A phenomenon-based approach to upslope contributing area and depressions in DEMs

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Abstract:
Description of the terrain surface through digital elevation models (DEMs) strongly depends on data collection methods and DEM data structures. For efficiency and availability reasons regular point distributions are most common, which yield artefacts such as depressions and preferential flow directions. These facts need to be considered when natural phenomena are modelled, as is shown for handling depressions and for estimation of flow paths and upslope contributing areas. Analysis of the main reasons for the occurrence of depressions shows that they usually better reflect the terrain than their surroundings. Thus, the most common remedial method of raising depressions is rejected. Algorithms that ‘cut’ a flow path from the depression through its bounding barrier are favoured instead. Several flow routing algorithms are evaluated for their behaviour in regular grids. It is shown that the multiple flow direction (mfd) algorithm that distributes water from a grid cell to the lower of its eight neighbours proportionally to their elevation differences (slope) exhibits correct flow directions and the best rotation invariance. It is suggested that the estimation of upslope contributing areas (TCAs) is undertaken in two steps: first, a high quality flow direction data set is derived by a well-behaved mfd algorithm or by subgrid modelling of flow paths; secondly, the upslope contributing areas are obtained by counting the upslope elements. © 1998 John Wiley & Sons, Ltd.

KEY WORDS DEM; upslope contributing area; depressions; flow routing

INTRODUCTION
It is nearly three decades since Freeze and Harlan (1969) outlined a blueprint for physically based hydrological response models. Their ideas have been partly implemented in the following decades, especially with the development of GIS (geographical information systems). However, GIS are mainly used to organize and visualize input and output data; they have not brought significant improvement in modelling strategies and methodology (Wilson, 1996). In particular, several well-known problems have not yet been resolved satisfactorily. Here, aspects of terrain surface modelling with digital elevation models (DEMs) are considered.

Although DEMs are by far the most accurate and dense data available for hydrological response or erosion models, there is still a need for better descriptions of the terrain surface. Several authors have found that a ground resolution of approximately 10 m, depending on the terrain characteristics, is sufficient to describe the variability of the relevant parameters for typical input elevation data (Zhang and Montgomery, 1994; Quinn et al., 1995). Higher resolution might eventually provide variability at even smaller scales (Montgomery, 1996). With conventional data collection methods such high resolution is the absolute limit obtainable at reasonable costs at the catchment scale. For large areas, the use of manual or analytical

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photogrammetry or GPS (Global Positioning System) are too expensive. Digital image matching allows much automation, but has problems with off-terrain objects. A very promising new data collection technology is laser altimetry for both its extremely high resolution (down to submetre) and its potential to penetrate the tree crown layer and obtain accurate elevation data even in densely forested areas (Lohr and Eibert, 1995; Kilian et al., 1996; Kraus et al., 1997; Næsset, 1997). The technique may well become an important elevation data source in the future, especially in forested areas.

Mark (1979) stated, nearly two decades ago, that the phenomena in question should determine model structures and algorithms in use rather than computational considerations. Today, even with high resolution data, it seems that major problems remain. Inappropriate surface descriptions follow not least from arbitrary point distributions during data collection. Grid data structures are the main DEM source in use for historical as well as practical reasons, and it is probable that these inappropriate structures will remain for a long time, especially with high resolution data. So the question is, how to achieve the best results with the existing limitations in data and structure.

It is necessary to address the weaknesses in all stages of the modelling process and to focus upon both the causes of problems and their consequences. Algorithms can be optimized and tuned to improve some of their major drawbacks and reduce quality loss. In the following discussion, these issues are addressed in relation to the handling of depressions and to flow routing with regard to estimating flow paths and upslope contributing area. The main focus here lies on geometric aspects rather than on flow physics. Grid structures are employed because of their frequent use and their persistent and possibly increasing importance in all kinds of terrain modelling.

DEPRESSIONS

General

Depressions in DEMs have been recognized as one of the major problems in hydrological applications, since they hinder flow routing (O’Callaghan and Mark, 1984; Jenson and Domingue, 1988). A depression is defined as a point or an area lower than all immediately surrounding points. Each depression has its own catchment area. The outlet of a depression is defined as a point through which water could leave the depression; normally the lowest point along the border of the depression catchment (e.g. Mark et al., 1984; O’Callaghan and Mark, 1984; Band 1986; Jenson and Domingue, 1988). The area within the depression that is lower than the outlet point is referred to as the ‘inner area’ of the depression (Figure 1).

