# MEASUREMENT-BASED GIS

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## Abstract

GIS designs have evolved over the past 30 years, and once adopted a design tends to persist as a legacy despite progress in fundamental research. A feature of most GIS designs is the representation of position by derived coordinates, rather than by original measurements. In such coordinate-based GIS it is impossible to apply traditional error analysis, or to estimate uncertainties in derived products. Thus concern for accuracy issues forces a rethinking of fundamental design. Measurement-based GIS is defined as retaining details of measurements, such that error analysis is possible, and such that corrections to positions can be appropriately propagated through the database. It is shown that measurement-based designs have major economic advantages, in addition to a more comprehensive approach to uncertainty.

## Introduction

The earliest geographic information system (GIS) was designed and developed in the 1960s, and since then much progress has been made in developing tools for working with digital geographic information, and in bringing these tools to a wider audience through commercialization (for reviews of the history of GIS see Coppock and Rhind, 1991; Foresman, 1998). Today, GIS has become a widely available approach to the solution of a large number of needs, from Earth science to local decision-making.

The literature on accuracy and uncertainty in GIS is much more recent. Although Maling and others were writing about analytic cartography from a statistical perspective in the 1960s and 1970s (Maling, 1989), and although the field of geometric probability traces its roots back to the work of Buffon and others in the 18th Century, the first reviews of these issues and their importance for GIS date only from the 1980s (see Burrough, 1986; Goodchild and Gopal, 1989). GIS designers have many options to choose from, since there are many ways of representing the same geographic phenomena in digital form. So if such design decisions were being made as much as two decades before the first discussions of accuracy, an interesting question arises: were the early design decisions appropriate, or should they be reexamined in the light of newer concerns for uncertainty?

Burrough and Frank (1996) have already identified one area that casts light on this issue: the representation of objects with uncertain boundaries. It is clearly difficult to characterize such objects entirely in the vector domain, since there are no simple vector-based models of uncertainty in the position of a boundary or of heterogeneity of the object's contained area, although use has been made of simple descriptive statistics such as the epsilon band (Mark and Csillag, 1989). Instead, it is necessary to switch into a raster representation (or more generally, a field-based view), so that every pixel can be

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assigned some measure of belonging, either to a general class or to a specific object. Goodchild (1989) provides an early discussion of the field/object dichotomy from this perspective.

The purpose of this paper is to discuss and explore another instance of the same general problem. Although maps and geographic databases are ultimately constructed from measurements and observations, it is common for such data to be processed, interpreted, and interpolated in creating a final product. Moreover, such early data are commonly not retained—a typical soil map, for example, contains no representation of the original observations from which the map was compiled. In this paper I explore the consequences of this fundamental design decision, within the context of uncertainty. I show that the decision was flawed, because it severely constrains the value of spatial databases to their users, particularly when the impacts of uncertainty must be assessed, or when the opportunity exists for update or the reduction of uncertainty. I propose an alternative design, termed *measurement-based* GIS to distinguish it from the traditional *coordinate-based* GIS. The paper shows how it avoids some of these problems, and explores some of its details and implications.

The idea is not entirely new. The field of adjustment in surveying is a well-developed area of theory connecting uncertainty in measurements to uncertainty in compiled maps. Some years ago Kjerne and Dueker (1988) showed how object-oriented ideas could be exploited to code these connections. Buyong and Frank (1989), Buyong *et al.* (1991), and Buyong and Kuhn (1992) have also written about the inheritance of measurement uncertainty. In this paper I have attempted to generalize some of these ideas, place them in a broader framework, and explore some of their wider implications.

## Measurements and coordinates

### *Geographic information defined*
The fundamental atom of geographic information is the tuple $<\mathbf{x},\mathbf{z}>$, which links a spatiotemporal location $\mathbf{x}$ to a set of attributes $\mathbf{z}$, drawn from the things that can be known about a location—its temperature, soil type, county, owner, the name of a geographic feature type known to be present at the location, etc. Because space and time are continuous, the creation of a spatially continuous map or image of an area would require an infinite number of tuples, even if it were limited to representing a single instant in time. In practice, we resort to numerous schemes for reducing or compressing what would otherwise be an infinite set of tuples, by ignoring areas that are outside the area of geographic coverage, or areas that are empty with respect to the topic of interest, or through various forms of generalization and abstraction. For example, we identify a set of locations with a region, represent the region as a polygon formed by a finite number of vertices, and assign attributes to the region. Goodchild *et al.* (1999) review many of these methods.

