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Interactive map projections and distortion[☆]

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Abstract

We introduce several new methods for visualizing map projections and their associated distortions. These methods are embodied in the Interactive Map Projections system which allows users to view a representation of the Earth simultaneously as a sphere and as a projection with the ability to interact with both images. The relationship between the globe and the projection is enhanced by the use of explicit visualization of the intermediate developable geometric shapes used in the projection distortion. The central idea is one or more floating rings on the globe that can be interactively positioned and scaled. As the rings are manipulated on the globe, the corresponding projection of the rings are distorted using the same map projection parameters. This method is applied to study areal and angular distortion and is particularly useful when analyzing large geographical extents (such as in global climate studies) where distortions are significant, as well as visualizations for which information is geo-referenced and perhaps scaled to the underlying map. The floating ring tool is further enhanced to study 3D data sets placed over or under map projections. Examples include atmospheric and oceanographic data, respectively. Here, the ring is extended into a cone with apex at the center of the sphere and emanating beyond the surface into the atmosphere. It serves as a reminder that distortion exists in maps and data overlayed over maps, and provides information about the degree, location, and type of distortion. © 2001 Elsevier Science Ltd. All rights reserved.

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1. Introduction

There is an urgent need to educate the public as well as to remind practitioners of the idiosyncrasies of map projections. The problem of map distortion is particularly acute when information (e.g. demographic) is overlayed on them. This problem is further exacerbated when 3D models are displayed over these maps. As access to information overlayed over maps become more prevalent and available to the general public through the web and other media, it is increasingly important to remind users of the areal and angular distortions (as well as distance distortion which gives rise to these two) in different map projections. Toward this end, we present a two-pronged approach involving (a) an interactive cartographic educational software with an easy to use interface that allows users to experiment with different types of map projections and to observe the distortions associated with each one, and (b) an interactive tool in the form of a floating ring for 2D overlayed information, and a floating cone for 3D overlayed information to highlight the nature and magnitude of distortions present in different map projections.

The union of computer technology with the earth sciences and cartography that has occurred in the past several decades, and especially since the late 1980s, has dramatically changed the ways in which humans interact

[★]Code and colour images available at http://www.cse.ucsc. edu/research/avis/map.html.

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with geographic information. Instead of limiting ourselves to the passive viewing of static images, we have been taking much more of an active role in the processes of map creation and cartographic visualization (Mac Eachren and Monmonier, 1992; MacEachren, 1994). Today's Geographic Information Systems (GIS) allow users to store, process and display geographic information with speed and flexibility, even on very modest desktop systems.

Cartography is much more widely accessible now than in what Robinson and Sale (1995) refer to as the "dark ages" before the computer revolution. The use, and misuse, of cartography by non-experts is a driving motivation for this work. The main contribution of this work is a new interactive and animated set of cartographic tools that provide users with methods of exploring map projections and distortion. The Interactive Map Projections (IMP) system (see Section 4) developed at the Advanced Visualization and Interactive Systems (AVIS) lab at the University of California, Santa Cruz allows a user to directly interact with an image of a globe and a map projection. Changes in the orientation of the globe, for instance, are reflected simultaneously in the image of the projection. Images of geometric objects are included which provide important visual cues to the user of the origin of the map projection. These shape images can also be manipulated by the user, and again the resulting change in the map projection occurs simultaneously. For example, a coneshaped image is presented with the Albers Conic projection. The user can change the shape of the cone so that it contacts the globe at different latitudes. The shape of the projection changes in reaction to the user's inputs, and provides a powerful visual connection between cylindrical, conic and planar projections (see Section 5.3 and Fig. 11).

The IMP system also provides a base for visualization tools to be built upon. In particular, an interactive tool for visualizing and analyzing map projection distortion is described (see Section 5), called the floating ring tool. As its name suggests, the tool is simply an image of a ring that "floats" on the globe, much like a contact lens on the surface of an eyeball. The ring can be moved and scaled directly via user input. The floating ring gives a fascinating range of distortion patterns depending on the map projection. Unlike other methods of representing distortion that produce static images, the user is able to interactively manipulate the floating ring to allow real-time viewing of shape deformations caused by the scaling or repositioning of the ring. The use of color enhances the visual cues of the ring shape deformations.

The floating ring tool is useful to people who use maps in a wide variety of applications and disciplines. Students can use the ring as an educational tool to gain insights into the subject of map projections. Map users can use the system as a guide in interpreting the visual information contained in maps. Finally, professional cartographers may find additional insights in an already well known projection, or a new projection.

An enhancement to the floating ring allows users to visualize map projection distortion for maps that contain data with a vertical component (see Section 5.5). Often scalar data that includes a vertical component is simply laid on top of an existing map. Usually there is no indication of the methods used in scaling and projecting this data. The purpose of this enhancement is to provide a visual reminder of the necessity of methods to prevent misleading or inaccurate geographic conversion of scalar height data.

We foresee the floating ring as an integral part of GIS to provide additional flexibility to its users. The system could be easily integrated into a GIS environment, either as a separate globe-and-projection pair, providing a general means for visualizing the distortion of a particular map projection, or as an integrated visual glyph for use directly on a data set, providing a more specific, and perhaps detailed, representation of the distortion in the particular area of interest.