The definition of the outlet may be problematic if there are several candidates along the catchment border whose elevations differ only within the height error. Therefore, Jenson and Domingue (1988) proposed an

![Figure 1. A depression: inner area, outlet and depression catchment](image)
algorithm that allows for several outlets by topologically splitting the area, especially for relatively flat areas. However, within depressions there is no geometric justification for splitting (i.e. from the elevation data). Generally, mere topological treatment of geometrically defined features ignores the broader geometric information that is available. For example, a more regional curvature analysis could be applied to find the most probable border position within depressions or flat areas, but the improvements achieved are probably small compared with the effort. Furthermore, whatever the approach, it is impossible to extract information reliably that is beyond the scope of the data (in terms of scale, resolution and accuracy).

Reasons for depressions in the DEM

Depressions may reflect genuine terrain forms; in such cases water would either fill the depressions and eventually overflow or would find subsurface paths for runoff. Assuming those subsurface paths correspond to the larger terrain structure, these depressions may be eliminated from the DEM in order to define flow paths within the wider area. The depression is thus seen as a local feature that is of minor interest for the delineation of flow paths. Few natural depressions cannot be treated this way, although karst terrain, for example, may provide some exceptions. However, the vast majority of depressions do not reflect genuine terrain features. There are two main reasons for such ‘artificial depressions’, data errors and the consequences of surface representation.

(a) Data errors. The type of errors depends mainly on the data collection method. Typical errors include:

(i) For manual profiling (analytical plotters, softcopy photogrammetry), the so-called ‘scan error’ (Kraus, 1984), resulting in (regularly) shifted profiles, depending on the scan direction. These can be eliminated by appropriate algorithms (Kraus, 1984).

(ii) For automatic DEM generation (image matching), off-terrain ‘objects’, such as vegetation or buildings, tend to raise the surface, frequently resulting in artificial gorges or depressions along areas with ground sight (roads, etc.). Manual correction is necessary in areas where such errors are too high (Walker and Petrie, 1996).

(iii) Random data errors may especially lead to depressions in flat areas where the mean elevation error exceeds the elevation difference between neighbouring points.

(b) Surface representation owing to point distribution. The surface representation depends on data collection methods and the DEM data structure used. Ideally, the terrain surface is collected through its main features (skeleton lines). However, this is a time-consuming task that requires high expertise, since the proper location and number of breaklines are neither easy to recognize nor to define and depend widely on the operator’s experience (Kubik, 1988). Collecting the terrain surface in (regularly spaced) profiles is much faster and easier. It does not require any information about the surface; thus, automatic DEM measurement techniques (image matching) usually produce grids (Hahn and Förstner, 1988; Helava, 1988; Krzystek, 1991; Ackermann, 1992; Kaiser et al., 1992; Miller and DeVenecia, 1992; Baltsavias et al., 1996), while automatic extraction of skeleton information has only recently been described (Wild and Krzystek, 1996). Hence, improper point distributions and data structures alone, mainly regular grids, are often used.

Owing to their inability to penetrate and recognize off-terrain objects, image matching techniques yield DEMs that ‘are not very good, but acceptable for orthoimage production’ (Gruen, 1996). Their main scope lies in small-scale applications. For use in runoff modelling the results in forested areas are not of acceptable quality.

Digitized contours as an elevation source are frequently used because of their relatively low costs. DEM generation is difficult from these data especially in areas of high contour curvature, such as valleys and ridges. These areas are most important for hydrology, since it is here that either channels or catchment borders are located. A particular problem is that general purpose interpolation techniques may yield flat
areas or depressions in these zones (Rieger, 1992a). Some success has been achieved in the generation of consistent DEMs (Aumann et al., 1991; Pilouk and Tempfli, 1992), but there are still inconsistencies because these algorithms use topological operations in areas of high contour curvature rather than fundamental geometric modelling. In flat areas, DEMs are generally weak owing to the arbitrary and coarse height resolution of contours. Yet, with proper constraints applied, the resulting DEMs at least provide consistent channel networks.

