World Population in a Grid of Spherical Quadrilaterals

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ABSTRACT

We report on a project that converted subnational population data to a raster of cells on the earth. We note that studies using satellites as collection devices yield results indexed by latitude and longitude. Thus it makes sense to assemble the terrestrial arrangement of people in a compatible manner. This alternative is explored here, using latitude/longitude quadrilaterals as bins for population information. This format also has considerable advantages for analytical studies. Ways of achieving the objective include, among others, simple centroid sorts, interpolation, or gridding of polygons. The results to date of putting world boundary coordinates together with estimates of the number of people are described. The estimated 1994 population of 219 countries, subdivided into 19,032 polygons, has been assigned to over six million five minute by five minute quadrilaterals covering the world. These results are available over the Internet. The grid extends from latitude 57°S to 72°N, and covers 360° of longitude. Just under 31% of the (1548 by 4320) grid cells are populated. The number of people in these countries is estimated to be 5.6 billion, spread over 132 million km^2 of land. Extensions needed include continuous updating, additional social variables, improved interpolation methods, correlation with global change studies, and more detailed information for some parts of the world.

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INTRODUCTION

Information on the world's population is usually provided on a national basis. But we know that countries are ephemeral phenomena, and administrative partitionings of a country are irrelevant to much scientific work. As an alternative scheme one might consider ecological zones rather than nation states, yet there is no agreement as to what these zones should be. By way of contrast global environmental studies using satellites as collection devices yield results indexed by latitude and longitude. Thus it makes sense to assemble information on the terrestrial arrangement of people in a compatible manner. A recent pilot study demonstrated some practical advantages of gridded population data (Clarke and Rhind, 1992), including reporting the potential impact of sea level rise on inhabitants of the coastal region of a Scandinavian country. The project described here extends the compilation to much of the entire Earth, using latitude/longitude quadrilaterals as bins for population information. In addition to its compatibility with environmental information this data format has considerable advantages for analytical studies, and the data

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can be converted to alternative formats with relative ease. The results of the study included an estimate of the population of over 6.5 million five minute by five minute quadrilaterals. This information was obtained by conversion from population censuses and boundary coordinates. The gridded population data are publicly available on the Internet in machine readable media for convenient use[1]. The complete report can be obtained from the National Center for Geographic Information and Analysis[2].

WHY

Many global environmental problems are human induced or affect human activities. One reason for the small number of studies that incorporate human factors in global change modelling is the lack of suitable population related data. In general terms these data encompass demographic variables (population numbers, components and dynamics), spatial arrangement, social factors (such as attitudes, education, income, health, etc.), and information relating to the economic activities of people. On the national level, many of these variables are collected and compiled on a regular basis by various agencies and institutions (e.g. the World Bank, World Resources Institute, United Nations, etc.), and macro-level economic and social analysis has been made easier by the increasing availability of these data in digital form either as a generic database or in a geographical information system (GIS) format.

On the sub-national level, however, efforts at compiling global socio-economic data have been scarce. Indeed, to date no consistent worldwide data-sets of socio-economic indicators for sub-national administrative units exist, other than the one we report on here. Partly this is due to the fact that data collection is rarely coordinated between countries. While great efforts have been made for national level data – for example, the national systems of product and income accounts designed by the UN Industrial Development Organization (UNIDO), or the standardized medical reporting encouraged by the World Health Organization (WHO) – there are no generic reporting schemes for the same information on a sub-national scale. A second factor is that the collection of socio-economic variables is a process which is often far more tedious than the collection of physical data. For instance, large scale monitoring techniques such as remote sensing have no equivalent in the human domain. Data collection therefore has to rely on expensive and time-consuming censuses (for a comprehensive overview of these problems, see Clarke and Rhind, 1992). Yet the range of potential applications for global socio-economic data is clearly very wide. The lack of consistent population related information was seen as one of the major impediments for strategic planning and targeting of development strategies for large areas at the recent meeting of the Coordinative Group for International Agricultural Research (CGIAR) and Global Resource Information Database of the UN Environment Programme (UNEP/GRID) on digital data requirements in the agricultural research community (Martin, 1992). socio-economic data on a regional or a continental level are used by the CGIAR centres for the selection of representative survey sample sites, for the generalization of relationships determined in micro level analysis to larger areas, and for the forecasting of future target areas. These efforts are all based on the relationship of population pressure to food availability. Further applications can be anticipated in global-level migration studies, large scale epidemiological research (e.g. Thomas, 1990, 1992), or global economic modelling (Hickman, 1983).

Many of the human impacts on the natural environment are directly related to the number and arrangement of people and the associated industrial metabolism. The project reported on here has attempted to address a small part of this problem by collecting population totals at a modest level of resolution, but for the entire Earth. There are two primary advantages to having global demographic data referenced to latitude and longitude. The first of these is the analytical capability which it provides. When given in this form, geographic observations are rendered susceptible to the same kinds of objective analyses as are performed on time series data, as pointed out by the Swedish geographer Hagerstrand in 1955. Viewing spatial data as a kind of (two-dimensional) regional series, we may undertake a variety of analyses. For example, population pressure might be defined analytically as the absolute value of the gradient of population density, and this is easily computed from gridded data (Muehrcke, 1966). Thus analytical and simulation models of several types are more conveniently formulated when one works with cellular geography.

The second advantage is the ability to relate demographic data to other global data. The focus of global change studies is often to link environmental data to socio-economic variables for the study of human induced degradation processes. A complement to this is the study of the social impact of environmental degradation. In many instances, such factors (e.g. vegetation cover or climatic variables) are stored in gridded form. Global monitoring systems using satellite sensors report data by pixels – small, usually square, pieces of the terrestrial surface. Population data are typically available only for political units, generally countries and their political subdivisions. There is thus a mismatch between the two spatial data collection and reporting schemes. The higher the resolution of the satellite system, the greater the mismatch. This discrepancy renders more difficult the inclusion of population information into global modelling efforts. Minor additional advantage accrues to spherical quadrilaterals because these are essentially invariant collection boxes. They have a permanence that does not apply to political units, since these are likely to change. A corresponding disadvantage is that political action is generally only available on a nation state or regional basis. There have also been suggestions that ecological zones are more appropriate units of study than countries or other political units. Unfortunately there does not seem to be agreement on which set of ecological units to use. Such units are also likely to change over time, although probably not as rapidly as political units. Having the information in small latitude/longitude boxes allows easy reaggregation to arbitrary partionings of the Earth's surface, including ecological zones. Watersheds, or a hierarchy of watersheds, have also been proposed as a useful way of organizing information about the Earth.

The idea of using the Earth grid to assemble data is not particularly new. In 1851 Maury published a map of whale sightings by five degree quadrilaterals. Of course the oceans were not then partitioned politically, so he had to use this method. In Sweden, population counts in 1855, 1917 and 1965 are known to have been assembled in 10×10 km cells. Since the introduction of computerized geographical storage and processing systems in the 1960s the process has accelerated. Many of these systems work on a raster or grid basis and much local data is now collected in this manner. Minnesota, for example, has an information system which uses the square-like Public Land Survey system of Sections, Townships and Ranges for many observations. A recent British Census reported data by kilometre squares on their national grid (Rhind *et al*, 1980). The recent *Population Atlas of China* (Chengrui *et al*, 1987) contains several maps with population density shown within 1′15″ by 1′52.5″ quadrilaterals. Japanese social data are now sold, for marketing studies, by a commercial firm on the basis of 500×500 metre cells for a portion of the country.

We must also address the question of which demographic data. The typical demographer wishes population totals, further broken down by gender, age, occupation, education, and so on. Ideally the number of births, by age of mother, and the number of deaths, by age groups, should also be known. Completing the demographic equation requires knowledge of the number of immigrants and of emigrants, optimally also by age group and social, occupational and educational status. In the migration category, refugees should also be included. The economist would want information on energy consumption, capital investment, and so on. The political scientist is interested in attitudes. The geographer would want tables of trade or of information exchange. The list seems endless, and all of the information should pertain to the same time period. Our effort represents only a very modest beginning and one of the concerns is the necessity to introduce mechanisms by which other groups, (international, national, and/or private) can be induced to continue to maintain and improve the database established. In order that this be done it is necessary to overcome the reluctance of many nations to release information that may be embarrassing or considered of value in potential conflict, including economic domination. It is important to emphasize that the indirect environmental impacts of population are even more difficult to assess, with even greater data lacunae, than simple population quantities (Stern *et al*, 1992; National Research Council, 1994). Socio-political data and attitudes may be as important as socio-economic or demographic information.

