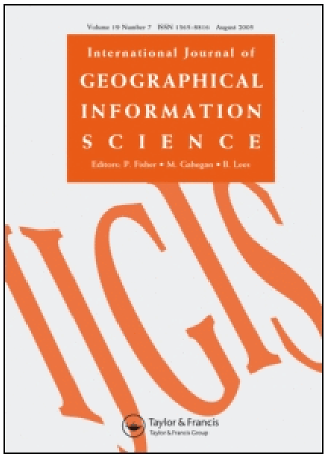

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Research Article

Towards a general theory of geographic representation in GIS

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Geographic representation has become more complex through time as researchers have added new concepts, leading to apparently endless proliferation and creating a need for simplification. We show that many of these concepts can be derived from a single foundation that we term the atomic form of geographic information. The familiar concepts of continuous fields and discrete objects can be derived under suitable rules applied to the properties and values of the atomic form. Fields and objects are further integrated through the concept of phase space, and in the form of field objects. A second atomic concept is introduced, termed the geo-dipole, and shown to provide a foundation for object fields, metamaps, and the association classes of object-oriented data modelling. Geographic dynamics are synthesized in a three-dimensional space defined by static or dynamic object shape, the possibility of movement, and the possibility of dynamic internal structure. The atomic form also provides a tentative argument that discrete objects and continuous fields are the only possible bases for geographic representation.

Keywords: Representation; Data model; Dynamic; Interaction

1. Introduction

The past four decades have witnessed a massive proliferation of geographical information system (GIS) data models, data structures, and discussions of geographic representation and ontology. Berry's geographic matrix (Berry 1964) and Sinton's three-dimensional schema (Sinton 1978) provided early models of the geographic world, as did vector and raster representations. In the 1970s, topological data structures (see, for example, Laboratory for Computer Graphics and Spatial Analysis 1978) were the subject of much discussion; in the 1980s, the relational model was found to offer a straightforward way of implementing topological structures in commercial GIS (Worboys 1995); and in the 1990s, the object-oriented model was widely adopted (Egenhofer and Frank 1992, Worboys 1995, Arctur and Zeiler 2004), although frequently by utilizing more conventional relational database technology. Recently, much effort has gone into extending these models to include time (Langran 1992, Peuquet 2002) and the third spatial dimension (Raper 1989,

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Breunig 1996). The distinction between discrete-object and continuous-field conceptualizations has received attention (Goodchild 1991, Couclelis 1992), and metamaps (Takeyama and Couclelis 1997) and object fields (Cova and Goodchild 2002) have been described.

Faced with such complexity, anyone approaching GIS in the early years of the 21st century might well ask whether geographic representation needs to be so complex; whether a general theory might bring all of these ideas under a single umbrella that includes all three spatial dimensions and time; and whether such a theory might eventually simplify the design and construction of GIS. We know that the geographic world is complex, but it does not follow that the rules that are built into our systems and used to represent the geographic world also need to be complex. A general theory that provided a simpler set of building blocks for geographic representation would give better support for the scientific investigation and management of the surface and near-surface of the Earth, including its description, representation, analysis, visualization, and simulation.

In this paper, we outline the elements of such a theory, responding to many of the *desiderata* of a spatio-temporal ontology identified by Galton (2003):

- 'To provide suitable forms of representation and manipulation to do justice to the rich network of interconnections between field-based and object-based views of the world;
- To extend the field-based and object-based views, and the forms of representation developed to handle them, into the temporal domain;
- To provide a means to develop different views of spatio-temporal extents and the phenomena that inhabit them, especially with reference to those phenomena which seem to present dual aspects as both object-like and field-like.'

The focus is on geographic information, and the representation of such information in a digital system. In practice, geographic data are acquired and compiled into information in many ways, through combinations of sampling, direct measurement, interpretation, interpolation, generalization, and inference—and in practice, many of these steps may be subjective, non-replicable, or poorly documented. Geographic data can range from records of single geo-referenced transactions or point observations to the complex content of maps, and to elaborate database structures. In general, the theory outlined in this paper is independent of the methods used to acquire geographic information, though comments are made on the acquisition process where appropriate. Thus, our central concept—the geo-atom—is an abstraction and not necessarily linked to any system of measurements in the real world, any model of human cognition, or any internal representation in a digital computer. We show how the geo-atom provides a central concept from which other concepts of continuous fields, discrete objects, and field objects (Yuan 1999, 2001) can be derived and show how relational tables and object-oriented classes implement this core concept. We introduce the concept of a geo-dipole, and show how it can be used as a theoretical underpinning for object fields (Cova and Goodchild 2002), metamaps (Takeyama and Couclelis 1997), and the association classes of the object-oriented approach to data modelling.

The purpose of the paper is to offer a conceptual and theoretical framework to clarify and integrate thinking on geographic representation in GIS. While we discuss practical implications at several points, the focus is more on theory than practice.

After a review of the literature on integration and simplification of data-model concepts, section 2 outlines the fundamental elements of the proposed theory. Section 3 describes the extension of the theory to include concepts of interaction. Section 4 presents a discussion of time-dependent concepts, and section 5 presents our conclusions.

1.1 *Relevant literature*

There have been several prior calls for simplification and integration of geographic data modelling concepts, and some successes (e.g. the effort by Winter and Frank 2000, to unify vector and raster perspectives on topology); most relevant to this discussion are those that have focused on the fundamental conceptual duality between discrete objects and continuous fields. That duality was described in a very early form in Peuquet's (1988) distinction between location-based and object-based representations. Moreover, 'The high degree of parallelism shown between location-based and object-based views and the limited number of types of relationships seem to indicate that a unified framework for representing geographic phenomena need not be as complex as had been previously anticipated.' She went on to add, however, that such a unified framework 'remains a multifaceted research issue.'

Terminology converged on (continuous) field and (discrete) object in the early 1990s. Goodchild (1989) saw the distinction between them as essential to an understanding of uncertainty and error in GIS, arguing for example that uncertainty in an area-class map was better understood as uncertainty in class c as a function of location $\mathbf{x}$ than as uncertainty in the positions and attributes of the discrete nodes, edges, and faces of the map. While the field approach leads to the possibility of simple models of how uncertainty distorts the value of $c(\mathbf{x})$, the object approach must create separate but necessarily interdependent models of uncertainty in nodes, edges, and faces, and must somehow deal with the fact that the topology of the boundary network is itself uncertain.

Couclelis (1992) argued that the field/object distinction was fundamental to our understanding of the world around us, and strongly related to human perception (see also Couclelis 1996). Humans clearly perceive the world around them as populated by discrete objects, to which they give names and ascribe behaviours. Other properties of the environment are perceived as continuously varying fields, however, including atmospheric pollution, noise, and crowding. Many physical laws concern continuous fields and are specified as partial differential equations, including the laws of hydrodynamics, while the motion of planets must be understood through laws governing the interactions of discrete objects.

It is helpful to examine the field/object distinction first in the context of *form* and then of *process*. If we consider first how the world *looks*, it is possible for any scene, whether in two, three, or four dimensions, to be conceived either as an empty space littered with discrete, countable objects, or as a collection of one or more variables that are functions of location (Longley *et al.* 2005). The discrete-object and continuous-field views are essentially interchangeable, in the sense that any scene can be represented in either way. For example, a map of land ownership can be seen either as a collection of non-overlapping, space-exhausting, discrete areas or as a function that maps location to a nominal variable *owner*.

The field/object distinction is perceived very differently, however, if the scene is manipulated or subjected to the operation of dynamic, form-modifying processes; in other words, if one examines how the scene *works*. While it might be possible to count the discrete clouds in a single snapshot of the sky, the frequent merging, splitting, and shape changes that occur in clouds render the discrete-object view impractical. On the other hand, vehicles and biological organisms by and large maintain their identity through time, making a discrete-object view the dominant conceptualization for modelling transportation or for certain aspects of biogeography.

These arguments and examples draw attention to the multiple perspectives that exist on the field/object distinction, and to the various arguments in favour of using one perspective or the other in approaching a given problem. In some instances, the choice is clear, but in other cases, such as the single snapshot of a cloudy sky, there are advantages and disadvantages to both conceptualizations. Clearly, it would be advantageous if both could be brought within a single framework, and several authors have commented on this possibility.

