

## Measurement

- Shape
- Miller
- Bunge
- Boyce-Clark
- Fourier measures
- Distribution
- Quadrat analysis
- Nearest neighbor analysis



## Nearest-Neighbor Analysis

- Unlike quadrat analysis uses distances between points as its basis
- The mean of the distance observed between each point and its nearest neighbor is compared with the expected mean distance that would occur if the distribution were random
- Also needs a reference area



## Distance to the nearest neighbor



## Advantages of Nearest Neighbor over Quadrat Analysis

- No quadrat size problem to be concerned with
- Takes distance into account
- Problems
- Related to the entire boundary size
- Must consider how to measure the boundary
- Arbitrary or some natural boundary
- May not consider a possible adjacent boundary



## Measurement

- Does A relate geographically to $B$
- Spatial Correspondence
- Coefficient of areal correspondence (Set theory, intersection / union
- Chi-square
- Yule's Q



## Example Test of Spatial Pattern

- Is there a relationship between the distribution of rainfall and the wheat yield in the area shown?
- NULL HYPOTHESIS: There is no relationship
- ALTERNATIVE HYPOTHESIS: There is a relationship


## Chi-square

- Make assumption that there is no relation between maps $A$ and $B$
- Compute statistics that allow the assumption to be rejected
- Chi-square is the sum of the (Observed value Expected value)^2/Expected value
- Can check value against table for actual likelihoods

| Calculating Chi-Squared : text p193 <br> Observed frequencies |  |  |  |
| :---: | :---: | :---: | :---: |
| Right: Wheat <br> Yield <br> Below: Rainfall High Low <br> High 8 2 <br> Total   <br> Low 5 13 <br> Total 13 15 |  |  |  |


| Calculating Chi-Squared : text p193 <br> Observed and Expected frequencies |  |  |  |
| :---: | :---: | :---: | :---: |
| Right: Wheat <br> Yeid <br> Below: Rainall High Low Total <br> High $8(5)$ $2(5)$ 10 <br> Low $5(8)$ $13(10)$ 18 <br> Total 13 15 28 |  |  |  |


| Calculating Chi-Squared : text p193 <br> Observed and Expected differences |  |  |  |
| :---: | :---: | :---: | :---: |
| Right: Wheat <br> Yield <br> Below: Rainfall High Low <br> High $8-5=3$ <br> $[9 / 5]$ $2-5=-3$ <br> $[9 / 5]$ <br> Low $5-8=-3$ <br> $[9 / 8]$ $13-10=3$ <br> $[9 / 10]$ <br> Total 13 15 |  |  |  |

## Chi-squared

$$
\text { Chi-square }=\Sigma\left[(O-E)^{2} / E\right]
$$

For the example $=9 / 5+9 / 5+9 / 8+9 / 10$

$$
1.8+1.8+1.125+0.9=5.625
$$

This value is then compared to a table of chi-squared to See if the value allows us to reject the null hypothesis that the observed values are not those expected based on proportions

## Chi-squared tables

| $v$ | 0.995 | 0.990 | 0.975 | 0.950 | 0.900 | 0.100 | 0.050 | 0.025 | 0.010 | 0.005 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 1 | 0.000 | 0.000 | 0.001 | 0.004 | 0.016 | 2.705 | 3.841 | 5.024 | 6.635 | 7.879 |
| 2 | 0.010 | 0.020 | 0.051 | 0.103 | 0.211 | 4.605 | 5.991 | 7.378 | 9.210 | 10.597 |
| 3 | 0.072 | 0.115 | 0.216 | 0.352 | 0.584 | 0.651 | 7.815 | 9.348 | 11.345 | 12.838 |
| 4 | 0.207 | 0.297 | 0.484 | 0.711 | 1.064 | 7.779 | 9.488 | 11.143 | 13.277 | 14.860 |
| 5 | 0.412 | 0.554 | 0.831 | 1.145 | 1.610 | 9.236 | 11.070 | 12.832 | 15.086 | 16.750 |
| 6 | 0.676 | 0.872 | 1.237 | 1.635 | 2.204 | 10.645 | 12.592 | 14.449 | 16.812 | 18.548 |
| 7 | 0.989 | 1.239 | 1.690 | 2.167 | 2.833 | 12.017 | 14.067 | 16.013 | 18.475 | 20.278 |
| 8 | 1344 | 1.646 | 2.180 | 2.733 | 3.490 | 13.362 | 15.507 | 17.535 | 20.090 | 21.955 |

Two by two table has four values so three degrees of freedom
Chi-squared of zero is no relationship.
Higher the value the stronger the relationship.

## Conclusion

- Using Chi-squared it is not possible to reject the NULL hypothesis that there is no relationship between wheat yield and high precipitation
- Test statistic fails, but only just
- Use another method?


## Yule's Q

- Divide world into high/low (2 classes)
- Overlay two maps gives four classes
- Count quadrats in the four classes in a $2 x$ 2 table (with cells a,b,c,d) (i.e. Observed only)
- $\mathrm{Q}=(\mathrm{ad}-\mathrm{bc}) /(\mathrm{ad}+\mathrm{bc})$
- Value lies between -1 and +1
- -1 is perfect inverse relationship, +1 is perfect positive

Calculating Yule's Q : text p193
Observed frequencies

| Right: Wheat <br> Yield <br> Below: Rainfall | High | Low | Total |
| :---: | :---: | :---: | :---: |
| High | 8 (a) | 2 (b) | 10 |
| Low | 5 (c) | 13 (d) | 18 |
| Total | 13 | 15 | 28 |

## Calculating Q

- $\mathrm{Q}=(\mathrm{ad}-\mathrm{bc}) /(\mathrm{ad}+\mathrm{bc})$
$(8 \times 13)-(2 \times 5)$
--------------------- = 94/114=0.82
$(8 \times 13)+(2 \times 5)$

Close to +1 , so can conclude that there is a positive relationship

## Testing spatial relationships

- Is there a relationship between geographical location and the price of gas?
- Are apartment rents less as distance from the campus increase?
- Are grocery store prices higher in poorer areas?
- Are the increased cancer death rates in a district caused by water contamination?
- Is there a relationship between hydrocarbon emissions and decreased upper atmosphere ozone in the polar regions?


## Summary

- Distributions can be quantified, using NNS or other means
- Maps can be compared using Chisquared, Yule's Q etc.
- Allows cartometry of higher order structures on maps: shape, distribution, arrangement and pattern

