RANDOMNESS AND ORDER IN THE TOPOLOGY OF SETTLEMENT SYSTEMS*

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The frequency distribution of settlement links forms the basis of a linear programming methodology which can be utilized to analyze the structure of settlement systems. The resulting index of system topology is compared to a more conventional measure, Geary's contiguity coefficient, which may be applied when the problem is conceptualized as one of spatial autocorrelation on a *k*-color map. The index of linkage similarity that is introduced appears to have a useful advantage over the Geary measure. The Geary and linkage similarity indices are used to examine the topology of four Canadian and two hypothetical settlement systems.

The term settlement system implies a set of population clusters and the set of interactions which occur between them. In many instances these interactions manifest themselves in the form of a readily identifiable transportation or communication infrastructure—a road network, for example. It has become conventional to apply graph theoretic techniques to the analysis of such a system; the connectivity between nodes undifferentiated on the basis of population size is of central concern [*9; 14; 23*].

A number of alternative approaches to the analysis of settlement systems have been employed by geographers, economists, and regional scientists. These include: settlement size distribution analysis [*1; 2; 20; 24*]; models of relative location [*3; 4; 15; 22*], the best known of which derives from central place theory; and mathematical process models [*6; 7; 11; 16; 18*], which test whether an areal distribution of points (settlements of unspecified population size) could have been generated by a particular mathematical process.

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No one of these approaches to settlement system analysis is able to satisfactorily deal with both the size differentiation of nodes in a system and the connectivity between nodes. Nodal connectivity is a particularly important consideration, for unless the nodes are connected in some manner they do not constitute a system. Similarly, the size differentiation of nodes is for most purposes an essential component of this type of analysis in that a settlement system is characterized by a functional hierarchy that is generally thought to be related to settlement size. Settlement size distribution analysis is clearly unsatisfactory in this context in that it is entirely aspatial. Both graph theory and mathematical process models fail to differentiate nodes on the basis of size. In addition, the latter are concerned only with the distribution of nodes in the plane and neglect the connectivity between them. A similar criticism can be made of models of relative location which, while considering the size and spacing of nodes, generally do not explicitly address connectivity. Further, relative location models may be principally regarded as descriptive frameworks having limited utility as analytical tools.

The purpose of this paper is to suggest a pair of parallel methods for analyzing the hierarchical spatial structure of settlement systems. Both methods provide readily interpretable indices of hierarchical connectivity in the topology of spatial systems and may be employed to determine whether such systems are hierarchical, non-hierarchical, or random in structure. The development of such a methodology represents a significant complement to the set of tools available to the spatial analyst; it now becomes possible to objectively measure the extent to which a set of settlements are hierarchically arranged over space. The implications of the utility of such a tool in the context of central place theory, for example, are obvious.

The first of the parallel methods is derived using a linear programming framework. The second method represents a novel application of a more conventional measure, Geary's contiguity coefficient, which may be applied when the problem is conceptualized as one of spatial autocorrelation on a k-color map. Although the two methods are closely related, the index of linkage similarity that is introduced appears to have a useful advantage over the Geary measure.

The basis of the approach adopted is conceptually simple. In a settlement system consisting of A links and N nodes, if nodes are assigned to k discrete size classes, it becomes possible to define a set of $k\ (k + 1)\ /2$ link types. It is the frequency distribution of settlement link types which is central to this method of analysis.

THE EXPECTED DISTRIBUTION OF LINK TYPES

In the absence of any detailed physical, socioeconomic, or historical information one would expect the relative frequency distribution of link types to be related in a probabilistic manner to the relative frequency distribution of the N nodes across the k classes. More precisely, the expectation is that a given settlement size distribution will result in a set of linkages having relative frequencies the same as if two settlements were drawn, with replacement, from an urn containing k different classes of settlement, where P_k is the probability that a settlement belongs to class k. This is a random mixing model which denies the systematic occurrence of linkage patterns, other than those which would occur by chance. Under this free sampling scheme, the expected number of links between two type k settlements is given by AP_k^2, and the expected number of links between two settlements of types i and j is given by $2AP_iP_j$. If, using notation introduced in a later section, the settlement size classes are R, H, V, T, U and occur with relative frequencies P_R, P_H, P_V, P_T, P_U, the link type frequencies have the multinomial distribution for two trials, the probabilities of which may be generated by expanding

$$(P_R + P_H + P_V + P_T + P_U)^2 = P_R^2 + P_H^2 + \ldots + 2P_RP_H + 2P_RP_V + \ldots + 2P_TP_U$$