Time optimization and quality improvement techniques for photogrammetric DEM data collection are known as progressive, selective and composite sampling in (incomplete) grids (Makarovic, 1973, 1977; Charif, 1992) or at irregularly distributed points (Mann, 1988). One step may even be the extraction of skeleton information during the sampling process in order to improve data collection (Charif, 1992). This information can be used directly for hydrological applications. However, these techniques are not widely used, and if skeleton information is not digitized manually, but rather extracted from grid data, it again reflects the grid structure.

Representing the terrain surface through regular grids frequently leads to chains of depressions along narrow valleys, since only a few grid points lie on the valley bottom. Points close to the valley bottom frequently describe depressions. (Figure 2). The same is true for triangulated irregular network (TIN) DEMs, if the collected data points are regularly distributed, because usually the original data are used to define the TIN nodes. Most depressions are located in flat areas and in valley bottoms, especially in deep or narrow valleys (Rieger, 1992a).

Handling depressions

(a) General. In this discussion it is assumed that depressions are to be eliminated either explicitly or implicitly, so generating a ‘depressionless DEM’. Flow can then be routed down to the DEM edges, with the exception of flagged (actual) depressions that are permitted to remain. Within valleys, depressions can normally be eliminated with little danger of gross errors, since runoff paths are well defined. For flat areas the situation is more difficult and gross misplacements of flow paths are possible.

Depressions can be eliminated at several stages. The optimum solution is that elevation data are collected in sympathy with the DEM generation process so that no artificial depressions are present. Alternatively, depressions can be eliminated during the DEM generation process (Hutchinson, 1989). However, in most cases depressions are still present after DEM generation and so they must be eliminated from it. Regardless of the stage at which the elimination process is included, it is necessary to understand and to take into account the reasons for the occurrence of depressions.

Figure 2. Chains of depressions in a narrow valley owing to grid structure

Methods to eliminate depressions. The first step in eliminating a depression is to define the outlet point, usually the lowest point along the edge of the catchment border. Most of the available algorithms then raise the inner region of the depression. The following methods of achieving this have been described.

1. Raising the inner area to the elevation of the outlet point (O’Callaghan and Mark, 1984; Jenson and Domingue, 1988; McCormack et al., 1993). The method has the great disadvantage that all form information within the depression is completely lost. Thus topological principles (rather than elevation information) are needed to define flow paths through the depression.

2. Rerouting the flow direction data set such that flow paths are defined (Chorowicz et al., 1992). The principle is the same as in point 1 above, if routing is done merely topologically (Jenson and Domingue, 1988).

3. Raising the inner area and retaining the form of the depression (Rieger, 1992b). The inner area of the depression is raised between the elevations of the outlet point and its lowest outer neighbour, retaining the height relationships between the points. The flow path is then lowered from the depression to the outlet point. The advantage of the method is that the form is retained, so that there is no necessity to define flow directions arbitrarily within the depression.

4. Qian et al. (1990) and, similarly, Hadipriono et al. (1990) calculated drainage paths for a DEM with depressions, relating in a lot of unconnected pieces of drainage lines for which they derived a set of geometric properties. They then used expert system shells describing drainage network properties such as curvature, relative elevation of line pieces, or segment lengths, in order to find the most probable connections. However, the resulting channel networks did not correspond to reality as well as one might expect. So, either some parameters of the expert system shell (weights) need to be changed or some additional criteria need to be introduced.

5. Smoothing has sometimes been used to eliminate many small depressions (O’Callaghan and Mark, 1984; Tarboton et al., 1990). This is, however, problematic since it flattens all curvature in a landscape and systematically shifts hillslopes.

None of the above methods takes into account the reasons for depressions. In fact, the methods of Qian et al. (1990) and Hadipriono et al. (1990) do not consider depressions explicitly. Chorowicz et al. (1992) come closest to what is suggested in the following discussion. As has been shown, depressions often exhibit correct terrain elevations compared with surrounding points that are too high, either because of systematically incorrect data or because of improper (arbitrary) point distributions. The following section provides a phenomenon-based approach that takes into account the main reasons for the occurrence of depressions.

