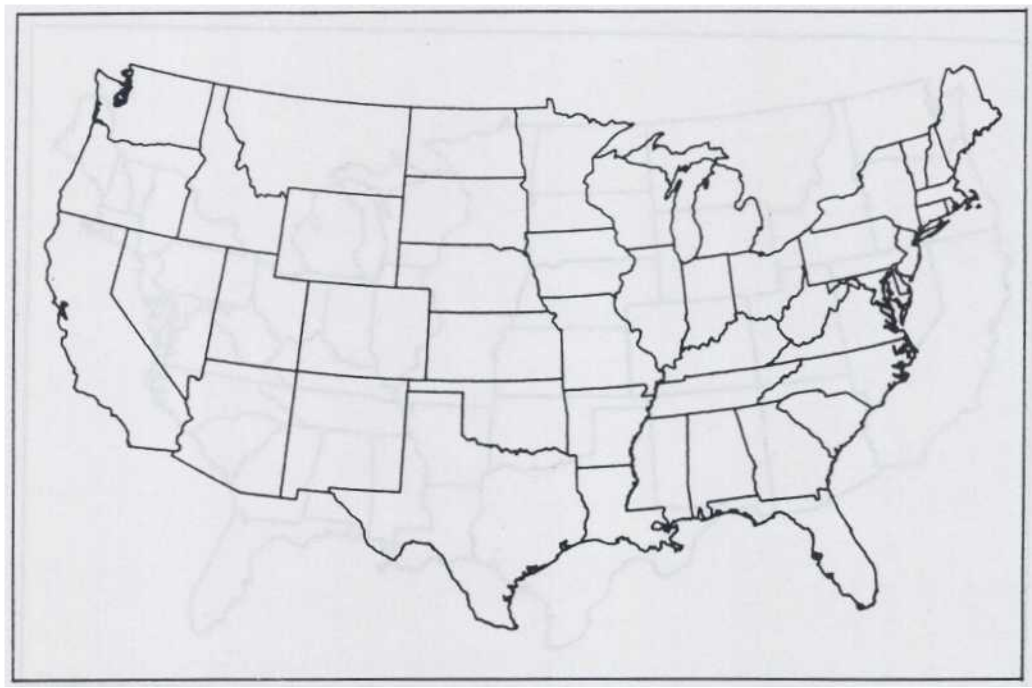

Fig. 8. State boundaries extracted from U.S. County DIME File: full detail of 11,541 points.

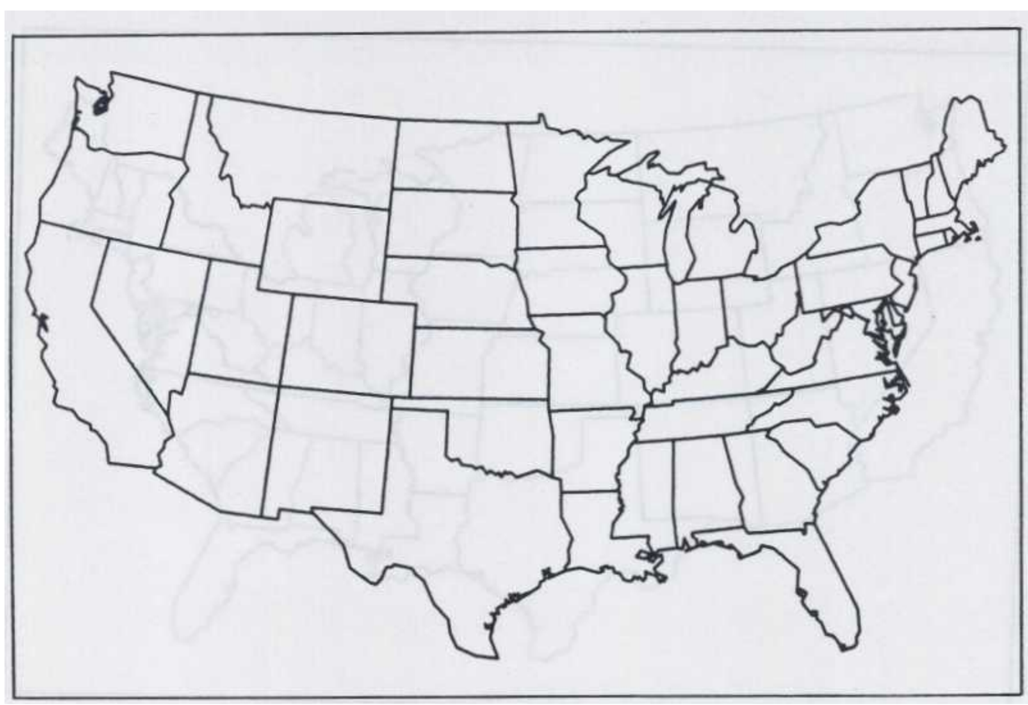

Fig. 9. Filtered state boundaries: 1,055 points.