This work initially grew out of our work in visualizing uncertainty in a number of applications (Pang et al., 1997). One of these applications is data assimilation (Djurcilov and Pang, 1997) and (Saxon et al., 1997) where model forecasts are correlated with readings from in-situ weather instruments such as meteorological stations, wind profilers, and radiosondes as well as remote sensing instruments such as satellite imagery, CODAR (ocean surface current) and NEXRAD (atmospheric reflectivity) moment data. The 3D weather models usually output their forecasts using a particular projection (e.g. Lambert Conformal). One of the key issues in fusing the forecasts with the actual data is the different coordinate systems in use and the need to account for the distortions in different map projections. See also Willmott et al. (1985) for work that addresses errors in small-scale climate maps caused by interpolating and contouring in Cartesian two-space. The work presented in this paper also seeks to answer some of the challenges presented in Beard et al. (1991) and reiterated in MacEachren and Kraak (1997) and van der Wel et al. (1994) on issues regarding visualization of data quality.

2. Background

2.1. Map projections and distortion

The problem of projecting a spherical object to a flat surface has captured the interest of cartographers, mathematicians and navigators for over 2500 yr (Snyder and Voxland, 1989). A sphere is not a developable surface, thus any system of conversion to a planar surface must include some "stretching" or "squashing" of some or all of the areas on the original surface. More precisely, distortion is based on the concept of scale at a point in a direction on a map. On a globe, scale can be considered unity everywhere and in every direction, except at the poles. The process of projection introduces changes in the scale at homologous (corresponding) points on the sphere and projection, referred to as distortion. The knowledge of the types, magnitudes and distribution of distortion on a projection is of paramount importance in understanding the relationships between information on a map and the "real world" objects being represented.

There are hundreds of projections in standard use, and often the most important consideration in map use is the type of projection to use for a particular purpose. For instance, the popular Mercator projection has been used heavily in navigation because true compass bearings are represented as straight lines on the map, a valuable characteristic. Other projections may be used to minimize the effects of distortion in a particular area of interest.

2.2. Classification of map projections

There are many classification systems for map projections (Tobler, 1962; Snyder, 1987; Dent, 1998). We briefly describe two that are relevant to this paper one based on the type of developable surface used in the projection, and the other based on the type of distortion introduced in the map projection.

Many map projections can be classified by the shape of the intermediate developable surface used for projection. Fig. 1 shows spheres inscribed in three such shapes. A cylindrical projection is created by projecting every point on the sphere onto the surface of the cylinder. The cylinder is then "cut" along the length of its body and laid flat to produce the map. Likewise for a cone and a plane, producing conic and planar projections, respectively. The point or loci of points of the developable shape tangent to the sphere define a region of constant scale. Note that not every projection can be associated



Fig. 1. Three different types of developable surfaces used in map projections: (a) cylindrical, (b) conic, and (c) planar.

with a geometric object. In fact there is an important class of projections that are constructed mathematically and have a shape similar to an oval. Some of these are called pseudocylindrical because of some similarities to the class of cylindrical projections (Robinson, 1988). Several pseudocylindrical projections have also been shown to be polycylindric. Many other projections are grouped as miscellaneous projections, and include interrupted projections. There is also the Van der Grinten projection, which is circular but not planar (Robinson, 1988). Yet other kinds of projections such as retroazimuthals, two point azimuthals, gnomonic, etc. exist but they are not of general interest. The user interface of the Interactive Map Projections system described in Section 4 uses this classification to aid users in selecting a map projection.

Another popular system for classifying map projections is by the type(s) of distortion present on the projection. Angular distortion occurs if the *ratio* of the scale factors in the principal directions at a point is not unity. According to Tissot, the principal directions are a pair of perpendicular directions on the sphere that are retained as perpendicular on the projection. At least one pair is guaranteed to exist at each point, and the maximum deviation of the scale factors occur in the principal directions. Therefore, the measurement of the scale factors for the purposes of measuring distortion is always along the principal directions.

A conformal or orthomorphic projection is one that preserves the ratio of scale factors in the principal directions at every point, and therefore contains no angular distortion. It must be stressed that this quality only applies to points and not areas of any extent. A conformal projection tends to preserve reasonable shape across small geographic extents but deforms the area. The Mercator projection, in Fig. 7, provides a clear example of the effects of the distortion of a conformal projection, especially in the higher latitudes. Notice that Greenland (2,175,600 km²) appears larger than the South American continent (18,291,928 km²), which is very misleading if comparing respective areas. The Mercator projection, incidentally, was never meant to be used as a general-purpose world map, even though it is often misused as such.

Area distortion occurs if the *product* of the scale factors in the principal directions at a point is not unity. An equal-area or equivalent projection preserves unity of the products of the scale factors at every point along the principal directions on a map, except at the poles. Equivalent maps therefore represent geographic areas in relative proportion, i.e. two objects of identical size on the sphere will retain that property on the projection. The tradeoff is that angular relationships will change because the ratios of perpendicular scale factors are not preserved. Thus equal-area maps tend to distort the shapes of land masses, but retain areal relationships. Conformality and equivalence require contradictory scale relationships so that no map projection can retain both characteristics, and in fact most retain neither.