The observed link type distribution for a settlement network may be compared with the network's expected distribution using a goodness of fit test. This makes it possible to determine whether a given network exhibits a random arrangement in terms of the direct connectivities between nodes belonging to the various classes. The appropriate null hypothesis is, of course, that there is no significant difference between the observed link type distribution and the random distribution given by the multinomial model.

A chi-square test using data from selected areas in the Prairie and Atlantic Regions of Canada indicates that all observed link type distributions differ significantly from their respective null hypothesis (random) distributions. While the knowledge that these particular settlement systems have topologies that are non-random is perhaps interesting in itself, a considerable amount of useful information is still obscured. Specifically, in what manner do these networks depart from a random arrangement of linkages? Is there a tendency for nodes to be linked to other nodes of a similar class, or to be linked to other nodes belonging to different classes? In order to answer this question, we introduce a measure of linkage similarity based upon a linear programming framework.

LINKAGE SIMILARITY INDEX

A settlement network consisting of A total links, N total nodes, k size classes, n_i nodes in class k, $k(k+1)/2$ link types, and having n_{ij} links between type i and type j nodes may be defined by the following constraints:

$$\sum_{i=1}^{k} \sum_{j=1}^{k} n_{ij} = A \qquad (1)$$

$$\sum_{j\geq i} n_{ij} + \sum_{j>i} n_{ij} \geq n_i \qquad i = 1, \ldots, k \qquad (2)$$

$$\sum_{j>i} n_{ij} + \sum_{j<i} n_{ij} \geq 1 \qquad i = 1, \ldots, k \qquad (3)$$

$$0 \leq n_{ij} \leq n_i n_j \qquad i = 1, \ldots, k \quad j = 1, \ldots, k \quad i \neq j \qquad (4)$$

$$0 \leq n_{ii} \leq n_i(n_i - 1)/2 \qquad i = 1, \ldots, k \qquad (5)$$

Constraint (1) indicates the observed total number of links in a network; constraint (2) requires that there be as many links within a class of nodes as there are nodes (this does not eliminate the possibility of isolated nodes, but only networks without isolated nodes are being dealt with); constraint (3) requires that there be at least one link between node types; and constraints (4) and (5) set the lower and upper bounds upon the number of links, n_{ij}, possible in the system. The collection of n_{ij} satisfying constraints (1)-(5) is denoted by $S(A, n_1, \ldots, n_k)$.

For the given parameters $A, n_1, \ldots, n_k$, there are many link distributions $X = (n_{ij})$ in the network system $S(A, n_1, \ldots, n_k)$. Let $X^* = (n^*_{ij})$ represent the random link distribution generated by free sampling in the multinomial model; X^* may be demonstrated to be in $S(A, n_1, \ldots, n_k)$, if $A \geq N$:

$$\sum_{j=1}^{k} n_{ij} = AP_i^2 + 2A \sum_{j\neq i} P_i P_j =$$

$$\frac{An_i^2}{N^2} + \frac{2An_i \sum_{j\neq i} n_j}{N^2} =$$

$$\frac{An_i^2}{N^2} + \frac{2An_i\,(N-n_i)}{N^2} = \frac{2ANn_i - An_i^2}{N^2}$$

$$\geq \frac{ANn_i}{N^2} \geq n_i$$

Thus, constraints (2) and (3) are satisfied; clearly, constraints (1), (4), and (5) are satisfied by construction.