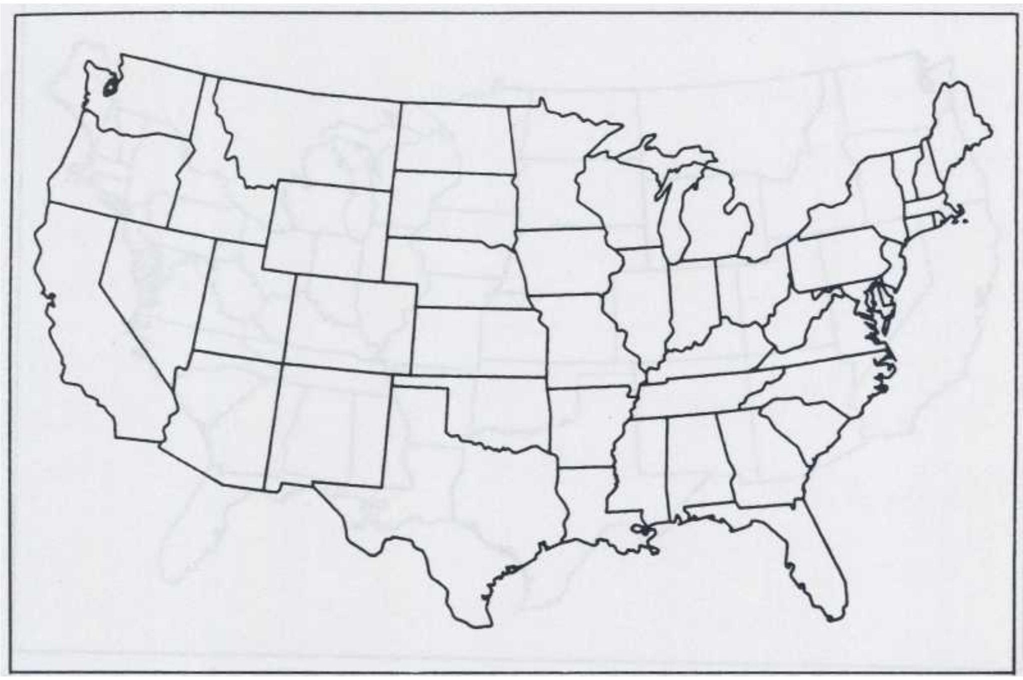

Fig. 10. One fractalization and smoothing of state boundaries: 3,193 points.

