Analytical and Computer Cartography

Lecture 2:
Review: Map Projections
Transformations in Mapping

• **T1: True earth to globe**
  - Lengths scaled by the Representative Fraction e.g. 1:1M
  - Real world objects become symbols (representations) e.g. coast to a blue line
  - Need to choose earth model (Datum)

• **T2: 3D earth to flat map**
  - Map projection transformation
  - Inherent distortion!
The North Korean Missile Threat

Extrema

Source: http://www.andrewt.net/blog/posts/fun-with-the-mercator-projection/
The graticule

- **Graticule**: The latitude and longitude grid drawn on a map or globe. The angle at which the graticule meets is the best first indicator of what projection has been used for the map.
The graticule shows increments of latitude and longitude.
Geographic Coordinates
For example: ArcGIS PRJ file

GEOGCS["Geographic Coordinate System",
DATUM["WGS84",
SPHEROID
["GRS1980",6378137,298.2572220960423]],
PRIMEM["Greenwich",0],
UNIT["degree",0.0174532925199433]]
The longitude problem

• Latitude has as a point of origin (the equator) and ends at the poles: distinct locations
• It can be measured directly by observing the sun or the stars
• Can also measure the tropics of Cancer and Capricorn
• Not so for longitude
• Hard to measure
At first, each country had its own prime meridian.
The longitude solution

John Harrison’s Chronometer Won longitude Prize 1773

Longitude

Dava Sobel
The International Meridian Conference (1884: Washington DC)

“That it is the opinion of this Congress that it is desirable to adopt a single prime meridian for all nations, in place of the multiplicity of initial meridians which now exist.”

“That the Conference proposes to the Governments here represented the adoption of the meridian passing through the center of the transit instrument at the Observatory of Greenwich as the initial meridian for longitude.”

“That from this meridian longitude shall be counted in two directions up to 180 degrees, east longitude being plus and west longitude minus.”
The Prime Meridian (1884)
Now, on to projections

Round

Flat
Advantages of projections

• Globes are hard to store and use
• Globes cannot show the whole world at once, at equal visual range
• Projection can be optimized to minimize distortion by region
• Computer screens are flat
• Projected map can be used for thematic mapping
Projection properties

• Form: cylindrical, planar, conic
• Aspect: Equatorial, polar, oblique
• Tangency: Tangent vs. Secant
• Distortion: Conformal vs. Equivalent
• Interrupted vs. Continuous
• Analog vs. mathematical
• Shape: Ellipse with flat poles preferred
• Purpose
• Extent: World vs. Country
• Mash-ups
Classes of Projections

A CLASSIFICATION OF MAP PROJECTIONS

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The desire for a classification of map projections stems from the fact that an infinite number of distinct projections are possible. Hence, the fundamental problem in classifying map projections is the partitioning of this infinite set into a comprehensible and useful finite number of all-inclusive and preferably non-overlapping classes.

Several classifications of map projections are to be found in the cartographic literature. The advantages and disadvantages of each, of course, depend on the purpose, just as the properties by which classes are to be distinguished depend on the purpose. The classification based on geometric models (perspectivities) separating projections into conic, cylindrical, planar, polyconic, polyconical, etc., is convenient and is often used. The major shortcoming of this system is that it is not all-inclusive. Another most important method of transformations and Euclidean geometry, continuous transformations and topology, etc.

Still other classifications are based on the appearance of the meridians and parallels on the map, i.e., whether these consist of circles, ellipses, quartics, etc. Projections can also be classified according to the form of the equations; whether these are linear, algebraic, transcendental, and so forth. Maurer, in his study of map projections, attempts to partition 237 projections into classes and subclasses based on combinations of these systems. Several of the problems of projection classification are discussed by Maurer, and a most interesting but very involved Venn diagram is presented showing the overlap and interrelations of the various classes. Tissot is another who recognized fine distinctions and obtained an elaborate classification of map projections. More recently, classifications have been pre-
Thinking about projections

Cylindrical

Azimuthal

Conical
Map projections: Form

Azimuthal

Conical

Cylindrical
Map projections: Aspect

Cylindrical

Equatorial  Transverse  Oblique

Conical
Secant vs. Tangent
Secant Cylindrical Projections

Equatorial

Oblique

Transverse
Thinking about projections

- Mathematical (and digital)
- Input is long/lat pair \((\lambda, \phi)\) (lambda, phi)
- Output is \((x, y)\) coordinates on paper
Again, mathematically

\[ x' = Rs (\lambda - \lambda_0) \]

\[ y' = Rs \log \left( \tan \left[ \frac{\pi}{4} + \frac{\phi}{2} \right] \right) \]

We already know R and s
Equatorial Mercator
Mercator world map of 1569 Nova et Aucta Orbis Terrae Descriptio ad Usum Navigantium Emendate Accommodata ("New and more complete representation of the terrestrial globe properly adapted for use in navigation")
Mercator formula: Edward Wright 1599
"the only Fellow of Caius ever to be granted sabbatical leave in order to engage in piracy"
The West Wing map projections
episode  Season 2 Episode 16