Equidistant projections maintain correct distances either between all points along one direction or from one or two points in all directions. Correct distance between points implies uniform scaling along the direction between the points.

Finally, azimuthal projections show some "true directions" as straight lines. True direction in the cartographic sense is the path along any great circle and is the shortest distance between two points on a sphere. In general, true direction is not equal to true compass bearing. A line of constant bearing (rhumb line) will coincide with a great circle if you are traveling north or south along a meridian, or east or west along the equator. Usually azimuthal maps represent true directions as straight lines only from one or two points or within a limited area.

3. Related work

3.1. Tissot's indicatrix

Formal methods for graphically representing distortions in map projections date back at least 100 yr. The most notable is the use of Tissot's indicatrix. The indicatrix is simply an ellipse whose shape and size represent the scale factors in the principal directions at a point on the sphere. Fig. 2 is a representation of the basic idea of the indicatrix. The circle centered at O of radius OA = OB represents an infinitesimal circle on the surface of the sphere. The radius is considered unity. The values a and b represent the scale factors of point O in the directions of OA and OB, respectively. It is important to keep in mind that the lengths shown in the diagram are only representative of scale factors at a point and do not correspond to any actual distance. Once projected, the circle will be deformed. If the projection is conformal, the shape will remain circular since the scale factors along the principal directions must be equal. The area, however, is not constrained and therefore will vary from point to point. An equal-area projection, on the other hand, must retain relative areas, so the product of a and b must be equal to unity, but a = b does not hold, so the shape of the indicatrix becomes elliptical and angular distortion is introduced. The point M on the circle and the corresponding point M' on the ellipse are points subject to the maximum angular deflection. The amount of angular distortion in a quadrant ω , is given by the relation $\omega = U - U'$ where $U = \angle MOA$ and $U' = \angle M'OA$. The maximum angular distortion at a point is given as 2ω , which represents the maximum angle deflection in two quadrants adjoining the major axis of the ellipse.



Fig. 2. Tissot's indicatrix: Smaller circle represents infinitesimal small circle on globe of unit radius. Ellipse represents same circle on projected image. Values a = OA' and b = OB' represent scale factors used to define angular and areal deformation at point in projection.

Conceived in 1881, Tissot's indicatrix is still considered the standard method for representing map projection distortion. It is simple, elegant, easy to comprehend and very effective in conveying the distortion characteristics of a projection. It is, however, limited in its usage. Since each ellipse is representative of the scale factors at an infinitesimal point, distortion over areal extents of a projection is generally achieved by placing ellipses on convenient intersections of the graticule, such as every 10 or 15 degrees latitude and longitude, as in Fig. 3. The distortion scale therefore cannot be known at arbitrary points on the projection, and the presence of many such ellipses on a static projection image may obscure geographic and overlaid data.

3.2. Other distortion visualization methods

A frequently used method of representing distortion on map projections is the use of isolines to show regions of equal angular or areal distortion. This method is useful because it gives the user an idea of the overall pattern of distortion on the projection, but the lines can interfere with the geographic information being represented, and is in practice limited to show discrete values.

Cartographers may use what Robinson and Sale (1995) refer to as Visual Analysis to determine overall distortion characteristics of an unfamiliar projection by examining graticule patterns. For instance, the intersection of parallels and meridians will always be at right angles in a conformal map, so angles other than 90 indicate a non-conformal map. This method is useful only for those with considerable knowledge of map projections, and may not make apparent some of the subtleties of the projection.



Fig. 3. Distortion diagram based on Tissot's indicatrix. Mercator projection graticule shown with ellipses plotted on line intersections. Notice each ellipse is circle, indicating conformality. Area of each ellipse is proportional to areal distortion at that point, and increases toward poles (top and bottom rows).

Another popular and useful device is the general method of "familiar shapes". As the name implies, the distortion characteristics of a projection can be demonstrated by the deformation of a projected image of a familiar shape. Examples of this method employ single shapes, such as a human's head (Robinson, 1988), and (Robinson and Sale, 1995), or a circle and compass rose (Mulcahy and Clarke, 1995), or multiple shapes, such as a network of equilateral triangles covering the extent of the globe (Fisher and Miller, 1944; Hsu, 1981; Robinson, 1988). In general, these methods obscure underlying geographic information and are limited by the static nature of the projected image. Of course, the deformation of familiar coastlines and other geospatial characteristics can often provide sufficient visual cues to allow a user to gain an overall sense of the distortion of a projection.

Clarke and Mulcahy (1995) introduced the concept of color differentiation, mapping three color scales to the x range, y range and angular convergence at a point on the projection. The distortion pattern of the entire projection is visualized without obscuring underlying uncolored geographic data. Projecting a checker-board pattern or grid square pattern has been used by Steinwand et al. (1995) to study the effects of distortion introduced by re-projecting raster data sets.

