TEMPORAL TRENDS IN THE SPATIAL ORGANIZATION OF A SERVICE AGENCY

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TERRITORIAL PATTERNS form a dominant feature in the spatial organization of societies, as has been noted in *The Science of Geography*.[1] The analysis of these patterns has attracted the attention of several disciplines and approaches.[2–7] It is the purpose of this paper to offer some objective measures of the evolution of territorial patterns in an attempt to understand the processes which operate when space is divided into functional areas. It is hoped that this study will complement earlier work[8] by measuring the form of the areas through time and by explaining changes in terms of the role of the decision-makers.

The Rural Operating Areas (ROAS) of the Ontario Hydro Electric Power Commission were set up over thirty years ago to service electrical distribution in rural areas of the province. In the southern part of Ontario, the study area, there were 86 areas in 1948 (Figure 1), and in every year of operation since 1948 there has been at least one boundary change or amalgamation. By 1967 (Figure 2) there were 53 areas. Table I summarizes the changes that have taken place in the period 1948–67. Each area is serviced from a centre which serves as the base for operations within the area and as an equipment storage depot. In this study the depot will be treated as a point location.

The data base was particularly suitable for several reasons. Decisions regarding boundary and service centre location are made by regional managers, who are assumed to have equal access to information and comparable levels of business acumen. In 1948 there were eight managers, but this number has since been reduced to five.

Further, the measures used in the study require certain conditions within each area, notably a constant population density and a dense transportation network. Because the population served in each ROA is purely rural, and demand for electrical service comparatively ubiquitous, the ROAS provide a useful and suitable body of data for this analysis.

Three assumptions are made concerning the characteristics of an ideal spatial organization against which real patterns can be measured. The first relates to the relative magnitude or scale of operation of each area. From an academic point of view, it embodies the concept of economy of scale, and the determination of an area's optimum magnitude from economic considerations. From a point of view probably more relevant to the decision-maker, the areas all perform the same function, and so will operate best when of roughly equal scale. How this vague notion of scale is perceived will be determined empirically. The other two characteristics

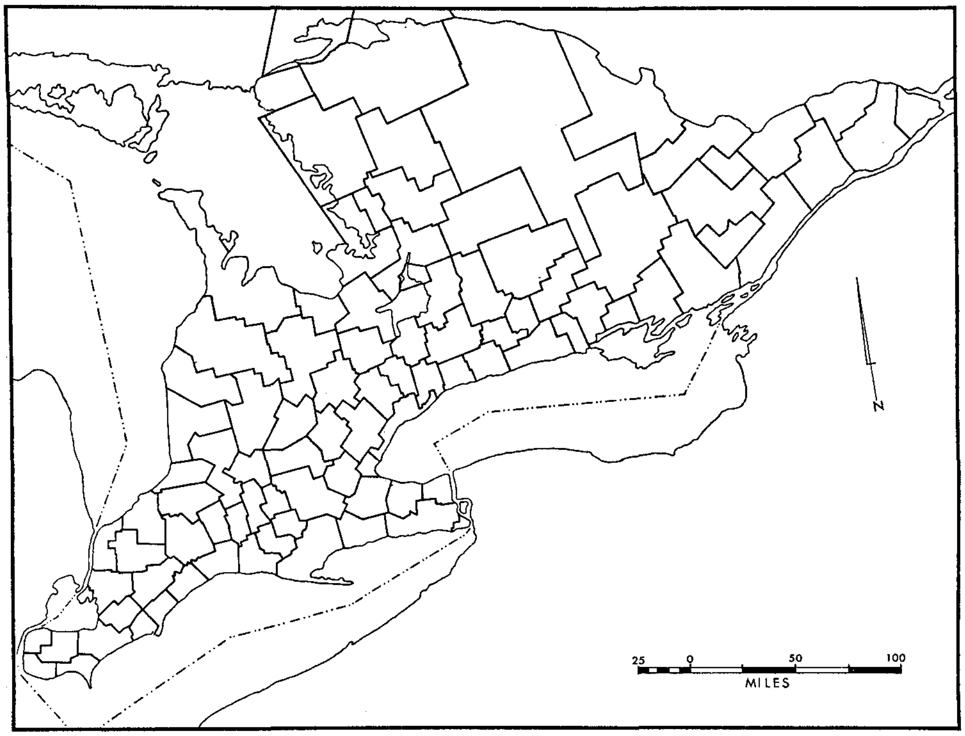

FIGURE 1. Ontario Hydro rural operating areas, 1948.