In this paper I focus on $\mathbf{x}$, and issues of positional uncertainty, though much of the discussion also applies to $\mathbf{z}$. I ignore also the issue of separability: whether it is possible to distinguish uncertainty in $\mathbf{x}$ from uncertainty in $\mathbf{z}$ (can one distinguish between correct attributes of the wrong location and wrong attributes of the correct location?).

The representation of geographic location (the spatial dimensions of $\mathbf{x}$) is almost always

absolute in traditional GIS—that is, location is with respect to the absolute Earth frame, through geographic coordinates (latitude and longitude) or some convenient planar coordinate system, such as UTM (Universal Transverse Mercator). Thus a GIS is able to associate attributes with geographic locations, and service queries of the form "Where is **z**", or "What is at **x**?" Without absolute location, it would be impossible to integrate different databases by location, a function that is often claimed to be one of GIS's greatest strengths (note, however, that this does not imply that all locations in the data structure are in absolute form, only that absolute location can be determined as a service of the GIS).

In this paper I distinguish two bases for determination of **x**—those in which **x** is *measured* directly, using GPS or geometric techniques, and those in which **x** is *interpolated* between measured locations. The latter occurs, for example, when the position of some feature recognizable on an aerial photograph is established with respect to registered tics or control points. It also occurs when a surveyor establishes the location of a boundary by linking two surveyed monuments with a mathematically straight line.

Let the set of measurements required to establish a measured location be denoted by **m**, and let the function linking these measurements to the location be denoted by $f$, that is:

$$\mathbf{x} = f(\mathbf{m})$$

The inverse of $f$ is denoted by $f^{-1}$, that is, the function that allows measurements to be determined from locations. In what follows this expression is also used to describe the derivation of an array of locations from a complex set of measurements.

### *The theory of measurement error*
Suppose that some scalar measurement, such as a measurement of temperature using a thermometer, is distorted by an error generated by the measuring instrument. The apparent value of temperature $x'$ can be represented as the sum of a true value $x$ and a distortion $\delta x$. If some manipulation of $x$ is required, the theory of measurement error provides a simple basis for estimating how error in $x$ will propagate through the manipulation, and thus for estimating error in the products of manipulation (Taylor, 1982; and see Heuvelink, 1998, and Heuvelink *et al.*, 1989, for discussions of this in the context of GIS). Suppose that the manipulation is a simple squaring, $y = x^2$, and write $\delta y$ as the distortion that results. Then:

$$y + \delta y = (x + \delta x)^2$$

$$y + \delta y = x^2 + 2x\delta x + \text{terms of order } \delta x^2$$

Ignoring higher-order terms, we have:

$$\delta y = 2x\delta x$$

More generally, given a measure of uncertainty in $x$ such as its standard error $\sigma_x$, the uncertainty in some $y=f(x)$, denoted by $\sigma_y$, is given by:

$$\sigma_y = df/dx\,\sigma_x$$

The analysis can be readily extended to the multivariate case and the associated partial

derivatives.

### *Errors in position*
Suppose that position has been distorted by error, such that the observed location $x'$ is distorted by a vector $\varepsilon(\mathbf{x})$ that is a function of location. Kiiveri (1997) and Hunter and Goodchild (1996) have discussed this model, and the conditions that must be imposed on $\varepsilon(\mathbf{x})$ to ensure that normal conditions are not violated—that the space is not torn or folded, ensuring that its basic topological properties are preserved. We also typically assume that $\varepsilon(\mathbf{x})$ varies smoothly in space, with continuity and strong spatial autocorrelation, in order to permit locations to be interpolated with reasonable accuracy, and to allow the use of rubber-sheeting methods in registration. That is, we assume:

$$\varepsilon(\mathbf{x} + \delta\mathbf{x}) - \varepsilon(\mathbf{x}) \text{ tends to } 0 \text{ as } \delta\mathbf{x} \text{ tends to } 0$$

and that strong covariances exist among $\varepsilon$ at different locations.

In practice a single database may contain objects with many different lineages. If two objects occupy the same location, it does not follow that $\varepsilon$ is the same for both objects. Instead, it may be necessary to model many different error fields, and to associate each object or even parts of objects with distinct fields. The implications of this are discussed in the next section.