The United Nations publishes a considerable quantity of statistics, and there are now several sources of machine-readable world population data. The number of countries in the world ranges from 165 to 311, depending on the source, and is partly a matter of definition (Freedman, 1993). Besides the United Nations and some other official (i.e. bureaucratic, governmental) public agencies, there are also several private (e.g. atlas publishers) and non-profit (Population Reference Bureau, etc.) organizations that have assembled such data. Generally they do this only by political unit or subunit, or in some cases by city. In addition there are several publications that focus on global problems, sponsored by the World Resources Institute, the Worldwatch Institute, the World Bank, and several others. These provide tabular data by country, and cite sources, including national and international documents. Recent books include Keyfitz and Flieger (1990), Willett (1988), the World Resources Institute (1990), the United Nations Development Programme (1990), and Zachariah and Vu (1988). A missing component in virtually all of these information assemblages is the geography, i.e. the precise specification of the coordinates of the boundaries of the units used. This is where geographers and geographical information and analysis systems can provide valuable assistance. Furthermore, the quality of the data given by these suppliers is extremely heterogeneous and of unknown validity. Internal strife often renders all census data inadequate. One need only recall what is happening in Rwanda, Yugoslavia, the former Soviet Union, Somalia, the Sudan and Angola to see the difficulty. On occasion, political considerations bias 'official' statistics to reflect favourably on the group in authority, or to justify action. At a later stage in this type of work, a necessary component should be studies to evaluate the problem of data accuracy. Sub-national data are usually the most difficult to obtain at a distance, and often may be less reliable.

HOW

Several methods exist to make compatible the disparate data bases. One can aggregate the environmental pixel data to the country or regional scale, which has the advantage that decisions are often made at this level, but this also often results in meaningless averages. Consider, as an example, one average temperature reading for the entire United States. Mixing the Alaskan climate with that of Florida hardly makes sense. Alternatively one can attempt to reallocate the national data to finer levels of resolution. This is what is described here, and has involved correspondence with many international and national agencies. We attempted to assemble global population counts (neglecting related parameters) and to convert them to spherical quadrilaterals: small latitude and longitude cells. Using one degree as the bins we would get 180×360 such cells. If only land areas are included, about 16,000 cells would result, each of approximately 7800 km^2. This contrasts with about 185 countries, with an average size of over 700,000 km^2. The resolution of these two schemes, the one degree cells or the countries, is very coarse for global modelling purposes. Finer data are generally available for political subdivisions of countries, although difficult to obtain outside of the individual nations. In the US there are 50 states, 3141 counties, and some 40,000 enumeration areas. Data are even available at the city block level. In France there are data for over 32,000 communes, in Switzerland there are 26 Cantons and 3029 Gemeinde, India has 337 districts, Mexico over 2400 Municipio, Italy records information on about 60 million people in 8090 municipalities, etc. The resolution also varies within different parts of each country. In the US the counties increase in size as one moves westward and across less densely occupied territory. The reason for this is a mixture of history, physical terrain, climate and technology. Of course, the resolution used by data collection agencies throughout the world to some extent reflects their perception of national problems, or population density, and is a type of adaptive grid. Some partial differential equation solvers use variable resolution grids, and these would seem to be possibilities for global modelling too. Forcing everything into a fixed grid carries some obvious disadvantages. But now it is time for some definitions.

For the purposes of the present study we define several levels of political partitioning. The nation state or country we refer to as the zero level. Then comes the first level, comparable in the US hierarchy to states. Below this is the second level, equivalent to US counties. In many countries, including the US, there are further levels. We have attempted to obtain data and boundary coordinates to only the second level, but have not been successful for the entire world. The second level for a country such as Switzerland consists of 3,029 Gemeinde, very nearly the same number as the 3142 units available for the US. Switzerland is a country of approximately 41,150 km^2 to the US at 9,282,840 km^2. The same number of units at the second level clearly yields a different level of spatial resolution in different parts of the world. We define the mean spatial resolution in kilometres by the following formula:

$$\text{Average resolution} = \sqrt{\frac{\text{Area (km}^2\text{)}}{\text{No. of observations}}} \quad (1)$$

Here the number of observations is the number of data collection units. Resolution measured in this manner is clearly the same as the length of the side of a square whose area is that of the average observation unit. Thus at the second level the Swiss data yield an average resolution of 3.69 km, whereas for the second level US counties one gets an average resolution of 54.45 km. A useful additional parameter would be the variance of this quantity for each set of units for which the resolution is calculated. For the sake of comparison the resolution (in metres) of a map is conveniently calculated by dividing the map scale by 2000. From the Nyquist sampling theorem we know that a spatial pattern can only be detected if it is greater in size than twice the resolution. This assumes data without error. When there is error in the data (is there ever no error?) then patterns or features can only be detected if they are several times this size. Using map scale again as a convenient analogy, the optimistically sized pattern that can be detected on a map is, in metres, the map scale divided by 1000. Thus on a map at a scale of 1:1,000,000 objects greater than 1 km in size might be detected. This neglects the fudging done by cartographers which confuses the situation.

The ultimate resolution needed for global population modelling is problem dependent, but a simple calculation shows that having data by 1° quadrilaterals of latitude and longitude is the equivalent of working with maps at an average scale of approximately 1/178,000,000. The mean quadrilateral size is then 7900 km^2. A sixty-fold reduction would result in quadrilaterals of one square minute of arc, roughly 1.5 km on a side. About 37 million cells of this resolution size would capture the world population, taking 3/4 of the Earth's surface as unoccupied water with Antarctica making up an additional 9% of the land area. This is a large amount of data and corresponds to a global map scale of approximately 1/3,000,000. A more ambitious effort would attempt 20" by 20" quadrilaterals, finer than the largest scale (1/1,000,000) world map for which a digital version now exists (Danko, 1992). A map coverage of the entire world at a scale of 1/250,000 would correspond to a demographic database of 3.4 trillion 5" by 5" quadrilaterals, with cells averaging roughly 125 m on a side. This is obviously impractical at present, and since human populations are mobile, hardly useful in any event. The average daily activity space of individuals is dependent upon culture, environment, social, and urban–rural status, but averages more than 15 km in western societies. This argues strongly for a macrogeographic approach, along the lines pioneered by Warntz (1965). In this paradigm populations are modulated by their distance apart, to yield a 'potential' field covering the earth. It seems reasonable that population potentials based on raw counts of people, or on births, deaths, migrations, or on income weighted or energy-usage weighted populations, could usefully be related to global models. This 'potential' approach is particularly useful when events depend on many neighbouring places, so that place-specific data capture only part of an impact. The population potentials (computed at each place as the sum of geographically discounted values) are more easily calculated from population data given by spherical quadrilaterals. Geodemographic models of the accounting type (i.e. future population at every geographic location equals current population plus births less deaths plus immigration less emigration) are also more easily modelled as systems of partial differential equations (Dorigo and Tobler, 1983) when each of these components is available in a system of regular tessellations rather than by political units. This approach makes use of the finite difference form of these equations, a relatively easy task on modern high speed computers.

At this time it is not possible to specify the optimum resolution for global demography. Tables and maps in the full report indicate what we have achieved. Of course, the resolution might have to vary in different parts of the world and for different purposes. Our method has been to collect world population data by political units from a variety of sources. We have then attempted to convert the population figures by one of the several possible methods detailed below. The basic data are the numbers of people, but the components of change (births, deaths, immigration, emigration), age distribution and energy consumption are also important; several of these are already available as estimates by country (or subunits). There are many problems with these data, synchronicity being the most obvious, and one might wish a time series for each latitude/longitude cell in order to do space–time modelling, simulations and animations. Data definition problems are also widespread.