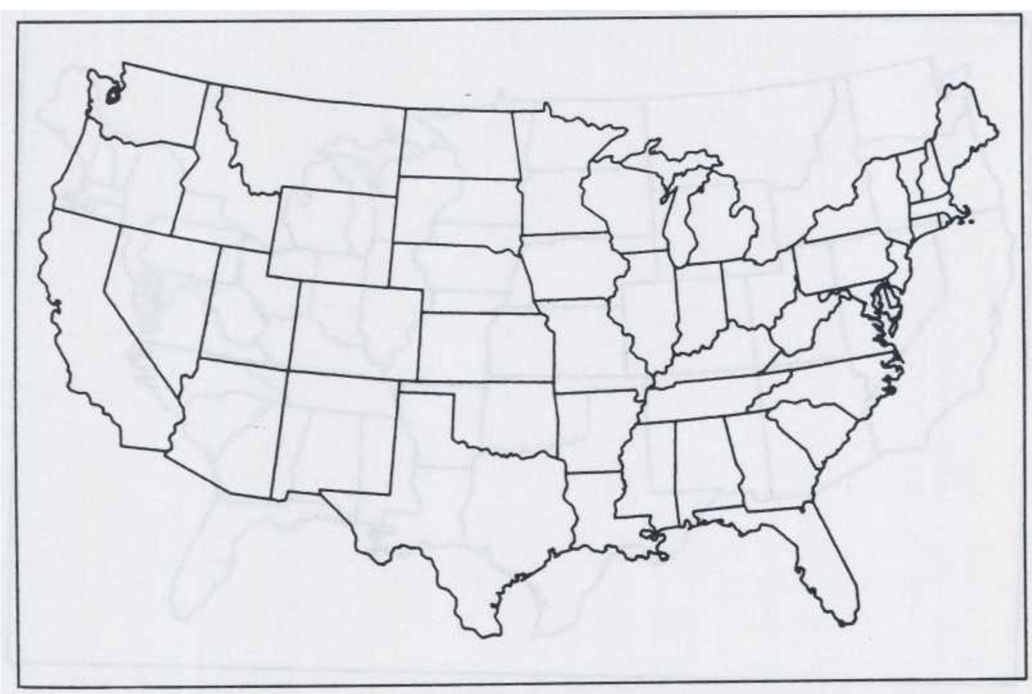

Fig. 11. Two fractalizations and one smoothing of state boundaries: 6,042 points.

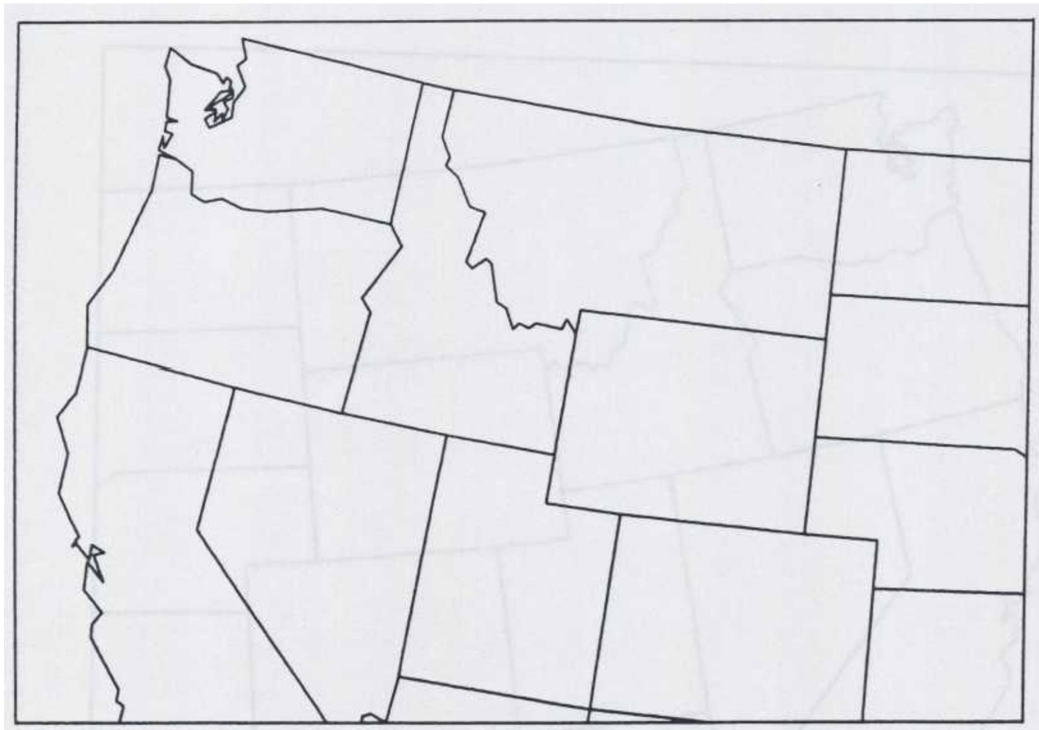

Fig. 12. 2x zoom of Fig. 9 (filtered boundaries).

tions. All are then given one iteration of smoothing. The parameters employed were held constant: SD = 1.5; UC = 0.5; SM = 0.1 mi.; ST = 50 mi.; Roundness = 0. 5. The value of 1.5 for SD (imposed fractal dimensionality) is probably excessive, even though UC at 0.5 tempers its effects considerably. This tends to exag-*gerate* the gross features still preserved in the filtered file but clearly demonstrates the results of enhancement. Fig. 18, which contains the full DIME File detail, provides the standard against which enhancements may be evaluated.

