Scale-Based Simulation of Topographic Relief

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ABSTRACT. An invertible model is proposed which uses the basis of scale-independence and scale-dependence to capture some of the essential cartographic elements of topographic relief. The model, which uses Fourier and fractal methods in combination with local surface operators, is invertible, and is used to simulate topography. The uses of such simulated topography are discussed, as are the advantages of the particular model used. The model is developed mathematically, and seems in its application to produce terrain with some desirable cartographic traits.

KEY WORDS: Terrain, topography, simulation, Fourier, fractal.

INTRODUCTION

Since the pioneering work of Mandelbrot (1967), considerable attention has been devoted in the literature of computer graphics to the computer simulation of realistic-looking landscapes. While many of the original applications were restricted to making graphics for the movie industry (Fournier et al., 1982), more recently realistic-looking cartographic depictions of simulated landscapes have found applications in flight simulators and war-gaming systems. While these computer-generated terrains have usually satisfied the use for which they were intended, much of the topography generated by simulators remains unsatisfactory from a cartographic standpoint. While the terrain is a good approximation of reality, it does not reflect the intrinsic terrain characteristics familiar to those who are professionally concerned with the cartographic representation of real terrain. For example, fractal terrain simulators usually produce terrain with many local maxima and minima, both of which are features that tend to be eliminated by mass wasting and stream processes in the real world.

The intent of this paper is to incorporate these intrinsic terrain characteristics into a model of terrain. This model can be calibrated using digital representations of actual terrain and can then be inverted to simulate terrain. The goal of this approach is to produce simulated terrain which a cartographer would recognize as having familiar properties and to increase the realism and utility of synthetic terrain for cartographic uses.

It is reasonable to ask why a cartographer should be concerned with the simulation of tography when a surplus of the real thing exists, much of it (especially the ocean floor) yet to be mapped. Cartographically, simulated terrain is invaluable in flight simulation and war games, and the proposed model is of value here for its near real-time simulation speed and ability to change scales. There are, however, numerous potential applications beyond these. For example, recently attention has been devoted to the perception testing of cartographic representational methods. This testing frequently requires a context-free "background" as a control. Morrison (1986) has suggested that simulated topography could be used to fill in background information on thematic maps where the need is merely to convey impressions. Geomorphologists and soil scientists frequently require terrains with specified characteristics for further simulation, for example, of the flow of surface water into drainage basins (Band, 1986). Cartographers and photogrammetrists are particularly interested in extracting ridge and channel structures from digital terrain models (Douglas, 1986), usually to allow conversion to the Triangulated Irregular Network (TIN) data structure, and could make use of simulated terrain with various pre-specified characteristics to test their methods. Finally, modeling terrain forces the modeler to examine the form of terrain and the processes which have created these forms, subjects of more than passing interest to geographers and geologists alike.

The advantages of the topography model presented are fourfold. First, the model borrows its
mathematical background from two substantive areas of theory, those of spectral and fractal analysis. Second, the model can be calibrated from highly generalized real terrain and is invertible across a large variety of larger scales allowing “zooming.” Third, since the model can be inverted without using stochastic or iterative methods, it is fast and efficient and produces terrain in a standard grid-based digital elevation model format. Finally, the inversion of the model allows parametric control over the appearance of the terrain structure and its texture, allowing the simulation of topography with “designer” characteristics.

SCALE AND THE STRUCTURE OF TOPOGRAPHIC RELIEF

Modeling topography depends greatly upon the separation of landforms into their component parts. For some time, geologists and geographers have separated regional trend or drift from local variations. This is the basis for the analytical methods which separate these components using least squares techniques, the most common of which is trend surface analysis (Unwin, 1975). In trend surface analysis, elevation is modeled as a polynomial expansion of eastings and northings, with the fit determined by least squares methods. More sophisticated methods have used trigonometric series and kriging (David, 1973). A review of both global and local surface models is contained in Bassett (1972). The least squares methods make the unreasonable assumption for terrain that residuals from a global model are randomly distributed, with zero mean and a given variance. These techniques are therefore generalization methods, providing at best a good regional or global model of terrain. Such models are simplistic in the extreme and fit such surfaces as linear trends (uniform dips) without breaks.

At this point it is appropriate to discuss the scale components of topography. Clearly global generalizations such as trend surfaces can give broad descriptive characteristics, but the inversion of these methods produces unrealistic terrain, even if a random model generates the residual. First, topography consists of slopes with fairly predictable characteristics, determined largely by gravity, structural geology, and the resistance characteristics of the various rocks in question. These characteristics apply at small scales, i.e., are large in magnitude on the earth’s surface and are therefore amenable to modeling with global surface models. In the following discussion this aspect of topography is referred to as topographic structure.