TABLE I

SUMMARY OF RURAL OPERATING AREAS*

I	II	III	IV	V	VI	VII	VII
1948	86	3,122	6.197	317.1	1,048	1,452	0
1949	86	3,533	22.065	346.6	1,157	1,645	0
1950	89	3,467	19.674	351.1	1,195	1,809	0
1951	89	3,456	16.261	379.0	1,274	1,959	0
1952	91	3,442	15.974	391.1	1,315	2,093	7
1953	92	3,385	14.870	395.6	1,338	2,290	5
1954	89	3,454	14.425	416.8	1,405	2,524	0
1955	89	3,468	14.458	428.6	1,433	2,772	0
1956	89	3,447	14.107	439.2	1,439	2,850	1
1957	88	3,483	14.167	446.3	1,471	3,078	3
1958	85	3,607	14.351	465.7	1,498	3,288	2
1959	84	3,646	14.400	478.9	1,521	3,486	2
1960	82	3,732	14.405	498.8	1,570	3,733	4
1961	78	3,882	14.818	518.6	1,617	3,693	4
1962	78	3,871	14.851	523.0	1,607	3,861	5
1963	74	4,072	15.514	555.9	1,687	4,224	5
1964	71	4,213	15.698	579.7	1,748	4,382	10
1965	65	4,585	18.339	630.9	1,877	4,879	11
1966	58	5,008	20.117	693.3	2,011	5,551	10
1967	53	5,386	22.147	745.6	2,112	6,191	–

*I = year of operation, II = number of rural operating areas, III = mean physical area (arbitrary units), IV = variance of physical area (arbitrary units), V = mean of miles of transmission line, VI = mean of number of farm customers, VII = mean of number of non-farm customers, VIII = number of eliminations of rural operating areas.

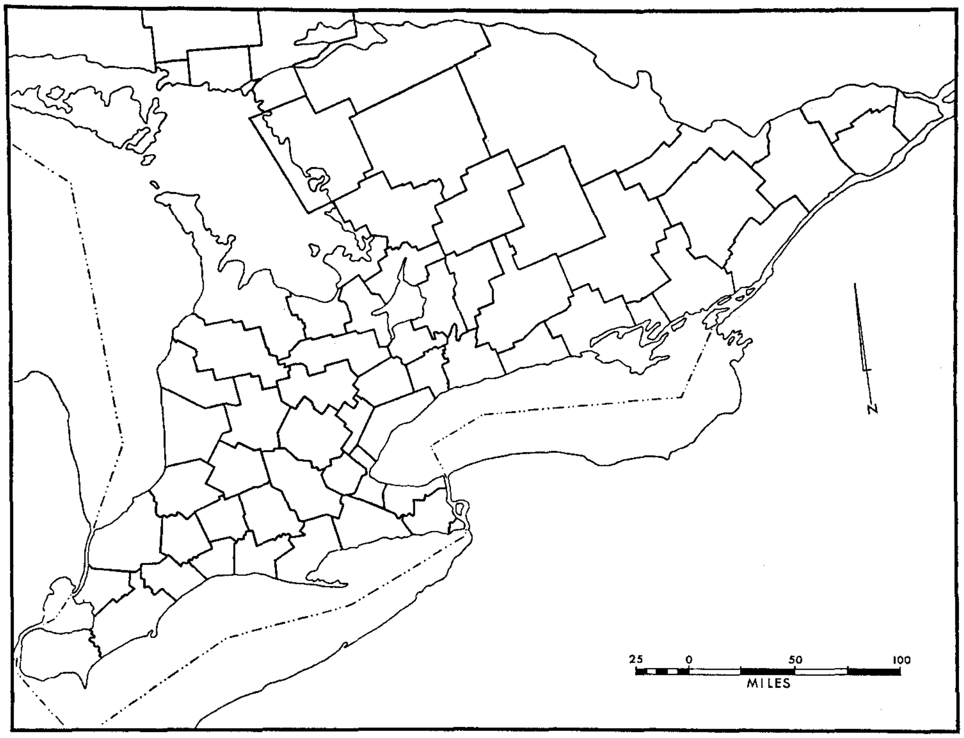

FIGURE 2. Ontario Hydro rural operating areas, 1967.

concern the geometry of an area once its scale has been determined. Minimization of operating cost requires that each area be as compact as possible, and that the service centre be centrally located. Operationalization of these concepts is discussed below.

Maps are available for each year in the study period showing the boundaries of the ROAs and the locations of the service centres.[9] The numbers of miles of line, farm customers, and non-farm customers are also indicated for each area. The boundary of each ROA was coded as a series of straight-line segments by defining two three-digit coordinates for each vertex. A Benson-Lehner digitizer was used, giving an accuracy of ±½ mile in the east-west direction and ±¼ mile in the north-south direction. All centres and boundaries were coded in this way on a standard grid. Throughout the study each area has been treated as a homogeneous two-dimensional shape by assuming that the demand for service is equally distributed throughout the area.

Measures of the degree of geometrical optimality were based upon the properties of the quantity known as moment of inertia. It is defined for any point x in the area as

$$I_x = \int_a r^2 da,$$

that is, the sum over the area of each minute segment of area multiplied by the square of the distance r separating it from the point x.