For any link distribution X satisfying the constraints (1)-(5) it is possible to define a measure of nodal affinity. If w_{ij} is a weight expressing the degree of similarity between type i and type j nodes, the total affinity of S is defined as

$$f\,(X) = \sum_{i=1}^{k} \sum_{j=i+1}^{k} w_{ij}\, n_{ij}$$

Returning to the random model, the network affinity is given by

$$f(X^*) = \sum_{i=1}^{k} \sum_{j=i+1}^{k} w_{ij}\,(2An_i n_j/N^2)$$

$$+ \sum_{i=1}^{k} w_{ij}\,(An_i^2/N^2) =$$

$$\frac{2A}{N^2}\left(\sum_{i=1}^{k} \sum_{j=i+1}^{k} w_{ij}\, n_i n_j + \frac{1}{2} \sum_{i=1}^{k} w_{ij}\, n_i^2\right) = \rho$$

This level of affinity ρ is achieved by chance. Networks in which there is a propensity for similar (or dissimilar) node types to link will show departures from this value. We can, in fact, determine the total affinity for the distribution of links X_m and X_M, which have the minimum and maximum values of f, respectively. So let

$$M = max \{\sum_i \sum_j w_{ij}\ n_{ij} \mid \sum_{i,j} n_{ij} = A,\ \sum_j n_{ij} \geq n_i,\ \sum_{j \neq i} n_{ij} \geq 1,\ 0 \leq n_{ij} \leq n_{ij} \geq n_i(n_i-1)/2,\ n_{ij}\ integer\}$$

$$m = min\ \{\sum_i \sum_j w_{ij}\ n_{ij} \mid \sum_{i,j} n_{ij} = A,\ \sum_j n_{ij} \geq n_i,\ \sum_{j \neq i} n_{ij} \geq 1,\ 0 \leq n_{ij} \leq n_i n_j,\ 0 \leq n_{ij} \leq n_i(n_i-1)/2,\ n_{ij}\ integer\}$$

The values M and m can easily be determined for a system $S(A, n_1, \ldots, n_k)$ with the following procedure:

1. In the case of the maximum, satisfy lower bounds given by constraint (2) with N links yielding maximum affinity. In the case of the minimum, satisfy lower bounds given by constraint (3) with k-1 links yielding minimum affinity.
2. Distribute the remaining A-N (or A-k+1) links satisfying upper bounds in (4) and (5) to get maximum (minimum) affinity.

If we have an observed distribution of links X in a connected network in the system $S(A, n_1, \ldots, n_k)$, then we can use similarity weights w_{ij} to determine:

- $f(X)$ = affinity of the observed network;
- ρ = indifference level for the system;
- M = maximum level of affinity for the system;
- m = minimum level of affinity for the system;

where $m \leq \rho \leq M$. Then the linkage similarity measure is defined as

$$J_1\ (X) = (f(X)-\rho)/(M-\rho) \qquad if\ \rho \leq f(X)$$

or

$$(f(X)-\rho)/(\rho-m) \qquad if\ \rho > f(X)$$

Clearly, $-1 \leq J_1(X) \leq 1$ and $J_1(\rho) = 0$, where $J_1(X) < 0$ or $J_1(X) > 0$ when there is an affinity for similar or for dissimilar node types to link, respectively. An alternative form of the index may be written as

$$J_2(X) = \frac{f(X)}{\rho}$$

One can see that if $J_1(X) > 0$, then

$$J_1(X) = \frac{f(X)-\rho}{M - \rho} = \frac{\frac{f(X)}{\rho} - 1}{\frac{M}{\rho} - 1} = \frac{J_2(X)-1}{\frac{M}{\rho} - 1}$$

