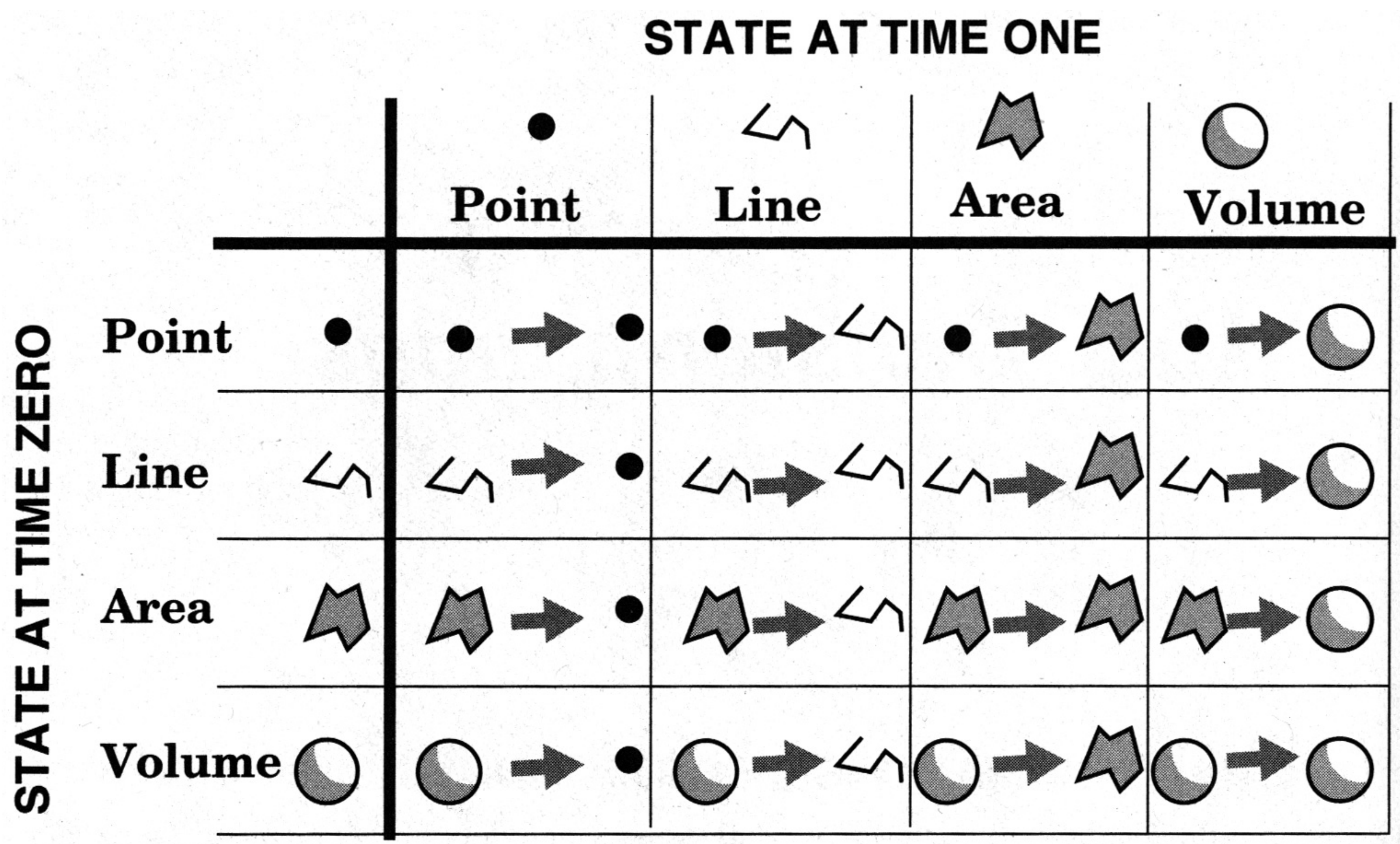
Analytical and Computer Cartography Winter 2017

Lecture 9:
Geometric Map Transformations

Cartographic Transformations

- Attribute Data (e.g. classification)
- Locational properties (e.g. projection)
- Graphics (e.g. symbolization)
- Information content of maps (e.g. data structure conversion)

Dimensional Transformations

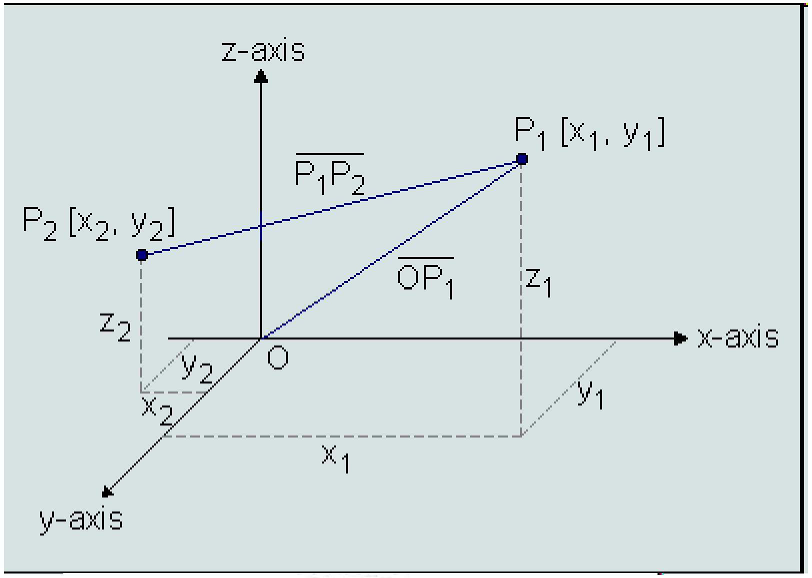


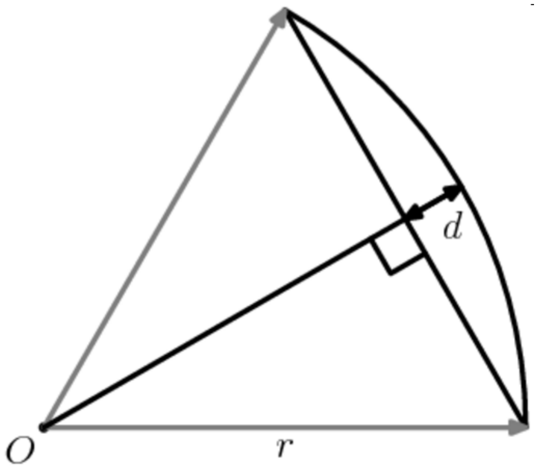
Geometric map transformations

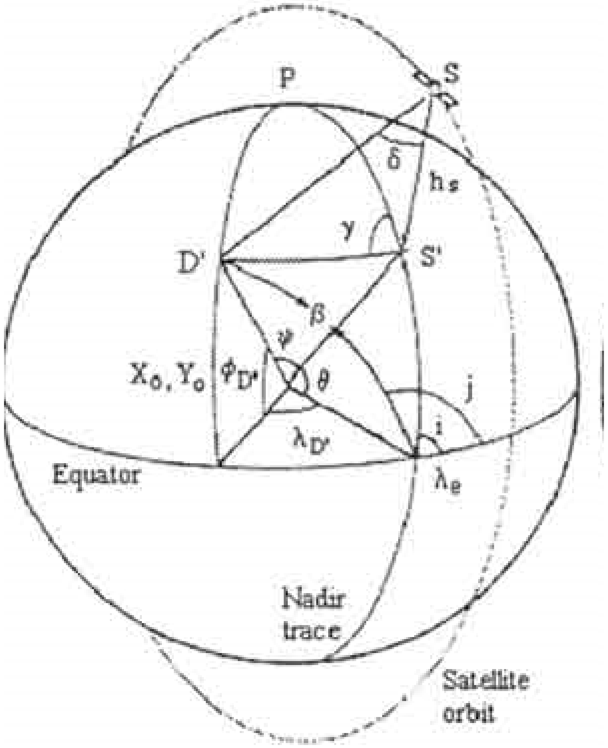
- Same dimension: e.g. point to point in a projection
- Change structure: e.g. TIN to grid in a DEM
- Change scale: e.g. Area to point as a city is generalized

Planar geometries

- Cartesian
- Spherical (ellipsoidal)
- Radial



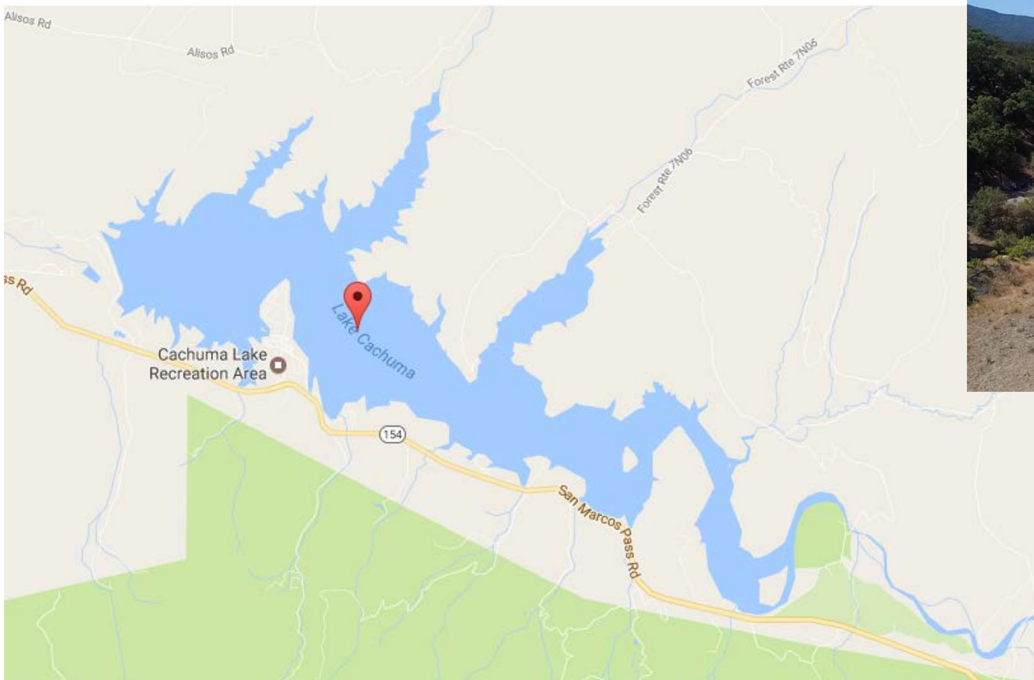




Analysis

- Input: geometric feature Point, line, Area, Surface, Volumes
- Output: Reduce feature to single dimension: e.g. centroid, length, perimeter, area in square unit lengths
- Output: A scalar, numerical value reflecting quantity “Collapse”
- E.g. shape, sinuosity, network metric
- Can compute using algorithms

Lake Cachuma



Lake Cachuma

Lake in California

Cachuma Lake is an artificial lake located in the San
central Santa Barbara County, California on the Sant
the north side of California State Route 154. Wikipec

Surface elevation: 753'

Area: 4.844 mi²

Outflow location: Santa Ynez River

Reservoir Information

Reservoir Elevations referenced to NGVD-29.
**Cachuma is full and subject to spilling at elevation 750 ft.
However, the lake is surcharged to 753 ft. for fish release water.
(Cachuma water storage is based on Dec 2013 capacity revision)

	Spillway Elev. (ft)	Current Elev. (ft)	Max. Storage (ac-ft)	Current Storage (ac-ft)	Current Capacity (%)	Storage Change Mo.(ac-ft)	Storage Change Year*(ac-ft)
Gibraltar Reservoir	1,400.00	1,399.93	5,272	5,255	99.7%	39	4,640
Cachuma Reservoir	753.**	669.75	193,305	33,079	17.1%	8,043	18,468
Jameson Reservoir	2,224.00	2,169.80	5,144	769	14.9%	52	211
Twitchell Reservoir	651.50	582.75	194,971	36,094	18.5%	20,454	36,093

Uncertainty in Geometric features

- Assume infinite thinness
- Assume exact location
- Assume unambiguous ontology (definition) e.g. wetlands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Low Graphics

Friday, 19 October, 2001, 10:51 GMT 11:51 UK

Geologist's clue to Bin Laden location

Western intelligence fears Bin Laden has moved on

An American geologist who spent years in Afghanistan believes he has narrowed down the location of terror suspect Osama Bin Laden to sandstone caves south of Kabul.

Jack Shroder, from the University of Nebraska at Omaha, reported his conclusions to US security agencies after assessing a cave that featured in a Bin Laden video released after the first US bombs fell on the Afghanistan on 7 October.