To Worboys (1995), while 'field-based and object-based models are in a sense inverse to each other' (p. 151), 'There is some level at which fields and objects can co-exist.' '(An) object-oriented approach may be taken to the specification of field models. Conversely, some of the types in an object-based approach may be fields. The field and object-based approaches to spatial data modelling are therefore not mutually exclusive' (p. 177). Galton (2001) provided the basis for a formal mathematical theory of objects and fields. On the other hand, Egenhofer *et al.* (1999) saw integration as an issue of interoperability, calling for systems that would allow 'users to perform analyses beyond the limitations of a single spatial conceptualisation', while Cova and Goodchild (2002) proposed a hybrid concept of object fields in which every point in geographic space mapped not to a value but to an entire discrete object. The 1998 report on the Specialist Meeting on the Ontology of Fields (Peuquet *et al.* 1999), conducted by the US National Center for Geographic Information and Analysis under its Varenius Project, contains one of the more extensive discussions of these and related issues.

Câmara *et al.* (2000) described a unified framework in which both fields and objects are special cases of a central concept of a *geographical object*, defined as a triple (S,A,f) where S is a subset of the Euclidean plane, A is a set of attribute domains, and the function f associates each location within the subset S with a value on the set of attribute domains. In this framework, a discrete object as traditionally defined is a *simple*, or *homogeneous* geographical object, while a continuous field is a *simple*, *non-homogeneous* geographical object.

The most substantial contribution to the literature on integrating field-based and object-based conceptualizations may be that of Kjenstad (in press). Using Unified Modelling Language (UML) as a framework, he develops schemata for both conceptualizations and then shows how they can be mapped to each other after some augmentation. Any object is defined by a set of *n* parameters that includes both attributes and geometry. Since an object can be a point, line, area, or volume, the number of parameters defining its geometry varies, and it can be as small as two in the case of point objects. The Parameterized Geographic Object Model (PGOModel) thus allows an integrated perspective. Among other concepts, Kjenstad introduces the PGOAtom, whose instances are defined by one specific combination of the *n* parameters of a PGOObject. When the object is a point, this concept, which plays a comparatively minor role in their model, is somewhat similar to what Goodchild *et al.* (1999) describe as the *atomic form* of geographic information. This atomic form is the foundation of our approach in this paper.

2. Primitives of geographic representation

Although numerous definitions of geographic information and GIS can be found in the literature, all focus on the concept of geo-referencing—the association of locations in the geographic domain with the properties of those locations. The geographic domain clearly includes the surface of the Earth, and will also likely include the near-surface, so as to include the domains of groundwater hydrology, geology, oceanography, and atmospheric science—perhaps as far as 10 km below the surface and 30 km above it. In the temporal dimension, the interests of geologists and palaeontologists might extend to several billions of years before present, and GIS-based models might be called on to forecast decades or more into the future, and millennia into the future in the case of models of physical process. Spatial and temporal resolution are defining characteristics of the geographic domain and essential to our theory, and each application will impose its own requirements. Spatial dimensions will likely not be resolved to finer than 1 cm, and the temporal dimension will likely not be resolved to finer than 1 s, in most applications.

Within these crude limits, a vast and potentially infinite number of properties might conceivably be found at a geographic location at any point in time (though agencies such as the US Federal Geographic Data Committee, www.fgdc.gov, and the European Commission's INSPIRE project, www.ec-gis.org/inspire, have sometimes limited the set for purposes of standardization and prioritization). For example, an agricultural scientist might be concerned with soil and crop types; a geotechnical engineer might also be concerned with soil but with an entirely different set of properties or types; a botanist might record trees; and an ecologist might record types of land cover. In all such cases, the semantic meaning of the properties must be understood by both creator and user of the geographic information if it is to be successfully shared. 'Geographic location' might refer to a point, a line, an area, a volume, or some indeterminate region with fuzzy boundaries (Burrough and Frank 1996); and in the temporal dimension, the property might be associated with an instant, an extended interval, or an indeterminate interval. A statistician might refer to this geometric basis for a statement about a property as its *support*. To simplify this complexity, we first reduce all geographic information to a very primitive form, which we term the *geo-atom*.

2.1 *Geo-atoms*

A geo-atom is defined as an association between a point location in space–time and a property. We write a geo-atom as a tuple $<\mathbf{x},Z,z(\mathbf{x})>$ where $\mathbf{x}$ defines a point in space–time, Z identifies a property, and $z(\mathbf{x})$ defines the particular value of the property at that point. For example, a geo-atom might indicate that at 120°W, 34°N, at 0 m above mean sea level, and at local noon on 11 July 2005 (a four-dimensional definition of $\mathbf{x}$), the Celsius temperature (the property Z) was 20° (the value $z(\mathbf{x})$). The number of dimensions of space–time depends on the application, since many applications of geographic information ignore the temporal dimension, and many also ignore the third (vertical) spatial dimension, and in practice GIS technologies often impose dimensional limitations as well.

Many applications of GIS, particularly in transportation, address phenomena distributed over one-dimensional structures embedded in two or three spatial dimensions. Such networks are particularly common in transportation and hydrology. Field-based perspectives can be adopted when variables such as elevation, slope, or traffic density are distributed continuously over the network; and object-like perspectives may be present when discrete zero- or one-dimensional landmarks, bridges, or tunnels are associated with the network. Rather than treat networks as a third case, therefore, we regard them as special cases of field-based and object-based conceptualizations in which the set of possible locations $\mathbf{x}$ is limited to the one-dimensional network structure. In the field case, variables are continuous functions of locations on the network, while in the object case, the network is conceptualized as empty except where occupied by discrete, countable, and potentially overlapping objects.

Goodchild *et al.* (1999) describe the geo-atom as the *atomic form* of geographic information, arguing that all geographic information can be reduced to it. For example, a polygon defining the outline of California can be regarded as a set of atomic statements about the points constituting (or filling) the state, and a polyline defining the centreline of a street can be regarded as a set of atomic statements about points along the centreline. In both cases, the sets are in principle infinite, though we return to this issue later when we consider spatial resolution, uncertainty, and spatial autocorrelation (note that the spatial, temporal, and value resolutions of the example tuple above are all limited).

As we noted earlier, the concept of a geo-atom is an abstraction. Point-like observations are often made in such disciplines as meteorology, but many other forms of geographic observations concern larger geographic aggregates, and while reducing them to atomic form as statements about points is in principle possible, in general it is much easier to store them in aggregate form. Even point-like observations must in practice have some level of granularity, since physical principles inevitably place lower limits on the spatial footprint of observations.

Earlier discussions of this atomic form (e.g. Goodchild *et al.* 1999) have omitted Z, the identification of the property. We include it here in the interests of generality, since it allows the tuple to exist in isolation from other tuples relating to the same property. In that sense, the atomic form is independent of any subsequent organization of geo-atoms into larger aggregates.

In our framework, statements about higher-dimensional objects—pixels, lines, areas, and volumes—are aggregations of statements about geo-atoms, based on exact or approximate uniformity or some other simple rule. For example, property lots are aggregates of points that have the same ownership, contours are aggregates of points based on identical elevation, and the State of Hawaii is an aggregation of points under the administrative authority of the State of Hawaii. Moreover, statements about such objects are often aggregated further and implemented in the tables of the relational model or the classes of the object-oriented model. Both strategies make sense, but only when there are many objects of the same type sharing the same properties.

While there can be many properties ascribed to the same location $\mathbf{x}$, we assume that any single property Z can take only one value at any instant and location in four-dimensional space–time. The issue of single values becomes more complex, of course, when dimensionality is reduced. Thus, there are clearly examples, in the form of overhanging cliffs, where ground elevation is not a single-valued function of the two horizontal dimensions; nevertheless, the binary property 'in air/in ground' can still be treated as a single-valued function of the three spatial dimensions. The issue of single values is also complicated by the possibility of measurement uncertainty.