Moment of inertia has two important properties, which permit the construction of simple measures of compactness and centrality of location of the service depot. First, for any given area, I_x is minimum for a point located at the centre of gravity of the area. This is clear from the parallel axes theorem[10] which relates moment of inertia of any point to that of the centre of gravity,

$$I_x = I_a + Ah^2,$$

where I_a is the moment of inertia of the centre of gravity G, A is the total area of the shape, and h is the distance from x to the centre of gravity. Since the term Ah^2 must always be positive, the minimum value of I_x occurs when h is zero.

The first geometrical measure, the efficiency index E, is defined as I_a/I_x. It has the maximum value 1.0 when the depot is centrally located, and decreases to zero as the depot location becomes more remote. In this study, values of the index varied from 0.3 to 1.0.

The second property of the moment of inertia of a point forms the basis for determining a shape index. For any shape of area A, we may calculate a moment of inertia at the centre of gravity, I_a. This will be minimum when the shape is a circle, that is, when it is ideally compact. The shape index S is defined as

$$A^2/2\pi I_a$$

or the moment of inertia of a circle of area A at its centre, divided by the moment of inertia of the actual shape at its centre of gravity. It varies between 1.0 for a circle to 0.0 for an infinitely extended strip. In this study values varied from 0.50 to 0.95.

Both indices have maximum values corresponding to the ideal location of service centres and boundaries. They have an additional advantage in that divergence from optimality is measured in terms to some extent representative of increased cost of service. In this way the shape index has advantages over purely geometrical measures proposed, for example, by Bunge.[11] Clearly the square of the distance is not the best functional representation of the transport component of service cost, and has been used simply for its mathematical convenience.

To calculate these indices, each area was first divided into an exhaustive set of mutually exclusive triangles, using the vertices of the area as the vertices of the triangles. The area, location of the centre of gravity, and moment of inertia about the centre of gravity were calculated for each triangle as follows. For a triangle with vertices (X_A, Y_A), (X_B, Y_B), and (X_C, Y_C) and sides of length a, b, and c, we have:

$$\text{area } \Delta = \sqrt{[s(s-a)(s-b)(s-c)]}, \quad \text{from Hero's formula,}$$

where $s = \frac{1}{2}(a+b+c)$;

$$\text{centre of gravity} = (\tfrac{1}{3}(X_A + X_B + X_C), \tfrac{1}{3}(Y_A + Y_B + Y_C)) = (X_g, Y_g).$$

The moment of inertia of a triangle about its centre of gravity can be shown to be $I_\Delta = \Delta(a^2 + b^2 + c^2)/36$.

Next, the entire area is found to be $A = \sum\Delta$, and the coordinates of the centre of gravity of the area are

$$X_G = \sum(X_g\Delta)/\sum\Delta \text{ and } Y_G = \sum(Y_g\Delta)/\sum\Delta.$$

The moment of inertia of the entire area about its centre of gravity is found by repeated application of the parallel axes theorem, thus:

$$I_G = \sum(I_\Delta + \Delta d^2),$$

where d is the distance between (X_g, Y_g) and (X_G, Y_G). Finally the moment of inertia of the entire area about any point X can be found by a further application of the parallel axes theorem:

$$I_X = I_G + Ah^2.$$

A, I_G, I_X, E, and S were calculated for each ROA for each time period using these principles.

ANALYSIS AT THE AGGREGATE LEVEL

Both the shape index and the efficiency index are independent of scale. A large area can be just as inefficient to service as a small area, since efficiency is defined as a ratio between an optimum and the actual level. We might assume, however, that a decision-maker has some idea of the importance of an area, based upon its size, or the number of customers it contains, and feels perhaps that it is more important to rectify an inefficiency in a large area than in a small one, although the degree of inefficiency may be the same.

Accordingly, in studying the trends in the territorial organization through time, six different weights have been used for each ROA. They are: (1) equal weight (all areas the same), (2) miles of transmission line, (3) number of farm customers, (4) number of non-farm customers, (5) total number of customers (3 + 4), (6) physical area. These weights were applied to the two measures of spatial organization, the efficiency and shape indices, to obtain 12 means for each year of the study period.

The trend in each mean was examined over the 20-year period. A numerical evaluation of the consistency of trend was made by calculating the Spearman rank correlation coefficient. Thus only the consistency of the trend was measured, or the degree to which the means approached an unspecified monotonically increasing function of time. As there is no particular reason to expect a linear trend, the product-moment correlation coefficient could not be effectively interpreted. The rank correlations are given in Table II. Their statistical significance was evaluated by reference to the null hypothesis that the 20 means were independently drawn from a population of means showing no relationship to time.