Each of these methods is effective in different circumstances. The primary benefits of the floating ring include its flexibility through interactivity and ability to convey distortion information effectively across varying areal extents. Color information is used to enhance the user's ability to gauge the rate of change of distortion over the extent of the ring and the magnitude of distortion at a point. The shape of the ring is circular, a shape that has many beneficial qualities for use in this capacity, notably that small deformations of the shape are easily recognizable as such, and do not depend on the user's geographic or cartographic knowledge. The ring is transformable through user interaction, which overcomes the shortcomings of static images. The use of the ring does not obscure the geographic data, and can be easily hidden. Finally, the floating ring tool is implemented in a system that provides interactive visualization of map projections, so that the effects of different projections on the ring and the underlying data can be easily compared.

3.3. Animated and interactive cartography

The use of animation in cartography is not a new phenomenon. Animated maps were proposed in Thrower (1959, 1961) and were applied to the analysis of traffic accidents (Moellering, 1973), and population growth in urban areas (Tobler, 1970; Rase, 1974). Later the computer replaced film as the medium for animated cartography.

Due to the computational demands of cartography, much of the early computerized animated cartography were limited to fixed-sequence animation of pre-computed images (Robinson and Sale, 1995). This method restricts user interaction to passive viewing. Even so, some very impressive and effective fixed-sequence cartographic animation has been produced, notably the weather satellite images from the Geostationary Operational Environmental Satellite (GOES) from the National Oceanic and Atmospheric Administration (NOAA).

Most cartographic animation focuses on temporal changes in map data. In one common method, values of an independent variable are plotted through time onto a static map image. This method can be used to visualize both natural and human processes such as population growth, income distribution, ozone depletion, and global warming. Another method shows environmental changes in the map itself. For instance, the path of the continental plates can be visualized, or the effects of soil erosion.

Non-temporal animation also exists. Often this is a fixed-sequence series of images that plot data values as some other variable (besides time) is varied. For example, percent of population for various age groups was animated for the Omaha, Nebraska region (Peterson, 1993). Non-temporal cartographic animation is also used to allow users to more effectively view a static map. For example, by varying the user's viewpoint, a "fly-through" effect can be achieved (Moellering, 1980a, b). Zooming capabilities with the appropriate level of generalization at each resolution gives a user geographical detail and context (Robinson and Sale, 1995). In Vanoosterom and Schenkelaars (1995) a method for interactively exploring data sets is described.

As computers have become increasingly powerful, methods of cartographic animation have allowed greater degrees of user interaction. For animations that display pre-computed images, the user is given more control over the length and content of the animation, including methods for scripting the animations (Monmonier, 1990). For animations that include a fly-through, the user may have control over the path. Other examples exist, but the potential for new interactive cartographic methods is largely untapped. The Interactive Map Projections system described in this work greatly extends the level of interactivity that is generally accessible in cartographic animation today. There is a commercial product, however, that is somewhat similar and deserves mention. Geocart (www.terradata.com/Mapthematics. html) is a popular commercial product that, like the Interactive Map Projections system, implements interactivity and animation of map projection parameters and distortion diagrams. Geocart allows generalized mapping with an impressive choice of map projections. Users can interactively display many visual tools including Tissot indicatrix diagrams, great circles, rhumb lines, and scale distortion parameters. One advantage of the Interactive Map Projections system over Geocart is the ability of the user to interact directly with elements on the projection, including the distortion diagram. Interaction in Geocart is indirect, usually accomplished by the user entering numbers into dialog boxes. Although many features are offered, the Geocart system is not designed for immediate or direct user interaction and manipulation.

The field of animated and interactive cartography has not been fully realized. Although the amount of work has been increasing rapidly in the last several years, contributors in this field have only scratched the surface. The reader is referred to several recent works that provide more detailed summaries of the work done in animated and interactive cartography, including Dorling (1992), DiBiase et al. (1992), Asche and Herrmann (1994), Peterson (1995), and Robinson and Sale (1995). A good description of existing software products implementing animated and interactive cartography can be found in Cartwright (1994), Slocum (1994), and Peterson (1995).

4. Interactive Map Projections

4.1. Overview of system

The Interactive Map Projections system was originally designed as an educational tool, and later converted to a front-end for the visualization component (SLVG) of the Real-Time Environmental Information Network and Analysis System (REINAS) at the University of California, Santa Cruz. REINAS is a system that

supports real-time data acquisition, data management, and data visualization of regional scale environmental science (Saxon et al., 1997). One of the recent uses of the Interactive Map Projection system is to support the floating ring tool [see Section 5 and Brainerd and Pang (1998)]. The system is written in C++, and each object on the screen is represented by a class. This provides a modularity that allows objects to be easily added to or removed from the system. Users can choose the objects to be displayed, allowing for customization. For instance, the user can choose to hide the coastline data, allowing an unobstructed view of the graticule. This might be beneficial in studying graticule patterns in different projections. The modular design also allows the Interactive Map Projections program to be quickly modified based on the user's desired purpose. The educational version of the Interactive Map Projections system is available for download via ftp from http:// www.cse.ucsc.edu/research/avis/map.html. Whereas the current software release has only been tested on SGI platforms, the code can be easily ported to other UNIX/ Linux platforms. In addition, efforts are underway to port both the user interface and application to a Windows environment.