In principle, geographic features of more than zero dimensions contain an infinite number of points and therefore an infinite number of geo-atoms whose positions can be resolved to infinite precision. In practice, however, the number of statistically *independent* observations that can be made in space–time is limited. Tobler's First Law of Geography (TFL) states that 'all things are related, but nearby things are more related than distant things' (Tobler 1970), although certain phenomena clearly constitute exceptions (Longley *et al.* 2005). We noted earlier that applications of geographic information always impose a limit on spatial and temporal resolution. Moreover, we are unable to measure location on the Earth's surface exactly, and similar constraints apply to time, implying that it is impossible to determine whether two points are exactly coincident, or whether a point lies exactly on a boundary or surface.

2.2 *Geo-fields*

A geo-field defines the variation of one or more properties over a domain of space–time D. As such, it constitutes an aggregation of geo-atoms, where each geo-atom defines the same set of properties and aggregation is over a defined domain. Thus, a geo-field aggregates geo-atoms over space by property Z, irrespective of value. A geo-field for a single property such as elevation is termed a *scalar* geo-field, while a *vector* geo-field might describe the spatial (and temporal) variation of direction of a phenomenon such as wind or aspect over a domain, and would have one value for each of the components of the directional vector (two for directions in the horizontal plane, three for directions in three dimensions). A variety of terms have been used for this concept in GIS practice: *coverage* and *surface* often convey the sense of geo-field.

It is convenient to think of geo-fields as functions of geographic location, since they map locations in space–time within the domain D to values of one or more properties, though not to more than one value. Thus, elevation might be conceptualized as a geo-field $z=z(\mathbf{x})$ where z denotes the value of elevation at the point $\mathbf{x}$ in two-dimensional space.

Many properties can be conceptualized as geo-fields (Longley *et al.* 2005). Elevation and Celsius temperature are examples of properties measured on interval scales, while county, owner, and soil type are examples of properties measured on nominal scales that are nevertheless conceptualized as mappings from location to value, and therefore as geo-fields. Note, however, that whereas by definition a point can lie in only one county, it is possible under the legal system of property ownership that exists in the United States for a point to lie in more than one parcel, or in none, until such disagreements are resolved by legal action. In such cases, the ownership property is no longer field-like and would not be conceptualized as a geo-field, unless it is transformed into a single-valued property such as 'number of owners'.

In principle, any geo-field over a finite domain D aggregates, an infinite number of geo-atoms. So, unless it can be represented accurately by a mathematical function, in practice it is necessary to sample, select, or approximate to represent a geo-field in a store of finite size, using methods that essentially rely on TFL. For properties measured on interval or ratio scales, the law implies that variance will increase with distance, a requirement that is reflected in the observed structure of geostatistical semi-variograms (Goovaerts 1997), as well as in the properties of the functions commonly used to model them. For properties measured on nominal or ordinal scales, this implies the existence of connected areas of equal class and measurable size.

In current GIS practice, which is dominated by the two-dimensional case, there appear to be six commonly used ways of representing geo-fields (Goodchild 1993), which we term *discretizations*:

- *F*1: piecewise constant, such that the variable is constant within each of a set of non-overlapping, space-exhausting polygons (the *choropleth* map, and the *area-class* map in the nominal case);
- *F*2: piecewise linear with continuity of value over triangular elements (the *triangulated irregular network* or *TIN*), and where there is continuity of value across triangle edges);
- *F*3: piecewise constant in a regular grid of cells (normally rectangular), a special case of *F1*;
- *F*4: sampled at a set of irregularly spaced points;
- *F*5: sampled at a set of points in a regular array (normally rectangular), a special case of *F4*;
- *F*6: sampled along a set of isolines of the property *Z*.

Thus, a discretization $F \in \{F1, F2, \ldots, F6\}$ is a required and inherent property of the digital representation of any two-dimensional field, along with a domain and a property. The two point-based discretizations, *F*4 and *F*5, constitute samples of geo-atoms. *F*3 and *F*5, the two 'raster' options, are duals of each other, and the distinction between them is sometimes ignored in practice.

In the first three types of discretization, the user is able to query the value of the field at any point—for *F*1 and *F*3, the value returned will be the value associated with the enclosing polygon or rectangle, while for *F*2, it will be a value obtained by linear interpolation within the enclosing triangle. In the remaining three cases, however, some (unspecified) form of interpolation procedure that is not inherent in the discretization is needed to obtain an estimate unless the query point coincides with a sample point or isoline. For this reason, Goodchild (1993) has termed cases *F*1 through *F*3 *complete* and cases *F*4 through *F*6 *incomplete* representations of fields on two dimensions, respectively.

In effect, these discretizations create representations of fields as properties of discrete objects—polygons, triangular polygons, rectangular polygons, points, points, and polylines, respectively. However, there is an important distinction, in that the discrete objects utilized to represent a field normally have no meaning in reality but exist solely for the purposes of the representation. Moreover, the precise selection of points, lines, areas, volumes, or hyper-volumes that are created in a discretization process ultimately impacts the accuracy of the representation of any geo-field with respect to the real world. For example, in representing an interval-scaled field on two dimensions, such as elevation, it is desirable that the vertices of the TIN (*F2*) be located on peaks, pits, channels, and ridges. Similarly, in representing a nominal-scaled field on two dimensions using *F*1, it is desirable that the boundaries between areas be located along lines of rapid change in class. It is virtually impossible for a representation, based on a discretization, to agree perfectly with the real world, so the choice of a comparatively accurate discretization is clearly important.

In practice, the current generation of two-dimensional GIS products makes no distinction between points, polylines, or polygons representing a field on the one hand, and points, polylines, or polygons representing discrete objects on the other. Thus, all methods are available for processing, and the user is not in any way protected from absurd choices. For example, nothing would prevent a user taking a set of points representing cities with associated populations, and interpolating a field of 'population' between them. Similarly, editing rules are not constrained by whether the objects represent discrete objects or a continuous field, so users are free for example to allow isolines to cross, or to overlap polygons representing a nominal field.

The six approaches are clearly not the only possible discretizations of a geo-field. Finite-element meshes (Topping *et al.* 2000) cover a two-dimensional domain with a mixture of triangles and quadrilaterals, modelling the variation of a geo-field within each element as a polynomial function of location, and imposing continuity constraints across element boundaries. An extension to the TIN model has been described that models variation within each triangle as a quintic polynomial rather than a linear function (Akima 1978), and finite-element meshes that are restricted to triangles are essentially identical to this extension. Splines provide a range of possibilities for discretizing fields, as do Fourier transforms and wavelets. However, options such as Fourier transforms or polynomials that assume homogeneity over the field's spatial domain run contrary to the observation that geographic phenomena are almost always spatially heterogeneous; wavelets and other spatially piecewise approaches thus tend to be more useful in practice.

Although, in principle, fields are constructed from the properties of points, in other words from geo-atoms, in practice the definitions of many properties require convolution over a neighbourhood around the point. Fields representing density properties, such as population density, must be defined in this way. In other cases, convolution may be inherent in the process of measurement, as it is with remote-sensing instruments, for example. Suppose that a domain D contains n individuals, the ith individual being located at $\mathbf{x}_i$. Then, population density $P(\mathbf{x})$ can be defined as the convolution:

$$P(\boldsymbol{x}) = \sum_{i=1}^{n} K(\|\boldsymbol{x}_i - \boldsymbol{x}\|)$$

where K denotes a kernel function (Silverman 1998). The rate of decrease in K with distance will be defined by a parameter with units of length, which thus reflects the *scale* of the density field. In general, the coarser the scale the smoother the density field. Note that an appropriate strategy must be adopted for dealing with the edge effects consequent on the use of a limited domain D.

2.3 *Geo-objects*

A geo-object is defined as an aggregation of points in space–time whose geo-atoms meet certain requirements, such as having specified values for certain properties. For example, the geo-object Jefferson County is composed of all geo-atoms having the value Jefferson County for the property County. Thus, geo-objects represent geo-atoms aggregated by the values of properties using specified rules, rather than by property alone as in the case of geo-fields. Again, several terms in common use in GIS practice convey similar meaning, including *entity* and *feature*. The dimensionality of geo-objects is constrained by the space in which they are embedded; for example, a geo-object embedded in a space of two horizontal dimensions and time may be a point, line, area, or volume. We include points as geo-objects, although a point with a single property is formed from a single geo-atom, rather than from many by a process of aggregation.