MIT International Review | web.mit.edu/mitir

Web-published essay installment for 17 February 2009

Finding Osama bin Laden:

An Application of Biogeographic Theories and Satellite Imagery

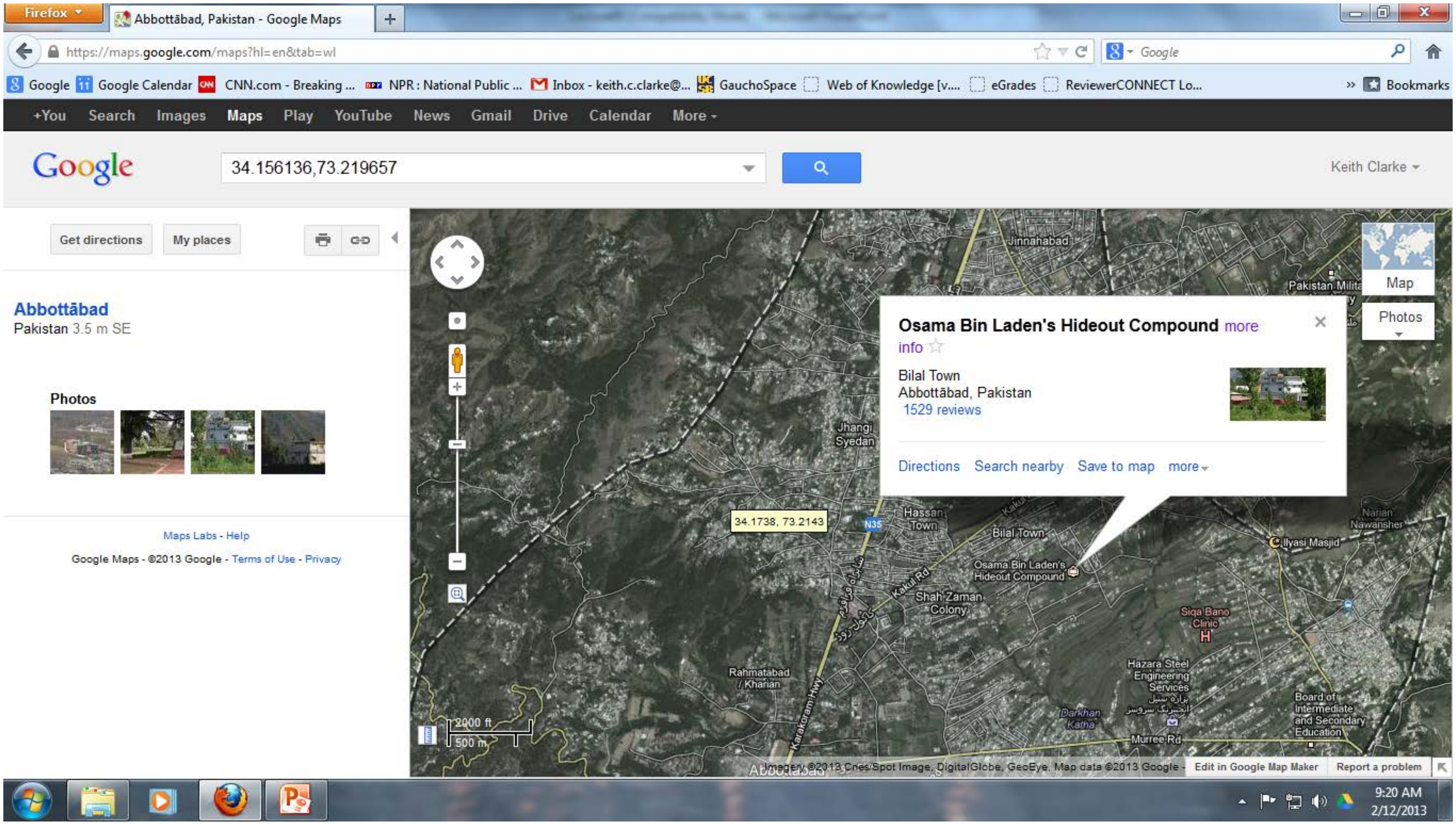
Thomas W. Gillespie and John A. Agnew are professors of geography at UCLA. They may be contacted respectively at tg@geog.ucla.edu and jagnew@geog.ucla.edu. Erika Mariano, Scott Mossler, Nolan Jones, Matt Braughton, and Jorge Gonzalez are undergraduates in UCLA's geography department. They may be contacted respectively at erikmari@ucla.edu, smossler@ucla.edu, nolanjones@ucla.edu, mbraught@ucla.edu, and jorgon@ucla.edu [1]

Likely location and setting (3 given)

Structure C
N 33.888207°
E 70.113308°

Actually at: 34.156136,73.219657

Abbottabad



Abbottābad, Pakistan - Google Maps
https://maps.google.com/maps?hl=en&tab=wl
+You Search Images Maps Play YouTube News Gmail Drive Calendar More
Google
34.156136,73.219657
Keith Clarke
Get directions
My places
Abbottābad
Pakistan 3.5 m SE
Photos
Maps Labs - Help
Google Maps - ©2013 Google - Terms of Use - Privacy
Osama Bin Laden's Hideout Compound more info
Bilal Town
Abbottābad, Pakistan
1529 reviews
Directions Search nearby Save to map more
34.1738, 73.2143
Jinnahabad
Jhangi Syedan
Hassan Town
Bilal Town
Osama Bin Laden's Hideout Compound
Shah Zaman Colony
Ilyasi Masjid
Siqa Bano Clinic
Rahmatabad / Kharian
Hazara Steel Engineering Services
Board of Intermediate and Secondary Education
Murree Rd
2000 ft
500 m
Map
Photos
Edit in Google Map Maker
Report a problem
9:20 AM
2/12/2013

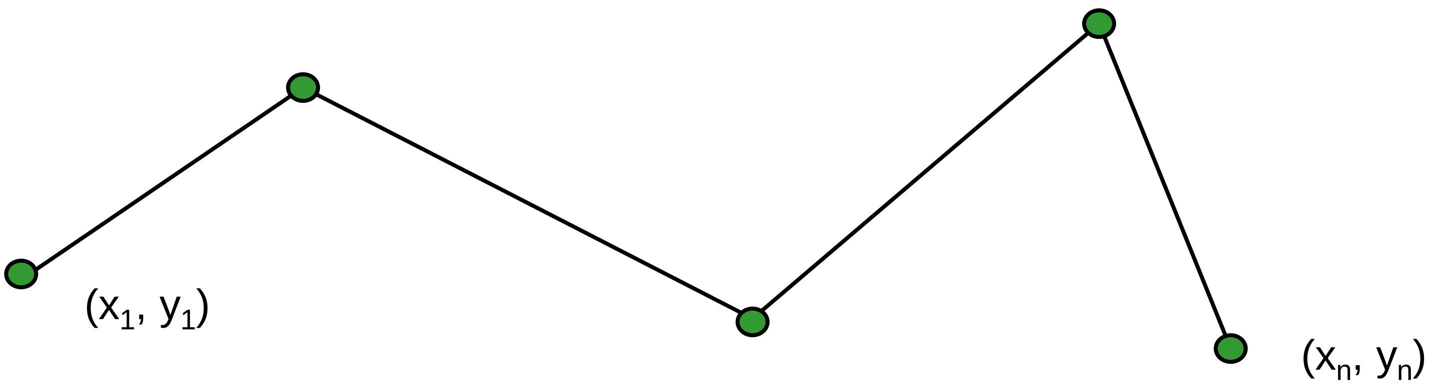
Assuming crisp features: Points, lines, areas, volumes

- What simple transformations can be performed on PLAV in Euclidean space?
- In C language
- typedef struct POINT { int point_id, double x, double y};
- POINT Point[100]; int npts;
- Create a loop
  - double sumx = 0.0
  - for (i=0; I <= npts; i++) sumx += Point[i].x;
  - meanx = sumx / npts;

Planar Map Transformations on Points
- Length of a line

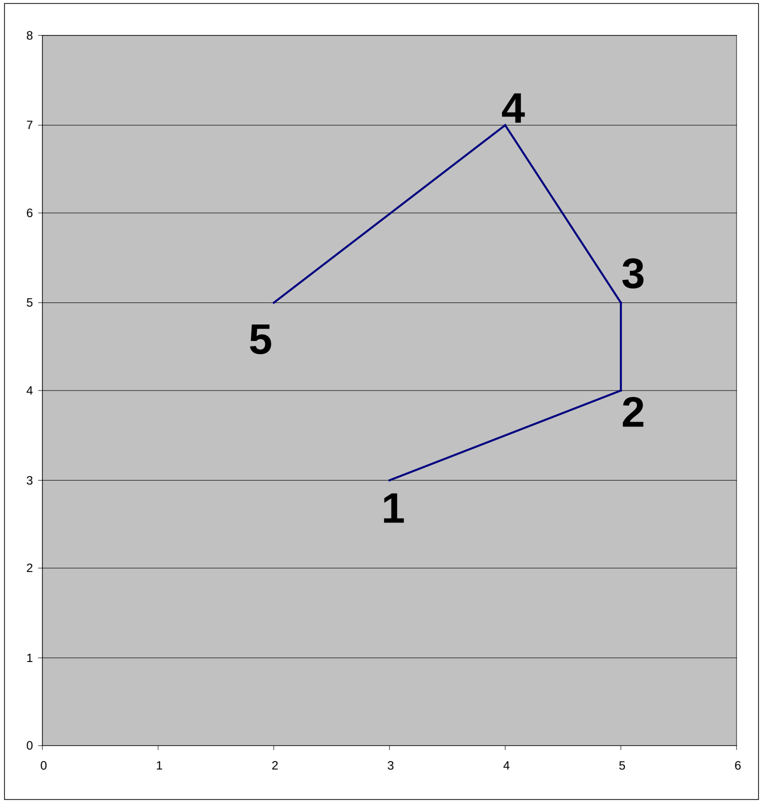
- Repetitive application of point-to-point distance calculation
- For n points, algorithm/formula uses n-1 segments

$$\text{length} = \sum_{i=1}^{npts} \sqrt{(x_i - x_{i-1})^2 + (y_i - y_{i-1})^2}$$



Length of a Line

Estimate?



Length of a generic line

i	x_i	y_i	x_i-x_{i-1}	y_i-y_{i-1}	$(x_i-x_{i-1})^2$	$(y_i-y_{i-1})^2$	$(x_i-x_{i-1})^2+(y_i-y_{i-1})^2$	$(x_i-x_{i-1})^2+(y_i-y_{i-1})^2$	$sqrt[(x_i-x_{i-1})^2+(y_i-y_{i-1})^2]$
1	3	3	--	--	--	--	--	--	--
2	5	4	2	1	4	1	5	5	2.236
3	5	5	0	1	0	1	1	1	1.000
4	4	7	-1	2	1	4	5	5	2.236
5	2	5	-2	-2	4	4	8	8	2.828

Sum =

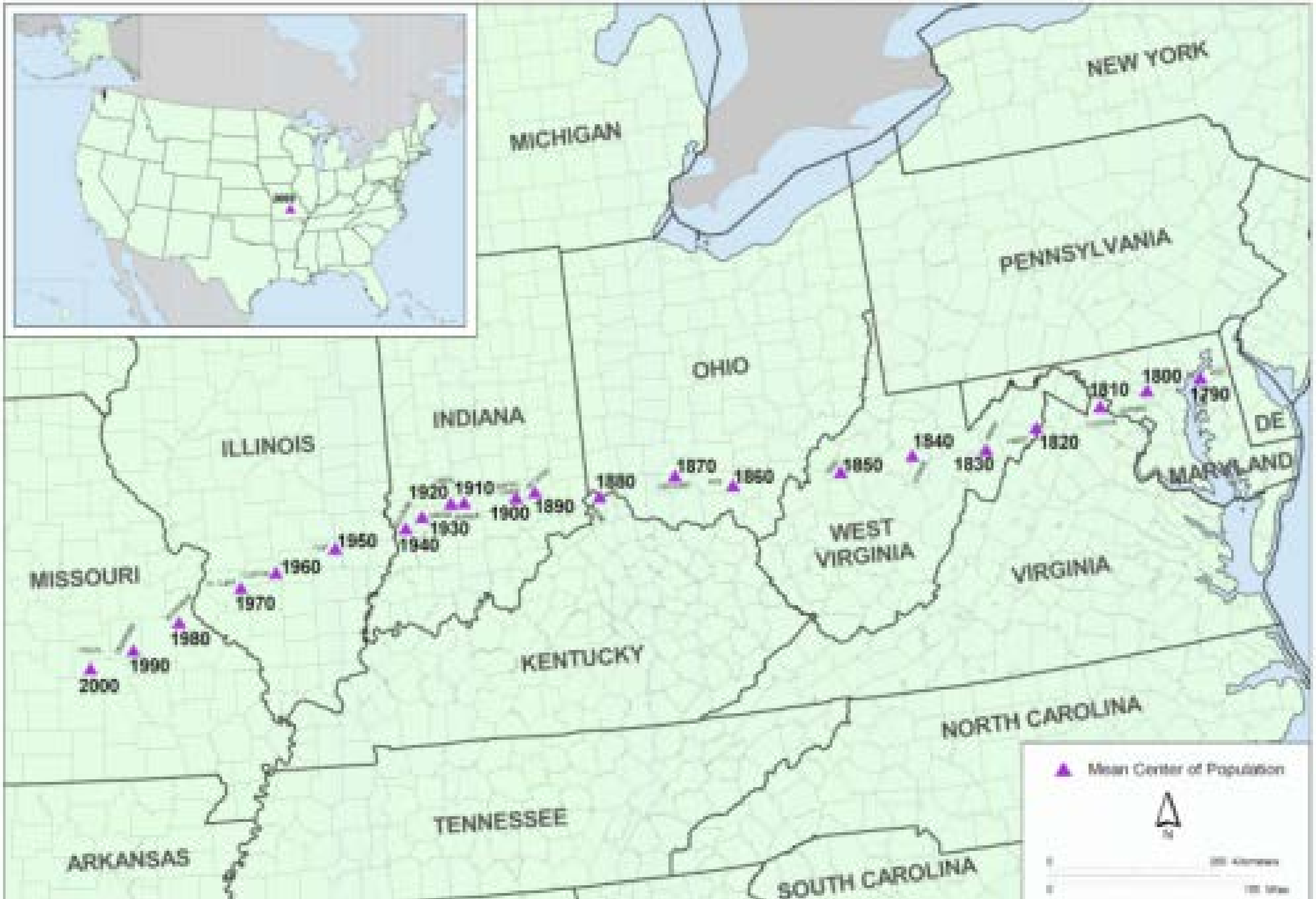
8.3 units

Planar Map Transformations on Points
- Centroids

- Multiple point or line or area to be transformed to single point
- Point can be "real" or representative
- Can use weightings or populations
- Mean center simple to compute but may fall outside point cluster or polygon
- Can use point-in-polygon to test for inclusion

$$\bar{x} = \frac{\sum_{i=1}^{npts} P_i x_i}{\sum_{i=1}^{npts} P_i} \qquad \bar{y} = \frac{\sum_{i=1}^{npts} P_i y_i}{\sum_{i=1}^{npts} P_i}$$