Precedents, Precautions, Related Applications

The notion of reducing the number of coordinates in a chain is generally understood to be useful and appropriate in storing and plotting digital maps. Very few cartographers would argue that all coordinates acquired through digitization are necessarily accurate or should be used in plotting. Fine detail often becomes redundant or distracting, given certain map scales, purposes, and plotting resolutions, and may be eliminated by a variety of line culling techniques. But when mapmakers or map readers are presented with algorithms that add or displace coordinates, eyebrows rise. Somehow the notion of creating detail seems arbitrary, inappropriate, or untruthful. In *careless* hands, it may be argued, such algorithms can yield maps conveying a false sense of reality.

But any map is an abstraction in which phenomena are selected, generalized, stylized, and emphasized by the mapmaker. A map is not even an abstraction from reality, it is an abstraction of ideas about reality (Robinson and Sale, 1969; Morrison, 1975). Both qualitative and quantitative rules of thumb are employed by cartographers in choosing which features to include, where to place them in relation to one another, what symbols to employ, and how large each should be. While their choices may be highly informed, and a consistent set of criteria can be inferred from a well-conceived

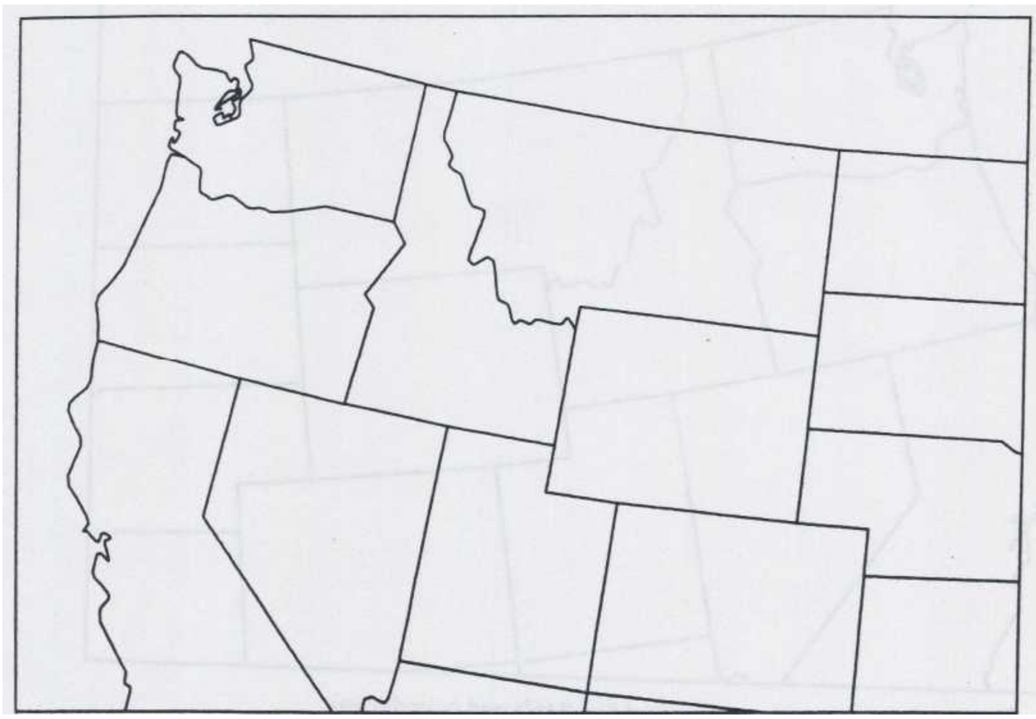

Fig. 13. 2x zoom of Fig. 10 (first enhancement).

map, rarely can a mapmaker or a map reader formalize the criteria into rigorous sets of rules, i.e., algorithms.

Sometimes cartographers must guess intelligently where to place *features on a* map, for example, when interpolating isolines. Elevation control points may be too sparse to define features known to exist by the cartographer, such as ridge lines and course lines. Mapmakers nevertheless bend contours "uphill" when crossing courses and "downhill" when crossing ridges. Similarly, and even when control points are. fairly dense, the "best" track for a contour line crossing an area of very low relief can become quite conjectural. Small errors in spot elevations can *translate* into quite visible horizontal displacements of contours, making their interpolation open to question.