Geographical locations can be specified in many ways. Different names given to the same place are referred to as aliases. The processing of geographical information often requires conversion between these aliases. A few of the geographic conversions which occur in practice are given in Table 1. Conversion of information given by political unit to a grid of latitude/longitude cells is only one such conversion task. Eventually one would expect nearly all of these conversions to be available on demand, using as input any data source with the output format tailored to the expected usage. It is also desirable to develop analysis methods that are invariant under alternative locational naming conventions (Tobler, 1990).

A simple method of converting national data to spherical quadrilaterals is to assign the average national density to all of the lat/lon cells covering the country, with simple prorating for partial cells. This yields a piecewise continuous function for the world, with abrupt jumps at the edges of countries, and a constant value within each country or unit. This method is clearly not appropriate for large countries, but must be used for such aggregates as gross national product (GNP), by definition only applying to the political unit, and a context variable for other properties. An alternative method of approximation assigns all of the data to the centroid of the country (or subunit) polygon. One then simply adds all of the values for those centroids that fall into each cell, a rather simple binning procedure.

Table 1. A few examples of geographic transformations.

Point>Point
(All aliases for the same place)
Street address < > State plane coordinates
Lat/Lon < > UTM coordinates
Digitizer x, y > Lat/Lon
Rectangular x, y < > Polar (distance, direction)
Point > Line
Highway Accident < > Highway segment
Point > Area
Street address > Census tract
Place name > Bingo coordinates
Lat/Lon < > Public Land Survey
Point > Field
Spot elevation < > Contour map
Dot map < > Density contours
Area > Point
Census tract < > Centroid
Public Land Survey < > Lat/Lon
Area > Line
Census tract > Street names
Political district < > Address list
Area > Area
Census tract < > School district
Country < > Grid cell
Political Unit < > Ecologic zone
Area > Field
County population < > Contour map
Census data > Choropleth map
Field > Point
Density contours < > Dot map
Elevation contours < > Spot height
Field > Area
Contours > Polygon total
Field > Field
Contour map < > Gradient map
Contour map < > Smoothed contours

For this, all that is needed is the centroid of each collection unit. This technique was used by Haaland and Heath (1974) with enumeration districts for the US and by Tobler (1992) for the entire world using five degrees of latitude and longitude. In this method there may result some cells with no population, and others with excessively large values. A light smoothing of this crude assignment helps to redistribute the population.

Another alternative is to interpolate from the known observations to the spherical quadrilaterals. Given the densities at centroids one can interpolate from these to a lattice or triangulation, making the usual assumptions needed in interpolation from data given at points. This is the method used by Bracken and Martin (1989, 1991) for areas in Great Britain. A better method is to use smooth pycnophylactic redistribution to reassign populations from one arbitrary set of regions to another (Tobler, 1975; Rylander, 1986; Park, 1990). This reassignment method has the advantage of always correctly summing to regional totals, using regional boundaries rather than centroids, and taking into account values in geographically adjacent areas. Coordinates for the boundaries of political units are more difficult to obtain than coordinates for centroids, but many such boundaries, at the national level, are now available in computerizedmap drawing systems (e.g. Taylor, 1989). The computational load for this method, involving a finite difference approximation to the solution of a partial differential equation, is greater than the summation methods previously cited, although the accuracy of the resulting interpolation should be better. Since this method has actually been used in this project it is summarized in the Appendix. An appendix in the full report gives details on how to adapt the published version (Tobler, 1979) to take into account the shape of the Earth, assumed spherical. In this approach it is not reasonable to use demographic rates; one must use counts (e.g. of people, births, deaths, migration, or diseases, etc). This also holds true for the ratio known as population density since, as with all such rates, the value obtained is highly dependent on the spatial extent of the chosen denominator. Rates can later be obtained by division of one cellular map (i.e. computer grid array) by another.

One can also make measurements on population density maps, where these are available (cf. Rhind *et al*, 1980; Liu, 1987). Using this method requires digitization of the isopleths followed by interpolation of the density values from the isopleths to a grid, then the use of a mathematical integration to get the 'volume' contained within each grid cell, taking into account the map projection used. This is rather like the 'cut and fill' computation used by the civil engineer when designing road construction. Urban populations on these maps are often represented by circles whose area is related to the population, and this can be inverted if class intervals (a form of discretization) have not been used. Population density maps are usually at a small scale, so that this technique can only result in a data file at a coarse resolution.

It is also well known that it is possible to make population estimates from aerial photographs and satellite images, although the accuracy of such estimates is disputed (Dureau *et al*., 1989). The procedures are well established for land use and agricultural crop estimates but also work for nucleated settlements (Tobler, 1975; Lee, 1989). Such imagery can also be used to mask empty districts. The method is best suited to high resolution estimates covering small areas, and is not well suited for complete world coverage in a short time; no one argues that birth and death rates, and similar important demographic components, can be obtained in this manner.

WORLD POPULATION AT MEDIUM RESOLUTION

As a first step towards a global sociopolitical–demographic–economic database indexed by latitude and longitude, we have produced a consistent data set of population numbers for the world. This required the collection of two major components. The first component was spatial boundary information describing administrative regions within the countries of the world, followed by assembly of the corresponding enumerations (or estimates) of the total population residing in each of these regions. In this initial attempt only information on the total number of people was collected. From these two components we have derived a gridded data set of population numbers for the globe at a modest resolution. We refer to this as modest or medium resolution because, as described earlier, finer data exist for several parts of the world. But we are not concerned here with data at the city block level, and we recognize that simply knowing the geographic arrangement of the world's population is not sufficient to determine its two-way interaction with the environment.

Information on 219 countries (or units; included are all major countries, but also small islands such as Aruba and Guam) has been obtained from various agencies and institutions. In some cases we have been able to assemble second level administrative unit boundaries but not second level populations. In other cases the converse situation holds. The exact details are in large tables in the full report. The primary product, an approximation to a globally continuous surface of population, required overlaying a grid on the polygons, followed by an assignment of the population to the cells of this grid. Essentially the problem is to distribute the published population count for a given administrative unit over the grid cells that fall within the unit, taking into account the population values in the adjacent units.

BOUNDARY DATA

The key to producing a spatial demographic database is the geographic boundaries that are used to reference socio-economic information. Virtually every country of the world has instituted an administrative hierarchy that partitions the country into units; often an attempt is made to have more or less homogeneous units, based on some criteria. The size of these units obviously varies greatly by country. Most first-level units in Zaire, for example, are still larger than the neighbouring countries of Rwanda and Burundi. For that reason, objective measures such as the average size of a unit at a given level (i.e. mean resolution as previously defined) or the number of people per unit in a country provide a basis for cross-national comparison. These measures are generally more appropriate than the position of the unit in the political hierarchy. Details for each country are given in the full report.

We attempted to obtain second-level boundaries, corresponding to counties or districts, except in the case of very small countries or islands. Owing to the limited time and resources available, the global demography project relied mostly on existing spatial boundary data sources. The primary producers of such data are national or supra-national public data collecting agencies (e.g. US Census Bureau, Eurostat), international institutions (e.g. UN-FAO, UNEP, CGIAR), universities engaged in research activities around the world, and private companies providing spatially referenced socio-economic data, chiefly for marketing applications. For a few countries for which no existing boundary data could be obtained, administrative boundaries were digitized at NCGIA using maps found in University of California libraries. We have included a detailed list of data sources by region in the full report.

The global database assembled at NCGIA contains 19,032 administrative units for some 219 countries of the world. The largest number of units were available for the US (3142), China (2422), Mexico (2402), and Brazil (2224). For very small countries, including some of the island states of the Caribbean and the South Pacific, no attempt was made to obtain subnational boundaries. Since the boundary information came from a wide range of sources, considerable effort had to be spent on rendering these data compatible in order to create a consistent global database. This typically involved coordinate transformation and projection change as the first step. Individual coverages for countries or groups of countries were merged into one of several regional coverages. The world map (Fig. 1) shows the boundaries of the administrative units used. This map also gives a good indication of the accuracy of the final result, since the redistribution of the total population within a unit will be more accurate if the units are very small.

In order to merge the individual regional coverages, the international boundaries needed to be matched to avoid sliver polygons or polygon overlaps caused by differences in data sources. These regional coverages formed the basis for setting up the population data and running the gridding routines. While this whole process is conceptually straightforward, several problems had to be dealt with, including the following.