Five of the trends in the efficiency index are statistically significant, but only one trend in shape index. All of the statistically significant trends are positive, suggesting that spatial reorganization has increased the level of spatial efficiency over time. Further, if the 1948 pattern is ignored, the most consistent trend is that of the indices weighted by area, as shown in Figure 3. This would suggest that area is the most

TABLE II

SPEARMAN RANK CORRELATION COEFFICIENT OF WEIGHTED MEAN EFFICIENCY AND SHAPE INDICES AGAINST TIME, 1948–67

Weighting	Efficiency index	Shape index
1. Equal (no weighting)	0.415*	0.224
2. Miles of transmission line	0.179	0.376
3. Number of farm customers	0.395*	0.012
4. Number of non-farm customers	0.824*	−0.102
5. Total number of customers	0.705*	−0.048
6. Physical area	0.656*	0.483*
Physical area (1949–67)	0.902*	

*Significant at 0.05 level.

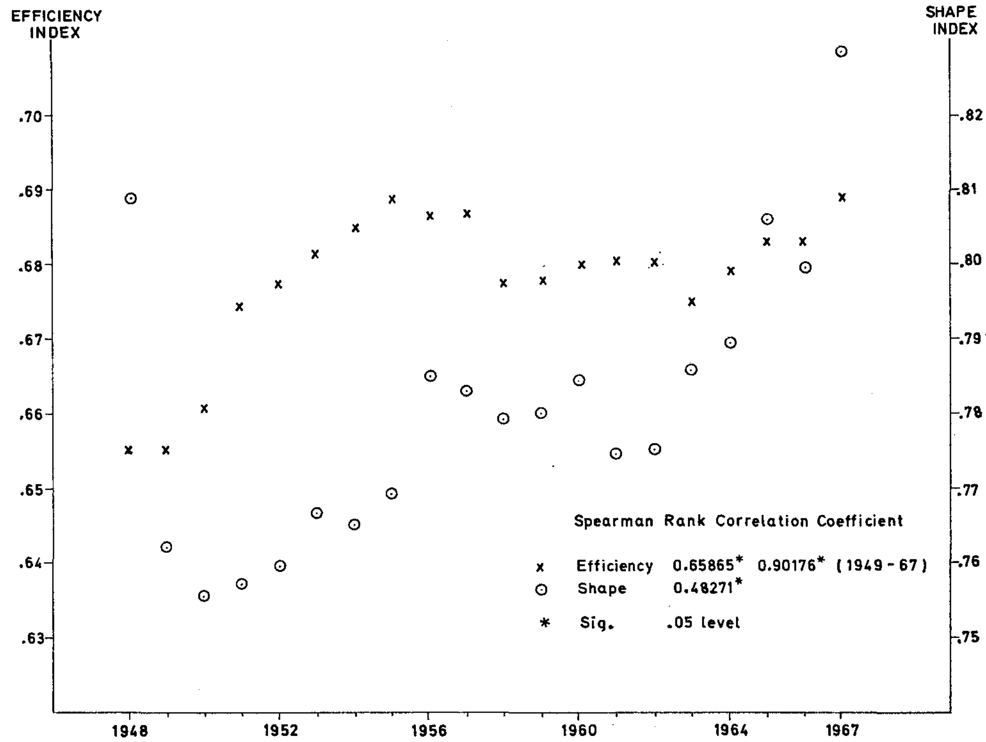

FIGURE 3. Efficiency and shape indices, weighted by physical area, 1948–67.

common measure of importance used by the decision-makers, a most acceptable conclusion since area is the one quantity visually apparent from a map of the system. The fact that the only significant trend of shape is also in the area-weighted index suggests that the decision-maker has incorporated shape into his criteria.

The analysis of trends in efficiency and shape has shown an apparent concern for these parameters on the part of the decision-maker. Amalgamations and boundary revisions have taken place in such a way that the system has become on the whole more compact and hence more efficient.

Both efficiency and shape are independent of magnitude, or scale of operation, and so do not reflect any trend towards greater uniformity in the ROAs. But this also is an attribute of organization. So far, the level of organization within each ROA has been studied: attention is now turned to the relationship between ROAs.

Four measures of scale for each ROA will be used in this section: (1) physical area, (2) miles of transmission line, (3) number of farm customers, (4) number of non-farm customers. As the territorial pattern approaches greater uniformity, the variation between ROAs will decrease. The conventional measure of variation is the variance, defined as the mean square deviation of each item from the mean,

$$\sigma^2 = \sum (x - \bar{x})^2 / n.$$

To allow comparison between years with different numbers of ROAs of different average size, the variance was divided by the mean squared to give a scale-independent quantity, or index of variation. Trends in this index were then investigated for the four measures. Table III gives the rank correlations with time. This table shows that in all cases except one there has been a trend towards greater uniformity between ROAS in those variables which relate to the magnitude of each operating area. Area again shows the most consistent trend. Removal of the initial year, 1948, from the period of analysis changes R_s from −0.712 to −0.999. It should be noted that coverage of the southern part of the province was not complete until 1949; in that year several large areas were designated to cover the sparsely populated shield country. It is notable that the three variables miles of line and number of farm and non-farm customers do not reveal any anomaly for 1948. Figures 4, 5, 6, and 7 show the trends of the index of variation for physical area, miles of transmission line, number of farm and non-farm customers respectively.