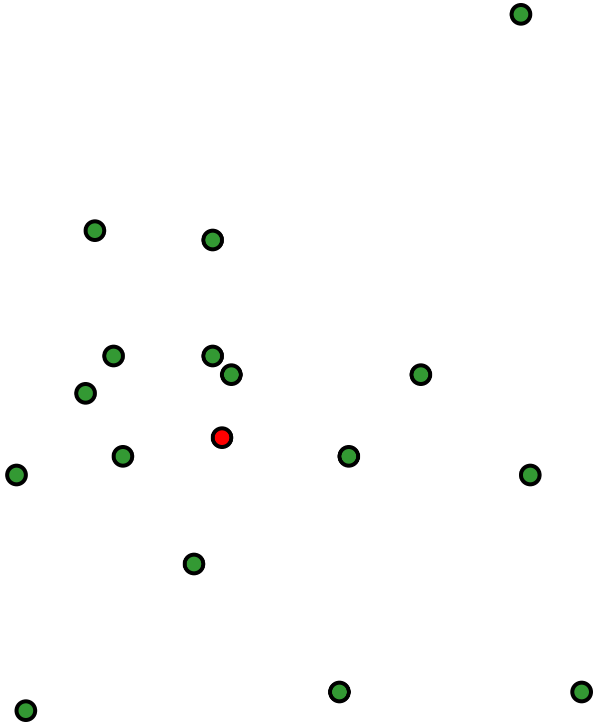
Mean Center of Population for the United States: 1790 to 2000



U.S. Department of Commerce Economics and Statistics Administration U.S. Census Bureau

Prepared by the Geography Division

Mean point vs. centroid



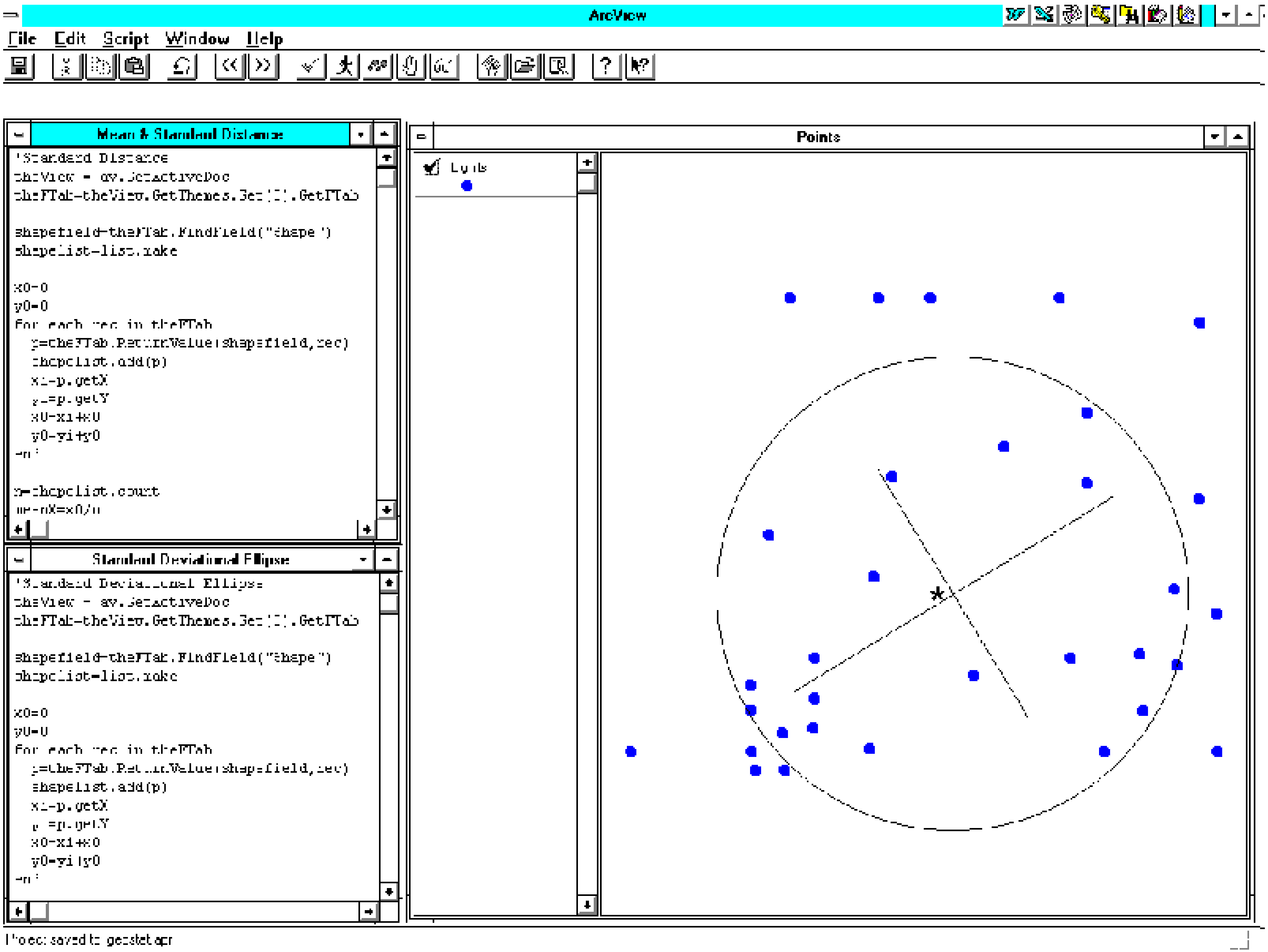
Planar Map Transformations on Points
- Standard Distance

- Just as centroid is an indication of representative location, standard distance is mean dispersion
- Equivalent of standard deviation for an attribute, mean variation from mean
- Around centroid, makes a "radius" tracing a circle

$$s_x = \sqrt{\frac{\sum_{i=1}^{npts} (x_i - \bar{x})^2}{npts}} \qquad s_y = \sqrt{\frac{\sum_{i=1}^{npts} (y - \bar{y})^2}{npts}}$$

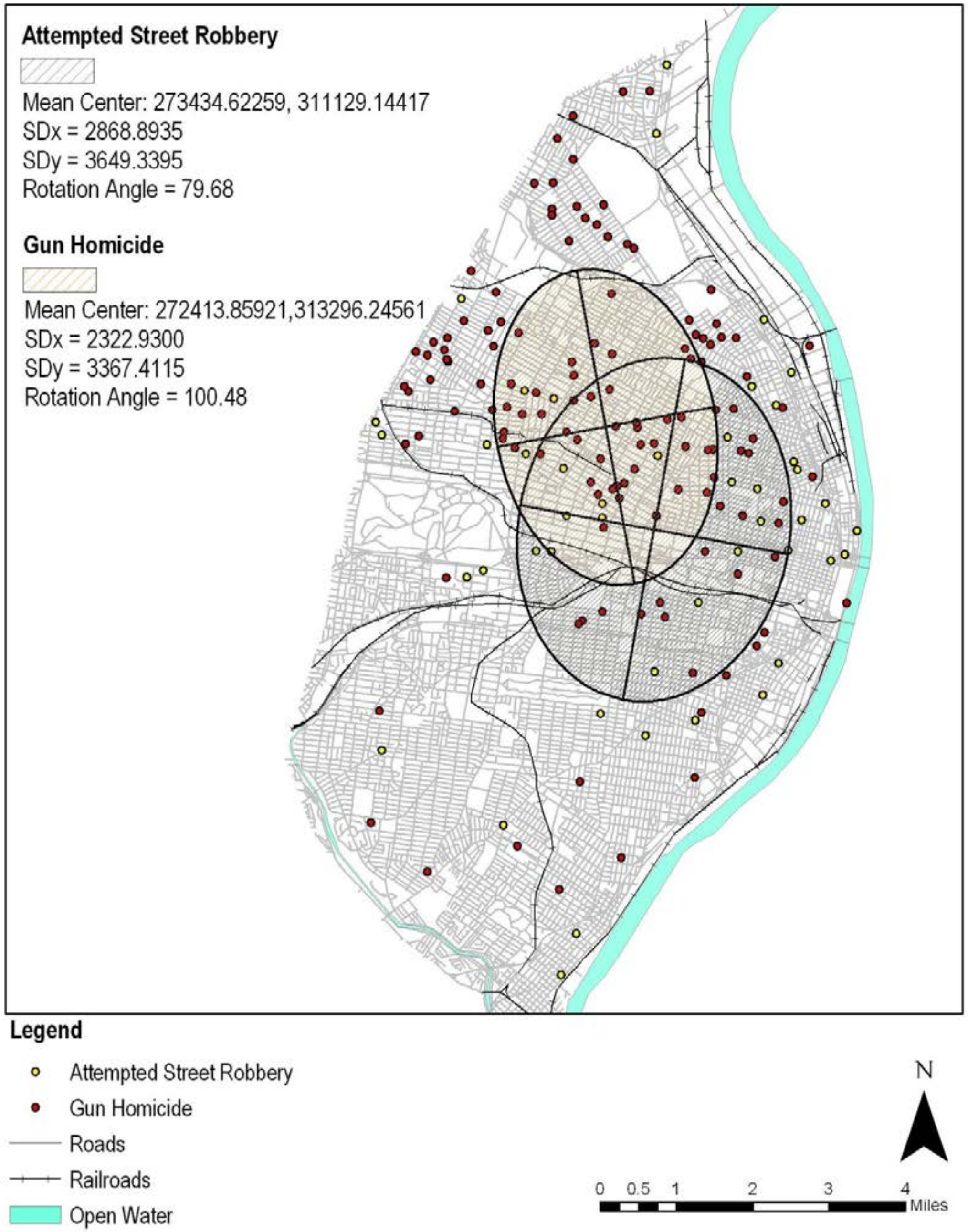
$$s = \sqrt{(s_x^2 + s_y^2)}$$

Standard distance



ATTEMPTED STREET ROBBERY AND GUN HOMICIDE POINT PATTERNS

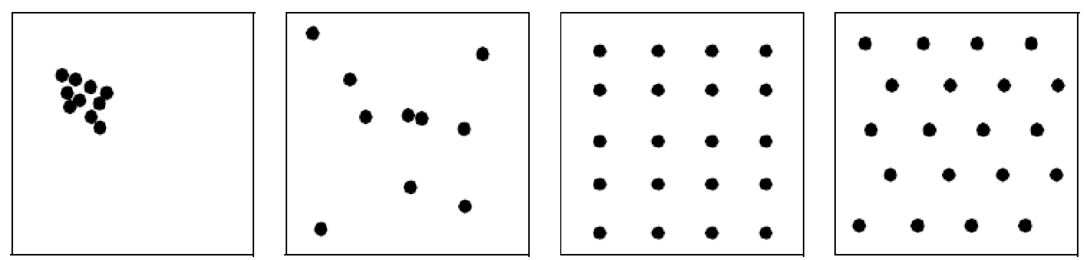
Standard Deviational Ellipses



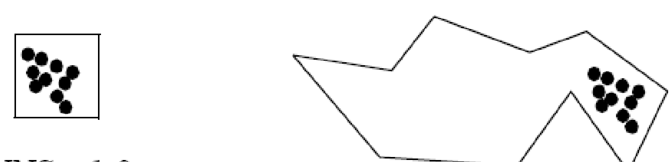
Planar Map Transformations on Points
- Nearest Neighbor Statistic