- Many of the boundary data contained no (or

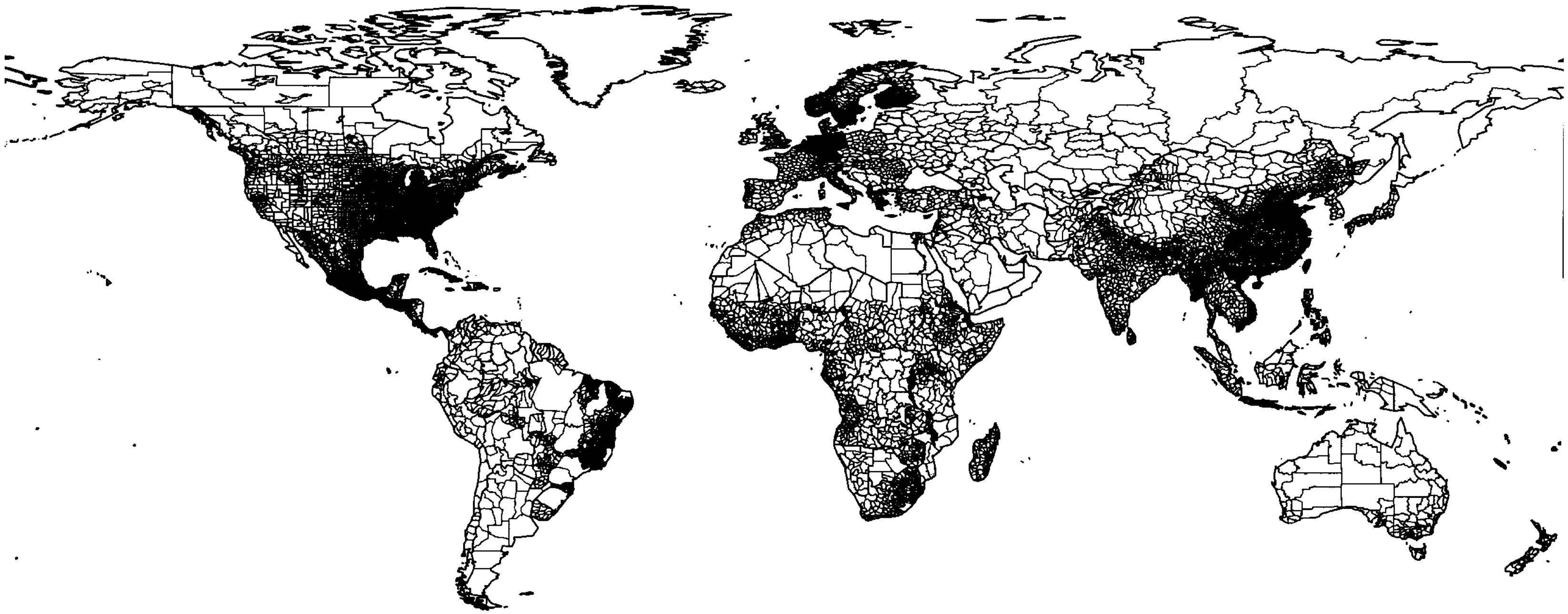

Figure 1. Map of 19,030 administrative districts. The blackened areas result from very fine units.

only minimal) documentation. Thus information on the scale and reference of the source maps, the time period to which the boundaries refer, or the accuracy of the information was often unavailable. For a number of boundaries, even the projection of the maps was not known. Administrative boundaries are rarely available on maps with precise geodetic referencing; particularly in developing countries, one needs to work with whatever is available, including 'cartoon maps', often page size, or hand-drawn sketch maps in census publications. Such information may be better than no information at all, and may indeed reflect the fact that at this level administrative boundaries are often not well defined. In cases where the boundaries were not spatially referenced, rubber sheeting using standard GIS tools was required to make the coverages compatible with the global database. The area of the polygonal units was obtained by converting the latitude and longitude coordinates of the boundary points to a local equal area map projection (by Carl Mollweide) with a subsequent polygon area computing routine. This introduces an error which is small as long as the units are small. The calculations used a sphere of radius 6,371.007178 km, equal in surface area to the World Geodetic System ellipsoid of 1984. The maximum differences between the sphere and ellipsoid for regions of this size do not exceed 0.5 km^2, and amount to less than one percent. Thus the spherical approximation is adequate for the present purpose. Summing all of the polygon areas yields an estimated 132,306,314 km^2 of occupied land, 25.94% of the world's surface area. These areal estimates may differ from published figures because some countries include or exclude internal water bodies, or have differing definitions of how large a lake must be to be excluded. Despite these source limitations we feel that the database is sufficiently accurate for global or regional scale applications, but not for high resolution applications which require more detailed boundary information.

- The source scale and the resolution at which administrative boundaries were digitized varied a great deal. Our estimate is that the boundary data sources ranged in scale from about 1:25,000 to about 1:5,000,000. This is an estimate because in many cases scale information was not indicated on the source maps.
- Due to the limited time available, no effort was made to make the international boundaries compatible with 'standard' global country databases such as the Digital Chart of the World at 1:1 million scale, or the World Boundary Database (WBDII) at 1:3 million scale. In practice, two neighbouring countries were merged into the same coverage and the resulting sliver polygons were eliminated using standard GIS tools. In retrospect this was a mistake, and in future it would be desirable to standardize the database by replacing all international boundaries with a standard data set. In this manner it would be possible to avoid problems encountered in combining nations or regions. An international effort may be required to achieve this. We have ignored problems of *de jure* versus *de facto* boundaries and of disputed boundaries.
- The spatial resolution of the administrative units acquired for different countries varies greatly. We normally used the most detailed level available to us. For a few countries more detail is available (e.g. block level data for the US), but the data volume involved then becomes impractically large. The average resolution by country is included in tables and on a PC diskette in the full report. Also listed there is the range in census unit sizes for each country, giving an impression of the resolution ranges. Taking into account the total occupied land area and the number of units, we calculate an average resolution for the world land area of 83.4 km, which is directly comparable to the size of the largest five minute spherical quadrilaterals used. A five minute quadrangle is aproximately 9.3 km wide at the equator with an area of 85.5 km^2. At 50° of latitude it is 5.95 km wide with an area of 55.3 km^2.
- Boundary data for several countries (e.g., Australia, Canada, Community of Independent States, Mexico, New Zealand) came from commercial sources or from public agencies that put restrictions on further distribution of the data. This boundary information is copyrighted and consequently cannot be distributed freely. The database documentation lists all sources and contact addresses for these data sets. Interested parties can contact these organizations directly.

POPULATION DATA

For each of the administrative units we attempted to obtain a recent estimate of the resident number of people. For many analytical purposes it would be useful to have other demographic indicators as well (e.g. population by age and sex, rural versus urban, and economic involvement, or time series data of population figures). The compilation of such data, in particular the effort involved in ensuring cross-national definitional compatibility, was beyond the scope of the project. In several cases the agencies from whom population and/or boundary data were obtained do also have these sorts of additional demographic or economic data.

Population figures were typically taken from the latest available census. A list of censuses by country is compiled by the UN Statistical Division and continually updated. Additional sources of demographic data are civil registration systems or estimates based on sample surveys. For this project we used the most recent enumerated or estimated population figure released by the national statistical offices, and published in official census publications, yearbooks or gazetteers, that we were able to locate given our resource limitations.