TABLE III

SPEARMAN RANK CORRELATIONS FOR INDEX OF VARIATION AGAINST TIME 1948–67

Variable	Rank correlation (R_s)
Physical area	−0.713*
Physical area (1949–67)	−0.999*
Miles of transmission line	−0.690*
Number of farm customers	+0.571*
Number of non-farm customers	−0.677*

*Significant at 0.05 level.

ANALYSIS AT THE INDIVIDUAL LEVEL

The aggregate trends identified in the previous section are the result of a large number of changes and modifications to the system of rural operating areas through the study period. In this section the actual changes are classified according to the types of modification made, and analysed for the information they give about the decision-makers' criteria.

Five types of changes are identified as follows. Type A changes result when an area is abolished by being incorporated into a neighbouring area. It is hypothesized that the criterion for this type of change is based upon the area's viability as an administrative unit. Type B changes are the converse in which an area engulfs a neighbour. It is hypothesized that these changes are prompted by the neighbour's properties and are unlikely to depend on the attributes of the area itself. Type C changes are those in which the location of the centre is changed, while boundaries remain constant. Type D changes are the converse in which boundaries are revised without relocation of the centre. Finally, type E changes result when previously unassigned areas are engulfed. This type occurred in 1948 when the northern part of the area was included in the system and has occurred on other occasions when boundaries have been redrawn to include urban areas (though the rural operating areas are not concerned with urban electrical distribution per se).

Table IV shows the number of areas in existence in each year, together with the number of changes of various types made in that year, and the total number. Type C changes only occurred in 1952, but the other types are more evenly spread.

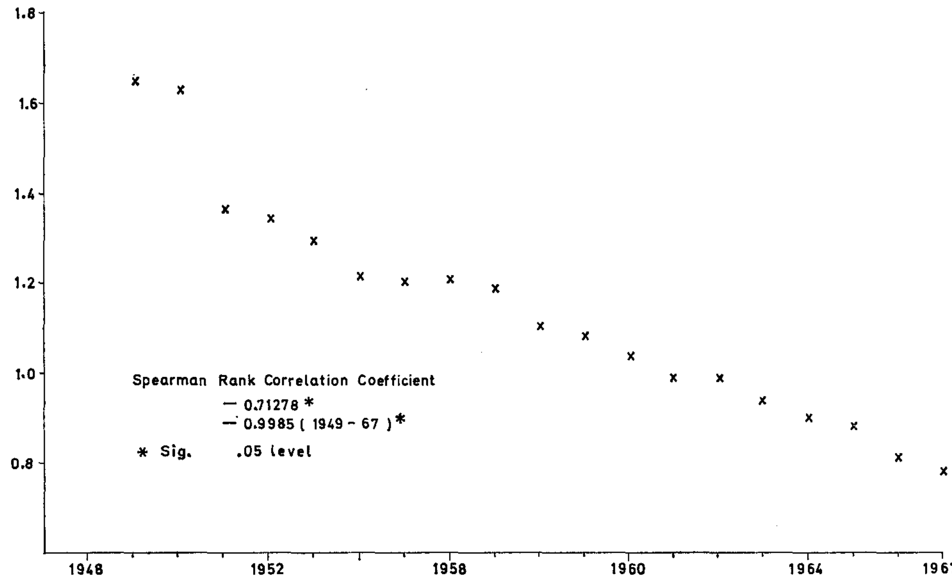

FIGURE 4. Trend of index of variation : physical area, 1949–67.