Therefore, we have the relationships

$$J_2(X) = 1 + (\frac{M}{\rho} - 1)J_1(X)\ \ if\ J_1(X) > 0$$

and

$$J_2(X) = 1 + (1-\frac{m}{\rho})J_1(X)\ \ if\ J_1(X) < 0$$

Now, as stated in the previous section, under the free sampling hypothesis, X has a multinomial distribution with the probability of an i to j link given by $\Pi_{ij} = p_i^2$ for i=j and $\Pi_{ij} = 2p_ip_j$, for $i \neq j$. Then $f(X)$, a linear combination of the frequencies in X, has a distribution which is asymptotically normal [*19*, p. 317] with mean

$$E(f(X)) = A \sum_{i,j} w_{ij}\ \Pi_{ij} = \frac{2A}{N^2} \left(\sum_{i=1}^{k} \sum_{j=i+1}^{k} w_{ij}\ n_i\ n_j + \frac{1}{2} \sum_{i=1}^{k} w_{ij} n_i^2 \right) = \rho$$

and variance

$$V(f(x)) = A[\sum_{i,j} w_{ij}^2 \Pi_{ij} - (\sum_{i,j} w_{ij} \Pi_{ij})^2]$$

$$= \frac{2A}{N^2} \sum_{i=1}^{k} \sum_{j=i+1}^{k} w_{ij}^2 n_i n_j$$

$$+ \frac{1}{2} \sum_{i=1}^{k} w_{ij}^2 n_i^2 - \frac{1}{2} \frac{N\rho^2}{A}$$

Since the indices $J_1(X)$ and $J_2(X)$ are simple rescalings of $f(X)$, we can test the randomness hypothesis for the indices ($E(J_1) = 0$ or $E(J_2) = 1$) with a normal test on $f(X)$.

SPATIAL AUTOCORRELATION

An alternative method of conceptualizing the problem of linkage similarity in a settlement network is suggested by considering the dual of the graph: each node is replaced by a polygon (the node may be thought of as the centroid of the polygon), and each link is replaced (intersected) by a boundary line between two polygons. The k settlement size classes, which have been used to give an ordinal measure to each node, can be represented as a coloring of each polygon. Since the problem of interest to us is dependent only upon the topology of the network, anything that is true of the original graph can also be derived from the dual.

Within this framework, the problem now becomes one of measuring spatial autocorrelation [5; 8; 10; 17] on a k-color map. For ordinal colors the conventional measures of randomness for the system as a whole are the Moran and Geary indices.[1] The Geary index has an expected value of 1.0 for a random pattern, and is less than 1.0 given a greater than random tendency for similar classes to be contiguous (linked) and greater than 1.0 for a greater than random tendency for dissimilar classes to be contiguous. One disadvantage of the Geary index, however, is its lack of general limits. The range of values that the index can assume depends upon the geometrical pattern of zones and upon the set of data values (colors) [12].

An alternative to the Geary index is Moran's I, to which c is related by an expression given in Geary [10]. The problem of undefined limits also extends to I, and since there are no clear grounds for choosing between them, we have limited the discussion to Geary's c. A spatial autocorrelation measure for ordinal data has been introduced by Royalty et al. [21], but it and the generalization by Hubert [13] are less appropriate to this application.

To calculate Geary's contiguity coefficient c, we first assign the integers 1 to k to the size classes (e.g., the class of smallest settlements is assigned 1, the largest is assigned k). We denote this by x_i, where $i = 1, \ldots, k$. The coefficient is then

$$c = \frac{N-1}{2A} \frac{\sum_i \sum_j n_{ij}(x_i - x_j)^2}{\sum_i n_i(x_i - x^*)^2}$$

where $x^* = \sum_i n_i x_i / N$. This notation is not the usual one for interval data. Note that $c = J_2(X)$ when $w_{ij} = (j-i)^2$.

It is possible to test c against a null hypothesis that the colors (size classes) have been arranged randomly. The expected value of c is 1.0; for a given settlement system, link distribution X^* generated by the multinomial model produces a value of 1.0. The distribution of c is demonstrated by Cliff and Ord [5] to be asymptotically normal; c may be tested for significance as a standard normal deviate under either of two assumptions: normality, in which the x_i are assumed to be the results of N independent drawings from a normal population; or, randomization, in which, whatever the underlying distribution of the population, the observed value of c is considered relative to

[1] Cliff and Ord [5] provide a useful review of these indices.