For most countries, data were available from the 1980s or early 1990s. Often there is a considerable time lag between a census and public dissemination of the data. Thus the most recently collected figures may not yet have been published. All limitations associated with published demographic data, which are well known in demographic research (e.g. Chapter 3 of UN Population Division, 1993), obviously apply to our collection of population figures as well. The main problem faced in compiling the data relates to synchronicity. Firstly, census dates vary. In our database, the reference years range from 1979 to 1994. We decided to derive estimates for 1994 for administrative units using a standard growth model based on average annual growth rates by country provided by the United Nations Population Division (1993):

$$P_t = P_c \exp\left(\sum_{i=c}^{t-1} \frac{r_i}{100}\right) \qquad (2)$$

(see Rogers, 1985) where P_t and P_c are the regional populations in the target year (1994) and in the census year respectively, and r_i is the annual percentage growth rate in year i for the particular country. While this method is admittedly simplistic (it assumes uniform growth in all regions of a country), the error introduced should be small for most countries that have had a recent census. Furthermore, the error is likely to be minor in relation to the overall uncertainty associated with census information. Ideally we would like to use historical census data to derive growth rates that are specific to each administrative unit in order to capture recent urbanization processes, migration, etc. This was not possible with our limited resources. We have however compared our estimates with the latest UN estimates and the disparity between these two suggests the magnitude of the possible errors; as another comparison the 1995 *World Almanac* lists 1994 estimates of the population of the major countries of the world. These numbers, which differ from the two cited lists, were allegedly obtained from the US Bureau of the Census – see the full report for details. The total 1994 population estimated for our 19,032 units is 5,617,519,139 people. This works out to a world average of 42.5 persons per square km^2 of occupied land.

The second problem related to synchronicity derives from the fact that boundary and population data often did not come from the same source. Administrative boundaries change frequently, and in some cases, boundaries were only available for a previous census, and did not match the units for which population figures were defined. In other cases, the opposite was true. In the few cases where a mismatch occurred, the population data had to be aggregated or disaggregated using simple proportional areal weighting, although more complex areal interpolation techniques could be applied (e.g. Goodchild *et al.*, 1993). Attempting to compile geographical time series exacerbates the problem of boundary consistency and it may be necessary to use pycnophylactic areal redistribution between changed zonations to make the data compatible over time.

GRIDS

The primary product, an approximation to a globally continuous surface of population, required the gridding of the polygons followed by an assignment of the population to the cells of this grid. Essentially the problem is to convert the published population count for a given administrative unit into a population within the grid cells that cover the unit. This grid assignment is followed by a smoothing to overcome the jumps between boundaries of the administrative units by taking into account the population values in the adjacent units. The smoothing technique used is the smooth pycnophylactic reallocation described earlier in this paper, modified to take into account the spherical shape of the planet. The routine has been implemented in the C programming language and is executed from within an Arc/Info macro. Thus, Arc/Info is used to do the basic tasks like the polygon to raster conversion and the establishment of a list of population totals by region. The Arc/Info export format has been used to prepare and deliver the compressed files. Copyright restrictions do not permit us to release all of the actual boundary coordinates.

While the preparation and interpolation of the raster data is conceptually fairly simple, several logistical problems arose as we implemented the gridding process in Arc/Info's GRID program. The two major problems – in the sense that they required a great deal of time – were:

(1) very small administrative units disappeared in the polygon to raster process, and were not assigned to any grid cell. Some of these had a large population; and
(2) gaps appeared between regional grids due to small differences in the country boundaries obtained from different data sources.

The reason for the omission of small administrative units is attributable to the discretizing effect which occurs in any polygon to raster procedure. It is generally impractical to choose a grid sufficiently fine to avoid this. If more than one polygon falls into a cell, the value for the polygon which takes up the largest areal proportion of the cell is taken as the value for the entire cell. The remaining polygons are ignored. All vector polygon to raster programs will have this problem. It is important that it is stated when small polygons are missed in this manner.

In the present instance the areas of the five minute quadrilaterals range from 85 km^2 at the equator to 22 km^2 at the extreme latitudes of our data. But Macao is only 9 km^2 in size, and in regions with fairly high resolution administrative units (e.g. some of North America and especially Central America), several census units are under 20 km^2 in area. These units do not occupy enough area in a cell for their value to be included in the gridded data. Since many of these units have high populations (i.e. the urban districts), it was necessary to derive a method for including them in the grids.

The first step is to grid the vector data at a smaller cell size. We chose 2.5 minutes for this process, twice the final resolution, because this size was small enough to include most of the units omitted at five minutes but still large enough so as not to be unwieldy. For example, in North America, 35 units are omitted at the five minute cell size but only 15 units are omitted at the 2.5 minute cell size. It is then possible to create a separate grid containing only the missing units. In these grids each of the units occupied only a part of a cell, but the data are taken to apply to the entire cell. The populations of the small units are then added to the main grid so that all of the population for every administrative unit is included. This process is fairly simple using the relational database capabilities of the program used. Since the average resolution of the original polygon units was close to 85 km^2 the 2.5 minute grid size was considered unrealistically precise and not appropriate for the final distribution.

Many gaps between regions occurred because the administrative boundaries came from a wide range of sources. When we assembled the countries into regions, or received data for an entire region, we addressed these mismatches. However, when we converted the polygons to grids for several regions at once, we had to ensure that there were no gaps or slivers when adjacent regions were joined. We identified where the boundaries did not match, and where empty or overlapping cells occurred. A certain amount of minor fudging was then necessary in these problem areas. The process seems simple but in fact required several tedious steps, detailed in the full report.

SURFACES

In the previous section we described some difficulties in the gridding process. Here we note some difficulties in the reallocation process used to create the spatially continuous population surface. For the smoothed redistribution, the pycnophylactic program performed the interpolative smoothing at the 2.5 minute resolution; the program then aggregated the resulting populations to the five minute cell size. The results of this pycnophylactic interpolation formed the final smoothed product.The difference between the smoothed and unsmoothed surfaces show on the maps for West Africa (Fig. 2) and southeast Asia (Fig. 3). Several limitations of the source information are apparent from these images. The resolution of the administrative units used is obviously dependent on availability. Consequently the resolution sometimes varies dramatically between, as well as within, countries; for example see Brazil on the world map. We tried to address this problem by varying the number of iterations in the smoothing reallocation algorithm, using the average resolution of the computational regions and a smoothing convergence criterion. The number of iterations varied from only 30 for Central America to 100 for the former Soviet Union and the Middle East, where the units are larger. Within many regions the variance in resolution was still too large to overcome this problem completely. In Brazil very large units are adjacent to many small ones and no single iteration count was able to yield entirely satisfactory results. A modification of the algorithm to be adaptive in the amount of smoothing might be more successful. This might require a grid size adapted to the resolution, with a corresponding small change in the computer program, followed by reaggregation to a uniform grid. A further unanticipated result appeared in the population reallocation method. When the units are relatively homogeneous in resolution the method works well and is appropriate. In cases where small units abut large ones, or the population density changes drastically from one administrative unit to the next, certain anomalies occur. For example, where urban districts are located next to rural areas, the objective of achieving maximum smoothness will 'pull' nearly all of the population of the rural region tightly towards the boundary with the urban region. This is apparent in the maps (e.g. in Sumatra and central India), but also occurred in Australia where it is probably very realistic. The problem seems exacerbated in areas where the resolution variance is greatest.

The smoothing redistribution used to create the population surface maintains the correct population totals for each polygon and includes the geographic insight that neighbouring places have similar values. But it is rather mechanical and, as previously noted, also has the curious consequence of pulling virtually all of the population of a sparsely populated region to the border adjacent to a densely populated unit, leaving the remainder of the sparsely populated unit completely devoid of population. Such an effect may represent the true situation, but it looks strange on density maps – see, for example, the region around Hyderabad in central India (Fig. 4). Thus the method used for the spatial disaggregation of the population needs to be improved. One approach is to obtain further guidance from geographic theory: for example, the cited interaction potential of population and the literature on urban density gradients. Thus we believe that the reallocation can be greatly improved by using additional data on factors that influence the population arrangement. These factors would include the location and size of towns and cities, major infrastructure (roads, railroads), and natural features (rivers, uninhabitable areas, protected and wilderness areas, etc.). The feasibility of this approach has been shown in a study by UNEP/GRID in which a population density surface for the African continent was produced and subsequently published in UNEP's *Global Atlas of Desertification*; see Deichmann and Eklundh (1991) for a description of the methodology. In that project, efficient use was made of the capabilities of GIS to integrate heterogeneous data within a consistent framework. This approach was subsequently adopted and modified for producing experimental population maps of the Baltic States at 1 km^2 resolution (Sweitzer and Langaas, 1994) and for the European continent at 10′ resolution (Veldhuizen *et al.*, 1995). To apply this to the entire world requires overlays of the infrastructure and physical geography, and these have not been used in the present project. But this work could now be facilitated by the availability of a consistent global base map, namely the Digital Chart of the World or the digital world topography (at 5′ by 5′), both now available on compact discs (CDs). Taking into account secondary information is referred to in