Not only derivative map elements like contour lines but visible features as well must sometimes be approximated. For example, in tracing a stream from an aerial photograph, one may encounter difficulty at locations where vegetation *obscures the stream* channel. In such cases the cartographer must rely on clues, such as changes in topography and vegetation type, to *estimate* best the course line. Although the technique for line *generalization* presented in this paper does not attempt to infer shapes from contextual data, it should be evident that synthesis of detail is not *foreign to cartography*. It is both held to be legitimate and practiced widely, but it should be done with restraint and only when appropriate.

The ability to manipulate the fractal dimensionality of cartographic objects is perhaps more useful for thematic mapping than for *other* cartographic applications. Certainly maps used for navigation or for displaying boundary surveys must respect actual measurements as closely as possible or practicable. For such purposes there is no substitute for careful surveying, drafting, and digitizing *of features,* and the algorithmic addition and deletion of coordinate data is done at some peril. But thematic maps, in which boundary

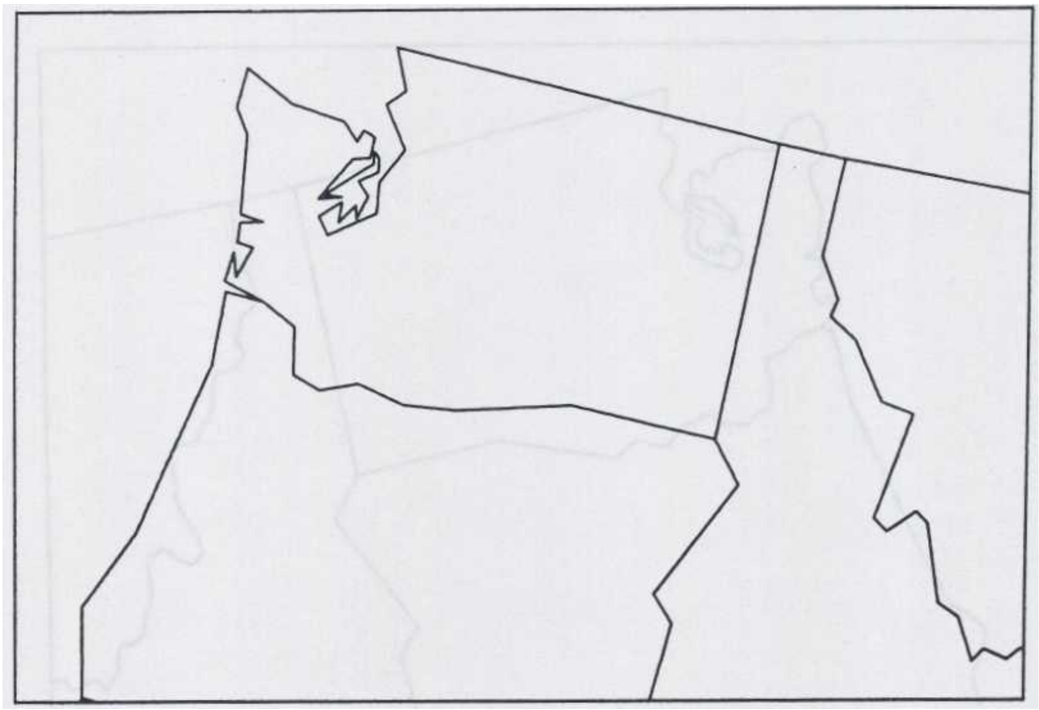

Fig. 14. 4x zoom of Fig. 9 (filtered boundaries).

data are principally a matrix for communicating other variables, can be enhanced without great risk of conveying false information. In such maps it is not the location of boundaries that is of prime concern but their appearance.

Although the author is not aware of attempts elsewhere to enhance cartographic lines using fractal methods, there have been some uses reported which synthesize and enhance digital terrain models (Carpenter, 1981; Fournier and Fussell, 1981). Both of these approaches interpolate cells in elevation grids using stochastic neighborhood operators. Such synthesis of surfaces will no doubt be eagerly exploited in flight and other environmental simulation applications; not only can they drastically reduce the bulk of a terrain data base required for generating realistic images, but such enhancement algorithms can be easily built into hardware, aiding real-time response. Fractal surface enhancement may also prove useful in generating specific textures for analytical hill shading.

Concluding Comments

There is no one "correct" or "best" method of fractal enhancement. As stated above, the approach presented here is but one of about half a dozen trial algorithms, and others have been outlined. Although its mechanics are inherently rigid, its parameters provide a good measure of control over its results. In any case, its sensitivity to local conditions could be considerably improved upon. That is, rather than imposing an often inappropriate fractal dimensionality at each point along each chain in an entire file, local dimensionality should be allowed to retain its variability.