At larger scales the impact of geomorphological processes is more noticeable. Probably the biggest impact at this scale concerns the typical forms resulting from stream erosion and deposition, i.e., the filling of depressions, the back-cutting of stream valleys and the dendritic structure of valley and ridge arranged within a basin. At this scale, both structural and local forces determine form. For example, within a drainage basin valleys and ridges may repeat at regular intervals as one makes a circuit around the main drainage channel. Locally, however, numerous factors can divert streams and their valleys. At the very large scale, down to the level where distances are in meters and tens of meters, individual boulders, mass-wasting, vegetation characteristics, human intervention, landslides and a myriad of other local processes tend to dominate. These processes result in small features with a larger stochastic component, but spatial structure nevertheless.

The implications of this scale-separation of forms and processes for simulating terrain are that scale itself must form the basis for a model of terrain. Above all, a model of terrain must be able to measure and separate forms with distinctive scales from other aspects of the terrain. This must be done globally, but should also apply to intermediate scales to catch systematic variations such as ridge and valley sequences. Separation of these elements allows the model to be inverted to depict solely these structural elements of terrain.

The basis of the proposal model is that two types of variations exist within topography. At small and intermediate scales, topography exhibits scale dependence. This means that geological and geomorphological processes have resulted in forms with distinctive spatial scales which have produced large features in the landscape. These features, the topographic structure, are scale-dependent and show significant scale and size relationships. As is developed below, the residual characteristics of terrain when the scale-dependent structure is extracted can be modeled effectively as scale-independent fea-

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tures, since all significant scales have been extracted. This implies that the much-discussed fractal model of topography (Goodchild, 1982; Pentland, 1984) is actually a special case for landscapes where structure is missing, as may be concluded from the empirical work of Mark and Aronson (1984). In addition, Burrough (1981) has suggested that terrains seem to exhibit fractal characteristics at discontinuous scale ranges. Both Mark and Aronson (1984) and, more recently, Roy et al. (1987) have noted that different fractal dimensions apply to different scales, apparently violating the requirements for self-similarity. It is arguable that the breaks in the fractal characteristics are attributable to scale-dependencies, an assertion which can only be tested by measurement over very large scale ranges.

In terms of the scale-dependencies, the model must be able to capture both smooth variations and the discontinuities often associated with structural geology such as fault lines, scarp slopes and precipices. While trend surfaces can only generalize these features, Fourier analysis is one method which has been used successfully to capture them (Bloomfield, 1976; Rayner, 1971; 1972). The method is applicable to statistical interpolation and data compression (Clarke, 1984; 1985) and has been used extensively in shape analysis (Moeller and Rayner, 1981; Christopher and Waters, 1974). Fourier analysis allows a terrain surface to be tested using sets of trigonometric series to examine the "goodness of fit" of expansions with particular amplitudes and wavelengths.

The method does not use least squares, but can compute a power spectrum showing the relative and absolute contribution of each set of two-dimensional harmonic sine and cosine waves to the explanation of the entire topographic surface (Rayner, 1971). The inverse Fourier transform can then be applied to the significant harmonics to yield a Fourier-based generalization of the terrain.

An additional benefit of using the method is that the power spectrum, computed from the Fourier coefficients, can be used to determine the precise size of the scale-components present in a particular piece of topography. A full description of the application of the method for three dimensional data and a computer program for their implementation are contained in Davis (1973). It should be noted that as a preliminary step the method computes and subtracts a linear trend surface from the terrain equivalent to the extraction of a scale component with an extremely large wavelength.

The model of terrain structure, therefore, is a simple one. It consists of stating that small-scale (large) features in topography can be modeled using sets of trigonometric functions, in particular the sums of pairs of sine and cosine waves with different wavelengths and amplitudes. Since the sampling theorem comes into play when a particular scale is chosen, the model can be made to incorporate the fact that scale components can be constrained in size to those which repeat within the size of the map used. A range of from the length (or width) of the map to three times the map resolution was chosen since extension beyond this limit was found to exhibit discontinuities at sharp breaks in the terrain, a characteristic known as the Gibbs phenomenon, which largely account for the severe banding effect in the images produced for a preliminary test of the proposal model (Clarke, 1987). Furthermore, since the inversion of the Fourier model produces fairly smooth surfaces, these digital terrain models can be split to double their resolution by simply adding new values using linear interpolation within the grid.