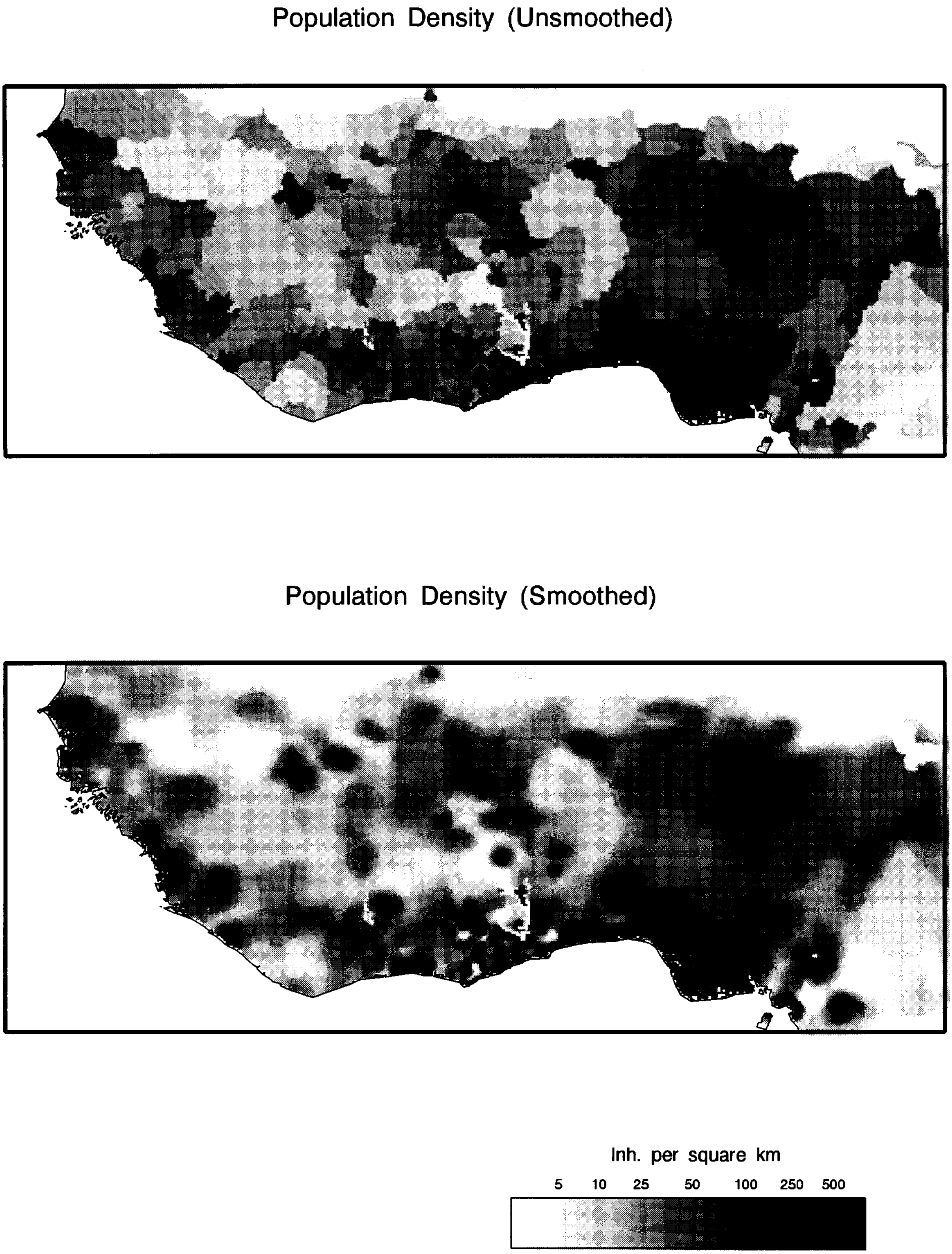

Figure 2. Unsmoothed and smoothed population density in West Africa.

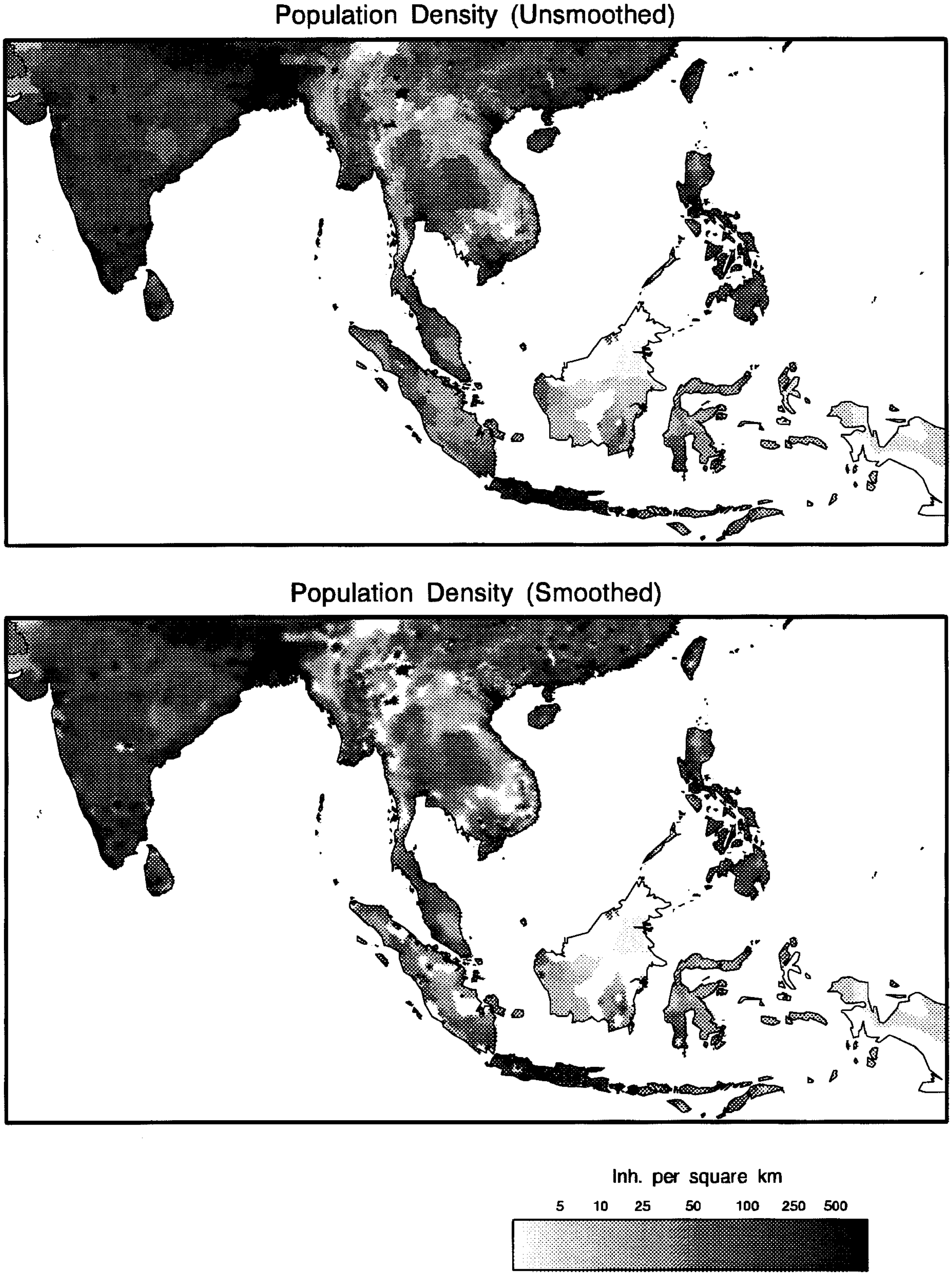

Figure 3. Unsmoothed and smoothed population density in southeast Asia.

some interpolation literature as co-Kriging, and is an obvious approach as long as the additional information is available and its relationship to the primary variable is known. Thus satellites could be used to delineate areas of sparse population, or sensors capable of recording visible emissions at night (used judiciously) may offer a means of detecting, correcting or calibrating global population concentrations.