• https://www.youtube.com/watch?v=vVX-PrBRtTY

• 892,048 views
During the projection transformation

- We have to distort the 3D earth
- We may reduce scale (generalize)
- We make break lines (interruptions)
- We distort scale
- We distort area
- We distort directions
No flat map can be both equivalent and conformal.
Classification by Independence

- \( x = f(\lambda) \) and \( y = f(\phi) \)
- \( x = f(\lambda, \phi) \) and \( y = f(\phi) \)
- \( x = f(\lambda, \phi) \) and \( y = f(\lambda, \phi) \)
- Partitioning into gores
- Invertible exactly or numerically
- Works for both sphere and ellipsoid
Mollweide: An equal area pseudocylindrical projection

$x$ and $y$ are not independent

$$x = \frac{2\sqrt{2}}{\pi} \lambda \cos\left(\frac{\theta}{2}\right)$$

$$y = \sqrt{2} \sin\left(\frac{\theta}{2}\right)$$

$$2\theta + \sin(2\theta) = \pi \sin(\phi)$$
The central meridian

- In the mathematical derivation, $x$ can map onto any meridian, by default we get $0^\circ$
- $x' = f (\lambda - \lambda_0)$
- We can change the center of the map
- Can even change the direction of the axis (by rotation)
Changing the central meridian
We can also choose the interruptions

Interrupted Mollweide
And mix projections together
Goode’s Homolosine=Mollweide (lobes) plus sinusoidal near the equator (J.P. Goode, 1923)
Many possible interruptions
Projection distortion: Tissot’s Indicatrix

Werner projection
Sinusoidal plus Tissot
Minimizing projection error

• Use a small area
• Choose a projection to limit error
• Compute expected error amount and location
• Use secant projection
• Customize: e.g. Chile
Distortion on the Secant Conic
Lambert conformal conic projection
Projections for the USA

Lambert conformal conic
USA on Mercator
Figure 3: A projection created by computing the weighted means of the Ginzburg VIII (35%) and the Eckert IV (65%) projections and scaling vertical coordinates by the factor 0.9.
GS-50 Snyder
Fun projections

Hammer retroazimuthal

Berghaus star
Bernhard Jenny: Adaptive Projections
goo.gl/ig4he6
Projection websites

- Geographers Craft: [http://www.colorado.edu/geography/gcraft/notes/mapproj/mapproj_f.html](http://www.colorado.edu/geography/gcraft/notes/mapproj/mapproj_f.html)
Miscellaneous

- Floating Ring [https://slvg.soe.ucsc.edu/map.html](https://slvg.soe.ucsc.edu/map.html)
- NOBLEED [http://noobeed.com/nb_ex_map_projection.htm](http://noobeed.com/nb_ex_map_projection.htm)
- R [https://cran.r-project.org/web/packages/mapproj/index.html](https://cran.r-project.org/web/packages/mapproj/index.html)
Software and Directories

• Projection selection tool http://projectionwizard.org/
• NASA Gprojector http://www.giss.nasa.gov/tools/gprojector/
• Hunter College page http://www.geo.hunter.cuny.edu/mp/software.html
• Map Projection gallery http://www.csiss.org/map-projections/
• Natural Earth http://www.shadedrelief.com/NE_proj/
• Mapthematics and Geocart3 https://www.mapthematics.com/
• Flex Projector http://www.flexprojector.com/
NASA: G.Projector—Global Map Projector (Raster only)
Code Libraries

- Matthew's Map Projection Software
  http://www.users.globalnet.co.uk/~arcus/mmps/
- PROJ.4 https://trac.osgeo.org/proj/
- GEOTRANS http://earth-info.nga.mil/GandG/geotrans/
- Java Map projection Library
  http://javamapprojlib.sourceforge.net/
Other Resources

• Geographers Craft
  http://www.colorado.edu/geography/gcraft/notes/mapproj/mapproj.html

• Gall-Peters
  https://sites.google.com/site/thepowerofcartography/gall-peters-map-projection

• MicroCAM: http://www.csiss.org/map-projections/microcam/
A Map Projections Information Clearinghouse?

Map Projections:

Azimuthal:
Azimuthal projections have the property that directions from a central point are preserved and therefore great circles through the central point are represented by straight lines on the map.

Conics:
Conics are any projection in which meridians are mapped to equally spaced lines radiating out from the apex and circles of latitude (parallels) are mapped to circular arcs centered on the apex.

Cylindrical:
Cylindrical are any projection in which meridians are mapped to equally spaced vertical lines and circles of latitude (parallels) are mapped to horizontal lines.
Map Projections: Summary (1)

• Cartographic transformations: scale, projection, symbolization
• Projections convert the surface of a 3D sphere (oid) into a plane
• There are 180 degrees of latitude and 360 of longitude
• It is easy to measure latitude, harder for longitude
• The prime meridian was standardized in 1884 at Greenwich
• Map projections can be created mechanically or mathematically
Map Projections : Summary (2)

- We can project onto a plane, a cylinder or a cone
- We can orient the projection as equatorial, oblique or transverse
- We can make the projection tangent or secant
- No flat map can be both equivalent and conformal
- Projection matters