TFL ensures the existence and representational efficiency of geo-objects. With TFL, rules applied to the values of geo-atoms, such as 'mean annual temperature 10–15 C and mean annual precipitation 500–700 mm' can be expected to identify connected areas of measurable size. But while the point sets formed by such aggregations are normally connected, they may sometimes contain holes or enclaves. The Open Geospatial Consortium (OGC) Simple Feature Specification (documents relating to the Simple Feature Specification are available at www.opengeospatia-l.org), which deals with static geo-objects, allows in the two-dimensional planar case for *multipart polygons* (collections of disconnected islands that share the identity property), *multipart polylines* (collections of disconnected polylines with shared identity), and *multipoints* (collections of points with shared identity). In addition, the geometry of geo-objects may be defined by curves, such as arcs of circles, arcs of ellipses, or Bézier curves, rather than by straight lines.

Most current GIS data models take a space-centred approach to geo-object identity that recognizes geo-objects primarily by their locations and associated geometries, and only secondarily by their attributes. In doing so, new geo-object identities are needed when changes occur to location or geometry, in other words to the point set defining the geo-object. Alternatively, the identity of a geo-object may derive from its attributes, such as its name, that may survive changes of location and even geometry. 'Portage', for example, was used as the county name for two spatially disjoint areas in Wisconsin at different times, and thus one might reasonably model both as versions of the same geo-object. When state and county names are used as county identifiers, the identity of a county is not necessarily tied to a particular geographic location.

In addition to properties that are assumed uniform over the geo-object (*spatially intensive* properties), properties at the set level are likely to emerge and may include both direct measures of the point set (e.g. length, area, volume, shape) and integrals of spatially intensive properties at the points (e.g. total population integrated from population density). Such set measures and integrals are termed *spatially extensive* (Longley *et al.* 2005), and it would make little sense to attach them to constituent geo-atoms. For example, the geo-atom <**x**,population density per km^2,300> based on a spatially intensive property is clearly more meaningful than the geo-atom <**x**,total population of containing census tract,3000>, since the latter confuses a property of the point with a property of its containing area. We note, however, that this approach is commonly used in Tomlin's map algebra (Tomlin 1990) in *zonal* operations, when it is necessary to store spatially extensive properties such as measures of area in raster-only systems. The distinction between spatially intensive and spatially extensive properties is reflected in the *split and merge rules* (Zeiler 1999), that is, the methods that apply whenever geo-objects are split or merged. For example, when a county is subdivided into two smaller units, a spatially extensive property must be partitioned between them such that the sum of the spatially extensive properties of the parts is equal to the property of the whole, while a spatially intensive property of the whole may also apply to both parts.

While homogeneity is clearly the basis for the definition of many geo-objects, others may arise through the aggregation of geo-atoms using more complex rules. For example, the *functional region* (Johnston *et al.* 2000) is defined by regional geographers based on interaction between its component parts. Thus, a metropolitan area is composed of an urban core and its inner and outer suburbs, held together by interaction and complementarity rather than by the uniformity of any one property.

The spatial extent of a geo-object may be established by *fiat*, as for example when counties are defined by administrative decision, or may be bona fide if the spatial extent reflects some form of internal cohesion or homogeneity, as for example when geo-objects represent individual organisms, houses, geographic regions, or severe storms (Smith and Varzi 2000). In the bona fide case, the rules defining membership in the constituent point set can be complex and vague, as for example in the rules defining membership in topographic features (e.g. mountain, valley) or meteorological features (e.g. severe storm, atmospheric high).

Geo-objects with indeterminate boundaries (Burrough and Frank 1996) can be modelled through partial membership, as collections of points with associated membership functions. Consider a geo-object and its constituent point set. Then, for each location $\mathbf{x}$, define a membership function $m(\mathbf{x})$ that gives the degree of membership of the point in the geo-object. Since $m(\mathbf{x})$ is a function of location, one can conceive of it as a geo-field. More generally, any geo-object can be conceptualized as a membership field that is continuous-scaled in the case of indeterminate boundaries, and binary in the case of determinate boundaries. In this view, the isolines of the membership field are related to the boundary of the geo-object, and the gradient of the field at any location is related to the boundary's local degree of indeterminacy.

2.4 *A theory of bona fide geo-objects*

This line of argument can be extended to reach other useful conclusions, and in this section we present a simple but general theory that integrates concepts of geo-fields and geo-objects by allowing the latter to be derived from the former. At the functional level, operations that create geo-fields from geo-objects and vice versa are well known (Galton 2003, and for a comprehensive review see Câmara *et al.* 1996); they include:

- *density estimation*, which creates a continuous field of density from a collection of discrete objects;
- *object extraction* algorithms in image processing and pattern recognition that extract discrete objects from a field of reflectance or radiation; and
- algorithms for identification of *surface-specific points and lines* (peaks, pits, passes, ridges, etc.).

While each of these provides *functional* ways of linking geo-objects and geo-fields, the two conceptualizations clearly interact in other ways as well. Bian (2000) discusses the modelling of dynamics, and the processes that determine the movement of objects, in ways that draw on both conceptualizations. In another context, ecologists might ask how discrete, homogeneous *patches* of a biogeographical landscape arise in a world that is defined by the field-like variation of physical properties such as temperature and precipitation. In this section, we review a simple but general model that provides one more link between geo-objects and geo-fields.

Consider a set of properties $\{Z_1, Z_2, \ldots, Z_m\}$ measured on interval or ratio scales. For example, the properties might be those relevant to the success of various plant species, such as mean annual temperature, mean annual precipitation, elevation, slope, and soil pH. Suppose these properties have been evaluated over a spatio-temporal domain D, forming a set of geo-fields. Under TFL, each property can be expected to vary spatially in a manner that exhibits strong positive autocorrelation, characterized by a semi-variogram.

Now, consider an m-dimensional space defined by these properties, and assume that the space is partitioned into C irregularly shaped zones, each zone corresponding to one habitat type, and defined by the range of conditions characteristic of that type. We might term this a *phase* space (Goodchild and Wang 1989) by analogy to the space defined by temperature and pressure that characterises the gaseous, liquid, and solid phases of a chemical compound. Any point in this space will map to one of the C types, depending on the zone in which the point falls. Thus, any point in geographic space, characterized by a set of values on the m properties, maps to a point in the m-dimensional space, which in turn maps to one of the C types in geographic space. In short, the model provides a function f linking the values at a point to the type at that point: $c(\mathbf{x})=f(z_1(\mathbf{x}), z_2(\mathbf{x}), \ldots, z_m(\mathbf{x}))$, where $z_1(\mathbf{x})$ through $z_m(\mathbf{x})$ are the values of the respective properties at point $\mathbf{x}$ and $c(\mathbf{x})$ is the type at point $\mathbf{x}$.

In this model, $c(\mathbf{x})$ can be regarded as a nominal-scaled field or *area-class* map (Mark and Csillag 1989); or alternatively connected areas of the same value of c can be regarded as bona fide geo-objects. TFL ensures that these connected areas are of measurable size, which they would not be if the input geo-fields lacked positive spatial autocorrelation. Moreover, because of TFL, only certain adjacencies can occur between types: only types that are adjacent in phase space can be adjacent in geographic space. When $m=1$, the boundaries of the types will be isolines of the input variable, and hence the map of $c(\mathbf{x})$ will have a characteristic appearance with no nodes in the boundary network.

The model just described can be identified in numerous sources in biogeography (e.g. Holdridge 1971) and is also the model used in remote sensing to classify multispectral data (Lillesand *et al.* 2004). This provides a convenient and general way of understanding how homogeneous patches and area-class maps arise in reality, and of linking concepts of geo-objects to concepts of geo-fields.