Another important feature of using this method is that the significant harmonics required for inversion usually fall within the first few sets of harmonic pairs, i.e., at the high wavelengths. This means that the Fourier analysis can be conducted on a highly generalized digital terrain model for any piece of terrain, normally less than 20 rows and columns in size.

**SCALE AND TEXTURE**

Given a set of scale-dependencies forming a structure for terrain, attention can be turned to the modeling and eventual simulation of the texture of terrain. Textural features have a relief far less in magnitude than the structural relief, and can be spatially patterned or not. Without any spatial structure, a Gaussian random noise generator could provide a good simulation, but the inclusion of this term in the model would be by the addition of an error term. Terrain is hardly shaped by errors and since no spatial structures would exist in random
texture, this seems a poor model of terrain. On the other hand, as the scale becomes larger the share contributed by process becomes increasingly influenced by larger and larger numbers of factors and therefore tends to seem more random.

In addition, texture is influenced by some of the structural components which define such things as the direction and magnitude of slope. A satisfactory model of texture, therefore, has to contain a random element, but must let non-randomness permeate the resultant distributions. The fractal model has these characteristics and has been used a great deal for the simulation of terrain. In this model, dimension (usually two for a surface) is fractional and is allowed to take values between 2.0 (uniformly flat) and 3.0 (indistinguishable from a solid). Various methods have been used to measure the fractal dimensions of terrain, including cross-sections, contours, variograms, surface area calculations and cell-counting (Shelberg et al., 1982; Clarke, 1986; Goodchild, 1982; Mark and Aronson, 1984). Few of these methods are invertible, with the notable exception of the Fourier power spectrum method (Burrough, 1981; Pentland, 1984). This method was suggested by Fourier et al. (1982) as one of three possible approaches to fractal modeling and was used by R. Voss for the illustrations in Mandelbrot (1977). Much like many of the other techniques, this method performs a log-log least squares fit of the sums of the squared amplitudes of the Fourier coefficients (the power) with resolution; in this case the wavelength determined as a root of the sum of the squared wavelengths east-west and north-south. Successive harmonic numbers of each wavelength have wavelengths given by the length of the map divided by increasing integers. Since the amplitudes associated with fractional Brownian noise are non-zero and decay with the inverse of the harmonic number to some power with a wavelength of the length of the map divided by h, the log-log fit of these data is a straight line with a slope (spectral density exponent) of:

$$\beta = 7 - 2f$$

where $f$ is the fractal dimension. Since the Fourier power spectrum has already been computed for the small generalized source data set in the structure phase of the method, only an additional regression is necessary to compute the fractal dimension. This regression is then inverted and used to produce Fourier coefficients with fractal characteristics. These new coefficients are given the same signs as the originals from which they were derived and are simply equated across the four coefficients defining each harmonic. An alternate strategy would be to assign the values at random with totals forced to equal the amounts predicted in the regression.

The Fourier transform can then be inverted to produce the texture which must be scaled down to be added to the structure. The amount of scaling is as a percentage of the structural relief. When the structure is split to interpolate new values, the texture is simply mirrored about its axis and duplicated since self-similarity allows replication of parts with scale changes.

**SCALE AND PROCESS**

The final part of the model has to do with local processes, in particular mass-wasting. Since this process applies only at the very largest scale, the operator has a small neighborhood (a grid cell's eight neighbors) and operates on the final, fully split grid. As has been noted, the fractal texture generates large numbers of local maxima and minima for the data which are not reflected in real terrain. In this case the worst offender is the pits, which are normally filled in by mass-wasting in real terrain except in a few special case terrain types (e.g., karst). The final phase of the model involves a local operator to find pits and fill them by averaging their elevations with the next-lowest neighboring grid cell. Few adjustments would be necessary to modify this operator to include erosion by smoothing to eliminate peaks as well and to allow back-cutting of slopes to erode stream valleys. The operator is invoked as many times as the user wishes, filling and smoothing more or less as required.

**THE MODEL**

The above-described model can now be expressed in summary form. Terrain is assumed to be represented as a set of discrete points in the form of a regular grid. Elevation $z$ at implicit location $(x,y)$ is given by:

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by mirroring the grid onto itself so that each new quadrant takes the form of the original.

The final stage of the inversion involves passing the pit filter over the terrain a number of times determined by the user. Zero passes of the filter leaves the pits as they were generated by the fractal texture simulator. As the number of passes increases, the pits become filled, eventually smoothing all depressions but leaving peaks untouched.