(c) A phenomenon-based approach. Taking into account the main reasons for the occurrence of depressions, it appears that depression points are likely to reflect correct elevations, while the outlet points are too high. In these cases — particularly in valleys — phenomenon-based handling of depressions should prevent the elevation of the depressions from being lost. Rieger (1992b) suggested lowering the flow path from the depression, starting from the outlet point. The inner flow path leads to the deepest point of the depression, and the outer flow path to the first valley point that is deeper than the depression. The connecting path is lowered from the deepest point of the depression (inner end) to the outer end so that it leads monotonically downwards (Rieger, 1992b). The algorithm may also be applied to nested depressions by handling them in decreasing order (Figure 3).

FLOW ROUTING AND UPSLOPE CONTRIBUTING AREA

Definitions

Flow paths are usually defined from surface topography, but they may also be appropriate to groundwater flow (Anderson and Burt, 1978; Zaslavsky and Sinai, 1981; Bonell, 1993). Flow paths are used for flow
routing (Quinn et al., 1991), delineation of catchments and estimation of upslope contributing areas (O’Callaghan and Mark, 1984; Jenson and Domingue, 1988; Rieger, 1993; and others), and, indirectly, for estimation of soil wetness indices (O’Loughlin, 1986).

There have been numerous attempts to estimate flow paths at the surface from DEMs, including grid models (O’Callaghan and Mark, 1984; Band, 1986; Jenson and Domingue, 1988; Skidmore, 1990; Quinn et al., 1991; Rieger, 1992a,b; Tribe, 1992; McCormack et al., 1993; Desmet and Govers, 1996; and others), TINs (Palacios-Vélez and Cuevas-Renaud, 1986) and contour DEMs (O’Loughlin, 1986; Moore, 1988; Moore and Grayson, 1989, 1991). Reviews are given by Moore et al. (1991, 1993), Tribe (1992), Beven and Moore (1993), and Desmet and Govers (1996). Here, only grid models are considered.

The total contributing area (TCA, also often denoted as $A$) is the plan area upslope of a surface element (normally, a contour line segment or grid cell) that drains to that element. The grid widths in the axis directions are denoted as $D_x$ and $D_y$, respectively. In many cases a square grid is used, denoted by $\Delta = \Delta x$ and $\Delta y$.

Flow routing and slope direction

For contour models the calculation of flow paths is straightforward (O’Loughlin, 1986; Moore and Grayson, 1991). For TINs the ease of definition depends on the terrain position of the individual triangles. For grid models the situation is fairly complex, since one has either to leave the grid structure or to design algorithms that account for the arbitrary nature of the structure.

There are two classes of flow path algorithms: one-step algorithms estimate the TCA directly; two-step algorithms calculate an intermediate data set, the flow directions, and use it to integrate the TCA in the second step. The advantage of two-step algorithms is the generation of the flow direction data set which can be used for several applications (Jenson and Domingue, 1988). The disadvantage is that the generation of the flow direction data set requires a unique decision for each grid cell, as to which grid cell (or cells) flow occurs to. In contrast, one-step algorithms can be more sensitive to the actual slope direction.

Another common classification identifies single (sfD) and multiple flow direction (mFD) algorithms. SfD algorithms route the accumulated TCA to one neighbouring grid cell, while mFD algorithms are able to split the TCA to several neighbours. The classical single flow algorithm (O’Callaghan and Mark, 1984), also known as ‘steepest descent’ and frequently named ‘D8’ (Costa-Cabral and Burges, 1994; Meijerink et al., 1994; Blöschl, 1996) has widely been recognized as behaving poorly, since it yields slope lines parallel to grid lines or diagonals rather than parallel to the actual slope direction, and it is unable to model divergent flow appropriately (e.g. Fairfield and Leymarie, 1991; Rieger, 1992a,b; Tribe, 1992; Costa-Cabral and Burges, 1994).

There have been several attempts to improve this behaviour. Lea (1992) used an interpolated plane to define the gradient, but the preferential directions problem remains. Another approach switches randomly
between deeper neighbours, resulting in a stochastically correct flow direction, called ‘Rho4’ or ‘Rho8’ for 4 or 8 neighbours, respectively (Fairfield and Leymarie, 1991). However, ‘the re-introduction of some randomness into a model equation which previously has been freed from random influences . . . appears as an illogical step backwards in the methodological research sequence’ (Ahnert, 1994) and actually ignores available elevation information in the particular case.