2.5 *Field objects (f-objects)*

As defined above, geo-objects are formed from points whose geo-atoms meet certain requirements. In the previous section, we argued that those requirements can be quite complex, based on substantial variability among the values of given properties, but nevertheless the outcome was a geo-object of homogeneous class. Yuan (1999) has argued that this approach is too restrictive for certain phenomena, and has defined the *field object* (f-object) as a geo-object with internal heterogeneity conceptualized as a field. For example, a severe storm may have a boundary defined by the limits of cloud cover, and an internal structure defined by the variation of such field-like properties as rainfall or atmospheric pressure. F-objects can be seen as generalizations of geo-fields in which the domain D is bona fide rather than fiat, or as generalizations of geo-objects to allow for internal variation. Both the internal structure and boundary of a f-object may be indicative of the physical dynamics that drive its development in space and time. For example, the winds, precipitation, temperature, and pressure fields of a convective storm characterize the dynamics and stability of the storm and how it may evolve under certain atmospheric conditions (Yuan 2001, McIntosh and Yuan 2005). As the f-object moves, it carries the embedded geo-fields with it, raising the possibility that its dynamics may be better understood within the moving (Lagrangian) coordinate frame of the bounding geo-object than in a fixed (Eulerian) frame.

2.6 *Tables and classes*

Thus far, we have discussed individual geo-objects. In practical applications, it is common to deal with many geo-objects, and to arrange them into classes based on shared sets of properties. In the relational model, these populate tables that are linked with keys (Codd 1970); in the object-oriented model, they form classes that are linked by inheritance, composition, aggregation (but not in the primary sense in which the word is used in this paper), and association relationships (Zeiler 1999). Such structures represent a comparatively advanced state of geographic knowledge, however, since they presume a set of rules for aggregating geo-atoms into geo-objects, a classification scheme for grouping geo-objects into tables and classes; and an expectation that the same set of attributes is known for each geo-object. In his classic book on databases, Date (2000) uses the term *atomic* to describe individual entries in such tables. In the geographic context, these entries record the attributes of geo-objects, or the attributes of objects formed by discretizing geo-fields. In our view, however, when such elements record geographic knowledge, they can almost always be decomposed further (with the obvious exception of knowledge about points), ultimately into geo-atoms—implying that Date's use of the term atomic would be inappropriate in this context.

The six discretizations of a geo-field discussed in section 2.2 all yield objects with common sets of properties that can be assembled into tables and classes. For example, in the case of the two horizontal dimensions, *F*1 yields either a set of polygons (e.g. ESRI's *shapefile* model) or sets of points (nodes), polylines (arcs), and polygons (e.g. ESRI's *coverage* model). *F*4, on the other hand, yields simply a set of points. Thus, the relational and object-oriented models are well suited to the representation of geo-fields; their use in the representation of geo-objects, on the other hand, may or may not make sense, depending on the number of such geo-objects present, the existence of a classification scheme, and complete sets of attributes for each geo-object.

3. Concepts of interaction: geo-dipoles

The previous section described the representation of distributions on the Earth's surface in terms of continuous geo-fields and discrete geo-objects. The processes that modify such distributions, however, must often be understood in terms of *interactions*. For example, demographic distributions are modified by flows of migrants between locations; the physical landscape is modified by flows of air, water, and sediment; and communities are created by social interaction. While interactions can be difficult to show cartographically, various methods have been devised for representing them in GIS in order to support analysis and modelling, and thus to improve our understanding of dynamic processes and their effects. These include *object fields* (Cova and Goodchild 2002), *metamaps* (Takeyama and Couclelis 1997), *object pairs* (Goodchild 1991), and *association classes* (Zeiler 1999). In this section, we introduce a new, fundamental concept that plays a similar role in relation to these approaches as the geo-atom plays in relation to geo-objects and geo-fields.

We define a geo-dipole as a tuple connecting a property and value not to one location in space–time as in the case of the geo-atom but to two: $<\mathbf{x}_1,\mathbf{x}_2,Z,z(\mathbf{x}_1,\mathbf{x}_2)>$. Geo-dipoles capture the properties of pairs of points, or properties that are associated with two points rather than one. For example, *Z* might represent such properties as distance or direction in space, interaction intensity, time interval, flow intensity, or flow direction, and $z(\mathbf{x}_1,\mathbf{x}_2)$ might represent their values for pairs $(\mathbf{x}_1,\mathbf{x}_2)$.

Like geo-atoms, geo-dipoles are a conceptual abstraction and not observed except under limited circumstances. For example, while distance and direction can be evaluated between points, the magnitude of a flow of migrants can only be observed between aggregate areas. Despite this abstract quality, however, geo-dipoles play an important role in unifying the various approaches that have been proposed to the representation of interaction, as we show in the following sections.

3.1 *Object fields (o-fields)*

Cova and Goodchild (2002) describe *object fields* (o-fields), in which each point maps not to a value but to a geo-object. For example, at every point on a topographic surface it is possible to define a *visible area*, or the area that can be seen from that point. Let $\mathbf{x}_1$ denote the point location of the observer, let Z be the binary property 'is visible from', and let $z(\mathbf{x}_1,\mathbf{x}_2)$ be the value of this property for the pair $(\mathbf{x}_1,\mathbf{x}_2)$—in other words, whether $\mathbf{x}_2$ is visible from $\mathbf{x}_1$. This defines a number of geo-dipoles of the form $<\mathbf{x}_1,\mathbf{x}_2,Z,z(\mathbf{x}_1,\mathbf{x}_2)>$. Now, form a geo-object from the set of points $\mathbf{x}_2$ for which the value with respect to $\mathbf{x}_1$ is 'visible', and call this geo-object $O(\mathbf{x}_1)$. We now have the central concept of object fields, a mapping from location $\mathbf{x}_1$ to a geo-object $O(\mathbf{x}_1)$. Cova and Goodchild (2002) identify a number of other instances of this concept, including watersheds (defined as the areas upstream of each point), and trade areas (the areas served by a hypothetical store located at each point). Fisher (1991) has investigated the indeterminate case, in which membership in the geo-object $O(\mathbf{x}_1)$ is affected by uncertainty regarding the exact values of the elevation field.

3.2 *Metamaps*

Takeyama and Couclelis (1997) describe the *metamap*, which they define as the Cartesian product of a raster. Consider an aggregation of geo-atoms into a raster of cells $\{O_i, i=1,n\}$ (compare *F*3 above). Now, consider a pair of such cells $\{O_1,O_2\}$, and the various properties that might characterize the pair, including interaction, connectivity, distance, direction, etc. Denote one such property as z_{12}, representing the flow of migrants from cell 1 to cell 2, for example (for a review of spatial interaction modelling based on pairs of geo-objects, see Fotheringham and O'Kelly 1989).

Just as in section 2.3, it is helpful to distinguish between spatially intensive and spatially extensive properties. Flow of migrants is a spatially extensive property, responding to the sizes of both O_1 and O_2, and must first be normalized for its decomposition to make sense, for example by dividing by the product of the populations of O_1 and O_2. Tobler (1988) describes one of several efforts to place interaction modelling on a spatially continuous basis using density functions defined on geo-atoms rather than aggregate measures defined on geo-objects. Other properties such as distance are spatially intensive and will not require normalization, though we note that the distance between representative points in O_1 and O_2 may be a poor estimate of the distances between pairs of points.

With this qualification, the tuple $<O_1,O_2,z_{12}>$ can be decomposed into atomic statements, or geo-dipoles, of the form $<\mathbf{x}_i,\mathbf{x}_j,I,z_{12}(\mathbf{x}_i,\mathbf{x}_j)>$, where I denotes a spatially intensive property and where $\mathbf{x}_i$ is contained in O_1, and $\mathbf{x}_j$ is contained in O_2.

3.3 *Association classes and object pairs*

Goodchild (1991) defines an object pair as a pair of geo-objects having properties that exist only for the pair, and not for the individual members of the pair. Distance, direction, interaction, and flow are all examples of properties that exist only for pairs of geo-objects, that is, for geo-objects taken two at a time. Other types of relationships between geo-objects have been studied extensively. For example, numerous papers have appeared that enumerate the various binary topological relationships that can exist between geo-objects, beginning with the work of Egenhofer and Franzosa (1991).