The following figures demonstrate the effect of varying the fundamental parameters of the model. Figure 1 shows an actual depiction of the Copake, New York 7.5-minute United States Geological Survey quadrangle, digitized with some 24,000 elevations selected from bench marks, significant points and along contours. These random elevations were gridded with a resolution of 50 meters and were generalized to 150 meters for Figure 1. The figures are perspectives, viewed from the SSW at 12 degrees above the horizon, and have been hyposometrically colored and hill-shaded.

The digital elevation model was then reduced to a 29 by 22 grid as input to the computer program which inverted the model. This corresponds to a ground resolution of 500 meters. The display images were split a total of twice to give a final terrain grid of 116 by 88. Figure 2 shows terrain simulated using the inversion of the model with no texture and no erosion, using a cutoff value of three percent of the total variance for inclusion of harmonics. The first stage involved computing and subtracting a weak linear trend surface (r-squared of 0.12) with a moderately steep dip to the west. The terrain model was inverted using only seven harmonics, with wavelengths between 15.5 kilometers and 6.07 kilometers. The blocky effect is due to the subdivision algorithm. Figure 3 shows the effect of reducing the value for the cutoff to one-quarter of one percent of the variance. The number of significant harmonics now becomes 32, with wavelengths from 15.5 to 2.78 kilometers. At this scale, the cyclical patterns of drainage systems at intermediate scales become visible.

Figure 4 shows the same structure as Figure 3, but has texture added and scaled to ten percent of the structural relief. Similarly, Figure 5 shows the same terrain with texture as 20 percent of relief. In this map the pit and peak problem of fractal texture becomes more obvious. The approximately equal number of local maxima and minima makes the terrain look unrealistically "spiky" compared to Figure 1. One obvious way to eliminate this effect is smoothing to simulate mass wasting. While this would obey the law of conservation of matter, it would also eliminate the local maxima. An alternative mass-wasting algorithm as described above preserves the maxima while filling in the pits. The log-log fit of the Fourier power spectrum with distance gave a slope value of 1.315 and an r-squared of 0.43. The low r-squared value is because the significant harmonics were included in the regression. The reason for this is that excluding them from the regression eliminates most of the data values in the higher wavelengths, which, as expected, have the higher powers. The effect is to bias the regression to decrease the slope, producing a higher fractal dimension. The actual corresponding fractal dimension for the surface was 2.84, corresponding with the 2.49 obtained from the variogram method (Mark and Aronson, 1984) and 2.14 from the Triangular Prism Surface Area method (Clarke, 1986).

Figure 6 shows the terrain in Figure 4 after three cycles of pit-filling. Figure 7 shows the result after nine such cycles. Too many cycles seem to make the terrain smoother than the actual terrain in Figure 1, as comparison with the detail of the texture on the actual terrain surface (Figure 8) reveals. A few pit-filling cycles seem to be all that is required to give the terrain a realistic appearance.

DISCUSSION

The invertible model proposed here has three distinct advantages over previous approaches to terrain modeling and simulation. First, the basis in scale means that both scale-dependency and scale-independency can be modeled independently to simulate different characteristics of terrain, most notably the structure and the texture. Also, the model incorporates local surface processes which can be used to introduce very large-scale variation. Thus both the model and the simulated terrain have a basis in the actual nature of real terrain rather than being merely stochastic in nature. In fact, the scale elements can be calibrated from actual terrains to give topography with specific characteristics. It is intriguing to think how the model's basic
\[ Z_{r,s} = T_{r,s} + cF_{r,s} + H_{r,s} + E_{r,s} \]  

where:

\[ T_{r,s} = \beta_0 + \beta_1 x + \beta_2 y \]  

defines a linear trend surface (very low frequency harmonic),

\[ F_{r,s} = \sum_{i=1}^{i-1} \sum_{j=1}^{j-1} (A_i C_i + B_j S_j + C_i S_i D_i) \]  

where the sine and cosine terms are given by:

\[ C_i = \frac{\cos(2i\pi x)}{L_r} \]  
\[ C_j = \frac{\cos(2j\pi y)}{L_s} \]  
\[ S_i = \frac{\sin(2i\pi x)}{L_r} \]  
\[ S_j = \frac{\sin(2j\pi y)}{L_s} \]

and where \( L \) is the length of the series in \( x \) and \( y \) as given by the subscripts. To avoid the problems of the Gibbs phenomenon, the maximum length of the series in \( x \) and \( y \), \( k \) and \( l \) above, is limited to one-third rather than one-half of the lengths of the map in \( x \) and \( y \). The power spectrum for this two-dimensional Fourier series is an array \( P \) given by:

\[ P_{ij} = A_{ij}^2 + B_{ij}^2 + C_{ij}^2 + D_{ij}^2 \]  