Mfd algorithms normally split the TCA to all or several deeper neighbours, using a specific criterion. The principle is to disperse the flow in order to allow for directions other than the major grid directions. Mfd algorithms ‘overspread’ the TCA values, which is normally compensated for by the overspread TCA values of the neighbouring cells. Several approaches to splitting have been developed. Beasley et al. (1980) divide a cell by a line in the slope direction through a corner point. The two areas are routed to the neighbouring cardinal grid cells. The algorithm only allows the TCA to be split to two lower neighbours. Quinn et al. (1991) and Wolock and McCabe (1995) routed the TCA weighted by the elevation differences to the lower of its eight neighbours, referred to as ‘DH8’. The same approach has been used for only four neighbours by Rieger (1992a,b; 1993), referred to as ‘DH4’. Quinn et al. (1995) have modified DH8 by an exponent to the slope. Other researchers have derived local curvature and combined the resulting pieces of the drainage network in several ways (Band, 1986; Chorowicz et al., 1992), and expert systems that use information from several geometric properties have been employed by Qian et al. (1990) and Hadipriono et al. (1990). Costa-Cabral and Burges (1994) were the first to model slope lines between grid cells (‘flowing tubes’). Flow is routed from each cell upwards [in order to compute TCA and specific upslope contributing area (SCA)] and downwards [to compute SDA (specific dispersal area) and SCA] along all possible paths. Within each cell, the portion of the respective flow path is modelled as a band parallel to the slope direction of that cell.

A key question is the number of neighbours involved in the routing process. For sfd algorithms normally the eight surrounding neighbours are used, allowing for the best resolution of direction. For mfd algorithms the question is more complex. Geometrically, only the flux to the four cardinal neighbours is defined. Nevertheless, it is common to route to eight neighbours even with a square grid, since this allows for a more sensible flow distribution.

Several studies have compared the results of sfd and mfd algorithms with one another (Quinn et al., 1991; Wolock and McCabe, 1995; Desmet and Govers, 1996). The mfd algorithms tend to yield smoother distributions of TCA values than the sfd algorithms. Costa-Cabral and Burges (1994) used synthetic surfaces, which allows comparison of the results with correct data. The same approach is adopted here.

One aim of flow algorithms is to estimate slope lines. As mentioned, sfd algorithms behave poorly except for the Rho8 algorithm (Fairfield and Leymarie, 1991). Mfd algorithms spread the area unrealistically and do not readily provide a single downslope path or connected flow direction data set, as can be created with sfd algorithms. They can, however, be used to estimate slope paths for single cells (Rieger, 1992b). Initializing only the source cell with a non-zero value, the flow algorithm yields a field of dispersion values that can easily be traced downwards, always stepping to the lower neighbour with the maximum dispersion value. The result accurately represents the flow path for those algorithms that spread the area proportional to elevation differences as shown for a tilted plane with the DH4 (Figure 4) and DH8 algorithm (Figure 5). The area-weighted algorithm of Beasley et al. (1980) and a modified version of DH8, which spreads proportionally to a power (other than one) of slope (Quinn et al., 1995), do not show the same property.

Accurate modelling of flow paths is only possible either by modelling subgrid flow channels (Costa-Cabral and Burges, 1994) or by mfd algorithms that distribute the upslope area such that the slope lines are represented, which is the case for both the DH4 and the DH8 algorithm. All these methods can be used to delineate single flow paths to specific seed points. The generation of a flow direction data set is also possible, yet very time-consuming. Flow paths need to be traced from all points downwards in order to define appropriate flow directions. For each point a flow accumulation data set must be initialized with zeroes, and a seed value has to be stored at the point’s position. The seed value is routed downwards (in ordered sequence; Rieger, 1992a,b). Finally, the flow path is traced until it reaches an already recorded flow direction value or the edge of the DEM, and the flow directions are stored in the flow direction data set. The whole process is
done in sequence of decreasing elevation for all points that do not yet have flow directions assigned. The resulting flow direction data set better reflects the DEM geometry than all known sfd algorithms.

The total upslope contributing area (TCA)

The TCA is the area that drains to a given surface element, the ‘reference element’. Although TCA values have been estimated in several different ways, little attention has been given to the significance of the associated surface element. For DEMs one would normally assume an element of the DEM structure (grid cell in grid models, triangle in TINs). Flow algorithms need to be designed such that the resulting TCA values reflect the reference element size.