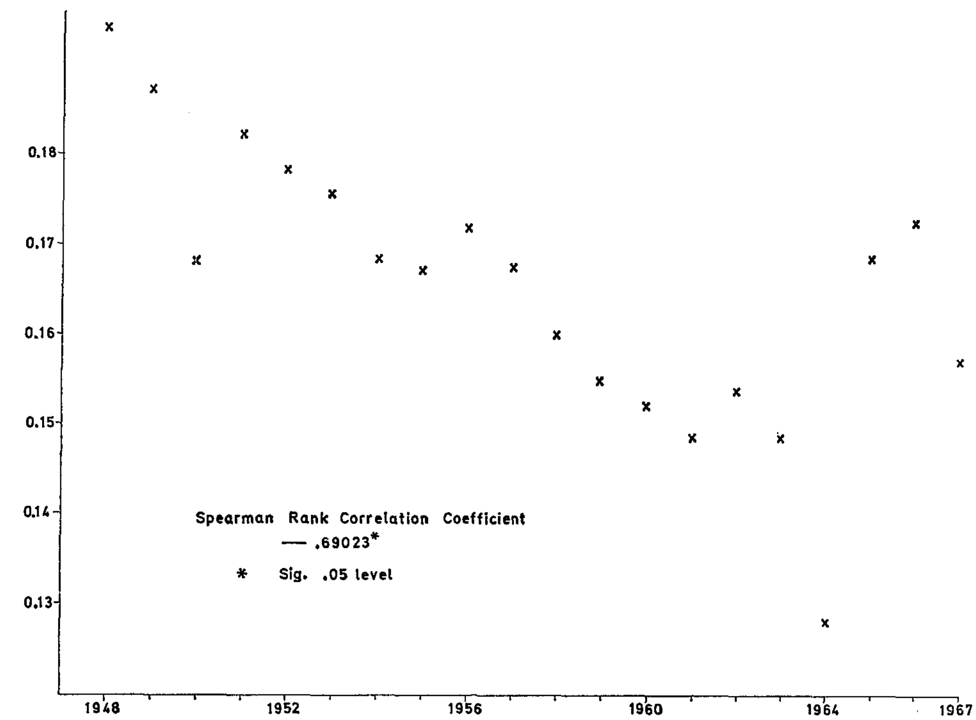

FIGURE 5. Trend of index of variation : miles of transmission line, 1948–67.

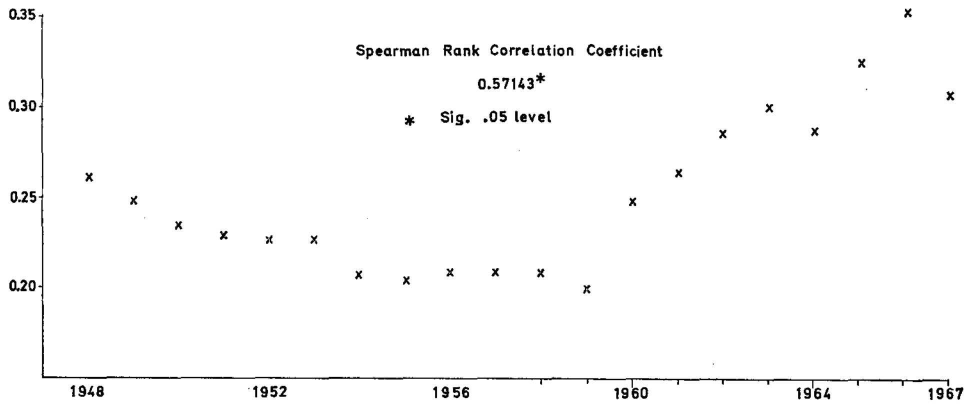

FIGURE 6. Trend of index of variation : number of farm customers, 1948–67.

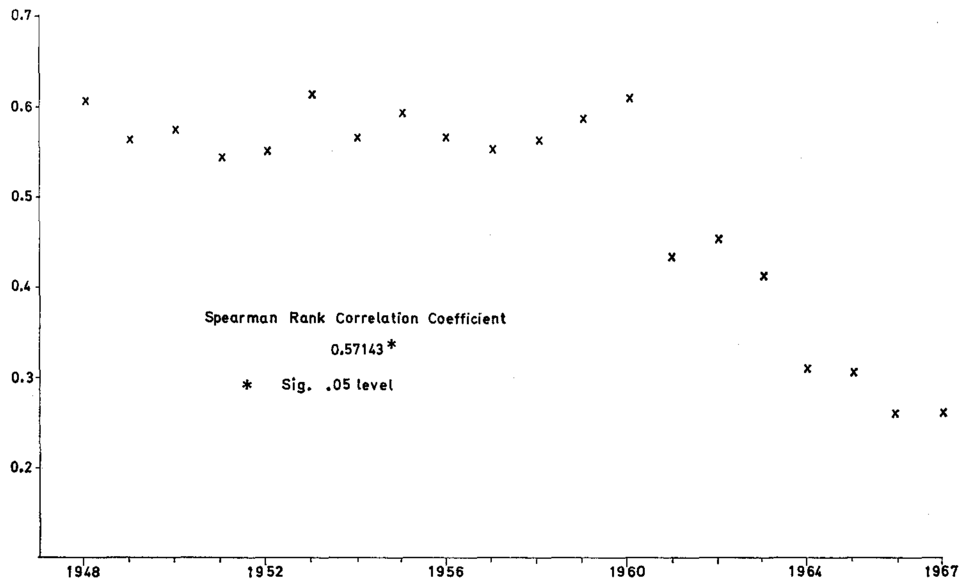

FIGURE 7. Trend of index of variation : number of non-farm customers, 1948–67.

Correlations between the incidences of various types of changes in each year are shown in Table v. Types A and B are highly correlated, as is to be expected since the presence of one implies the presence of the other. Types D and E are also correlated. There is, however, little correlation between the two groups. These regularities were formally identified by an R-mode principal component analysis of the data in Table IV. Two eigenvalues were greater than 1, and after varimax rotation the loadings were as given in Table VI. The factor scores enabled the years in the study period to be classified according to the strategy employed by the decision-makers. In the early period 1948–56, changes were primarily by adjustment of boundaries and centre locations; whereas, since 1957, changes have been made by abolition of certain areas and enlargement of others, with consequent attrition of the number of areas in existence.