4.2. Interactive Map Projections system

The Interactive Map Projections system performs interactive visualization of map projections, (see Fig. 4). The user is initially presented with an image of a globe consisting of the standard graticule and world coastline data. Through the use of mouse controls, the user is able to rotate the globe along the polar axis and an equatorial axis. The user can choose to view one of three developable geometric shapes in this window (Globe Window): a cylinder, cone, or plane. Each of these represents a class of map projections that can be viewed simultaneously in another window (Projection Window). Once a geometric shape has been chosen, a projection is shown in the Projection Window. The projection corresponds to the type and orientation of the selected geometric shape. The geometric shapes can be independently manipulated through rotation. Rotating the geometric shape or the globe, seen in the Globe Window, changes the projection in the Projection Window appropriately. Currently thirteen different standard projections are supported, and are listed in Table 1 along with the corresponding geometric and distortion characteristics. For the category of distortion characteristics, one of three labels is used: conformal, equal-area or compromise. Here, a compromise projection means one that is neither conformal nor equal-area, but exhibits both angular and areal distortion. Although compromise projections have both angular and areal distortion, the magnitude of the distortion tends to be less, in general, than either conformal or equal-area



Fig. 4. Interactive Map Projections system for interactively visualizing map projections. It is shown here with floating ring tool for interactive visualization of map projection distortion.

maps. Hence, compromise maps often look "correct", and are therefore often used as general purpose world maps.

In choosing which projections to include, an equal representation of distortion patterns and geometric classification in the projections was desired. Both old and contemporary projections were chosen, some that are in use today and others that are obsolete, but are included for historical and educational purposes. Routines from the General Cartographic Transformation Package (GCTP) from the United States Geologic Survey (1995) are used for the projections. This package is a general-purpose program for converting map projection coordinates between different projections. Although somewhat difficult to use, the package provides a fast and versatile means for converting coordinates from one projection to another.

An additional tool included in the system deserves mention. A bounding box tool is included that can be used to define regions of interest for geographic database queries. Like the floating ring tool, the bounding box floats on the surface of the globe and is

Table 1 Map projections used in Interactive Map Projections system

Projection	Geometric classification	Distortion
Mercator	Cylindrical	Conformal
Transverse Mercator	Cylindrical	Conformal
Miller	Cylindrical	Compromise
Lambert	Conic	Conformal
Polyconic	Conic	Compromise
Albers	Conic	Equal-area
Orthographic	Planar	Compromise
Stereographic	Planar	Conformal
Gnomonic	Planar	Compromise
Mollweide	Pseudocylindrical	Equal-area
Robinson	Pseudocylindrical	Compromise
Putnins P'2	Pseudocylindrical	Equal-area
Wagner VII	Modified Azimuthal	Equal-area

projected onto the map projection image. The bounding box is a "rectangular graticule" as it lines up on latitude and longitudinal lines. The bounding box's rectangular shape requires that it be deformed to match the curvature of the globe. Each corner of the box is moved separately through mouse picking, and motion is restricted so that every point on a side of the box remains along a meridian or a parallel (see Fig. 5).

5. Interactive distortion visualization

5.1. Floating ring tool

In this section a new type of distortion diagram is described that uses a visual tool called the floating ring tool. The floating ring tool is simple to comprehend and use, yet provides powerful visual cues of the types and amounts of distortion in a projection. The ring floats on top of the sphere, and can be directly manipulated by the user with simple mouse interaction. Fig. 6 shows a simplified schematic demonstrating the concept of the floating ring. Both the position and scale of the ring can be changed, and the results of those changes are shown in both the globe and the projection images in real time. The ability to see the changes made to both images immediately is one of the key strengths of this system.

The circular shape of the floating ring is integral to effectiveness of the tool for the following reasons. Every point on a circle is in contact with the sphere. A planar rectangular or triangular object does not have this property, unless the shape is deformed to match the curvature of the spherical surface using spherical geometry. Part of the effectiveness of the floating ring is that the circular shape remains constant and undeformed on the globe regardless of its position or scale. This gives the ring a simplicity and symmetry that greatly enhances the user's ability to comprehend the



Fig. 5. Bounding box tool for selecting regions of interest in geographic database query application.



Fig. 6. Schematic of floating ring on globe. Ring can be moved in any direction and scaled with simple mouse interaction. Ring is projected in real time.

deformation of the projected ring caused by the projection. The visual cues offered by the deformed ring provide a link to understanding the distortion characteristics of a projection, in both localized areas or over large regions of the map.

5.2. Map distortion and shape deformation

In this document, we will mostly refer to two projections for illustrative purposes, the standard Mercator and the American polyconic, shown in Fig. 7. The Mercator is a conformal, cylindrical projection used in navigation and conformal mapping of equatorial regions (Snyder and Voxland, 1989). It is almost universally classified as cylindrical, although technically it is not a direct geometric projection (Deetz



Fig. 7. Mercator (left) and Polyconic (right) projections. Central meridian is represented by yellow-green longitude line. Mercator projection preserves angles but not areas, whereas the polyconic projection preserves neither angles nor areas.