- NNS is a single dimensionless scalar that measures the pattern of a set of point (point-> scalar)
- Computes nearest point-to-point separation as a ratio of expected given the area
- Highly sensitive to the area chosen
- Available in ArcGIS etc

$$NNS = 2 \times \frac{\sum_{i=1}^{npts} d_i}{npts \times \sqrt{\frac{A}{npts}}}$$

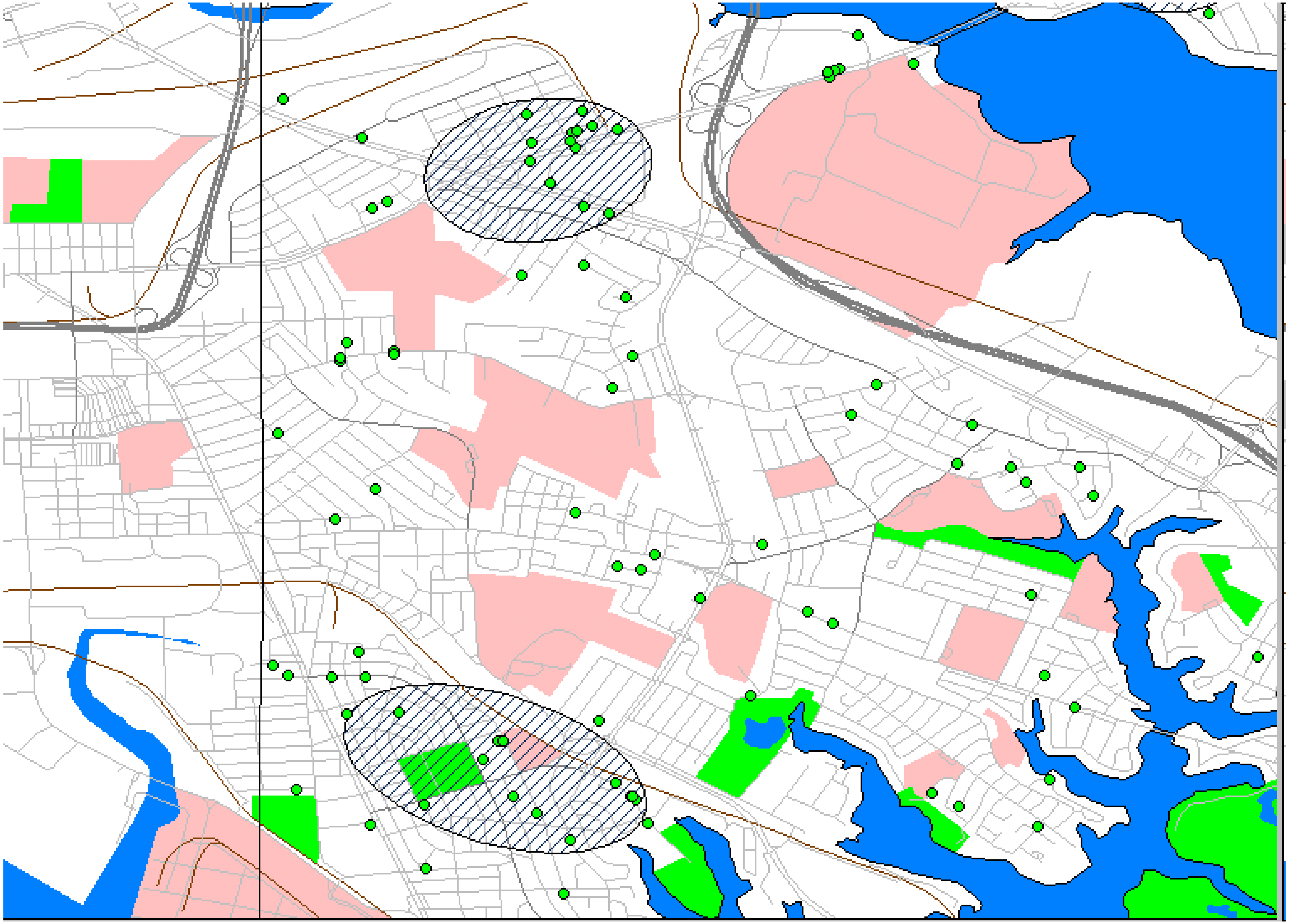


$NNS = 0.1$ $NNS = 1.0$ $NNS = 2.0$ $NNS = 2.14$



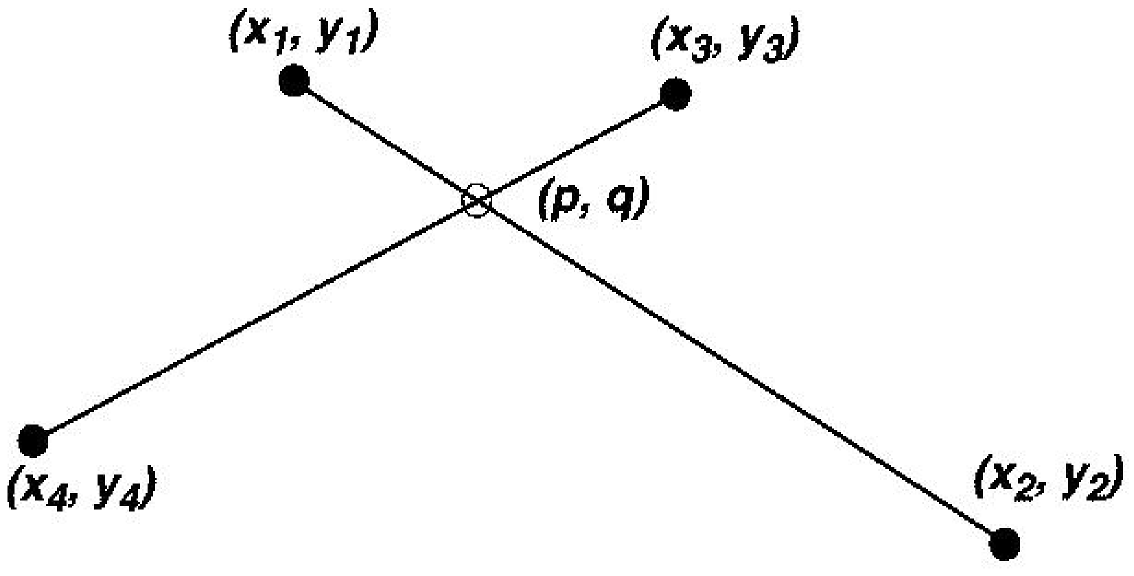
$NNS = 1.0$ $NNS =$ clustered or random?

Local NNS: Cluster busting

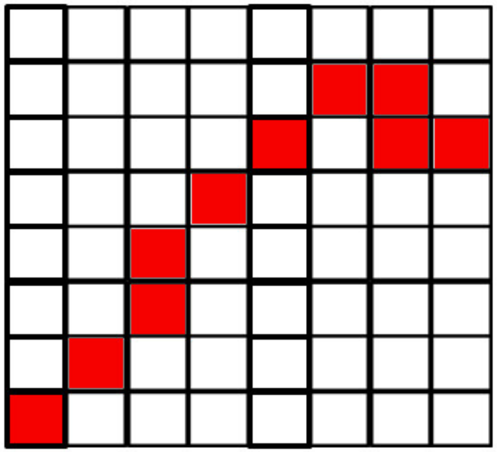


Planar Map Transformations Based on Lines
- Intersection of two lines

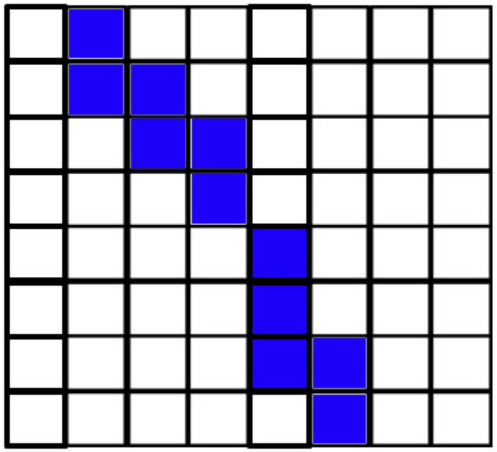
- Absolutely fundamental to many mapping operations, such as overlay and clipping.
- In raster mode it can be solved by layer overlay.
- In vector mode it must be solved geometrically.
- Lines (2) to point transformation



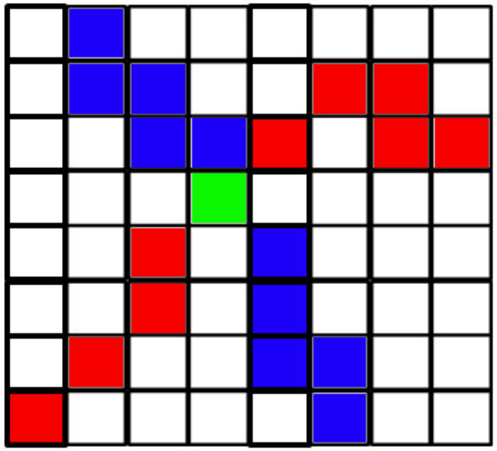
Raster solution: add binary arrays



Red = 1

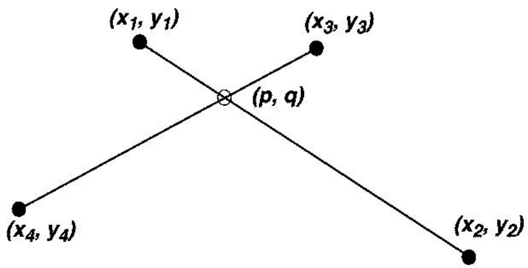


Blue = 2



Green = 3

Planar Map Transformations Based on Lines - Intersection of two lines (cnt.)



- When using this algorithm, a problem exists when b2 - b1 = 0 (divide by zero)
- Special case solutions or tests must be used
- These can increase computation time greatly
- Computation time can be reduced by pre-testing, e.g. based on bounding box.

If (x_1, y_1) and (x_2, y_2) lie on the same line, then

$$y_1 = a_1 + b_1 x_1$$

$$y_2 = a_1 + b_1 x_2$$

Similarly, if (x_3, y_3) and (x_4, y_4) lie on the same line, then

$$y_3 = a_2 + b_2 x_3$$

$$y_4 = a_2 + b_2 x_4$$

If there exists an intersection point, (p, q) that lies on both lines, then

$$q = a_1 + b_1 p$$

$$q = a_2 + b_2 p$$

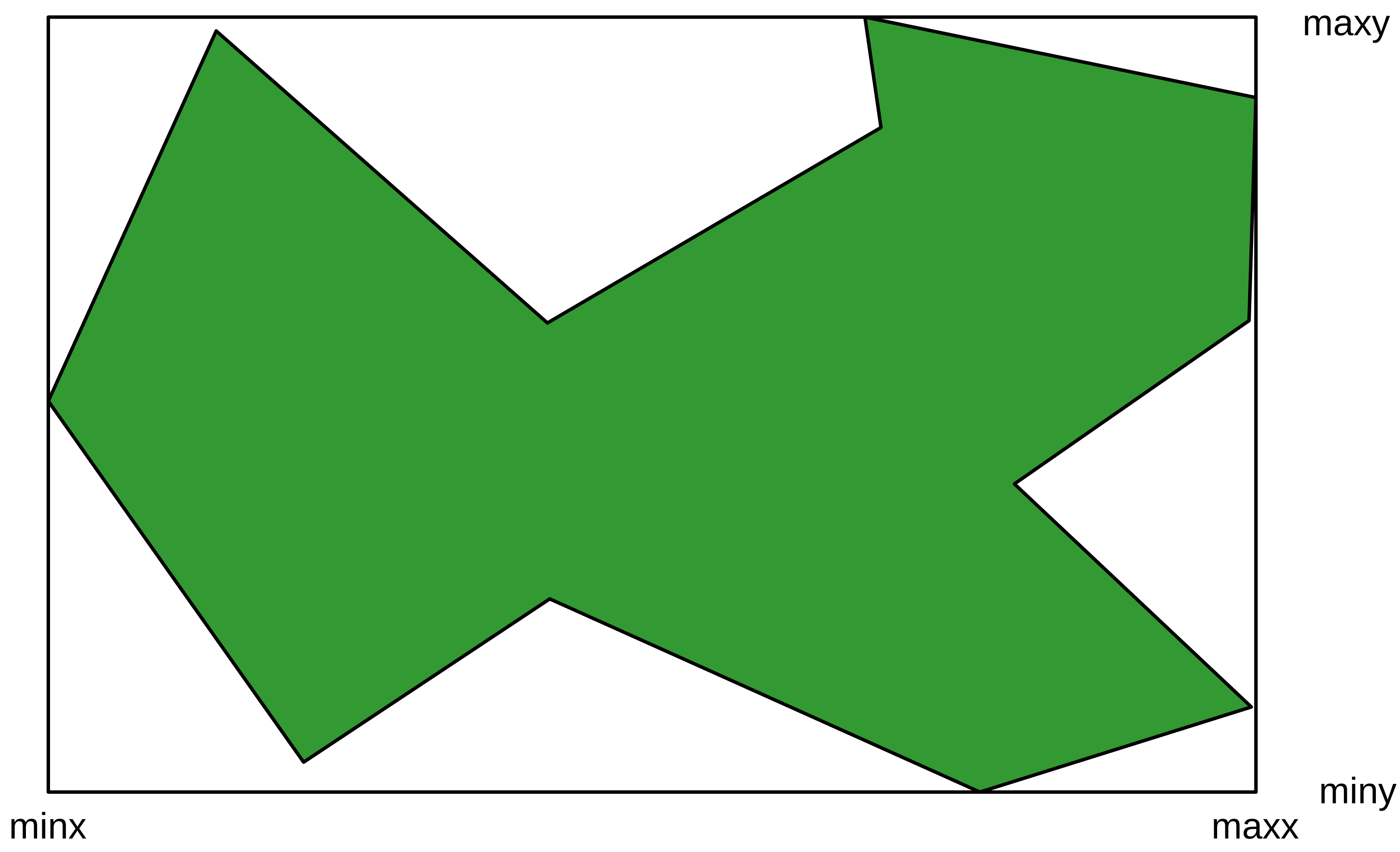
By subtracting the former from the latter,