TABLE IV

NUMBERS OF CHANGES

Year	Number of areas	A	B	C	D	E	Total change
1948	86	0	0	0	19	6	25
1949	86	0	0	0	4	0	4
1950	89	0	0	0	4	0	4
1951	89	0	0	0	1	0	1
1952	91	0	0	7	2	0	9
1953	92	4	3	0	11	1	19
1954	89	0	0	0	3	0	3
1955	89	0	0	0	4	0	4
1956	89	1	1	0	0	0	2
1957	88	2	3	0	0	0	5
1958	85	1	1	0	0	0	2
1959	84	4	1	0	5	0	10
1960	82	3	5	0	0	0	8
1961	78	0	0	0	0	1	1
1962	78	2	9	0	2	0	13
1963	74	3	5	0	1	1	10
1964	71	7	9	0	1	0	17
1965	65	7	6	0	2	0	15
1966	58	4	9	0	0	4	17
Total		38	52	7	59	13	169
Percentage		22.4	30.8	4.1	34.9	7.7	100.0

TABLE V

CORRELATIONS BETWEEN INCIDENCES OF TYPES OF CHANGES

	A	B	C	D	E
A	1.000	0.7640	−0.2076	−0.1320	−0.0446
B	0.7640	1.000	−0.1951	−0.2772	0.0861
C	−0.2076	−0.1951	1.000	−0.0571	−0.1035
D	−0.1320	−0.2772	−0.0571	1.000	0.6339
E	−0.0446	0.0861	−0.1035	0.6339	1.000

To analyse these changes in a systematic fashion multiple discriminant analysis is used. Discriminant analysis is concerned with the problem of grouping, both in a predictive and in an explanatory sense. In the latter, it is concerned with finding a quality, or combination of qualities, of a set of objects that best distinguishes between the groups into which these objects have previously been placed; and in the former, it is concerned with the identification of criteria by which a new object may be classified into an existing set of groups.

In this paper, the technique is applied in a purely deductive sense, to answer the question: given a set of objects that have been placed into groups, is it possible, by examination of the qualities of the objects, to discover the rules and criteria which led to the initial grouping?

Let the attributes of the objects be the variables $X_1, ..., X_n$, and let the number of groups be m. It is useful to distinguish two types of analysis, simple discriminant analysis for $m = 2$ and multiple for $m > 2$. The former problem is conventionally formulated in the following manner. Find a linear function of the variables $X_1, ..., X_n$, or $Y = \sum a_i X_i$, the discriminant function, such that the ratio of the difference between group means of this new variable to the standard error within the two groups is maximized.[13] The problem may be solved by analogy to the technique of multiple

TABLE VI

LOADINGS AFTER ROTATION

	Component 1	Component 2
1	0.8952	−0.1151
2	0.9149	−0.1245
3	−0.4535	−0.2349
4	−0.1588	0.8942
5	0.1251	0.8909

regression. If we consider Y to be the dependent, and $X_1, ..., X_n$ the independent variables, and assign Y a value of 1 for members of one group and 0 for members of the other, then the solution of the multiple regression problem is also a solution of the discriminant problem.

The method applied in this paper is due to the work of Cooley and Lohnes,[14] as implemented by Veldman.[15] Conceptually multiple discriminant analysis is similar to principal component analysis, but whereas the latter extracts components, or eigenvectors, that explain a maximal amount of variance in the data, the former identifies vectors, or discriminant functions, that explain a maximal amount of between-group variance, or identifies directions along which the groups are as separated as possible. Both discriminant functions and principal components are orthogonal and are extracted in order of the amount of variance, or between-group variance, that they explain.

Six groups are identifiable in this study; the five change types A to E, and the group formed of those areas to which no change was made in each annual decision-making process. The latter group comprises some 1,394 instances.

Eight attribute variables were used, as follows:

X_1	Physical area	A
X_2	Moment of inertia about centre of gravity	I_G
X_3	Moment of inertia about central depot location	I_X
X_4	Efficiency index	E
X_5	Shape index	S
X_6	Number of miles of transmission line in area	ML
X_7	Number of farm customers	F
X_8	Number of non-farm customers	NF

The area served from Renfrew and later Cobden was excluded since its physical area, an order of magnitude greater than any other, would have made an overwhelming contribution to the various groups to which it has been allocated at different times.

The first two roots extracted the greater part of the between-group variance. Further roots were so small as to be dominated by round-off errors in the calculations. The over-all group differentiation was tested by an F test of Wilk's Lambda[16] and found to be significant at the 99.9 per cent level. The significance of the two roots individually can be tested by a χ^2 test;[17] both roots were significant at the 99.9 per cent level.