To achieve this, SD must become a variable, although useful lower and upper limits might be imposed. Likewise, UC can also be locally determined. Local dimensionality, however, has no specific value; it will vary according to the extent of the locality used to calculate it and is dependent on the consistency, resolution, and conditioning of coordinate data. But

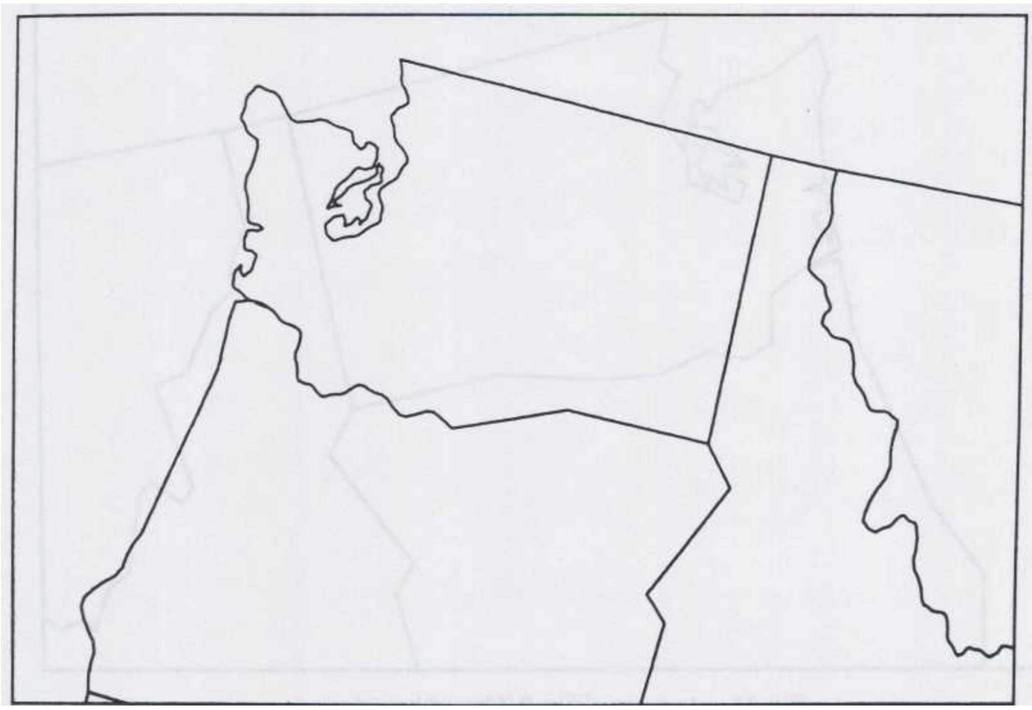

Fig. 15. 4x zoom of Fig. 11 (second enhancement).