When both geo-objects are of the same class, then such properties can be visualized as entries in a square table, or as properties of the Cartesian product of the members of the class. Such data types often arise in GIS in dealing with interactions over space but are not typically recognized as generic types in commercial GIS. For example, they arise whenever a distance matrix is calculated or when a $\mathbf{W}$ matrix is defined for many types of spatial analysis (Haining 2003; $\mathbf{W}$ is defined as a square matrix whose elements measure the relative proximity of pairs of features), and are found in the *turntable* construct in ESRI products, which is a table used to store whether it is possible to turn from one link into another link at a network junction, along with other relevant attributes of the turn.

In object-oriented modelling, such properties could be stored in an *association class*, which is defined as a class whose instances record properties of an association between two existing classes (Zeiler 1999). For example, an association class could be defined between the classes 'neighbourhood' and 'school'; and its instances could record distance, travel time, travel mode, and number of students travelling between a given neighbourhood and a given school.

As with object fields and metamaps, object pairs and instances of association classes can be conceptualized as aggregations of geo-dipoles. They generalize the concept of a metamap to include interactions between arbitrarily shaped geo-objects, and between pairs formed from geo-objects of different classes.

4. Time dependence and dynamics

In this section, we consider the implications of the temporal dimension for each of these concepts. The geo-atom has already been defined as linking a location in four-dimensional space–time to properties and values, and we have noted the common lack of support for time and the third geographic dimension in GIS practice, implying that in many cases, $\mathbf{x}$ will be no more than two-dimensional. We now consider each of the other concepts introduced in the previous sections.

A geo-field is defined on as many as four dimensions, including time. More often, however, a GIS user is faced with treating time as a series of snapshots or *occurrents* (the SNAP ontology of Grenon and Smith 2004), in other words as an ordered sequence of separate fields defined over the spatial dimensions only. Assume first that the discretizations of the snapshots are identical. For example, weather stations that are fixed at irregularly spaced sample points ($F4$) might generate time series of measurements; irregular reporting zones such as counties ($F1$) might be used to create longitudinal statistics on population; a sequence of Earth images might use congruent rectangular pixels (*F*3) in each time series; and changes in elevation through time might be reported for a constant grid of points (*F*5). In two cases, however (*F*2 and *F*6, the TIN and digitized isoline representations, respectively), the discretization will likely change between snapshots. TIN triangles may be repositioned to capture the new peaks, pits, ridges, and channels of each new surface; and isolines will by definition be repositioned as the surface changes through time. Thus, *F*2 and *F*6 make little sense as discretizations of sequences of snapshot geo-fields, though they are in principle valid discretizations of a single geo-field over a space for which one dimension is time. In the remaining four cases, a common strategy is to resample each snapshot to a shared discretization, either to a single set of polygons (*F*1; see, for example, the National Historic GIS project, www.nhgis.org) using some method of areal interpolation (Goodchild *et al*. 1993), to a shared set of sample points (*F*4), or to a shared raster (commonly by bilinear interpolation, reducing the rectangles in *F*3 to central points as in *F*5).

We now turn to the case of dynamic geo-objects and assume that the requirements or rules used to define geo-objects are persistent through time (they are *continuants* in the SPAN ontology of Grenon and Smith 2004). By dynamic, we mean that change in the geo-object through time is more than simply a change in values of its attributes. Figure 1 shows some of the more commonly observed characteristics of geo-objects through time, based on three conditions. First, the geo-object may be static or may move (front-to-back dimension of the cube). Second, the geo-object may change shape through time (vertical dimension of the cube). Finally, we extend

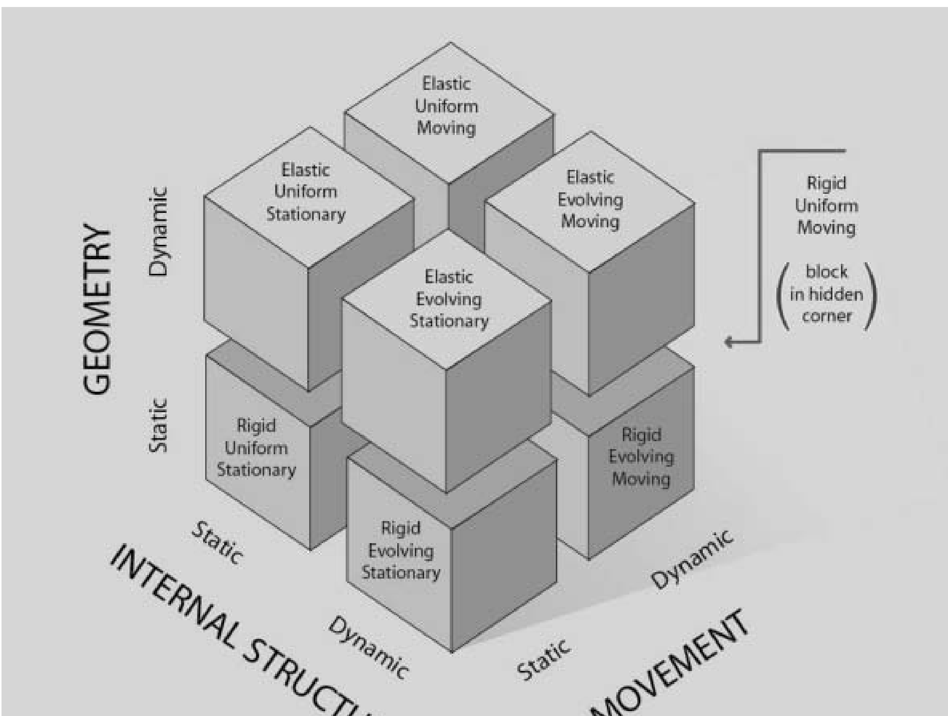

Figure 1. Three dimensions of temporal variability in geo-objects.

the concept of geo-objects to include internal structure, as discussed in section 2.5, where we introduced the term *field object*, and in the left-to-right dimension record whether the object is internally and constantly uniform, or whether it has heterogeneous and evolving internal structure. A geo-object with attributes that change through time, but without movement, change in shape, or a changing internal structure would be assigned to the first category in this taxonomy.

These three sets of conditions produce eight combinations, for which we provide examples as follows:

- Uniform Stationary Rigid: buildings and streets in a city.
- Uniform Stationary Elastic: the seasonal expansion or contraction of a lake when only the extent of the lake is considered.
- Uniform Moving Rigid: moving vehicles, and the *lifelines* through space created by human life histories.
- Uniform Moving Elastic: a spreading wildfire when only the burn scar is considered.
- Evolving Stationary Rigid: soils in a watershed and digital elevation models.
- Evolving Stationary Elastic: heat-island effects in an urban area, and vegetation cover during desertification.
- Evolving Moving Rigid: changing landscapes on moving, rigid tectonic plates.
- Evolving Moving Elastic: oil spills and hurricanes.

Stefanidis *et al*. (2003) have described the representation of moving, elastic geo-objects as *helixes* by adapting the image-processing concept of *snakes*. Assume that the geo-object is captured in a sequence of snapshots. The movement of the geo-object's centre of mass provides a polyline in three dimensions (two horizontal dimensions plus time), while its rotation and changing shape can be described by tracking its principal axes.

5. Conclusions

The concepts of discrete objects and continuous fields were introduced into the GIScience literature in the late 1980s and early 1990s, and have since come to dominate thinking about human conceptualizations of geographic space. Humans appear more comfortable describing the world in terms of discrete objects, while many physical processes are modelled in terms of continuous fields through the solution of partial differential equations. While some success has been achieved at integrating the two concepts, and several methods result in transformations between them, many questions remain: are there only two ways of thinking about the world; and why are they so distinct?

In this paper, we introduced the concept of the geo-atom and showed that it could provide the foundation for both discrete-object and continuous-field conceptualizations. Both aggregate the locations that are the first element of the geo-atom tuple. Geo-fields are formed by aggregating the geo-atoms for a single property, that is, the second element of the geo-atom tuple; and geo-objects are formed by aggregating geo-atoms according to rules defined on the third element, the geo-atom's value. Since these are the only available elements within the proposed theory, we can infer that discrete objects and continuous fields are indeed the only possible bases for conceptualization of the geographic world, if such conceptualizations are limited to aggregations of point sets. We also examined the concept of a field object and showed that it could be defined as a geo-field whose domain is a geo-object. We introduced the concept of phase space to provide one general theory of how geo-objects can be derived from geo-fields using rules that can be expressed as partitions of an *m*-dimensional space.