### *Relative and absolute accuracy*
In practice, it is common to distinguish two forms of positional error, though only informally. In this section I attempt to formalize their definitions.

Consider two locations $x_1$ and $x_2$, and suppose that distance must be measured between them. The error in the distance will be determined by the variance–covariance matrix of their positional errors. If $\varepsilon(x_1)=\varepsilon(x_2)$, in other words perfect correlation exists between the two errors, then covariances will equal the products of the square roots of the respective variances, and the error in distance will be 0. But if correlation is zero (errors are independent), then covariances will be zero, and the error in distance will show the combined effects of both positional errors. *Absolute* error is defined for a single location as $\varepsilon(\mathbf{x})$. *Relative* error is defined only for pairs of points, and describes error in the determination of distance. Moreover, a continuum of levels of relative error exist depending on the degree of correlation between the two positional errors. In principle it is possible for negative correlations to exist, such that relative error can exceed the errors inherent in the independent case, but in practice we suspect that correlations are almost always non-negative. Since relative and absolute error are not commensurate, one being a function of two locations and the other of one, and since one term, *relative*, describes an entire continuum, the dichotomy does not seem to provide much basis for formal treatment.

Conceptually, however, the distinction may help. Consider the objects shown in Figure 1. Suppose the building's form is described by four vertices, generated by interpretation of an aerial photograph and the use of a standard template that enforces parallel edges and rectangular corners. Assume that the building might have been located in various positions, described by an error model. The error model might be used to simulate equally likely observed locations (Openshaw, 1989). Because of correlations among the

errors, it is easiest to think of the entire ensemble as a single sample from a population of equally likely realizations of the entire ensemble, rather than as a collection of error models for each individual object or vertex.
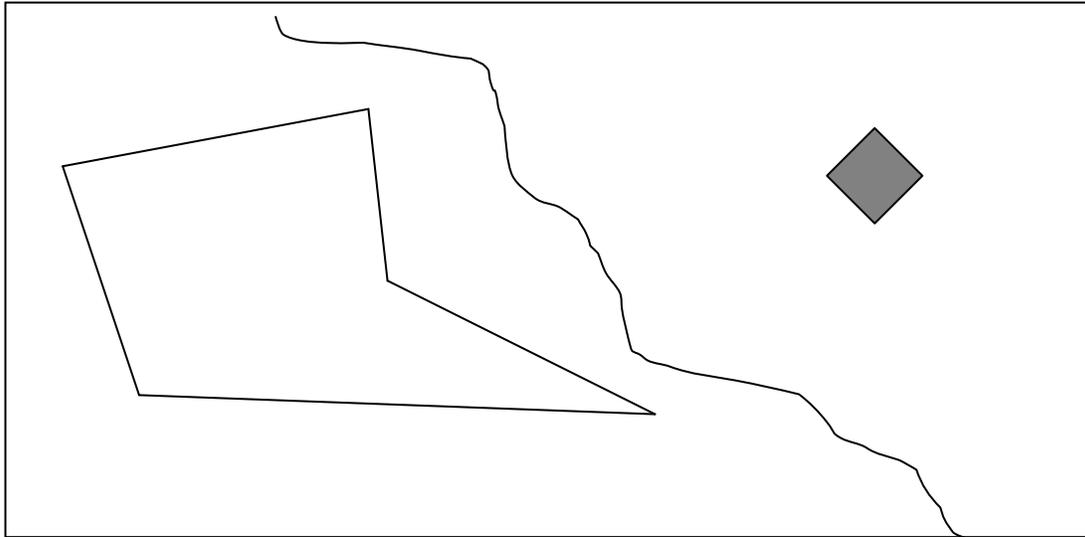


**Figure 1:** A parcel of land, a river, and a building. Each object and all of their parts are subject to distortion due to errors in positioning, but complex covariances exist between these errors. Because it was created using a template, the building behaves under positional distortion as a rigid frame able to move and rotate, but not to change shape.