RESULTS

The final useful products from the several processing steps include:

Figure 4. Smoothed population in India, obtained by pycnophylatic interpolation. Five minute cell size map using 150 iterations of smoothing.

- A piecewise continuous population surface, without any smoothing (i.e. a constant population within each 5′ by 5′ quadrilateral) taking into account the Earth's shape and the convergence of the meridians. Cells nearer the equator can have a slightly larger population because of the Earth's shape. This gives a constant density to the unit.
- Smoothly reallocated population values, by $5' \times 5'$ quadrilateral.
- Gridded population density surfaces, obtained by dividing the value in each cell in the preceding two files by the spherical area, in km^2. Deflation of the true density values may occur in this process since the cells could contain ocean or other water bodies as well as occupied land. No correction for this was applied.
- The administrative unit name(s) for each grid cell. This is particularly useful if one wishes to incorporate additional data by subdivisions of a country and automatically assign the values to the quadrilaterals, thus avoiding a new interpolation. Financial or nutritional information could thus be assigned to the five minute grid from sub-national observations.
- The centroid coordinates, in latitude and longitude, for all of the 19,032 polygons used. This form of the data does not allow recovery of the copyrighted administrative unit boundaries with any precision, but is useful for more generalized tasks. Included with the coordinates are the estimated population and calculated area for each polygon, and the administrative unit name(s). Population density at spot locations can therefore be calculated. This is a small data set of 19,032 records (928 kbytes) and has been put onto an IBM PC diskette accompanying the original report, along with a program to display the points on a PC screen. The display can be used to investigate the resolution of the assembled data in greater detail.
- Several regional maps are given in the full report which further illustrate our results. The more important result is the digital data which can be used for analysis, correlation with other quadrilaterized data,for analytical modeling, or from which the user can produce alternate outputs and aggregations. It is easy, for example, to summarize and cumulate population by latitude or longitude strip, or to produce population maps at medium scales.

Gridding the entire world by five minutes of latitude and longitude yields a raster of $180 \times 12 \times 360 \times 12$ which is 2160 rows by 4320 columns, or 9,331,200 cells. But less than a third of the Earth is people-occupied land. Consequently we have truncated our grid at 72°N and 57°S since there are essentially no permanent residents at higher latitudes. The resulting grid is 1548 rows by 4320 columns of four bytes each (before compression). Approximately 30.9% of the 6,687,360 cells contain people. The final global rasters range in size from 35.6 Mb to 46.3 Mb before compression. Since a good proportion of the world is unoccupied, compression reduces the storage volume by over 89%.

CONCLUSION AND EXTENSIONS

The global gridded population information described here should be considered an exploratory reconnaissance designed to stimulate interest worldwide. Now that this preliminary but basic database has been established it can be widely circulated and its usefulness determined. It will hopefully lead to follow-up projects aimed at improving and expanding the current effort. Improvements can be made by incorporating more up-to-date information as it becomes available, and by filling in areas for which we have not been able to obtain sufficient detail. Higher resolution data are desirable for several parts of the world, particularly the former Soviet Union where we have obtained only oblast data, and several countries in South America. A critical component is a maintenance organization. One model would be to have local authorities or researchers add data to the 5′ by 5′ cells and to keep them up to date, using a worldwide system of social observatories.

We describe here several extensions which we have considered. These include error estimation, the incorporation of urban populations, extension to include movement modelling, and data compaction schemes. We do not comment on the need for correlation with other items of global concern such as environmental degradation: etc. this need is rather obvious.

Data-Set Comparisons

Given the difficulties of obtaining population data at sub-national levels in many countries of the world, we have made no attempt to estimate the magnitude of the likely errors. But during the course of this study we have discovered a few alternative collections of demographic data which might help alleviate some of these concerns. We believe that comparison of these with those we have obtained might shed light on the reliability of our methodology. In particular, the Center for International Research in the US Bureau of the Census has recently made available their estimates of population by 10′ by 20′ cells for some 20 countries. Our data can easily be aggregated to this level. Of course the problem of synchronicity remains. In addition, the Goddard Institute for Space Systems (Matthews, 1983) has assembled world population by one degree quadrilaterals. Some enhancements to this tabulation have been performed by the Oak Ridge National Laboratory, and these are available, but nearly a decade old. Several countries, including the US, have population data at a much finer resolution than we have attempted. Some countries have demographic data by dwelling unit, recorded in coordinates to the metre or decametre level! These figures could be aggregated to the medium resolution we have assembled and used as an estimate of the errors introduced by our redistribution. A few countries even already have such aggregations and these can be compared directly with our assemblage in future work. A useful investigation would be to examine ways in which data assembled at such micro levels have been used, and whether this is warranted for larger areas of the world.

Urban Populations

In some cases it may be possible to distinguish between urban and rural populations, treating them separately. When an urban population count is available by city and if the latitude and longitude of the centre of a city is known, then it is possible to make an estimate of the population at various distances from the centre of the city. Thus the people can be assigned to the cells of a lattice around the city, after which the rural population portion can be allocated separately. This should provide a better estimate of the population arrangement than simply using the city centroid as a point of large population. It would be even better to have the urban area digitized as an individual polygon; this was in fact often the case in our second-level data. The bad news is that we have been unable to discover any source of comprehensive worldwide city demographic information with latitude and longitude coordinates for our use in those cases where this would have been helpful. We have found one file, apparently based on United Nations data, pre-dating 1992, of 2763 cities with populations and coordinates; the file is on the diskette accompanying the full report. The population in these cities sums to 1,334,783,002 people. We have not used these data in our compilation because much of the same information is captured in the administrative units. A more comprehensive file, if it existed, could be used to compare with our assemblage, or even to enhance it. The good news is that there is empirical evidence telling us how this might be done. It seems that relating the population of a city to the geographic area which it covers can be done with some degree of confidence. Although the correlations are high this must still be used with caution since the functional city and the political city often do not coincide. The land area estimate pertains to the functional city, but population counts are most often based on political cities.

The two empirical regularities on which the following discussion is based pertain to city sizes, as a function of population, and to the arrangement of people within a city. An idealized circular city size can be based on the observation that the land area of a city can be predicted with considerable accuracy by knowing the number of its residents: city area = a (population)^b, or taking logarithms, ln area = ln a + b ln P. From this an assumed circular city size of equivalent area can be computed. The total population density of a city, in persons/km^2, can also be estimated from this empirical formula for city area. We also find that the change in area, or radius assuming a circular city, due to an increase of one person is $dA = a\, dp^{\beta}$ or $dR = \theta\, dP^{\gamma}$, where the new coefficients α, β, θ, and γ can easily be determined from the above formula. It is then seen that the average density increases with city size, but that the influence of an individual on the total area or radius of a city decreases with increasing city size. Perhaps the individual's political influence also varies in this manner.

For the US, using *P* for the total population and *R* to denote a city radius in km, the empirical relationship is $R = 0.035\ P^{0.44}$. In some other cultures the scaling coefficient, a, is generally smaller, resulting in more compact towns, but the exponent, b, appears stable. The empirical observations from which this relation was inferred cover settlement sizes ranging from 150 people to over a million persons with very high degrees of correlation. A similar rule is used, in inverted form, by some archaeologists to estimate the former population of excavation sites. The constant of proportionality, a, may also change with time. In the US it appears to have changed since 1945 when the impact of the automobile affected suburbanization (Tobler, 1975).

The next step is to choose a model of the arrangement of the population around the centre of the city, assumed circular. Once this is done the population can be reallocated to the geographic area of the city. The density at the centre of the urban area can also be estimated from the models by setting the distance from the centre to zero. The models suggested include the exponential decay model suggested by Clark (1951) and the Gaussian model suggested by others. There is disagreement in the literature as to which model is most appropriate, and, as models, none can be expected to be perfect. Nor are cities really circular, although the equations predict the land area and total density quite well. Other models including gamma functions and more complicated types yielding lower populations in the city centre exist (e.g. Vaughn, 1987); some versions result in different urban densities in different azimuths (Medvedkov, 1965; Haynes and Rube, 1973) and with variations over time (Bussiere, 1972; Newling, 1973). To get the total population numbers it is necessary to convert the densities to a count, inverting the model equations. Using these models one can assign the urban population to a restricted (circular) subset of the area and spread the rural population over the remainder. Adjustments may need to be made at political boundaries, and overlaying a coastline file would be useful in avoiding the assignment of people to water areas and might require a corresponding adjustment to obtain the total land area suggested by the model. In these cases the theoretical radius might also suggest areas for potential future city growth by infilling.