Correlations between the two roots and the eight attribute variables, analogous to factor loadings, are given in Table VII. Root 1 has a high positive correlation with area, second moment about the centre of gravity, second moment about the central depot, number of miles of line, and perhaps number of farm customers. For these

TABLE VII

CORRELATIONS BETWEEN DISCRIMINANT ROOTS AND ATTRIBUTE VARIABLES

	Root 1	Root 2
Area	0.8208	−0.2051
Moment of inertia of centre of gravity	0.7997	−0.7917
Moment of inertia, service centre	0.7519	−0.8336
Efficiency index	−0.4506	−0.2091
Shape index	−0.4034	0.6965
Miles of line	0.8386	0.0361
Farm customers	0.5948	0.1072
Non-farm customers	0.3179	−0.0727

reasons, this root is identified as the scale dimension, measuring the size of an area's operation, its level of activity.

Root 2 has large negative correlations with the second moment measures, I_G and I_X, and positive correlation with the shape index S. I_G and I_X by themselves measure the dispersion of customers around the centre of gravity and central depot, respectively. Dispersion can be identified, to some extent, with the transport component in the cost of operation of the area. A high score on this dimension can be identified, then, with a low cost of transportation and shape index close to 1.0.

The roots have been selected by the analysis so that the six groups are as separated as possible in the directions that the dimensions represent in eight-dimensional attribute space. The location of each group can be represented by its centroid, or the arithmetic means of the attribute variables for each group. In Figure 8, the six group centroids are plotted in terms of the two discriminant roots. This, then, is the plane on which the groups are maximally separated.

In order to identify the criteria which led to the selection of specific areas for specific types of change, the centroid of that group is compared with the centroid of the "no change," or 0 group. The most distant, possessing therefore the most distinct and consistent criterion, is the type A group, areas abolished and engulfed by neighbours. The location of this centroid indicates that the criterion was one of small scale, poor shape, and high dispersion, as hypothesized.

The centroid of group B lies high on the scale dimension, indicating that the criterion for this type of change was one of large scale. The position indicates that a piece of territory previously belonging to an abolished area was likely to be assigned

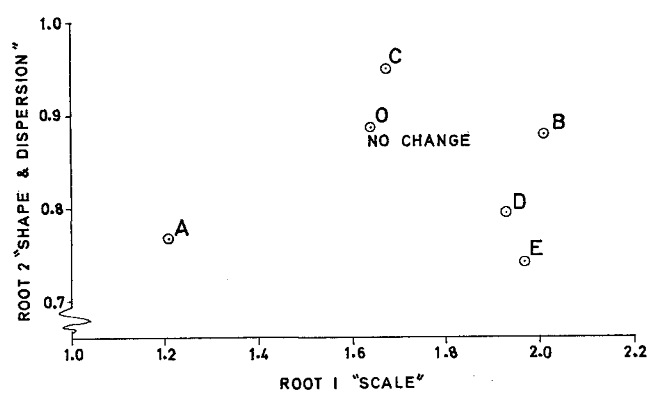

FIGURE 8

to a larger neighbour. This result is unexpected; it was hypothesized that the decision to engulf a neighbouring area would depend upon the attributes of the neighbour only. The B centroid would, according to this hypothesis, lie close to the 0 centroid.

Group C has only seven observations, all decisions being made in the same year, so it is most unlikely that this type of change was the result of the application of any consistent criterion to the attribute variables. The closeness of the centroid to the 0 centroid bears this out.

The D centroid is identified by high scale and low shape. Thus decisions to make type D changes, to revise boundaries, were based upon the shape index of each area, modifications being made to areas of poor shape, and in those cases where the scale of operation was large. This criterion reflects a feeling that it is more important to correct poor shape in large-scale administrative areas than small. The type E centroid lies close to D. Thus, if an unassigned piece of territory was to be allocated to a neighbouring area, the allocation was likely to be made in such a way as to correct poor shape in a large area.

CONCLUSIONS

This study has attempted to make operational some methods for evaluating the evolution of the spatial pattern of service areas. With reference to rural operating areas in Ontario, three trends have been identified: a tendency to more efficient location of the service centre for each area; a tendency for the shape of each area to approach optimum compactness; and a trend toward uniformity in the scale of each area. The analysis suggests that decisions to amalgamate areas are based primarily upon a visual impression of the map of ROAs and the objective of making the set of ROAs as uniform and efficient as possible. However, using discriminant analysis, several conclusions can be made regarding the decisions behind the evolution of this spatial system. First, two major groups of change types can be identified, the A, B group and the D, E group. The latter were predominant during the period 1948–56, and reflect a policy of improvement of efficiency without creation or abolition of operating areas. Beginning in 1957, however, A and B types predominate, as a result of a policy decision to reduce the number of areas. The analysis reveals that this has been achieved by elimination of small, inefficient areas, thus at the same time revising the general efficiency of the system.

Type C changes, relocation of service centres, were only made in one year. These changes are costly since the capital investment in a service centre is considerable, and must also be politically less desirable than boundary relocation, or perhaps even outright abolition.

The administrators involved in these decisions show a considerable sensitivity to derived measures of spatial efficiency, in addition to their concern for the direct scale quantities, as is indicated by the size of the loadings of the two discriminant roots on these variables.

ACKNOWLEDGMENTS

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