the set of all possible values which c could take on if the x_i were repeatedly randomly permuted around the system (there are $N!$ such values). We have chosen the randomization assumption. The variance in c is thus

$$\frac{1}{N(N-2)\ (N-3)\ 2A^2}\{2A^2[-(N-1)^2\ b_2$$

$$+ (N^2 - 3)]$$

$$+ 2A(N-1)\ [-(N-1)\ b_2 + N^2 - 3N + 3]$$

$$+ (D+A)\ (N-1)\ [(N^2-N+2)\ b^2$$

$$- (N^2+3N-6)]\}$$

where b_2 is the sample kurtosis coefficient

$$b_2 = (\sum_i n_i(x_i - x^*)^4 / \sum_i n_i) / (\sum_i n_i(x_i - x^*)^2 / \sum_i n_i)^2$$

and

$$D = \tfrac{1}{2}\sum_{i=1}^{N} L_i(L_i - 1)$$

where L_i denotes the number of polygons (nodes) contiguous to (linked to) the ith polygon (node).

A more simplified statistic of settlement linkage similarity may be defined by taking the link distribution data as nominal (binary) rather than ordinal. The total number of observed links between different settlement classes may be expressed as a proportion of the total number of expected links between different settlement classes $(2A \sum_i \sum_{j>i} P_iP_j)$. This gives an index comparable to Geary's contiguity coefficient and has the same interpretation. We will denote this index by c_0. Note that

$$c_0 = \frac{f(X)}{\rho}$$

when $w_{ij} = \{0,1\}$. The values of this index will be less extreme than the Geary coefficient because the links are biased in favor of big differences in settlement classes; this is ignored by the nominal approach. To test the significance of this cofficient a chi-square test may be used with the data grouped into two classes only—links between settlements of the same type, and links between dissimilar types.

SELECTED CANADIAN SETTLEMENT SYSTEMS

The above methods of measuring nodal linkage similarity (affinity) in a network are now applied to four selected Canadian settlement systems; for purposes of illustration two hypothetical systems representing linkage extremes are also included. A link between two settlements is defined by a direct road connection. Five settlement size classes are defined and the distributions given in Table 1. The observed and random link type distributions are shown in Table 2.

Table 3 presents the linkage similarity index ($J_1(X)$—written as L.S.I.), Geary's c

TABLE 1

SETTLEMENT SIZE DISTRIBUTION IN SELECTED SYSTEMS, 1976

System	50–249 R	250–999 H	1000–2499 V	2500–9999 T	10000+ U	Total
Prairies*	275	189	45	35	10	554
Cape Breton Island	218	47	9	5	2	281
Newfoundland	196	179	49	20	4	448
Prince Edward Island	319	41	7	4	1	372
Hypothetical A	275	189	45	35	10	554
Hypothetical B	275	189	45	35	10	554

* 16 Federal Electoral Districts in Southern Alberta and Saskatchewan.

TABLE 2

LINK TYPE DISTRIBUTIONS

Type	Prairies		Cape Breton Island		Newfoundland		Prince Edward Island		Hypothetical A		Hypothetical B	
	O	E	O	E	O	E	O	E	O	E	O	E
R–R	192	243	718	618	131	146	464	436	709	243	0	243
R–H	286	334	208	267	204	266	74	112	1	334	0	334
R–V	91	79	28	51	48	73	0	19	0	79	0	79
R–T	89	62	14	28	26	30	0	11	0	62	0	62
R–U	31	18	0	11	8	6	2	3	0	18	707	18
H–H	94	115	24	29	144	122	8	7	187	115	0	115
H–V	82	55	14	11	106	67	7	2	1	55	0	55
H–T	52	42	5	6	35	27	6	1	0	42	0	42
H–U	24	12	4	2	9	5	6	0	0	12	189	12
V–V	7	6	1	1	21	9	3	0	43	6	0	6
V–T	16	10	4	1	14	7	6	0	1	10	0	10
V–U	6	3	5	0	6	1	6	0	0	3	45	3
T–T	10	4	1	0	4	2	3	0	33	4	0	4
T–U	5	2	1	0	3	1	6	0	1	2	35	2
U–U	0	0	0	0	2	0	0	0	9	0	9	0
Total	985	985	1027	1027	761	761	591	591	985	985	985	985