For the D8 (and other) sfd algorithm the TCA values correspond to a reference element length of grid width, $\Delta$, when flow occurs in a cardinal (axis) direction, and of $\Delta/2$ in a diagonal direction (e.g. Figure 6). Since there are no other flow directions possible, no other reference element widths occur. In zones of convergence (valleys) the TCA is well represented and fairly direction independent, since the sfd algorithms
simply count the upslope cells; so the correctness of the results mainly depends on the correctness of the catchment borders.

For mfd algorithms the situation is more complex. The number of possible directions is not discrete, so there is a continuously changing relation to the aspect angle. Figure 7 shows the contour lines of a cone with a vertical axis and upwards peak. Figure 8 shows lines of equal TCA values for the DH4 algorithm on that cone. These lines exactly correspond to TCA values that are defined for the width of a grid cell seen in aspect direction $\alpha$. With grid widths $\Delta x$ and $\Delta y$, the width $b_\alpha$ of a grid cell viewed at a certain aspect angle $\alpha$ can be expressed as (Figure 9),

$$b_\alpha = \Delta x \cdot | \cos \alpha | + \Delta y \cdot | \sin \alpha |$$  \hspace{1cm} (1)

or, with equal grid sizes in both axis directions, $\Delta x = \Delta y = \Delta$, and $b_0 = \Delta$ as reference element size,

$$b_\alpha = \Delta \cdot (| \cos \alpha | + | \sin \alpha |) = b_0 \cdot (| \cos \alpha | + | \sin \alpha |)$$  \hspace{1cm} (2)

The scaling factor in Equation (2) varies between 1.0 in the axis direction ($\alpha = 0, \pi/2, \pi, \ldots$) and the square root of 2 (1.414) in the diagonal direction, ($\alpha = \pi/4, 3\pi/4, \ldots$). The theoretical TCA to a contour piece $b$ on a cone at a distance $R$ from the centre point is calculated according to (Figure 10),

$$A = b \cdot R/2$$  \hspace{1cm} (3)
resulting in circular lines of equal TCA values that increase linearly with distance from the centre point.

Changing the reference element size, $b$, to $b_x$ as above, yields direction-dependent TCA values at a constant distance $R$

$$A_x = b_x \cdot \frac{R}{2} \quad (4)$$

The lines of equal TCA values in that case are concentric squares rather than circles, rotated by $\pi/4$, since the distance $R_x$ of any point along the edge of such a square depends on the direction (Figure 11),

$$R_x = R_0 \cdot \frac{\sin\pi/4}{\sin(\pi/4 + \alpha)} = R_0 \cdot \frac{1}{\cos\alpha + \sin\alpha} \quad (5)$$

Figure 9. Width of grid cell seen in direction $\alpha$

Figure 10. TCA to contour line segment $b$ at a distance $R$ from the centre point of a cone with its peak pointing upwards (out of the page)

Figure 11. Distance from the centre point to a straight diagonal line in comparison with a circle

for example, in quadrant $Q_1$. Generalization to the other three quadrants introduces the absolutes of sine and cosine, resulting in the same factors as between $b$ and $b_x$ [Equation (2)]. The DH4 algorithm produces the same results, and so normalization takes place by division of the calculated TCA values by the factor of Equation (2),

$$A_{\text{norm}} = A / (|\cos \alpha| + |\sin \alpha|) \quad (6)$$
Figure 12 shows the normalized TCA values of the cone of Figure 7, which are exact circles. Figure 13 shows results for DH4 on a cone with its top pointing down. The corrected values exhibit systematic deviations from the theoretical TCA values close to the centre point.

Figure 14 shows TCA-values for the DH8 algorithm on a cone with its peak pointing up. Figure 15 illustrates the same algorithm applied to a cone with its peak pointing down. The direction dependence is less marked than with the DH4, especially in the convergent case (Figure 15 vs. Figure 13). For the DH8 algorithm the correction factor is more complex; it corresponds to the width of an octagonal pseudo-grid cell seen in direction \( z \), and is similar to Equation (5), with \( 3\pi/8 \) instead of \( \pi/4 \)

\[
b_{x,8} = b_0 \cdot \frac{\sin \left( \frac{3\pi}{8} + z^* \right)}{\sin \frac{3\pi}{8}} = b_0 \cdot \left[ (\sqrt{2} - 1) \cdot \sin z^* + \cos z^* \right]
\]