Figure 2 summarizes the proposed theory, showing the aggregation of geo-atoms into geo-fields and geo-objects, and the different implications of dynamics for both. The three binary dimensions identified in figure 1 lead to the eight cases of dynamic geo-objects at the lowest level of figure 2.

The theory outlined in this paper is limited by its focus on conceptualizations based on point sets, and thus on the aggregation of geo-atoms into geo-fields and geo-objects. The question of whether conceptualizations might be possible based on

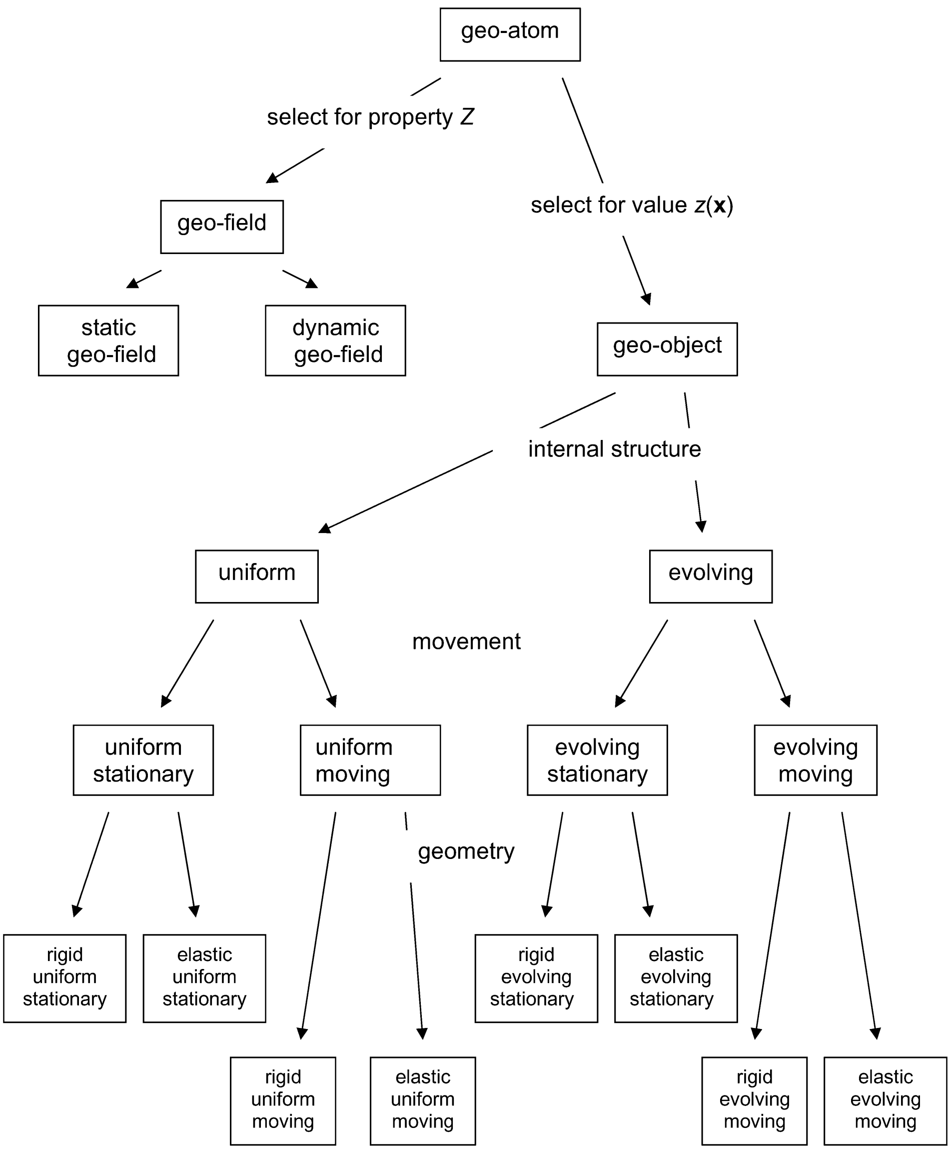

Figure 2. Basic elements of the theory, including the possibility of static geo-fields and the eight types of dynamic geo-objects.

operations other than aggregation remains open but is beyond the scope of this paper.

The concept of a geo-dipole was introduced, and shown to provide a foundation for a set of concepts dealing with such properties as distance, direction, interaction, and flow, including object fields and metamaps. While such concepts clearly play an important role in our understanding of the processes that dominate the evolution of social and physical landscapes, to date they have found little support in a GIS technology that has tended to emphasize aspects of the static *form* of the Earth's surface (Goodchild 2004). One might ask whether properties of locations taken *three* at a time (or more generally $n>2$ at a time) also have practical application.

We argued that relational tables and object-oriented classes implement the results of two stages of aggregation of geo-atoms. In the case of geo-objects, they are useful only when such objects can be grouped into classes, and are sufficient in number to populate them. In the case of geo-fields, they result from a process of discretization that converts a geo-field into a collection of geo-objects and is unsatisfactory in several respects. First, it assumes an ability to define a set of geo-objects that provides an accurate representation of the geo-field. Second, it fails to capture the behaviours that such geo-objects must exhibit as representations of a geo-field, such as non-crossing of isolines. Third, it raises problems when dealing with dynamic geo-fields, as shown in section 4. Finally, by transforming a geo-field into what appears to be a collection of geo-objects, it fails to prevent the user from performing inappropriate operations. This process of discretization of phenomena that are essentially continuous on the Earth's surface remains perhaps the most problematic area of geographic data modelling in current GIS practice, and one that might eventually motivate a new approach that is neither relational nor object-oriented.

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References

AKIMA, H., 1978, A method of bivariate interpolation and smooth surface fitting for irregularly distributed data points. *ACM Transactions on Mathematical Software*, **4**, pp. 148–159.

ARCTUR, D. and ZEILER, M., 2004, *Designing Geodatabases: Case Studies in GIS Data Modeling* (Redlands, CA: ESRI Press).

BERRY, B.J.L., 1964, Approaches to regional analysis: a synthesis. *Annals of the Association of American Geographers*, **54**, pp. 2–11.

BIAN, L., 2000, Object-oriented representation for modelling mobile objects in an aquatic environment. *International Journal of Geographical Information Science*, **14**, pp. 603–624.

BREUNIG, M., 1996, *Integration of Spatial Information for Geo-Information Systems* (Berlin: Springer).

BURROUGH, P.A., and FRANK, A.U. (Eds), 1996, *Geographic Objects with Indeterminate Boundaries* (Bristol, PA: Taylor & Francis).

CÂMARA, G., MONTERO, A.M.V., PAIVA, J.A., GOMES, J. and VELHO, L., 2000, Towards a unified framework for geographical data models. *Geoinfo 2000: Workshop Brasileiro de Geoinformatica, Sao Paulo*.

Câmara, G., Souza, R.C., Freitas, U.M. and Garrido, J.C.P., 1996, SPRING: integrating remote sensing and GIS with object-oriented data modelling. *Computers and Graphics*, **15**, pp. 13–22.

Codd, E.F., 1970, A relational model of data for large shared data banks. *Communications of the ACM*, **13**, pp. 377–387.

Couclelis, H., 1992, People manipulate objects (but cultivate fields): beyond the raster-vector debate in GIS. In *Theory and Methods of Spatio-Temporal Reasoning in Geographic Space*, A.U. Frank, I. Campari and U. Formentini (Eds), pp. 65–77 (Berlin: Springer).

Couclelis, H., 1996, Towards an operational typology of geographical entities with ill-defined boundaries. In *Geographic Objects with Indeterminate Boundaries*, P.A. Burrough and A.U. Frank (Eds), pp. 45–56 (Bristol, PA: Taylor & Francis).

Cova, T.J. and Goodchild, M.F., 2002, Extending geographical representation to include fields of spatial objects. *International Journal of Geographical Information Science*, **16**, pp. 509–532.

Date, C.J., 2000, *An Introduction to Database Systems*, 7th ed (Reading, MA: Addison-Wesley).