TABLE 3

MEASURES OF NODAL AFFINITY[a]

	L.S.I.[b] $(j\text{-}i)^2$	L.S.I.[b] $\{0,1\}$	c[c]	c_0[c]
Prairies	0.055	0.177	1.350	1.105
Cape Breton Island	–0.434	–0.251	0.557	0.751
Newfoundland	–0.030	–0.050	0.970	0.950
Prince Edward Island	–0.337	–0.241	0.663	0.764
Hypothetical A	–1.000	–1.000	0.002	0.007
Hypothetical B	1.000	1.000	7.148	1.582

[a] All values except the four for Newfoundland are significant at the .01 level.
[b] 0.0 = random arrangement.
[c] 1.0 = random arrangement.

coefficient, and the ratio of total observed to total expected links between different settlement classes (c_0). Note that the linkage similarity index is shown for two different weighting schemes: $w_{ij} = (j\text{-}i)^2$ and $w_{ij} = \{0, i{=}j; 1, i{\neq}j\}$. These two weighting schemes are directly comparable to the ordinal (c) and nominal (c_0) methods of treating the data, respectively. Recall that a random arrangement of size classes is indicated by a value of 0 in the case of the linkage similarity index and by 1 for the Geary and c_0 indices.

Table 3 indicates that in the Prairies there is a general tendency for dissimilar classes of settlements to be linked to one another. (If the Prairie network is disaggregated into Alberta and Saskatchewan networks, the tendency is stronger in the latter.) In the selected Atlantic Canada systems, the tendency is for similar classes of settlements to be linked;

Newfoundland most closely approaches the randomly derived distribution of linkages. The contrast in the settlement topology between the Atlantic and Prairies regions seems intuitively reasonable. One would expect the relatively isotropic Prairies landscape to manifest an hierarchical spatial distribution of settlements (links between dissimilar nodes) while such an arrangement would not be as readily expected in the case of the primarily linear coastal development of the Atlantic region, with its generally more homogeneous settlement sizes.

CONCLUSION

Two general methods of measuring one aspect of the topological structure of a settlement system have been presented —one based upon a linear programming framework and one utilizing the concept of spatial autocorrelation. Although derived in different ways, these measures have been demonstrated to be closely related to one another. Specifically, when the weighting scheme $w_{ij} = (j-i)^2$ is em-employed, linkage similarity index $J_2(X)$ is equivalent to Geary's c coefficient; $J_1(X)$ then represents a scaling of the index such that the final value falls within the range -1.0 to 1.0, with 0.0 denoting a random arrangement. When the data are regarded as nominal ($w_{ij} = \{0,1\}$), the linkage similarity index represents a scaled version of the c_0 measure. The general limits upon the value of the linkage similarity index provide a useful advantage over the autocorrelation measures whose limits depend upon the values and arrangement of the data.

The two weighting schemes that have been employed were selected because of their direct comparability to the weightings implicit in the autocorrelation indices c and c_0; alternative schemes could be considered and may yield further interesting and useful results.

The measures of linkage similarity have been applied to four selected Canadian settlement systems and demonstrate a basic difference in the topological structure of the Prairie and Atlantic regions, the former being characterized by a tendency for settlements of dissimilar size to be linked and the latter by a tendency for similar sized settlements to be linked. This tendency is stronger for Cape Breton and Prince Edward Islands than for Newfoundland. These findings are in agreement with expectations that may be derived from central place theory concerning the settlement topology of the two regions; an hierarchical arrangement is found in the relatively isotropic area but not in the coastal region.

Although the methodology developed here has focused upon the simple road connectivity between classes of nodes distinguished on the basis of population size, its extension may prove useful and instructive. For example, the differentiation of road connectivity on the basis of capacity, the use of other types of connectivity, or the introduction of other criteria for assigning nodes to classes might be considered. In particular, the functional differentiation of settlements may provide further insight into the central place hierarchy. Finally, this methodology has implications for the broader questions of the role of nodal affinity in a socioeconomic system and the structural changes in such a system over time.

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