Fig. 8. Mercator projection and floating ring: (a) View of globe inscribed in cylinder, (b) resulting projection. Image of floating ring on globe is shown on projection. Area of projected ring is larger at poles than if placed near equator indicating larger degree of distortion near poles. Also note that center of projected ring appears lower.

and Adams, 1944). Angular relationships are preserved at every point but area is distorted as one moves toward the poles. The polyconic projection, shown in equatorial aspect, is a conic projection, but is neither conformal nor equal-area. Other polyconic projections exist, some of which are conformal and some of which are equal-area. It shows increasing angular and areal distortion from the central meridian (yellow–green line running north– south through Africa in Fig. 7). It has been historically used to map regions of primarily north–south extent and limited east–west extent (Raisz, 1948). These projections were chosen partly because they have very different distortion characteristics, and thus together they provide contrast in demonstrating the floating ring tool.

The floating ring is effective in representing both angular and areal distortion. Areal distortion in a projection is accompanied by an enlarging or shrinking of the projected ring shape. In the Mercator projection, this is best seen as an enlargement of the ring as it is moved into higher latitudes, as illustrated in Fig. 8. One subtle point that the floating ring makes evident in Fig. 8 is that conformal maps do not preserve shape over large areas. In contrast, a standard distortion diagram based on Tissot's indicatrix, as in Fig. 3, hides this point. In the polyconic case, the area of the ring changes with longitude, becoming enlarged as it is moved away from the central meridian. Also effective is a scaling of the stationary disk, which allows the user to see the effect of distortion over a large area of the projection.

Angular distortion is accompanied by a deformation of the ring in the polyconic projection away from the central meridian, as in Fig. 9. This graphic representation is a simplification of the polyconic projection. The construction of the polyconic projection includes an



Fig. 9. Polyconic projection and floating ring: (a) View of globe inscribed in cone, (b) resulting projection. Image of floating ring on globe is shown on projection with both increased angular and areal distortion.

infinite number of cones tangent at infinitesimally spaced latitudinal steps. In Fig. 9, the ring becomes unsymmetrically elongated and rotated with respect to its orthogonal axes based on its position and scale.

The point at the center of the ring is also displayed. This serves as a reference point, and provides an additional visual cue of the effects of the distortion on the ring. In Fig. 8, for example, the ring's center point on the projection sits low in the ring, reinforcing the visual perception of increased distortion toward the poles.

5.2.1. Multiple rings

The use of multiple rings in a single projection can greatly enhance the ability of the user to gain an understanding of the distortion characteristics of the projection through simple visual comparison. Comparing multiple rings in different positions or with different scales on the same projection provides many interesting scenarios in addition to the single ring configuration. In Fig. 10, three examples of multiple rings are used with three separate but similar oval projections. In each case, the two rings are of the exact same size, but one is left near the center of the projection, where there is relatively little distortion, and the other is moved to the periphery of the projection, and as such is deformed to a greater degree due to the larger distortion. In each case we have a visual comparison of the difference between the two rings that provides an excellent basis for describing the distortion characteristics. This figure also illustrates the use of another method of analysis using the floating ring tool and the underlying system by changing the



Fig. 10. Comparison of similar projections using multiple rings.

projection under the floating ring. This method is described in Section 5.3.

Adding or deleting rings from the display is accomplished with graphical user interface (GUI) controls (see Fig. 4). Managing multiple rings on the display adds very little complexity to the user interaction. Rings can be moved and scaled through mouse picking, so the user controls remain intuitive and uncluttered. In principle there is no limit on the number of rings that can be added, but in practice having two or three is sufficient.

5.3. Using animation and the Interactive Map Projections system

The floating ring tool is embedded in the Interactive Map Projections system and the operation of the floating ring is closely tied to the workings of the underlying system. Two methods that take advantage of this to increase the effectiveness of the floating ring tool are described in this section. Both methods use the technique of moving the underlying projection instead of moving the ring. In the first method, the user switches between different map projections and in the second, one map projection is animated while the ring floats on top.

5.3.1. Different map projections

Fig. 10 is an example that uses different map projections without directly changing the floating ring. The Interactive Map Projections system allows the user to switch between map projections without discernible lag time. Thus a comparison of the effects of distortion from different projections on the shape of the ring can easily be seen. This method can be used for projections that are very similar to one another. Often it is difficult for the non-expert to discern the differences between similar projections and this method offers an effective method of direct comparison.

5.3.2. Animating map projections

Fig. 11 shows a series of screen shots representing the use of animation of the underlying map projection as a method of analysis. In this series, a single conic map projection (Albers Equal-area) is used. To the left of each of the screen shots is an illustration representing the approximate configuration of the cone and globe that produce the projection on the right. The parameter that is manipulated in these shots is the latitude at which the cone is tangent to the globe. In (a), at the top of the figure, the cone is tangent at its most northern position, the north pole. In this position the cone is tangent at a single point. Notice that in this position, the cone is essentially flattened out into a plane. In other words the tip and the base of the cone have become co-planar. The resulting projection is round, as is typical of a planar projection. As the cone is "pulled" down, i.e. the latitude at which the cone is tangent to the globe decreases, it begins to assume the more traditional shape of a cone, as in (b). Here the projection also takes on a shape associated with a conic projection. As the cone is pulled down to the equator, it resembles a cylinder as

the tip of the cone becomes infinitely far away to accommodate the parallel slopes at different points on the base. The corresponding projection in (c) resembles a cylindrical projection, as we would expect. (d) and (e) mirror (b) and (a), respectively, in the southern hemisphere. As the underlying map projection changes according to the description above, the floating rings become deformed based on this underlying projection. In this series the rings are exactly the same size and are not moved. One is situated in the northern hemisphere and the other in the southern hemisphere. In the first couple of shots, one notices that the southern ring is very deformed while the northern one more closely retains its circular shape. Likewise in the final two shots, the southern ring appears less deformed than the northern one. This animation is very effective in reinforcing the concept that the line of tangency is the line of least distortion, and distortion will increase with distance from that line. Another, perhaps more important insight that can be drawn from this animation sequence is the relationship between three major classes of map projections that are based on the geometric shapes, cylindrical, conic and planar. The smooth animation as the cone resembles each of the different shapes, and the corresponding change in the projections is an extremely powerful visualization that links all three classes of projections.

