The Trade Area of a Displaced Hexagonal Lattice Point

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Consider the classic central place system of uniform demand density on an infinite plain. Let us suppose that a single entrepreneur is, for some undefined reason, unable to locate at his optimal hexagonal lattice point. He is forced instead to locate at a distance \( r \) away in a line bisecting the angle between two of his nearest neighbours (Figure 1). We adopt a coordinate geometry in which the distance separating adjacent lattice points is 1, the optimal point \((0,0)\) and the six neighbours are located at \((\pm \sqrt{3}/2 \pm 1/2)\) and \((0 \pm 1)\). Our unfortunate friend has been forced to locate at \((r,0)\).

The upper half of the modified trade area of point 0 is found by connecting the perpendicular bisectors of \(0A, 0B\) and \(0C\) to form \(KLMN\). The lower half is symmetrical. By a little application of the principles of coordinate geometry, we find that the trade area is

\[
\frac{9\sqrt{3}}{2} \left(1 - \frac{r^2}{3-r^2}\right) \left(3-4r^2\right)
\]

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This equation is valid for \( r < 0.577 \), above which the trade area shares a boundary with that of point D, and thus assumes another side.

The hexagonal trade area associated with point (0, 0) is \( \sqrt{3}/2 \). Figure 2 shows the above function plotted as a percentage of this value against \( r \). Note that even at \( r = 0.17 \), that is, 17% of the distance to the nearest lattice point, the trade area is still 99% of its full value. It is still 92% at \( r = 0.5 \). We have, of course, deliberately chosen the worst direction. If 0 had been
displaced along the line directly towards $A$, for example, the drop in trade area would have been more rapid.

The implications are clear. The economic advantages of locating precisely on the nodes of a hexagonal lattice are quite weak. It is difficult to conceive of a real-world entrepreneur going out of business or being forced to relocate because his market area was only 92% of his neighbour's.