The Fourier coefficients \( A, B, C, \) and \( D \) are given by:

\[ A = B = C = D = \sqrt{P_{ij} / 4r^2} \]  

where \( r \) is 1 if \( i \) and \( j \) are zero, 2 if one but not both are zero, and 4 otherwise. The power associated with harmonic pair \((i,j)\) is given by:

\[ \log P_{ij} = \alpha + \beta \log d_{ij} \]  

where \( d \) is the ground resolution distance corresponding to harmonic pair \((i,j)\). The values of \( A, B, C, D \) and \( D \) in equation [v] can be either positive or negative, and if desired could be assigned random values such that their sum fell within limits. In the implementation of the model here, they were assigned values derived from an empirical Fourier analysis used to yield the \( H \) component.

The equation for \( H \) is the same as equation [iii], with the constraint that \( A, B, C, \) and \( D \) are zero unless:

\[ \tau < 100P_{ij} \sum_{i=1}^{i-1} \sum_{j=1}^{j-1} P_{ij} \]  

where \( \tau \) is a critical cutoff percentage of the total variance attributable to any harmonic, i.e., the level defining significance. The value \( c \) in equation [i] is a constant given as a proportion of the topographic relief into which the self-similar texture is to be scaled.

Finally, \( E \) is an erosion value determined by iterating a local operator. This operator applies only to pits in the terrain, i.e., cells where the center cell value is the minimum of its eight neighbors. In these cases the pit cell is replaced by an average of the current value and the next lowest elevation plus one.

**INVERTING THE MODEL**

Inversion of the model assumes the input of a small, highly generalized piece of terrain. This segment is used to compute the two-dimensional Fourier coefficients and the power spectrum and from them to estimate the fractal dimension and the log-log power to distance relationship. From these data the two major components of the landscape can be calculated. In the case of the scale-dependencies, the Fourier coefficients are zeroed out except in the cases when the harmonic is deemed to be significant, i.e., the harmonic explains more than some cutoff level of the total surface variance. The Fourier transformation is then inverted, generating elevation values as summations of the Fourier expansion summed over significant harmonics. The resultant array is then split until it reaches a desired maximum size, the number of times being determined either by the scale at which the terrain is desired to be sampled or by the size of a given display device.

The texture is generated by the same inverse Fourier transform, except that the Fourier coefficients are given by values from the log-log regression of power and distance, equalized across the four values and use the same signs as the original Fourier coefficients. The texture is scaled by a percentage of the relief in the structure and is split.
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parameters could be adjusted for different terrain "types," perhaps glacial, karst, tectonic, desert, sub-marine or even extraterrestrial (Woronow, 1981).

Second, the model is applicable at all scales above the size of the map. Although it would be possible to repeat the structural parameters for areas larger than the map, the model would generate essentially identical maps at smaller scales. At very large scales, the structural component would disappear and become a broad regional trend and the fractal texture with its local erosion and mass-wasting operators would predominate. Again, it is intriguing to imagine how the model's parameters would change with scale, perhaps even down to scales beyond those normally considered "cartography."

Third, the model is simple and efficient and the inversion is fast. The terrain models shown in the diagrams were all generated on a heavily-loaded VAX 11/730 running Berkeley UNIX 4.2 in less than two minutes of CPU time. The efficiency is largely due to the ability to split the structural and textural parts of the terrain separately. When surfaces are processed without this generalization, large numbers of harmonics are involved and CPU time increases from minutes to hours without justifiable improvements in the simulated terrain's realism. In the applications here, the Fourier analysis of the terrain and the computation of the fractal Fourier coefficients make up about one-eighth of the time, the two inverse Fourier transforms about one-half of the time, the splitting and input/output about one-eighth of the time and the remaining quarter of the time is taken up with pit-filling. Clearly this last element is a function of how many cycles are applied. More complex erosion algorithms may add considerably to the time. However, the order of magnitude of the simulation is within the capabilities of all mini- and even some micro-computers.

Finally, the resultant simulated terrain using the proposed model appears to give "realistic" terrain. During display of the images and occasional contouring of the maps, several colleagues, unaware that the terrain was simulated, attempted physical interpretations of the landscape features or asked where the mapped terrain was located. If this is a preliminary "perceptual" test of the cartographic acceptibility of the simulated terrain, then perhaps this model will indeed allow terrain simulation to enter the repertoire of tools of the digital cartographer.

REFERENCES