$$q - q = a_1 - a_2 + p\,(b_1 - b_2)$$

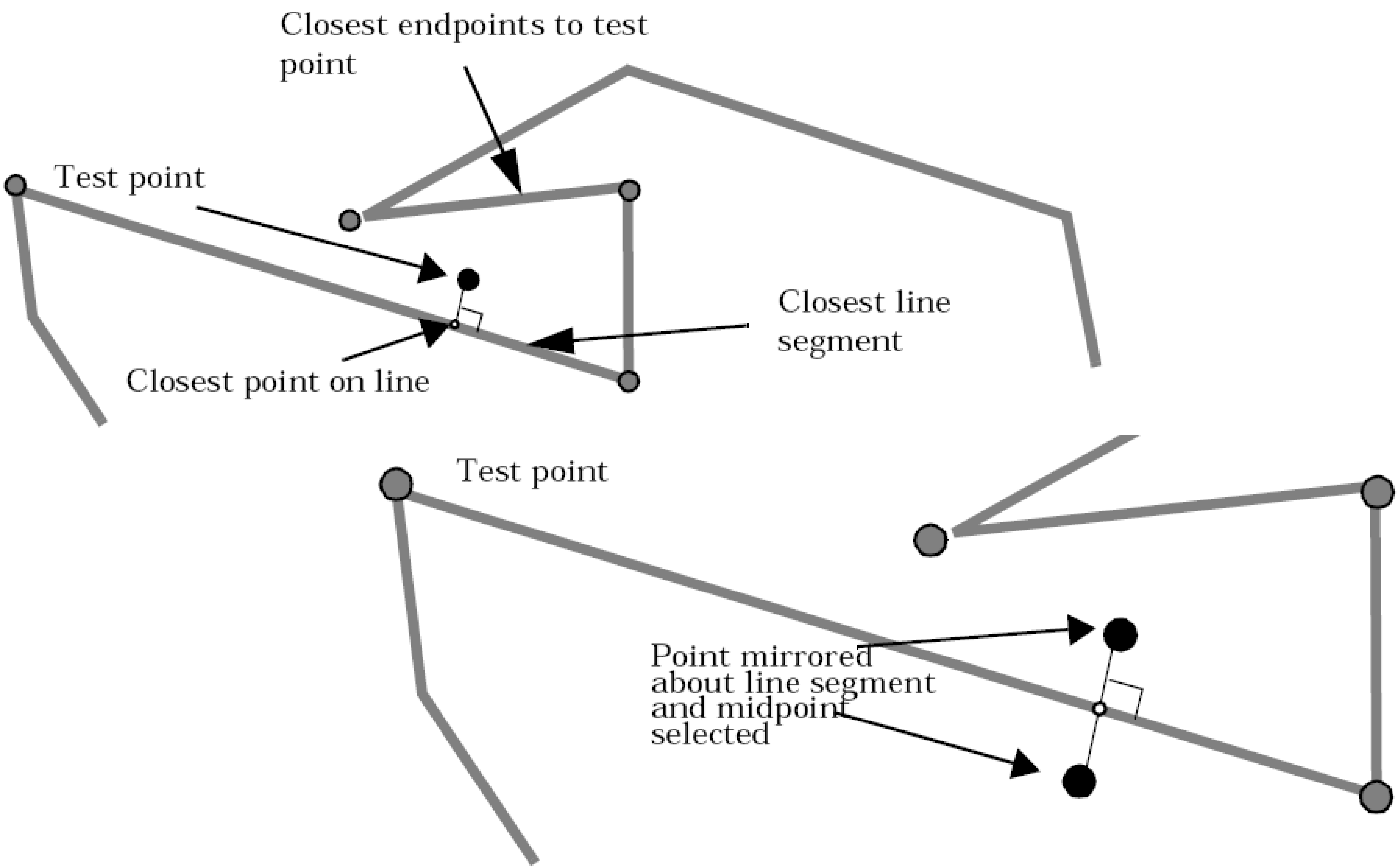
and rearranging, we obtain

$$a_1 - a_2 = p\,(b_2 - b_1)$$

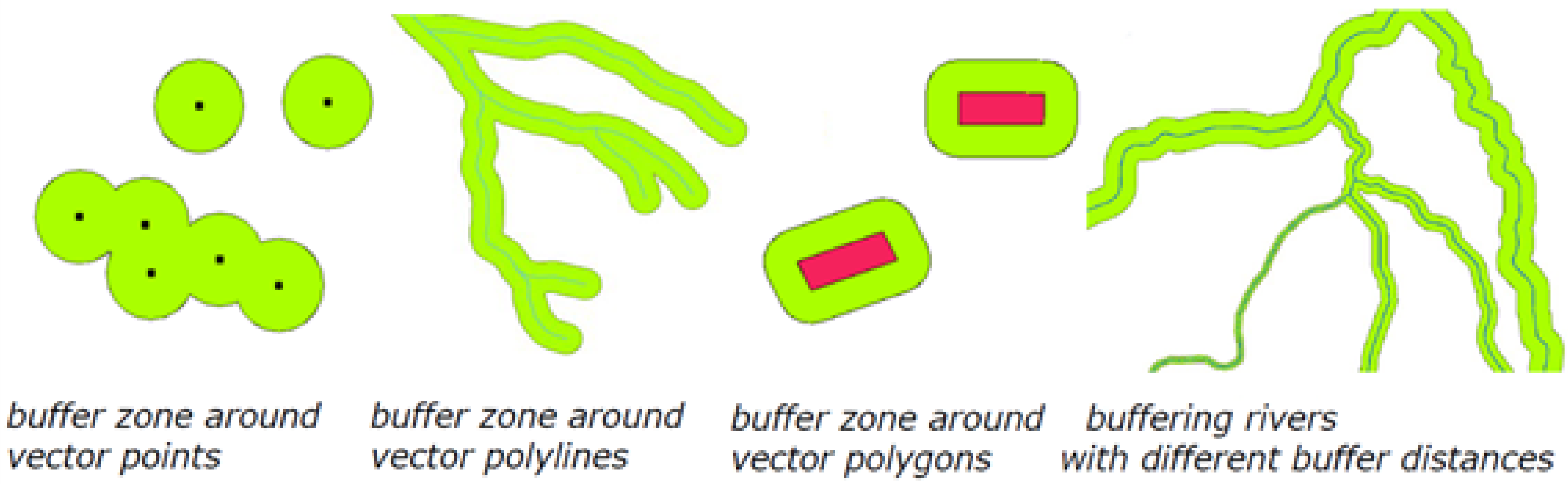
Bounding Box
if (x < minx) or (x > maxx) or (y < miny) or (y > maxy) then point is outside

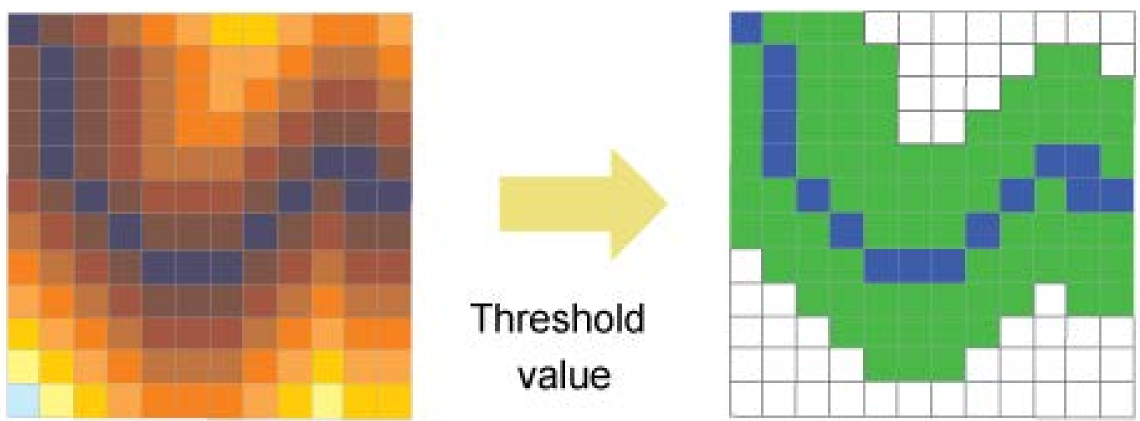


Planar Map Transformations Based on Lines - Distance from a Point to a Line



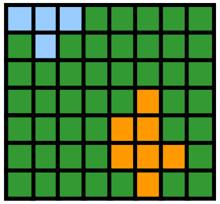
Used to create a buffer





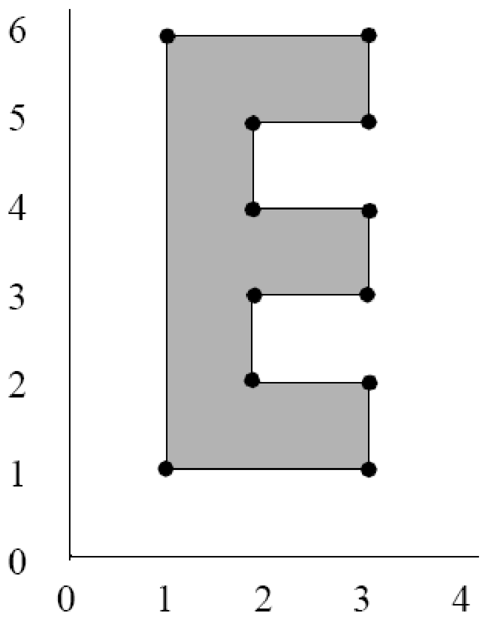
Planar Map Transformations Based on Areas

- Computing the area of a vector polygon (closed)
- Manually, many methods are used, e.g. cell counts, points on grid.
- For a raster, simply count the interior pixels
- Vector Mode more complex



$$A = \frac{1}{2}\left|\sum_{i=1}^{npts+1} (x_i y_{i-1}) - (x_{i-1} y_i)\right|$$

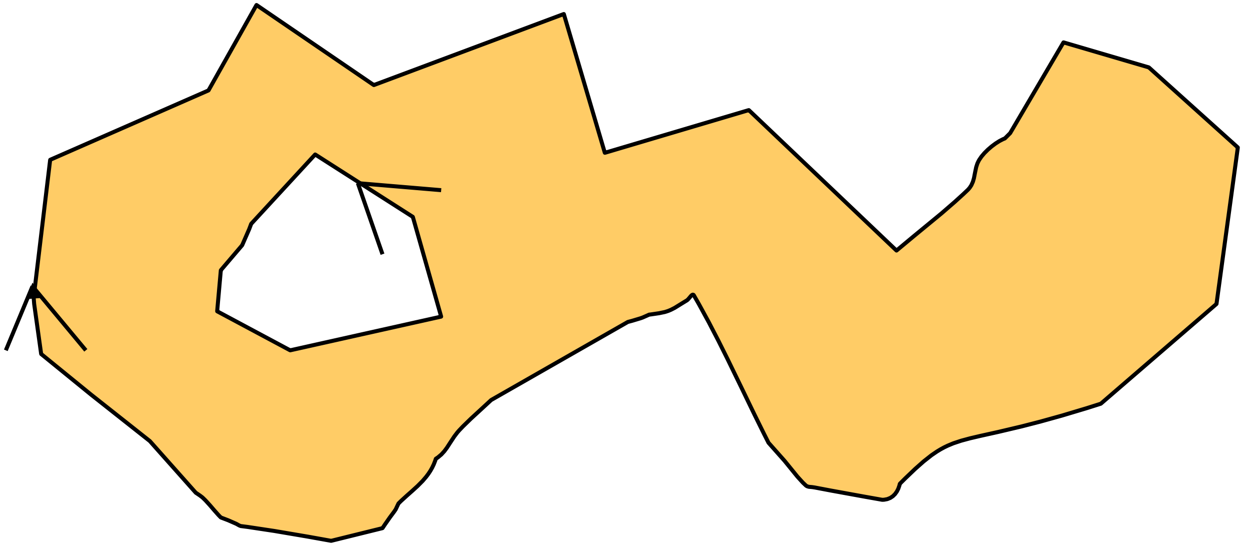
Planar Map Transformations Based on Areas