This method of reallocating urban population, using a kernel function to convolve the data, is quite analogous to the one used in statistics to assign two dimensional densities to scattered observations (e.g. Silverman, 1986; Scott, 1992; Wand and Jones, 1995). Examples used to distribute urban populations are available in the work of Honeycutt and Wojcik (1990) for the US, and Bracken and Martin (1989) for a region in Great Britain. The kernel is generally chosen to mimic the (theoretical) distribution of population within a city using one of the above models. Often the urban centroid coordinates are not available, although they may be estimated with sufficient accuracy by examining maps, or by choosing a dominant city within each region.

Movement Data

The immigration–emigration component of demography has not been included in the foregoing discussion. Since most change requires movement (of information, of people, of credit, or of material) more stress should be placed on the global measurement of movement. Most international agency data collections are of state variables rather than of flow variables, to use the language of systems analysis. That is, they give the sums of the volume of movement but not the actual path of the movements. Complete from–to tables for the world by country (circa 200 by 200 countries in size) are very difficult to obtain, in part because of definitional differences. And countries usually do not reconize subdivisions of foreign countries. Using sub-national data on movement at the same level as the data now assembled for population would require a table of 19,032 rows by 19,032 columns, although it would be quite sparse. Tables of industrial or agricultural commodity trade, when available, can be treated in a manner similar to migration tables, as can more general communication information. To be consistent with the present report the information should be given by latitude/longitude quadrilaterals (using the 6,687,360 five minute lat/lon cells would require a whopping 4.47×10^{13} two-way table entries). As mentioned in the introduction, partial differential equation models of movement could then be used and, as abstract models, should work for refugees and for global trade too. Realistically, given the computer and data constraints, meteorologists today are able to work only with cells about half a degree on a side. But the geographic movement of people or global monetary flow, is as important as wind and weather movements. Getting the data promptly and continuously is the hard part. Aside from this the details are not difficult.

Data Compaction

Verified theory is the best data compaction method, but for numerical data the classical method used by geophysicists for describing continuous variables distributed over the surface of the Earth is that of spherical surface harmonics, the spherical equivalent of Fourier series. This has also been reported for the arrangement of population on the Earth (Tobler, 1992). Non-scalar demographic change components, such as migration, may be represented by vector spherical harmonic fields, and similar techniques may be used for trade data (Raskin, 1994). But it is not apparent that the arrangement of population density satisfies the harmonic assumption. Consequently, less restrictive spherical wavelets have been investigated (Tobler, unpublished). Spherical splines have also been used to fit spherical distributions, including population (Wahba, 1981; Dierckx, 1984; Slater, 1993). This remains an area for further development and research applied to human relations.

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NOTES

(1) The Center for International Earth Science Information Network (CIESIN) makes the quadrilaterized population data publicly available at no cost on an ftp site, ftp.ciesin.org, placing the data in a directory called pub/data. For information contact CIESIN at the addresses below. The Environmental Systems Research Institute (ESRI) also distributes the data on their ArcWorld 1996 CD supplement. CIESIN, ESRI, and Microsoft have also published population density maps based on the gridded world population estimate. The NCGIA site NCGIA@NCGIA.UCSB.EDU may also be visited for further discussion.

CIESIN: 2250 Pierce Road, University Center, MI 48710, US. (517) 797-2700; fax: (517) 797-2622; e-mail: ciesin.info@ciesin.org

(2) Technical Report TR-95-6, *The Global Demography Project* (75 pp + PC diskette), can be obtained for $25 from the National Center for Geographic Information and Analysis, Geography Department, University of California, Santa Barbara, CA 93106-4060, US. E-mail: ncgiapub@ncgia.ucsb.edu

APPENDIX

Mass Preserving Reallocation from Areal Data

First define the primary condition for mass preservation. This is the required invertibility condition for any method of areal information redistribution:

$$\int_{R_i}\int f(x, y)\, dx\, dy = V_i \qquad \text{for all } i$$

where V_i denotes the value (population in the present context) in region R_i (a sub-national polygon).

Next constrain the resulting surface to be smooth by requiring neighbouring places to have similar values. This is an assumption about spatial demographic processes, a form of geographic insight capturing the notion that most people are gregarious and congregate. Densities in neighbouring areas therefore tend to resemble each other unless there is a physical barrier. Thus there should be no abrupt jumps between densities in adjacent administrative units. Laplacian smoothness, the simplest kind, is obtained by minimizing:

$$\int_{R}\int \left(\frac{\partial f^2}{\partial x}+\frac{\partial f^2}{\partial y}\right) dx\, dy$$

where R is the set of all regions. The boundary

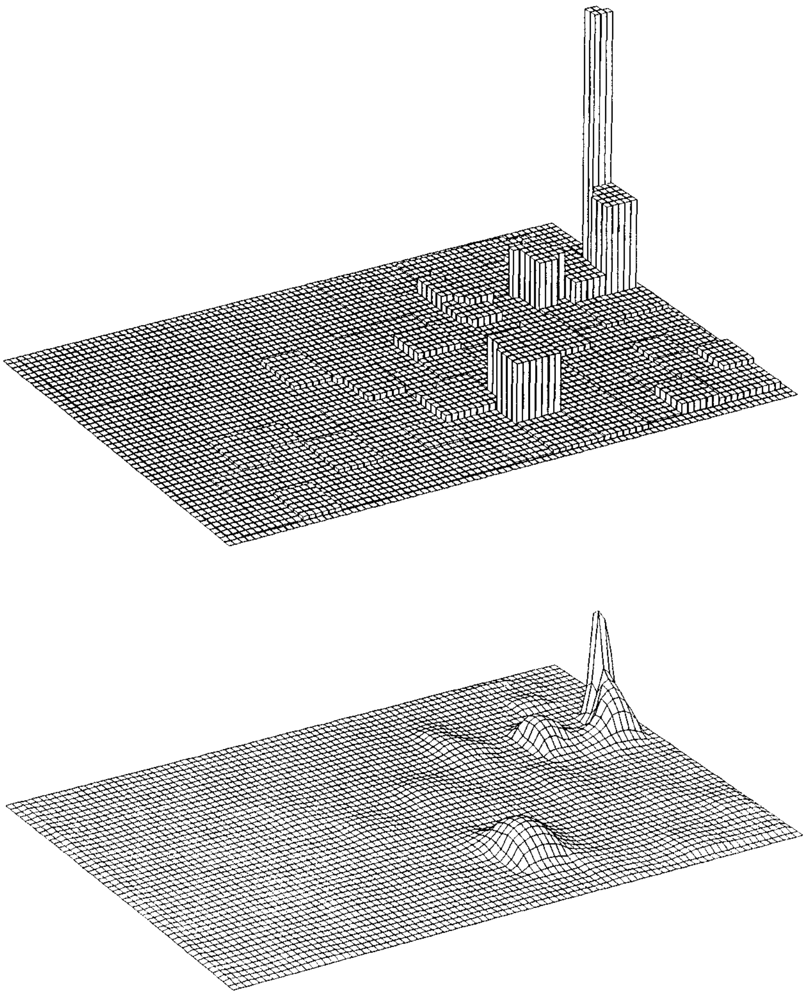

Figure 5. Illustration of mass preserving reallocation method for 1990 population of county for Kansas (data courtesy of T. Slocum).

condition generally used is

$$\frac{\partial f}{\partial \eta}=0$$

This says that the gradient normal to the edge of the region is flat; it is one of several possible choices. More detail is given in the original paper (Tobler, 1979). Also see the appendix to the full report for the spherical version. The procedure is illustrated graphically in Fig. 5.

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