Locating System,
Washington, D. C., 1970.

The Trade Area of a Displaced Hexagonal Lattice Point

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Consider the classic central place system of uniform demand density on an infinite plain. Let us suppose that a single entrepreneur is, for some undefined reason, unable to locate at his optimal hexagonal lattice point. He is forced instead to locate at a distance r away in a line bisecting the angle between two of his nearest neighbours (Figure 1). We adopt a coordinate geometry in which the distance separating adjacent lattice points is 1, the optimal point (0, 0,) and the six neighbours are located at $(\pm\sqrt{3}/2 \pm 1/2)$ and (0 ± 1). Our unfortunate friend has been forced to locate at $(r,0)$.

The upper half of the modified trade area of point 0 is found by connecting the perpendicular bisectors of 0A, 0B and 0C to form *KLMN*. The lower half is symmetrical. By a little application of the principles of coordinate geometry, we find that the trade area is

$$\frac{9\sqrt{3}\quad(1-r^2)^2}{2(3-r^2)\quad(3-4r^2)}$$

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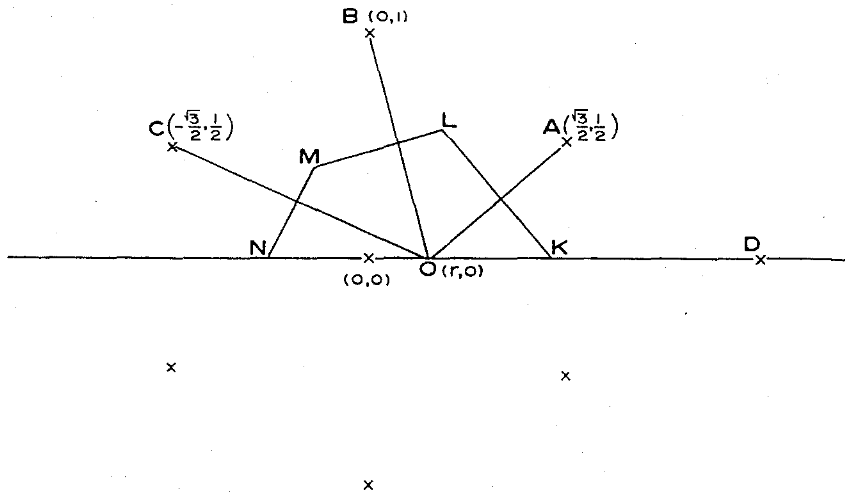

FIG. 1. Coordinate Geometry of a Displaced Central Facility.

This equation is valid for $r < 0.577$, above which the trade area shares a boundary with that of point D, and thus assumes another side.

The hexagonal trade area associated with point (0, 0) is $\sqrt{3}/2$. Figure 2 shows the above function plotted as a percentage of this value against r. Note that even at $r = .17$, that is, 17% of the distance to the nearest lattice point, the trade area is still 99% of its full value. It is still 92% at $r = 0.5$. We have, of course, deliberately chosen the worst direction. If 0 had been

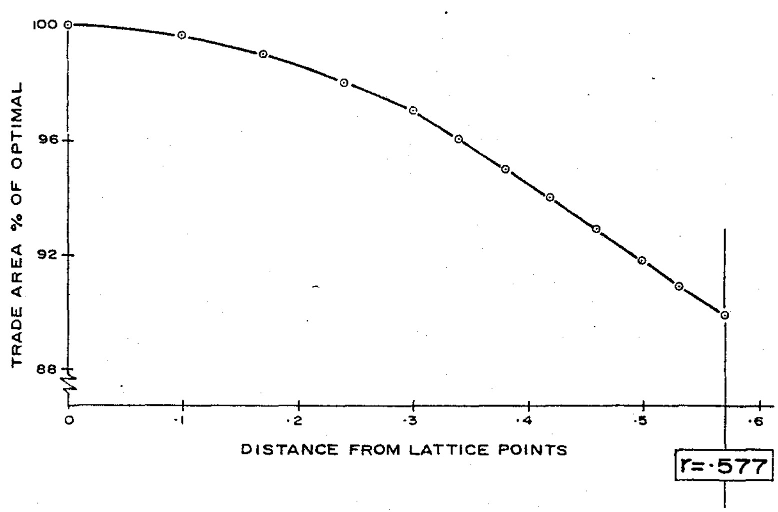

FIG. 2. Decline in Trade Area with Increasing Displacement.

displaced along the line directly towards *A*, for example, the drop in trade area would have been more rapid.

The implications are clear. The economic advantages of locating precisely on the nodes of a hexagonal lattice are quite weak. It is difficult to conceive of a real-world entrepreneur going out of business or being forced to relocate because his market area was only 92% of his neighbour's.