Vertex	x	y	A	B	Difference (A–B)
1	1	1	—	—	—
2	1	6	1	6	–5
3	3	6	18	6	12
4	3	5	18	15	3
5	2	5	10	15	–5
6	2	4	10	8	2
7	3	4	12	8	4
8	3	3	12	9	3
9	2	3	6	9	–3
10	2	2	6	4	2
11	3	2	6	4	2
12	3	1	6	3	3
13	1	1	1	3	–2
					16

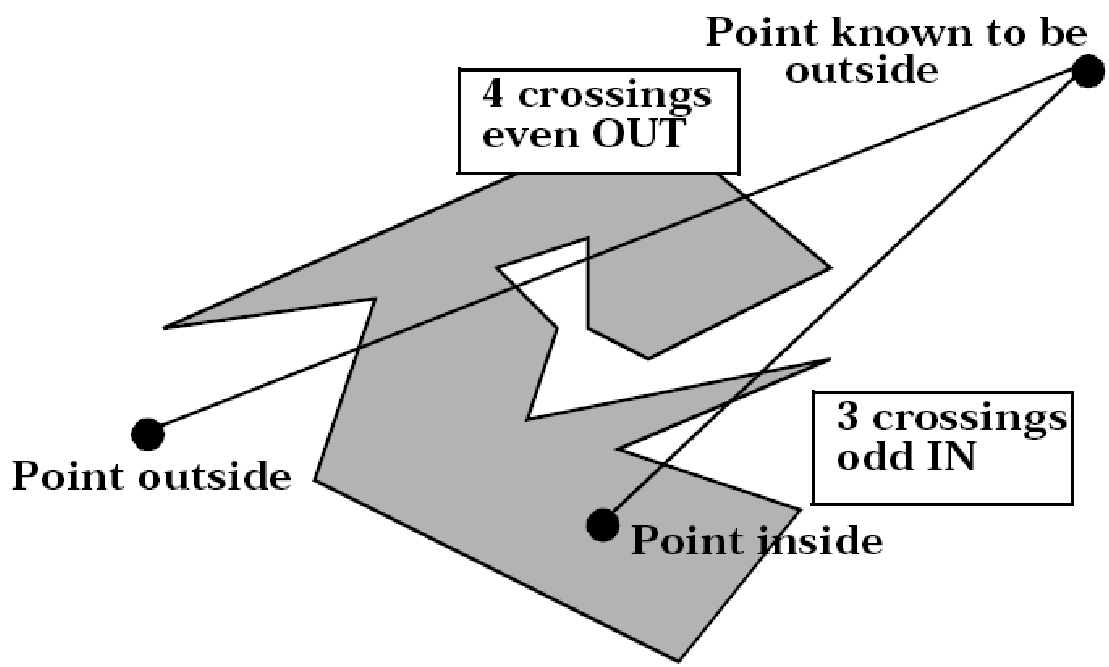
Polygon direction

- Clockwise: Sum is positive
- Counter-clockwise: Negative
- So make holes counter clockwise, include in area calculation, then add



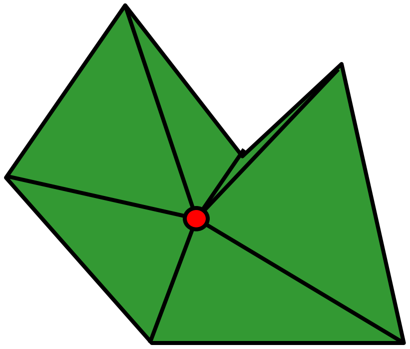
Planar Map Transformations Based on Areas - Point-in-Polygon

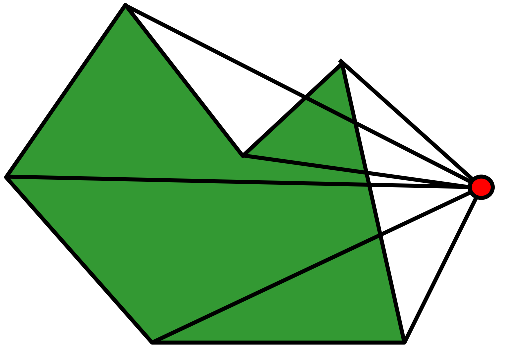
- Again, a basic and fundamental test, used in many algorithms.
- For raster mode, use overlay.
- For vector mode, many solutions.
- Most commonly used is the Jordan Arc Theorem
- Tests every segment for line intersection
- Test point selected to be outside polygon



Another Point-in-Polygon Algorithm

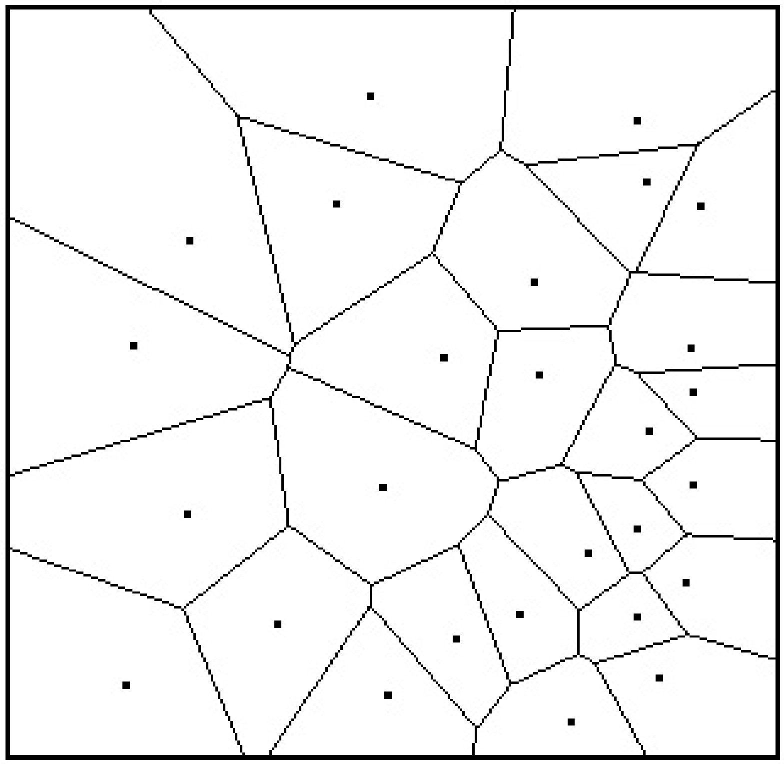
- Calculate area of polygon
- Find test point
- Calculate area for all triangles made by center and two sequential exterior points
- If sum of areas is the area of the polygon, point is inside polygon

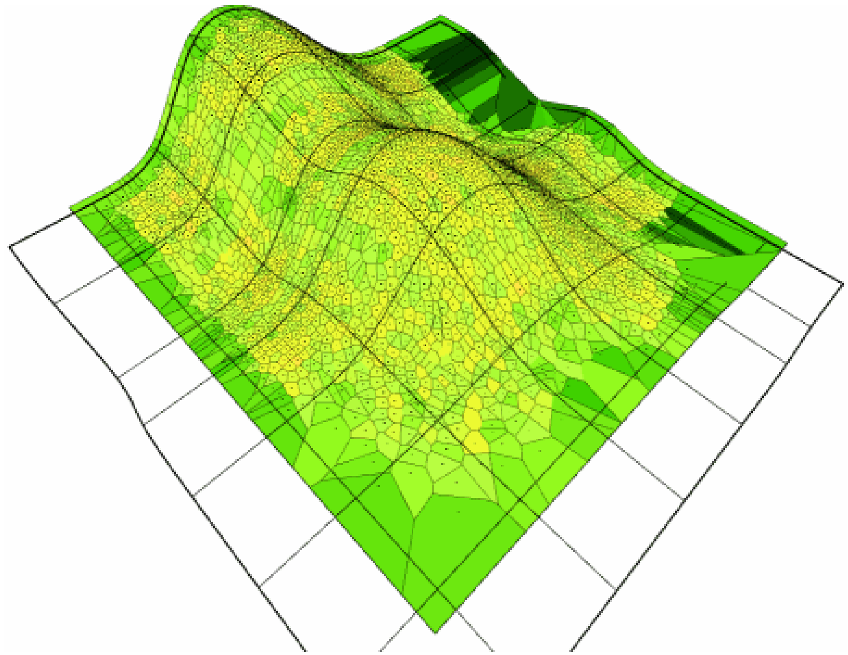




Planar Map Transformations Based on Areas - Theissen Polygons

- Often called proximal regions or Voronoi diagrams
- Often used for contouring terrain, climate, interpolation, etc





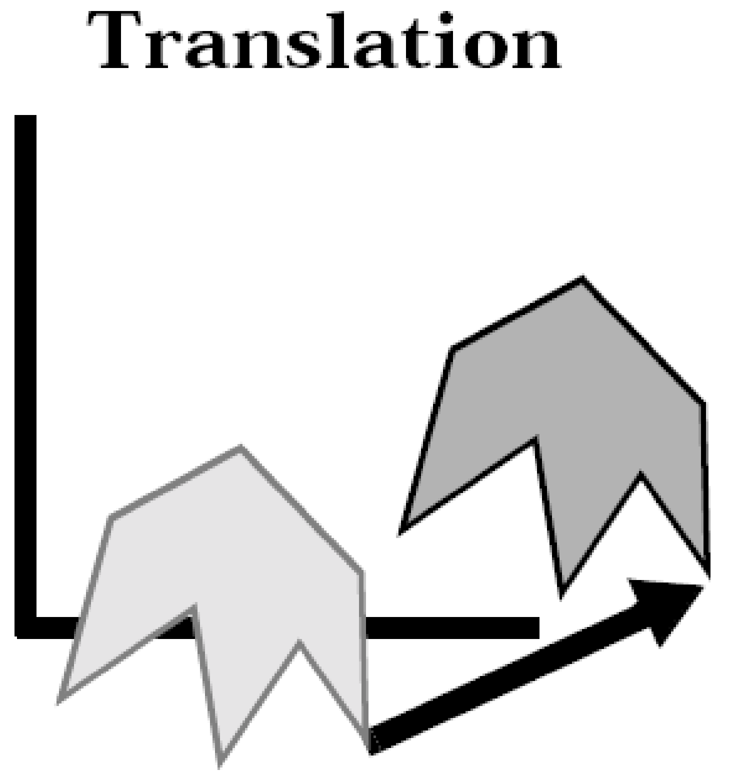
http://en.wiki.mcneel.com/default.aspx/McNeel/PointsetReconstruction.html

Affine Transformations

- These are transformation of the fundamental geometric attributes, i.e. location.
- Influence absolute location, not relative or topological
- Necessary for many operations, e.g. digitizing, scanning, geo-registration, and display
- Affine Transformations take place in three steps (TRS) in order
  - Translation
  - Rotation
  - Scaling

Affine Transformations - Translation

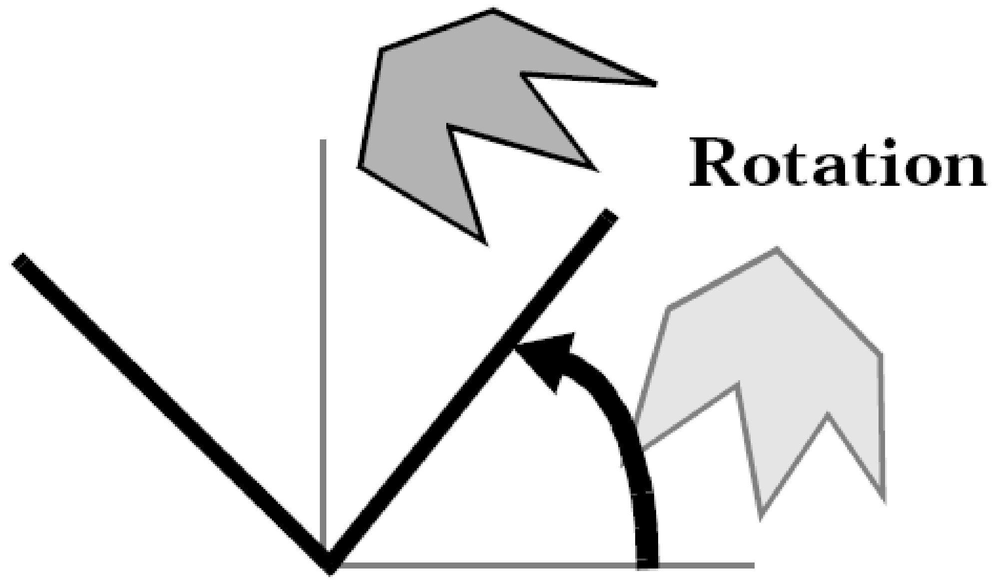
- Movement of the origin between coordinate systems



$$\begin{bmatrix} x & y & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -x_0 & -y_0 & 1 \end{bmatrix} = \begin{bmatrix} x - x_0 & ,y - y_0 & ,1 \end{bmatrix}$$