5.4. Color information

In addition to shape deformation, color is used to represent magnitudes and changes in distortion, both at single points and across areal extents. In the standard mode just the outline of the ring is presented. This gives an unobstructed view of the underlying geographic information and the graticule on the projected image. The ring is colored proportional to the magnitude of distortion at each point and this is discussed in more detail below. The user can also choose to color the entire area within the ring. Coloring the entire area can be useful in determining the magnitude of distortion at various points within the ring, or as an additional aid in understanding how the distortion changes over area. The use of color is in some instances preferable to the use of isolines because a visual representation of the continual change of the magnitude of distortion provides more complete information than discrete lines representing equal magnitude. In addition, the coloring does not introduce additional lines that might be confused for graticule lines or other geographic information.

Two separate colormaps are used, one to represent angular distortion and the other areal distortion. The user simply selects a colormap from the GUI (Fig. 4). The color model we use is based on the scale factors at a point in the principal directions, represented on Tissot's



Fig. 11. Use of animation and Floating Ring Tool.

indicatrix (Fig. 2) as a = OA' and b = OB', respectively. These values are simply the semi-axes of the indicatrix. To determine areal distortion at a point, we multiply aand b. This is proportional to the area of the indicatrix at that point, πab . Since the map scale factors are unity at every point on the globe, the product ab of a point on the projection determines the magnitude of the areal distortion. In all situations the lengths of a and b are considered positive, so increases in areal distortion occur when ab > 1 (here the scale constant has been assumed) and decreases in areal distortion occur when ab < 1. A color value is then assigned based on this value. Two colors are specified in the colormap, one for values less than unity and one for values greater than unity, as shown in Fig. 12. In a similar way, we use the ratio of scale values, a/b, as the measure of angular distortion. This value can be thought of as the ellipsoidal eccentricity of Tissot's indicatrix. Angular distortion occurs when $a/b \neq 1$ If a > b, the ellipse is "squashed", and if a < b, the ellipse is "stretched" vertically. As in the situation for



Fig. 12. Two possible colormaps used to color floating ring: (a) $Magenta \rightarrow Cyan$ and (b) $Cyan \rightarrow Magenta$. Unity represents scale factors on globe. Difference in angular or areal distortion is expressed with respect to unity. Both colormaps can be used for either angular or areal distortions. Fig. 13 uses colormap in (a) and Fig. 14 uses colormap in (b).

areal distortion, a color value is assigned based on the relationship of the distortion magnitude to unity.

Fig. 12 is a graphic representation of two separate colormaps. Either colormap can be specified by the user to represent either angular or areal distortion. This gives the user flexibility in determining how the map image should look.

In both the Mercator and polyconic projections, none of the scale values fall below unity, so only the color above unity dominates. In the Mercator projection, there is no angular distortion, so the color of the ring is constant (when the angular distortion colormap is chosen). The polyconic projection, however, includes substantial angular distortion. Conceptually, Tissot's ellipses become stretched with longitudinal distance from the central meridian, implying $b \ge a$ or $a/b \le 1$. Again, one color dominates, but at the minimum end of the colormap, i.e. all values are ≤ 1 . Fig. 13 shows a filled ring on the Mercator graticule, colored as in Fig. 12(a). Fig. 14 shows a filled ring on the Mercator graticule, colored as in Fig. 12(b).

Having two separate colormaps representing different types of distortion is an excellent means of comparing distortion information. For instance, the ring in a conformal projection has constant angular distortion, and will be colored uniformly in the angular distortion colormap, but not with the areal distortion colormap. The polyconic projection, on the other hand, will be colored differently in both colormaps since it is neither conformal nor equal-area. The user can switch back and forth between the two colormaps to gain nearly instantaneous comparative information on the types and magnitudes of the distortion.

5.5. 3D floating ring

An enhancement to the original floating ring tool is implemented that allows users to visualize distortion in three dimensions. The motivation for this enhancement is that distortion along the vertical dimension is usually ignored and can be significant in some applications such as meteorology. The 3D floating ring tool in the globe window, as shown in Fig. 15(a), is actually two rings, the



Fig. 13. Mercator projection showing colored disk. Colors inside ring correspond to amount of areal distortion according to Fig. 12(a). Areal distortion can be noted to increase closer to poles.