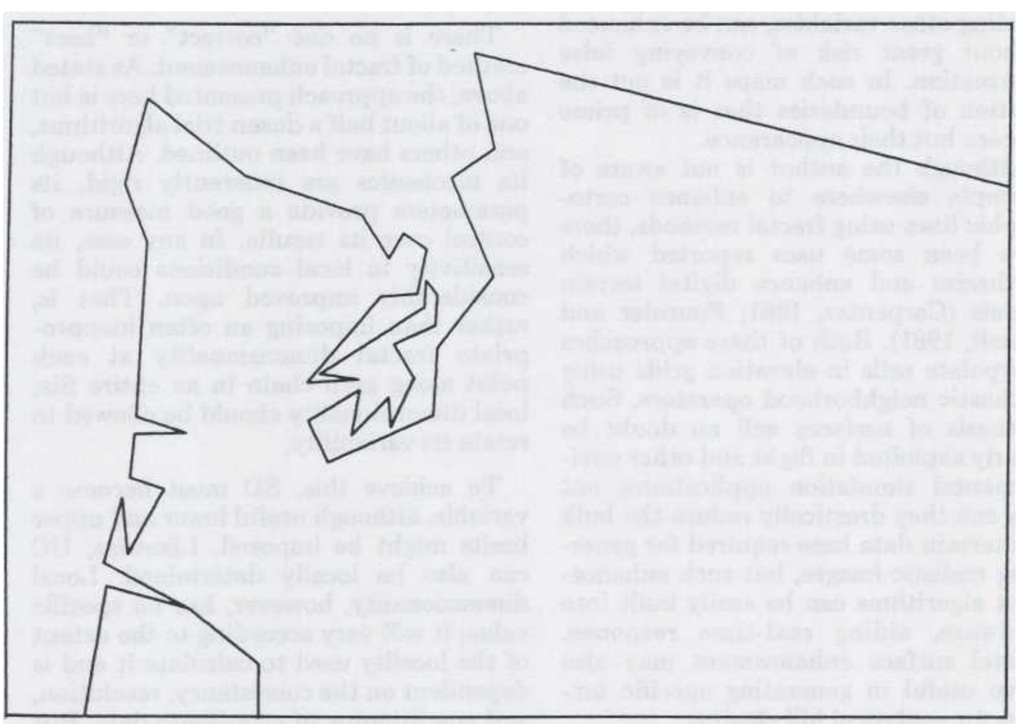

Fig. 16. IOx zoom of Fig. 9 (filtered boundaries).

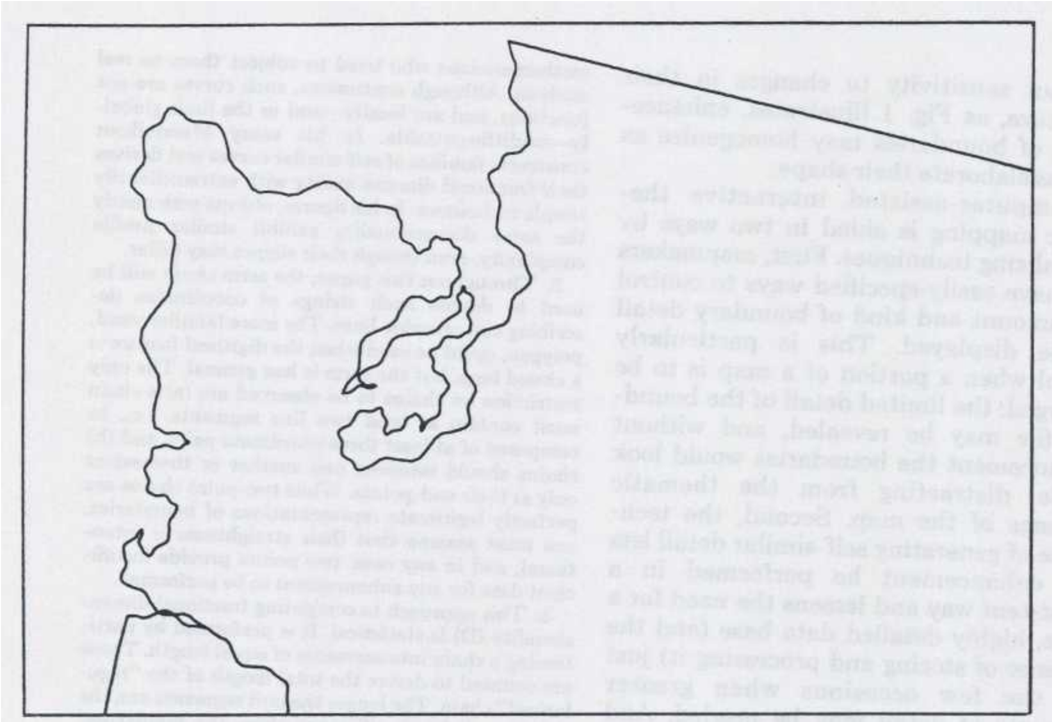

Fig. 17. lOx zoom of third enhancement (3 fractalizations, 1 smoothing: c. 9,000 points).

