Lecture 12: Grids, interpolation and extrapolation

The power of the grid
- Easiest representation of a field variable, measurable at all places
- Can vary spacing to suit task
- Interpolation necessary when data are not at grid intersections, or are irregular or sparse
- Interpolation becomes extrapolation
  - When areas deficient of points are interpolated
  - When interpolation is carried outside the data area

Models for terrain
- Contours: Vector
- Regular point samples
- Irregular point samples
- DEMs
- Surface patches
- TIN
- Voxel
- 3D point cloud
3D Transformations

- Not yet considered in a transformational context
- 3D data often for land surface or bottom of ocean
- Single valued function
- Need three coordinates to determine location $(X, Y, Z)$ or $(\lambda, \Phi, h)$

Terrain (surface) analysis

- Part of analytical cartography concerned with analysis of fields is terrain analysis
- Include terrain representation and symbolization issues as they relate to data
- Points, TIN and grids are used to store terrain
- How do we do transformations?
- In all cases we must fill in data where none exists in space = interpolation
- True even when data are dense, such as with LiDAR
Interpolation to a Grid

- Given a set of point elevations \((x, y, z)\) generate a new set of points at the nodes of a regular grid so that the interpolated surface is a reasonable representation of the surface sampled by the points.
- Imposes a model of the true surface on the sample
- "Model" is a mathematical model of the neighborhood relationship
- Influence of a single point = \(f(1/d)\)
- Can be constrained to fit all points
- Should contain \(z\) extremes, and local extrema
- Most models are algorithmic local operators
- Work cell-to-cell. Operative cell = kernel

Weighting Methods

- Impose \(z = f(1/d)\)
- Computationally rather intensive
- e.g. 200 \times 200\) cells \(1000\) points = \(40 \times 10^6\) distance calculations
- If all points are used and sorted by distance, called "brute force" method
- Possible to use sorted search and tiling (Hodgson, ERDAS)
- Distance can be weighted and powered by \(n = \) friction of distance
- Can be refined with break lines
- Use \(\cos\) (angle) to prevent shadowing

Inverse Distance Weighting

\[
Z_{i,j} = \frac{\sum_{p=1}^{R} Z_p d_p^{-n}}{\sum_{p=1}^{R} d_p^{-n}}
\]

- \(Z=\)height
- \(D=\)distance
- \(P=1, \ldots, R\)
- \(n=\)?
Inverse Distance Weighting: Clarke Algorithm

- inverse_d
- Assigns points to cells
- Averages multiples
- For all unfilled cells
  - search outward using an increasingly large square neighborhood
  - until at least npts are found
  - apply inverse distance weighting
- Has been parallelized (and found highly efficient) by Armstrong

Trend Projection Methods

- Way to overcome high/low constraint
- Assumes that sampling missed extrema
- Locally fits trend, trend surface or bicubic spline
- Least squares solution
- Useful when data are sparse, texture required

Search Patterns

- Many possible ways to define interpolated "region" R
- Can use # points or distance
- Problems in
  - Sparse areas
  - Dense areas
  - Edges
- Bias can be reduced by changing search strategy
Search patterns

- Assign points to cells
- Assign cells average of 4 neighboring cells
- Keep doubling cell size until all cells filled
- Looks “blocky” but works quickly
- Unbiased

Kriging

- "Optimal interpolation method" by D.G. Krigie
- Origin in geology (geostatistics, gold mining)
- Spatial variation = f(drift, random-correlated, random noise)
- To use Kriging:
  - Model and extract drift
  - Compute variogram
  - Model variogram
  - Compute expected variance at d, and so best estimate of local mean
- Several alternative methods
- Universal Kriging best when local trends are well defined
- Kriging produces best estimate and estimate of variance at all places on map

e.g. Quicklook gridding

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Variogram
Alternative Methods

- Many ways to make the point-to-grid interpolation
- Invertibility?
- Can results be compared and tested analytically
- Use portion of points and test results with remainder
- Examining spatial distribution of difference between methods
- Best results are obtained when field is sampled with knowledge of the terrain structure and the method to be used

Algorithm matters: IDW (5) vs. Splines (12, 0.1)

Error analysis

Surface measurement

- Can create DEMs directly from stereo imagery
- LIDAR creates a dense point cloud that must be sampled
- Traditional surveying creates points, often along profiles
- GPS gives points (but note z st.dev.)
- Can digitize lines from maps: Usually contours!
- Tiling errors possible
- Averaging in blocks common
Florinsky: Analysis of DEM error

Impact on aspect

Tiling error

Wedding cake effect: Visible in height histogram
Surface-Specific Point Sampling

- Terrain "Skeleton"
- Wedding Cake effect
- Specific problem when grids are made from contours (e.g., 3 arc second DEMs)

Wedding cake effect

Terrain filtering

Terrain Analysis

- Surface network extraction
- Surface network character, e.g., Strahler order
- Profile and Line-of-Sight
- Intervisibility and Viewshed
- Terrain modeling
- Visualization
Warntz Network

Terrain “skeleton” Warntz network Surface network

Visualization

Back to the Future; J. Mower 2009
Summary

- Analytical cartography also deals with surfaces or fields
- Terrain is the classic example, giving terrain analysis
- Surfaces can be transformed into other surfaces, e.g. slope, aspect
- Surfaces can be transformed into lines and networks, e.g. Warnitz network, contours
- Interpolation is almost always necessary, often point to surface
- Interpolation results are highly sensitive to method
- Terrain representation and visualization also important