In practice, because a template was used there will be very strong correlations between the errors distorting the positions of the four vertices of the building. Thus the model would have only three degrees of freedom—for example, distortions of one vertex in two spatial dimensions, plus distortion by rotation of the building about that vertex as a rigid body. It is possible to think of positional error in terms of the *movements* of objects and their parts that result from resampling of the error distribution. Some objects will change shape under resampling, if the larger errors occurred during independent determination of the locations of their vertices, while others will remain rigid. Some objects will move independently of each other, if their positions were determined by independent processes, whereas others that were produced by the same process and share substantial lineage will move in concert. Such objects may retain some degree of correctness in their relative positions while their absolute positions change. Thus a display of many distinct realizations of the ensemble error model in rapid succession will convey a fairly accurate impression of the error correlation structure.

### Adjustment and update
In principle, the variance–covariance matrix of positional errors in an ensemble of locations can be derived from knowledge of the characteristics of measurement errors, through extensions of the theoretical framework outlined above. Moreover, if the actual

error present in a point's location can be determined, it should be possible to correct the associated measurements, and then to adjust the locations of other points appropriately. Of course, if all covariances are zero then the point's location can be corrected independently. But in the normal case of strong covariances, especially within objects and between objects that share aspects of their lineage, correction of one location without simultaneous correction of locations with correlated errors will not be helpful. For example, correction of one vertex of the building without simultaneous correction of the remaining three vertices, based on knowledge of the variance–covariance matrix of errors, will change the building's shape.

### *The geodetic model*
These issues are to some extent resolved by use of what will be termed here the *geodetic model*. In this model locations are arranged in a hierarchy, as shown in Figure 2. At the top are a small number of locations termed control points or *monuments* that are established with great accuracy by geodetic survey. From these a much larger number of locations are established by measurement, through a process of *densification*. Since these measurements are not as accurate as those used to establish the monuments, the second tier of locations is also less accurately known. Further measurements using even less accurate instruments are used to register aerial photographs, lay out boundary lines, and determine the contents of geographic databases.
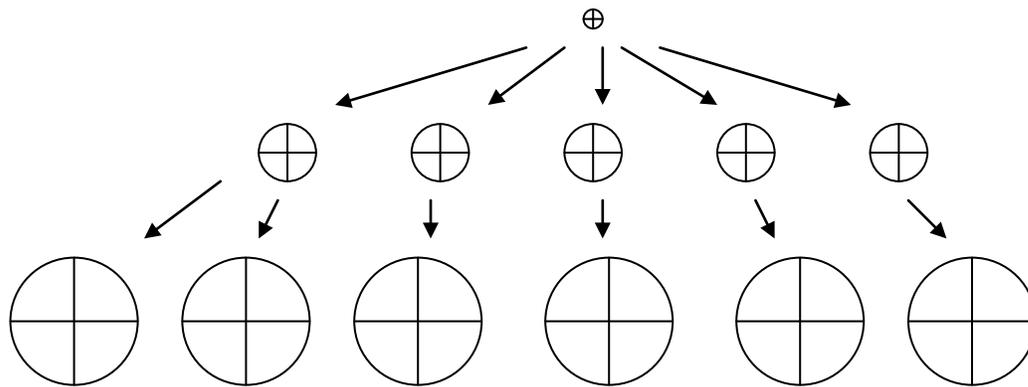


**Figure 2:** Inheritance hierarchy of the geodetic model. A single monument, located with high accuracy, is used to determine the positions of a denser network of less-accurate points, which are in turn used to determine the positions of denser and even-less-accurate points. Further locations may be interpolated between these points.

Following earlier arguments, there will be strong correlations in errors between any locations whose lineages share part or all of the tree. All points inherit the errors present in the monuments, but distances between points that share the same monument are not affected by errors in the location of the monument itself. Thus it is possible to achieve comparatively high accuracy in the results of simple GIS operations like area measurement despite inaccuracies in positioning. However, if the structure of the hierarchy is not known, it is not possible to know how much shared lineage exists between pairs of objects that are the subject of analysis, *even though such objects may be*

*in the same layers, or may be parts of the same complex object*. The internal structures used by the organization of the spatial database may mask the hierarchical structure of the geodetic model.

## Towards measurement-based GIS

### *Definitions*
I define a *coordinate-based GIS* as one that provides access to the locations of measured objects **x**, but not to the measurements **m** from which those locations were derived, or to the function *f* used to derive **x** from **m**. The GIS may or may not provide access to the rules used to determine the locations of interpolated objects from measured objects (for example, the rule defining the edge of a parcel as mathematically straight may be implicit in the data structure, but the rule defining a tree relative to the control points of an aerial photograph may not).