(7)
where $\alpha^*$ denotes angle $\alpha$ reduced to the interval $[0 \ldots \pi/4]$

$$\alpha^* = \alpha - \left[ \frac{4\alpha}{\pi} \right] \cdot \frac{\pi}{4}$$

(8)

with $[x]$ meaning the highest integer value that is lower than or equal to $x$. Quinn et al. (1995) use a different approach, distributing a TCA value to cell $i$ by

$$A_i = A_0 \cdot \sum_j (\tan \beta_j)^p$$

(9)

with an exponent $p = 1.1$. Figure 16 shows the corresponding lines of equal TCA on cones with their peaks pointing up and down, respectively. The direction dependency is reduced but is still present. The slope direction on a tilted plane, when calculated as line of maximum TCA values downwards from a seed point, is slightly wrong.
For the simple case of cones or tilted planes, a scaling factor can be applied in order to obtain direction independent results. However, with any change in direction along any flow path the length of the reference contour element changes within the extremes of $b_z$, resulting in erroneous TCA values. The error in these estimates attributable to slope direction depends on the flow algorithm used. Table I summarizes the maximum errors of several flow algorithms, compared with a contour length of grid width $D$. When TCA values are corrected by the factor of Equation (7), the remaining deviations resulting from grid direction of the DH8 algorithm will normally lie within $\pm 4\%$ of the actual TCA values.

As an alternative to the grid-dependent calculation of the TCAs, a flow direction data set can be used. If it has been calculated as described above, a fairly direction-independent estimate for the TCAs can be obtained by simply counting the number of grid cells upslope of the reference element (e.g. Jenson and Domingue, 1988). In this case, the flow algorithms are used to create the flow direction data set rather than to estimate the TCAs. To obtain an even more accurate measure, the ‘catchment affiliation values’ can be used (Rieger, 1992b). These values result from applying the flow routing algorithm in an upslope direction and yield the percentage of area in any cell that is routed to a given catchment area. The area of cells along the border of two catchments can be split according to these values, while the inner areas of the catchments are estimated by the counting process.

One problem that has not been dealt with in the above considerations is the assumption of linearity between the TCA and the contour length

$$TCA(k \cdot h) = k \cdot TCA(h) \tag{10}$$

This relation is strictly fulfilled only on special surfaces, such as tilted planes or cones. If linearity is not given, the application of a scaling factor on the TCA values will not yield correct results. However, the influence will

<table>
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<th>Code</th>
<th>Description</th>
<th>Type</th>
<th>Number of neighbours</th>
<th>Correction factor $b_y/b_z$</th>
<th>Deviation from actual TCA (%)</th>
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<td>sfd</td>
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<td>mfd</td>
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<td>1-000 ... 0-707</td>
<td>0... + 41</td>
</tr>
<tr>
<td>DH8</td>
<td>$\Delta h$ weighted</td>
<td>mfd</td>
<td>8</td>
<td>1-000 ... 0-924</td>
<td>0... + 8</td>
</tr>
</tbody>
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Figure 16. DH8 with exponent $p = 1-1$. Lines of equal log of TCA on a cone (left) with its top pointing upwards; (right) with its top pointing downwards. Only the grey area is part of the DEM.
normally be small. On hillslopes, significant changes in flow direction are rare; convex areas are normally of minor interest, since the TCAs are small, resulting in low erosion and soil wetness as well as in small absolute errors of the TCAs; within concave areas (valleys) the linearity criterion is likely not to be fulfilled and the influence of the local flow direction (i.e. valley direction) is small compared with the much larger areas of surrounding hillslopes, which may exhibit very different flow directions. In these areas the DH8 algorithm will yield the best results.

CONCLUSION

Given the main causes of depressions in grid DEMs, raising the inner areas is frequently inappropriate as a method of depression removal, since depression points, particularly in valleys, tend to reflect the channel position better than the surrounding points. In these cases, cutting flow channels is suggested as an alternative.

The effects of the grid direction on the estimation of flow paths have also been evaluated. The DH8 algorithm has been shown to behave best in terms of rotation invariance and correctness of flow direction. The application of a direction-dependent scaling factor further improves the TCA values. Further research is needed on how changes in upslope flow direction affect the TCA, especially in valleys.

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