Affine Transformations - Rotation

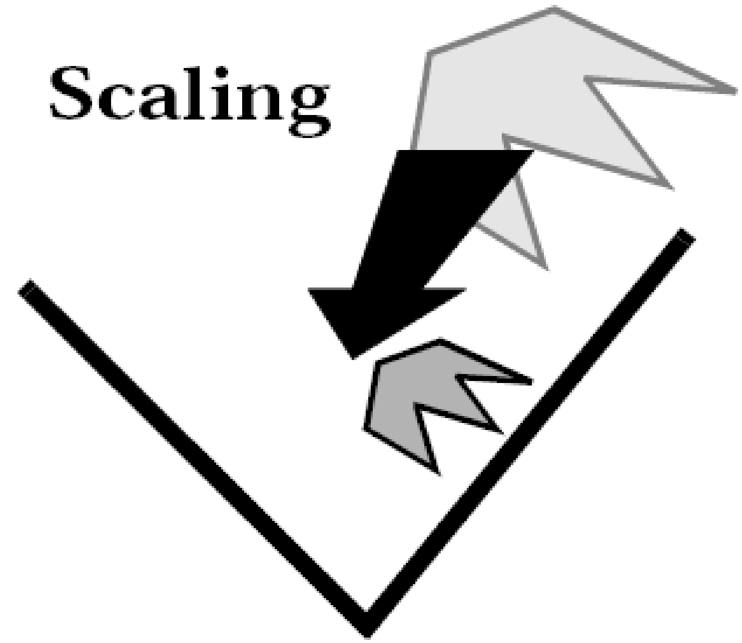
- Rotation of axes by an angle theta



$$\begin{bmatrix} x - x_0 & y - y_0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} \cos\theta(x - x_0) - \sin\theta(y - y_0) & \sin\theta(x - x_0) - \cos\theta(y - y_0) & 1 \end{bmatrix}$$

Affine Transformations - Scaling



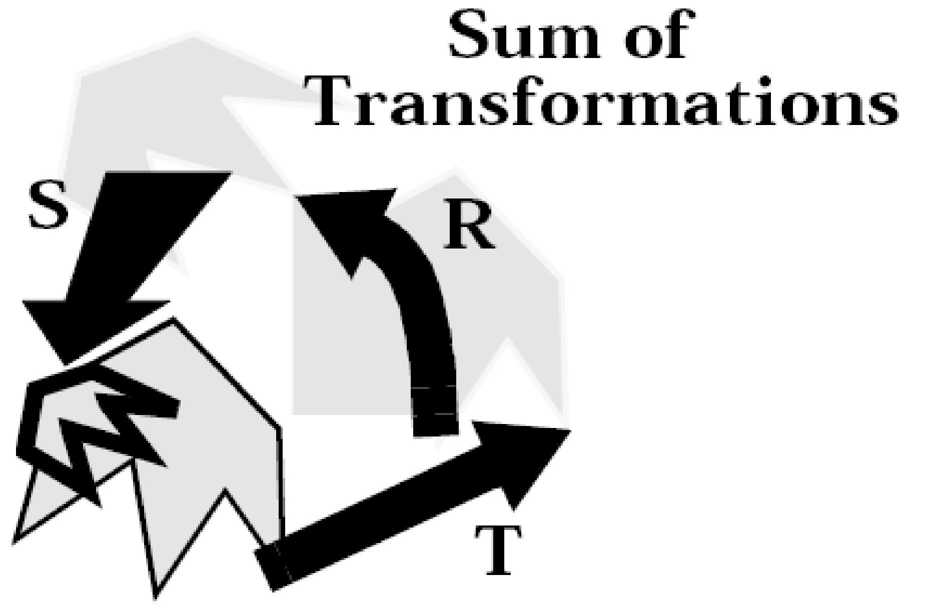
- The numbers along the axes are scaled to represent the new space scale

$$\begin{bmatrix} \cos\theta(x-x_0)-\sin\theta(y-y_0) & \sin\theta(x-x_0)-\cos\theta(y-y_0) & 1 \end{bmatrix} \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} S_x[\cos\theta(x-x_0)-\sin\theta(y-y_0)] & S_y[\sin\theta(x-x_0)-\cos\theta(y-y_0)] & 1 \end{bmatrix}$$

Affine Transformations

- Possible to use matrix algebra to combine the whole transformation into one matrix multiplication.
- Step must then be applied to every point



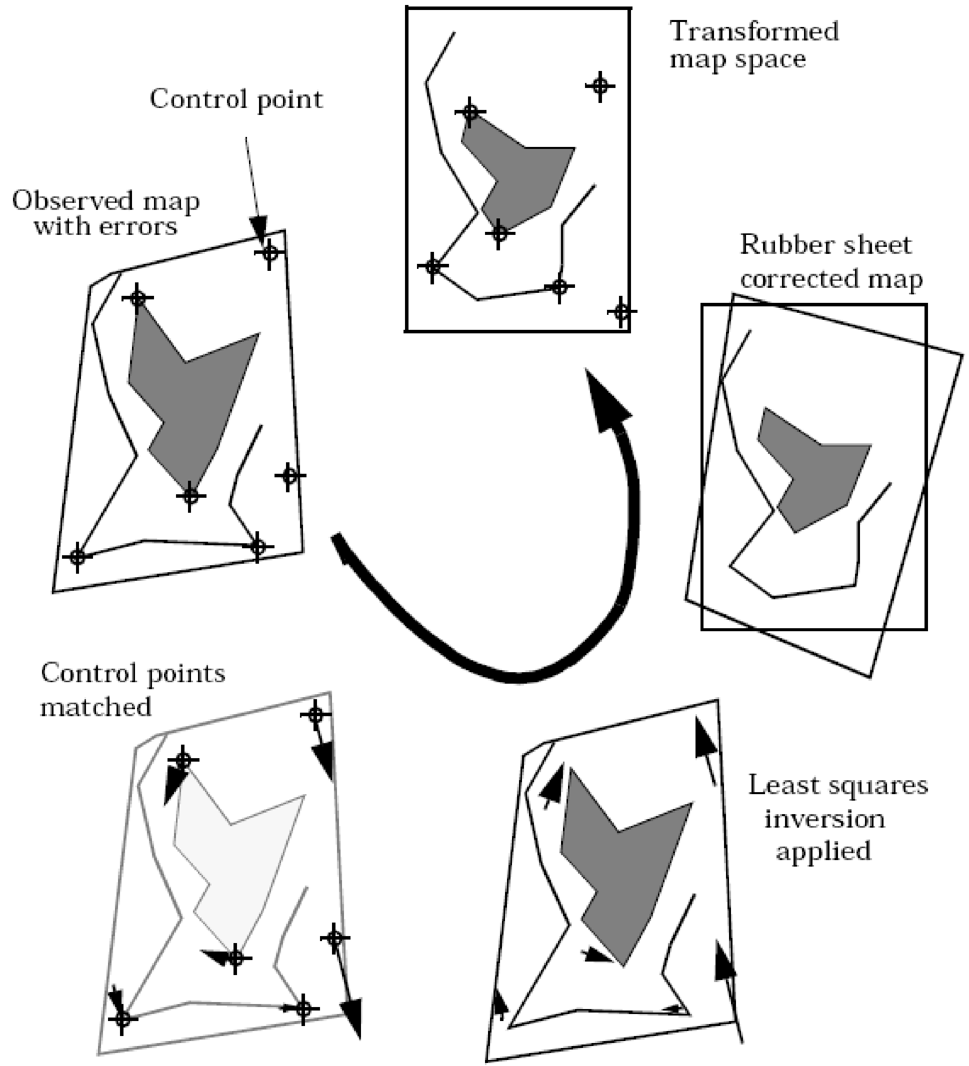
$$\begin{bmatrix} x & y & 1 \end{bmatrix} TRS = \begin{bmatrix} x' & y' & 1 \end{bmatrix}$$

$$x' = S_x[\cos\theta(x - x_0) - \sin\theta(y - y_0)]$$

$$y' = S_y[\sin\theta(x - x_0) + \cos\theta(y - y_0)]$$

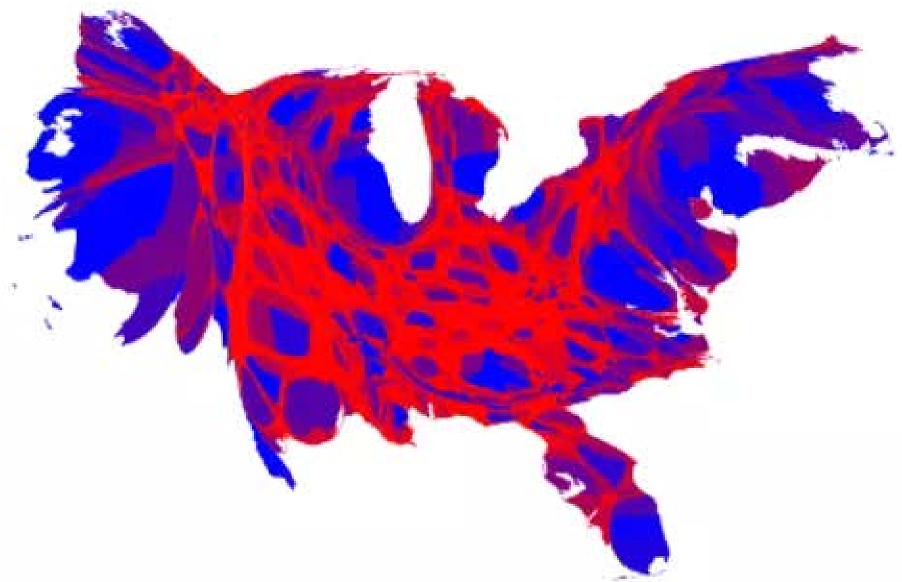
Statistical Space Transformations - Rubber Sheeting

- Select points in two geometries that match
- Suitable points are targets, e.g. road intersections, runways etc
- Use least squares transformation to fit image to map
- Involves tolerance and error distribution
- [x y] = T [u v] then applied to all pixels
- May require resampling to higher or lower density



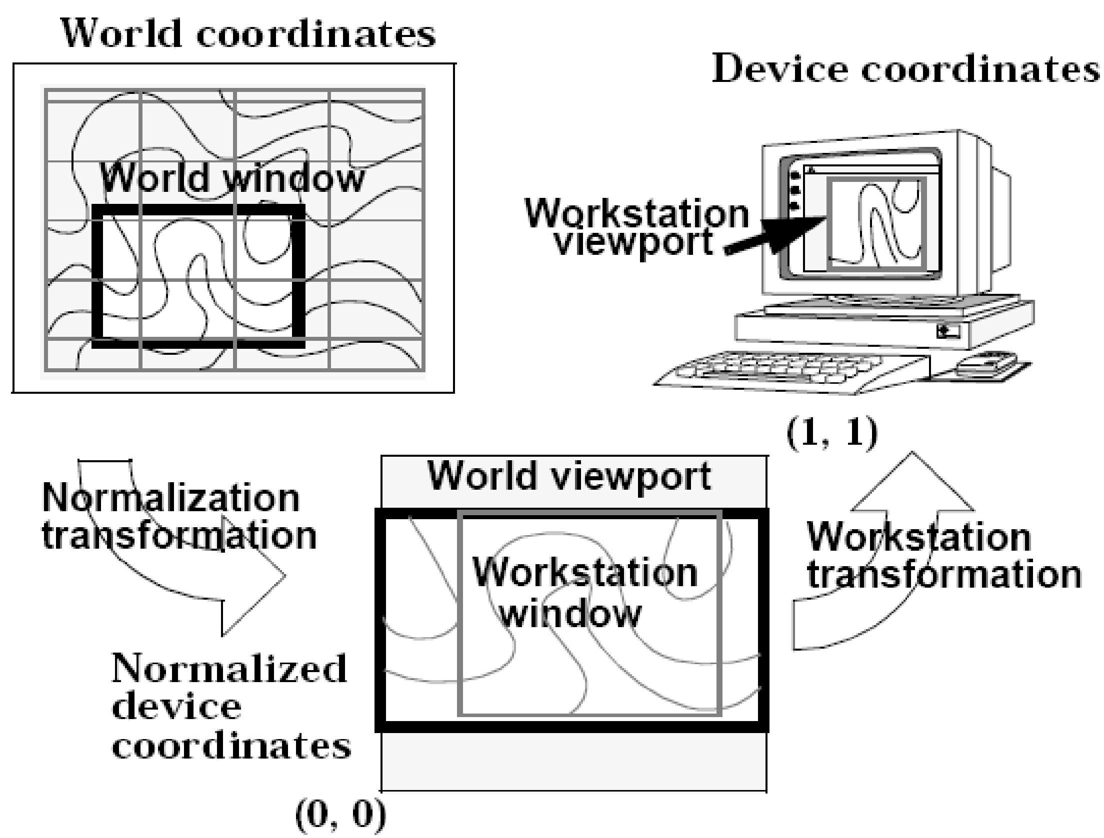
Statistical Space Transformations - Cartograms