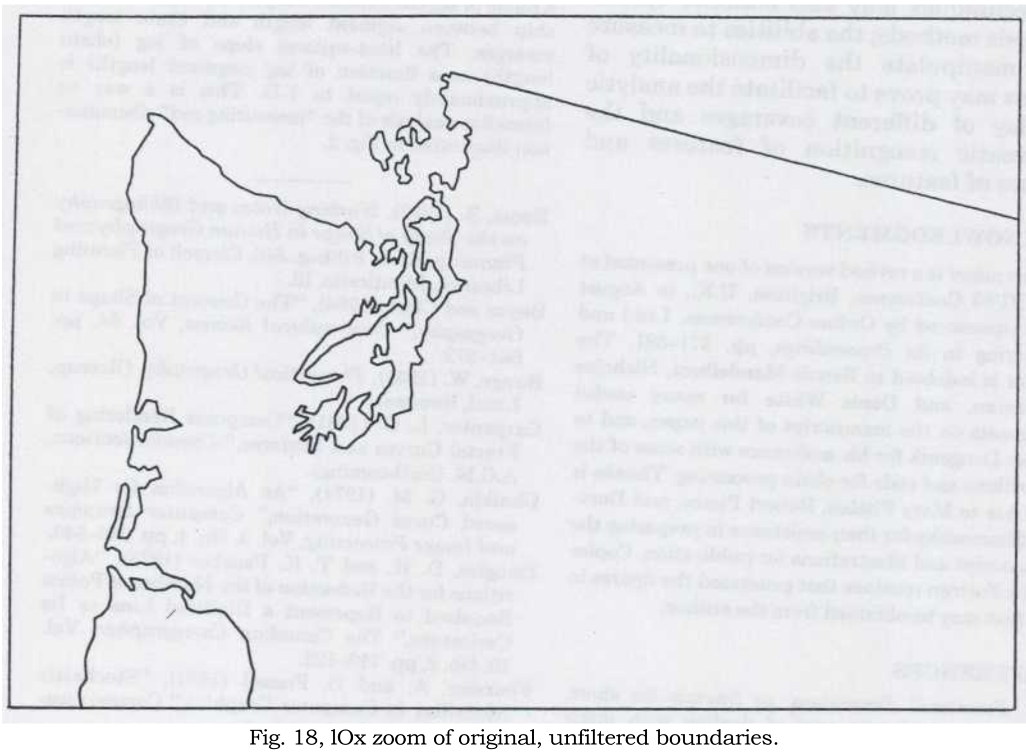

Fig. 18, lOx zoom of original, unfiltered boundaries.

without sensitivity to changes in their structure, as Fig. 1 illustrates, enhancement of boundaries may homogenize as well as elaborate their shape.

Computer-assisted interactive thematic mapping is aided in two ways by fractalizing techniques. First, mapmakers can have easily-specified ways to control the amount and kind of boundary detail to be displayed. This is particularly useful when a portion of a map is to be enlarged; the limited detail of the boundary file may be revealed, and without enhancement the boundaries would look crude, distracting from the thematic message of the map. Second, the technique of generating self-similar detail lets the enhancement be performed in a consistent way and *lessens the* need for a large, highly detailed data base (and the *expense of storing* and processing it) just for the few occasions when greater amounts of detail may be needed. And going beyond cartographic display, fractal techniques may also enhance spatial analysis methods; the abilities to measure and manipulate the dimensionality of chains may prove to facilitate the analytic overlay of different coverages and the automatic recognition of features and classes of features.

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1. Fractional dimensions, or *fractals* for short, represent an elegant way of dealing with many geometric "monstrosities," such as Peano curves, Koch constructions, and Cantor sets. Such mathematical constructions have frustrated generations of mathematicians who tried to subject them to real analysis. Although continuous, such curves are not functions, and are locally-and in the limit globally-undifferentiable. In his essay Mandelbrot constructs families of self-similar curves and derives their fractional dimensionality with extraordinarily simple techniques. In his figures, objects with nearly the same dimensionality exhibit similar profile complexity, even though their shapes may differ.

2. Throughout this paper, the term chain will be used to denote such strings of coordinates describing cartographic lines. The more familiar word, polygon, could he used when the digitized feature is a closed loop, but the term is less general. The only restriction on chains to be observed are (a) a chain must contain at least two line segments, i.e., be composed of at least three coordinate pairs, and (b) chains should intersect one another or themselves only at their end-points. While two-point chains are perfectly legitimate representations of boundaries, one must assume that their straightness is intentional, and in any case, two points provide insufficient data for any enhancement to be performed.

3. This approach to computing fractional dimensionality (D) is statistical. It is performed by partitioning a chain into segments of equal length. These are counted to derive the total length of the "regularized" chain. The longer the unit segments are, the shorter will be the distance along the regularized chain. By repeating this procedure stepping through a range of segmentations, a regular, inverse relationship between segment length and chain length emerges. The least-squares slope of log (chain length) as a function of log (segment length) is approximately equal to €-D. This is a way to formalize analysis of the "measuring rod" phenomenon illustrated in Fig. 2.

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