Fig. 14. Polyconic projection showing colored disk. Colors inside ring correspond to amount of angular distortion according to Fig. 12(b). It can be observed that angular distortion increases further away from central meridian (yellow–green longitude line).

original ring located on the surface of the globe, and an elevated ring that hovers above sea level. The elevated ring represents the ring at a user-specified elevation. The two rings are connected so that the entire visual representation is that of a cylinder, enlarged at the elevated end to accurately reflect the appropriate size, based on the size of the original ring and the elevation. In the projection window, Fig. 15(b), the 3D floating ring behaves exactly as its standard counterpart. Both of the projected rings are deformed based on the projection parameters. Again, the elevated ring is enlarged appropriately.

The primary contribution of the 3D floating ring is a visual representation of map projection distortion for maps that contain vertical scalar data. An arbitrary data point that lies above the earth's surface can be thought







Fig. 15. 3D floating ring tool on (a) globe and (b) projection.

of as intersecting a sphere that is concentric to the earth. The size of this sphere is of course dependent on its elevation. This sphere defines a space that is separate from, but dependent on, the space of the original globe for the purposes of mapping the data and points on the globe. Thus any portion of the globe to be mapped has a corresponding portion of the elevation space to which it is correlated. Every point on the globe has a corresponding point in elevation space. The two spaces, however, are not of equal size. Thus the data points in elevation space must be projected differently, and possibly scaled to the underlying map projection. The 3D floating ring tool does not seek methods to correct this. Rather the purpose is to provide a visual tool that clearly defines the two different spaces, globe space and elevation space, and provides a visual link between them.

An important visual cue used in this method is a clear separation between globe space and elevation space. On the globe window (Fig. 15(a)), elevation space is delineated via a translucent sphere around the globe. The user controls the elevation level, and thus the size. of the sphere. On the projection window (Fig. 15(b)), the elevation is implied based on the 3D floating ring tool itself. The elevation space is correctly projected, but not scaled, so as to retain a close connection to the globe, as well as reality. Traditionally, scalar data points with a vertical component are simply scaled to the underlying map and overlaid directly onto the map. This method has the disadvantage of hiding important information, such as the method of projection and scaling. By separating globe space and elevation space, the 3D floating ring tool, along with the IMP system, creates an alternative viewpoint for the user, one that avoids the ambiguities common in traditional methods.

6. Conclusions

The goal of the Interactive Map Projections system is to enhance the ability of users to understand map projections by providing a highly interactive environment from which map projections can be viewed and manipulated. The visual link between the generating globe and the resulting projection is the key strength of the system. This system is an example of the expansion of traditional cartography into an interactive experience accessible to those with varying levels of cartographic expertise, made possible by today's computer technology and in particular, the visualization and graphical power available in modern computers.

The floating ring tool represents a new type of distortion diagram. The primary strength of the tool is the ability of the user to directly interact and manipulate the tool. Both angular and areal distortion are visualized based on the size and shape of the deformed ring as well as color mapping. The floating ring tool allows qualitative and quantitative distortion analysis of points as well as arbitrary areal extents. The floating ring overcomes many of the shortcomings of traditional methods of map projection distortion visualization, mostly through the use of powerful, intuitive user interactivity, and the subtle effectiveness of the circular shape in conveying distortion information through shape deformation and color.

The field of animated and interactive cartography has incredible potential for expansion. Many of the restrictions to the development of animated cartography, such as enormous storage and computational power requirements, are lifting as faster computers become more affordable and accessible. The ability to perform very large numbers of calculations and display complex graphics is within the reach of today's basic desktop systems. Hopefully much more research effort will be expended to push the envelope of cartographic visualization to create new tools and methods to aid users in reaching insight into complex geographical data.

The use of computers and Geographic Information Systems to manage, store and visualize geographic information opens up many new avenues, not only for what we view but how we view the information. Interactively viewing and manipulating map projections and map distortion tools represents just one of those avenues, and is an attempt to use visualization methods to allow cartographers, students and others in the geosciences communities to enhance their understanding of spatial geographic information and hopefully stimulate creative insight from that information.

7. Future directions

There is much work that can be done to enhance the functionality of the IMP system. Creating better methods of comparison might increase the effectiveness of the system. For example, creating a method for animating smoothly between two projections would aid users in their ability to see and perhaps understand the differences between similar projections. This could be expanded to somehow quantify the differences between the projections. One method for achieving this is through the use of color to differentiate between areas where the two projections are similar and where they differ. If a particular line in the graticule moved very little from one projection to another, it would be colored a cool color. Likewise areas of the projection that differed greatly would be colored a warm color. Here, the use of animation would emphasize the connections between the color and the graticule pattern. Transparency could be used as an alternative to color as a method of visualizing the similarities between projections. Areas that are similar would remain opaque. In other areas, the level of transparency would be proportional to the degree of difference between two projections. The use of transparency would provide an effective and simple way to compare projections.

Another exciting area of future research, and one that could be integrated into the Interactive Map Projections system, is the ability of users to interactively create arbitrary map projections. In Canters (1989), a generalized method for creating map projections by representing the generic projection functions as a pair of fourth-order polynomials is described. Different projections are created by changing the thirty coefficients of the polynomials. This process could be implemented as an interactive visual process, whereby the user could have immediate visual feedback from changes made to the coefficients. Such a tool would be exciting and an important contribution to the field of cartography in exploring and perhaps discovering new map projections.

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