- also known as value-by-area maps and varivalent projections (Tobler, 1986)
- Deliberate distortion of geometry to new "space"
- Type of non-invertible map projection



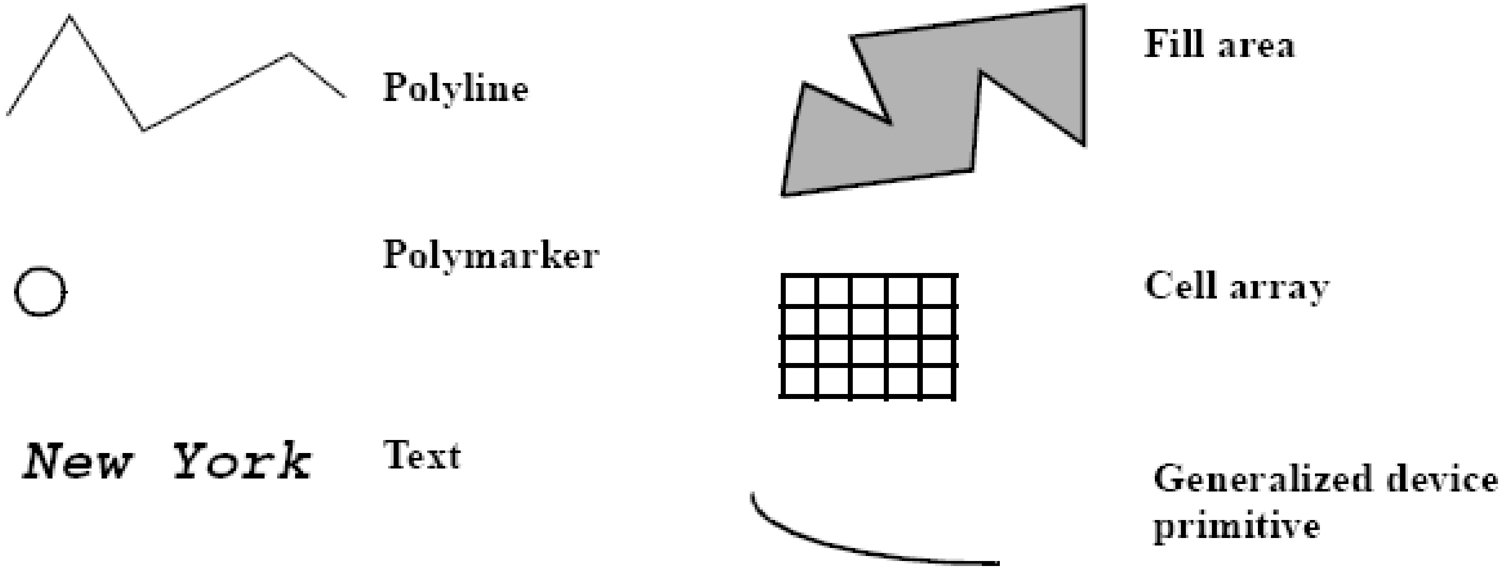
Symbolization Transformations

- Screen coordinates are often reduced to a "standard" device – Normalization Transformation
- Standard Device display dimensions are (0,0) to (1,1)
- World Coordinates-> Normalized Device Coordinates > Device Coordinates



Drawing Objects: Primitives

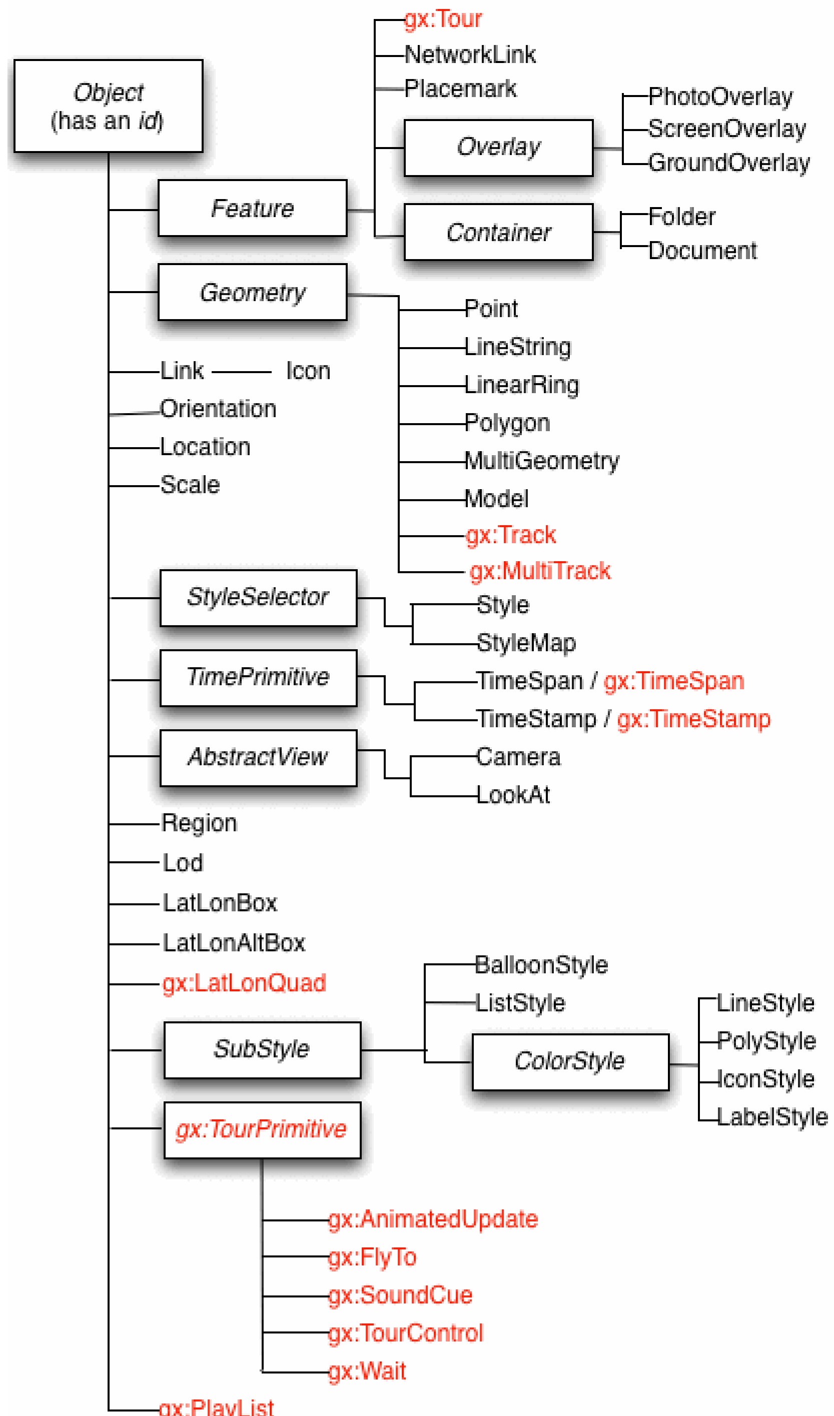
- Most use model of primitives and attributes
- The Graphical Kernel System (GKS) has six primitives, each has multiple attributes.



GEOJSON
Objects : Point, LineString, Polygon, MultiPoint, MultiLineString, and MultiPolygon


```
{
  "type": "Feature",
  "geometry": {
    "type": "Point",
    "coordinates": [125.6, 10.1]
  },
  "properties": {
    "name": "Dinagat Islands"
  }
}
```

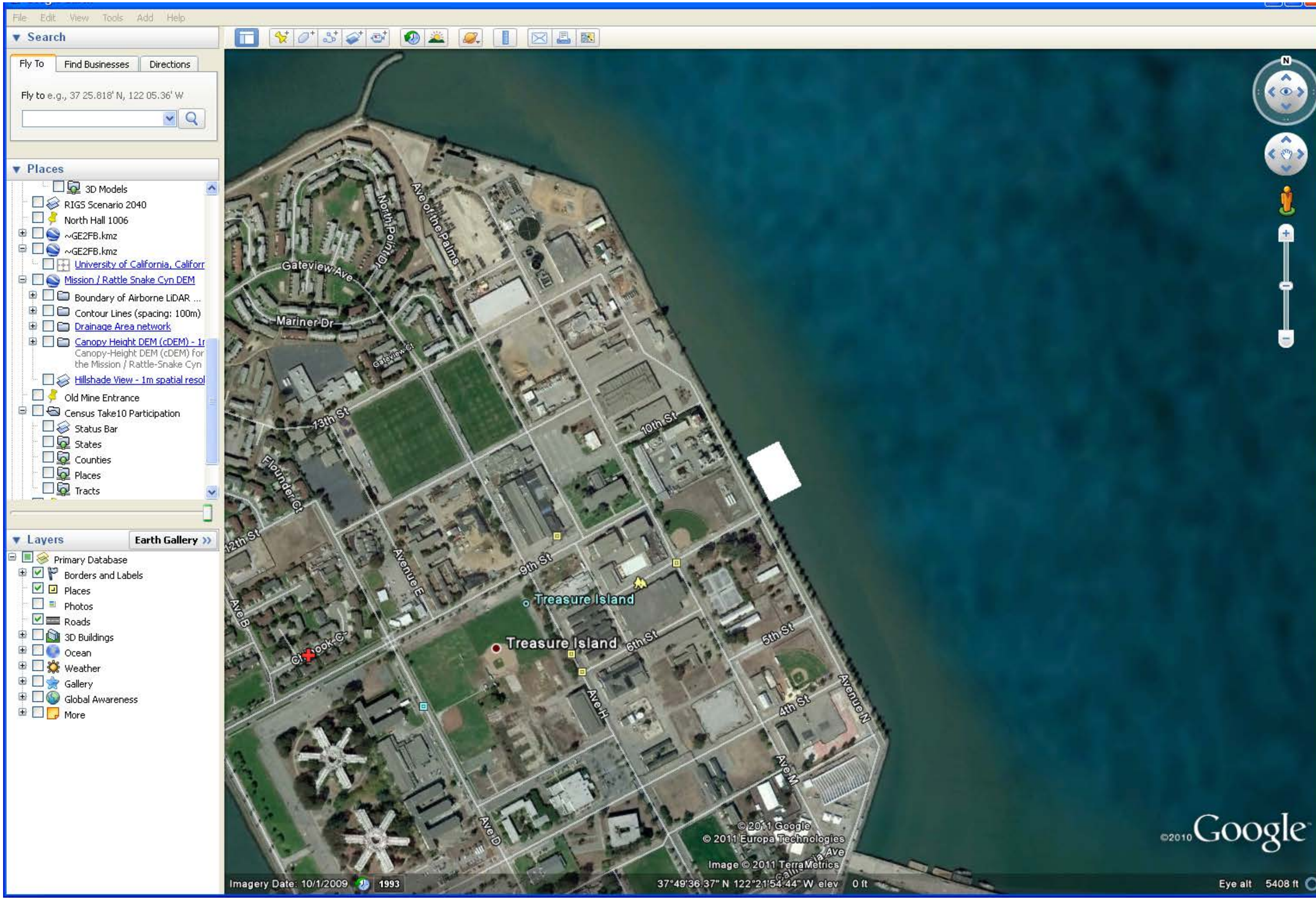
GoogleEarth KML Objects



For example, a LinearRing in KML

- <?xml version="1.0" encoding="UTF-8"?>
  <kml xmlns="http://www.opengis.net/kml/2.2">
  <Placemark>
  <name>LinearRing.kml</name>
  <Polygon>
  <outerBoundaryIs>
  <LinearRing>
  <coordinates> -122.365662,37.826988,0 -122.365202,37.826302,0 -122.364581,37.82655,0 -122.365038,37.827237,0 -122.365662,37.826988,0 </coordinates>
  </LinearRing>
  </outerBoundaryIs>
  </Polygon>
  </Placemark>
  </kml>

Launch GE, bring in KML file



Summary

- Geometry can be crisp or vague, raster or vector
- Operations can work even when information is vague
- Algorithms can apply basic transformations for essential measurements, such as clustering, length, area, density
- Basic features can be ingested into simple programming language data structures
- Functions then can measure length, mean center, dispersal, area
- Can also transform geometry in non-invertible ways, e.g. cartograms
- Last transformation is into normalized device coordinates, then device/viewport coordinates
- Gave examples of simple objects and their rendering objects