Egenhofer, M.J. and Frank, A.U., 1992, Object-oriented modeling for GIS. *Journal of the Urban and Regional Information Systems Association*, **4**, pp. 3–19.

Egenhofer, M. and Franzosa, R., 1991, Point-set topological spatial relations. *International Journal of Geographical Information Systems*, **5**, pp. 161–174.

Egenhofer, M.J., Glasgow, J., Günther, O., Herring, J.R. and Peuquet, D.J., 1999, Progress in computational methods for representing geographical objects. *International Journal of Geographical Information Science*, **13**, pp. 775–796.

Fisher, P.F., 1991, First experiments in viewshed uncertainty: the accuracy of the viewshed area. *Photogrammetric Engineering and Remote Sensing*, **57**, pp. 1321–1327.

Fotheringham, A.S. and O'Kelly, M.E., 1989, *Spatial Interaction Models: Formulations and Applications* (Boston, MA: Kluwer).

Galton, A.P., 2001, A formal theory of objects and fields. In *Spatial Information Theory: Foundations of Geographic Information Science*, D.R. Montello (Ed.), pp. 458–473, Lecture Notes in Computer Science Vol. 2205 (Berlin: Springer).

Galton, A.P., 2003, Desiderata for a spatio-temporal geo-ontology. In *Spatial Information Theory: Foundations of Geographic Information Science*, W. Kuhn, M.F. Worboys and S. Timpf (Eds), pp. 1–12, Lecture Notes in Computer Science Vol. 2825 (Berlin: Springer).

Goodchild, M.F., 1989, Modeling error in objects and fields. In *Accuracy of Spatial Databases*, M.F. Goodchild and S. Gopal (Eds), pp. 107–14 (Basingstoke: Taylor and Francis).

Goodchild, M.F., 1991, Geographical data modeling. *Computers and Geosciences*, **18**, pp. 400–408.

Goodchild, M.F., 1993, The state of GIS for environmental problem solving. In *Environmental Modeling with GIS*, M.F. Goodchild, B.O. Parks and L.T. Steyaert (Eds), pp. 8–15 (New York: Oxford University Press).

Goodchild, M.F., 2004, GIScience: geography, form, and process. *Annals of the Association of American Geographers*, **94**, pp. 709–714.

Goodchild, M.F., Anselin, L. and Deichmann, U., 1993, A framework for the areal interpolation of socioeconomic data. *Environment and Planning A*, **25**, pp. 383–397.

Goodchild, M.F., Egenhofer, M.J., Kemp, K.K., Mark, D.M. and Sheppard, E., 1999, Introduction to the Varenius project. *International Journal of Geographical Information Science*, **13**, pp. 731–745.

Goodchild, M.F. and Wang, M.-H., 1989, Modeling error for remotely sensed data input to GIS. *Proceedings, AutoCarto 9*, pp. 530–537 (Falls Church, VA: ASPRS/ACSM).

Goovaerts, P., 1997, *Geostatistics for Natural Resources Evaluation* (New York: Oxford University Press).

GRENON, P. and SMITH, B., 2004, SNAP and SPAN: towards dynamic spatial ontology. *Spatial Cognition and Computation*, **4**, pp. 69–103.

HAINING, R.P., 2003, *Spatial Data Analysis: Theory and Practice* (Cambridge: Cambridge University Press).

HOLDRIDGE, L.R., 1971, *Forest Environments in Tropical Life Zones: A Pilot Study* (New York: Pergamon Press).

JOHNSTON, R.J., GREGORY, D., PRATT, G. and WATTS, M., 2000, *The Dictionary of Human Geography*, 4th ed (Oxford: Blackwell).

KJENSTAD, K., 2006, On the integration of object-based models and field-based models in GIS. *International Journal of Geographical Information Science*, **20**, pp. 491–509.

LABORATORY FOR COMPUTER GRAPHICS AND SPATIAL ANALYSIS, 1978, *Proceedings of the First International Advanced Study Symposium on Topological Data Structures for Geographic Information Systems*, Harvard Papers on Geographic Information Systems Volume 5 (Cambridge, MA: Laboratory for Computer Graphics and Spatial Analysis).

LANGRAN, G., 1992, *Time in Geographic Information Systems* (London: Taylor & Francis).

LILLESAND, T.M., KIEFER, R.W. and CHIPMAN, J.W., 2004, *Remote Sensing and Image Interpretation* (New York: Wiley).

LONGLEY, P.A., GOODCHILD, M.F., MAGUIRE, D.J. and RHIND, D.W., 2005, *Geographic Information Systems and Science*, Second Edition (New York: Wiley).

MARK, D.M. and CSILLAG, F., 1989, The nature of boundaries in 'area-class' maps. *Cartographica*, **26**, pp. 65–78.

MCINTOSH, J. and YUAN, M., 2005, A framework to enhance semantic flexibility for analysis of distributed phenomena. *International Journal of Geographical Information Science*, **19**, pp. 999–1018.

PEUQUET, D.J., 1988, Representations of geographic space: toward a conceptual synthesis. *Annals of the Association of American Geographers*, **78**, pp. 375–394.

PEUQUET, D.J., 2002, *Representations of Space and Time* (New York: Guilford Press).

PEUQUET, D.J., SMITH, B. and BROGAARD, B., 1999, *The Ontology of Fields. Report of a Specialist Meeting Held under the Auspices of the Varenius Project* (Santa Barbara, CA: National Center for Geographic Information and Analysis).

RAPER, J. (Ed.), 1989, *Three Dimensional Applications in Geographical Information Systems* (London: Taylor & Francis).

SILVERMAN, B.W., 1998, *Density Estimation for Statistics and Data Analysis* (Boca Raton, FL: Chapman & Hall/CRC).

SINTON, D., 1978, The inherent structure of information as a constraint to analysis: mapped thematic data as a case study. In *Harvard Papers on Geographic Information Systems*, G. Dutton (Ed.). Volume 6 (Reading, MA: Addison-Wesley).

SMITH, B. and VARZI, A.C., 2000, Fiat and bona fide boundaries. *Philosophy and Phenomenological Research*, **60**, pp. 401–420.

STEFANIDIS, A., EICKHORST, K., AGOURIS, P. and PARTSINEVELOS, P., 2003, Modeling and comparing change using spatiotemporal helixes. In *Proceedings, ACM-GIS '03*, E. Hoel and P. Rigaux (Eds), pp. 86–93 (New Orleans, LA: ACM Press).

TAKEYAMA, M. and COUCLELIS, H., 1997, Map dynamics: integrating cellular automata and GIS through Geo-Algebra. *International Journal of Geographical Information Science*, **11**, pp. 73–91.

TOBLER, W.R., 1970, A computer movie simulating urban growth in the Detroit Region. *Economic Geography*, **46**, pp. 234–240.

TOBLER, W.R., 1988, The quadratic transportation problem as a model of spatial interaction patterns. In *Geographical Systems and Systems of Geography: Essays in Honor of William Warntz*, W.J. Coffey (Ed.), pp. 75–88 (London, Ontario: University of Western Ontario, Department of Geography).

TOMLIN, C.D., 1990, *Geographic Information Systems and Cartographic Modeling* (Englewood Cliffs, NJ: Prentice-Hall).

TOPPING, B.H.V., MUYLLE, J., PUTANOWICZ, R. and CHENG, B., 2000, *Finite Element Mesh Generation* (Edinburgh: Saxe-Coburg).

WINTER, S. and FRANK, A.U., 2000, Topology in raster and vector representation. *GeoInformatica*, **4**, pp. 35–65.

WORBOYS, M.F., 1995, *GIS: A Computing Perspective* (Bristol, PA: Taylor & Francis).

YUAN, M., 1999, Representing geographic information to enhance GIS support for complex spatiotemporal queries. *Transactions in GIS*, **3**, pp. 137–160.

YUAN, M., 2001, Representing complex geographic phenomena with both object- and field-like properties. *Cartography and Geographic Information Science*, **28**, pp. 83–96.

ZEILER, M., 1999, *Modeling Our World: The ESRI Guide to Geodatabase Design* (Redlands, CA: ESRI Press).