A *measurement-based GIS* is defined as one that provides access to the measurements **m** used to determine the locations of objects, to the function *f*, and to the rules used to determine interpolated positions. It also provides access to the locations, which may either be stored, or derived on the fly from measurements.

In the following, it is assumed that the spatial database is of sufficient complexity that multiple object types exist, with complex lineage. More specifically, it is assumed that covariances between errors in the positions of pairs of measured locations are positive. It follows from the nature of interpolation that covariances are also frequently positive between pairs of interpolated locations.

In a coordinate-based GIS it is not possible to correct positions for part of the database, since the knowledge of error covariances needed to adjust other positions is not available. Partial correction may improve the absolute positions of corrected points, but will affect the relative positions of corrected and uncorrected points in unknown ways. These impacts include changes of shape and other geometric inconsistencies, such as non-existent bends or offsets in linear features, and violations of topological constraints.

In the annals of GIS there are many anecdotes about the costs of attempting partial correction of coordinate-based databases. For example, Goodchild and Kemp (1990, Unit 64) describe the costs to a utility company when partial update moved a large proportion of features across land ownership boundaries. In such situations many agencies have resorted to recompilation, abandoning old, low-accuracy data completely because of the problems of partial correction.

By contrast, no such problems exist in measurement-based GIS. If a location is observed to be distorted, the means exist to determine the offending measurements, correct them, and propagate the effects of correction to all other dependent positions, because **m** and *f* are known.

In addition, it is possible in measurement-based GIS to calibrate error models fully, allowing determination of the impacts of propagating positional errors through GIS operations. The properties of the error field $\varepsilon$ could be determined, allowing interoperation between two distorted maps of the same area (for a practical motivation see Church *et al.*, 1998). In the attribute domain, knowledge of measurements could

allow the spatial dependence parameters identified by Goodchild, Sun, and Yang (1992), Heuvelink (1998), and Hunter and Goodchild (1997) to be defined and utilized in Monte Carlo simulations.

*Hierarchy*

A measurement-based GIS is structured as a hierarchy, as outlined in the discussion of the geodetic model above. Let $\mathbf{x}^{(i)}$ denote a location at level $i$ in the hierarchy. Then locations at level $i+1$ are derived from level $i$ locations through equations of the form:

$$\mathbf{x}^{(i+1)} = f(\mathbf{m}, \mathbf{x}^{(i)})$$

At the top (or root) of the tree are locations $\mathbf{x}^{(0)}$ which *anchor* the tree. At each level the measurements $\mathbf{m}$ and function $f$ are stored, and the locations $\mathbf{x}$ are either stored or derived as needed.

Consider, for example, a utility database in which locations of underground pipes are stored. In such examples the locations of pipes are typically recorded by measurement from other features of known location, such as property lines, or street kerbs. A pipe might be recorded as 3 ft from a given property line, offset to the left looking in the direction in which the property line is recorded. In this case $\mathbf{m}$ would be recorded as {3.0,L} or in some other suitable notation. If the pipe is resurveyed, or moved, its position can be reestablished by correcting the measurement, or by changing other aspects of the measurement data. But since the dependence is explicit, there will be no need to worry about corrupting the relative positions of pipe and property line, as there would in a coordinate-based GIS.

*Beyond the geodetic model*

Situations often arise in GIS where the root of the tree is not determined with great accuracy. Suppose, for example, that a national database of major highways is built, and anchored to no better than 100m accuracy (according to the U.S. National Map Accuracy Standards such a database could be described as having a *scale* of 1:200,000, but see Goodchild and Proctor, 1997). It follows that all other locations in the database are absolutely located to no better than 100m. However it is likely that relative accuracies are higher, since independent distortion of as much as 100m in the elements of such a database would be unacceptable because of the geometric and topological distortions it would produce. Again, the metaphor of a semi-rigid frame floating in space is helpful in conceptualizing situations like this.

Suppose now that a local agency wishes to link its own database of streets to the national database. This database is likely to be much more accurate, perhaps anchored to 1m to the geodetic frame. This database could be conceptualized as a second tree, but in this case the positional standard error associated with $\mathbf{x}^{(0)}$ would be only 1m. In essence, the example can be characterized as two trees, with no common root, and with one tree having a standard error that is much larger than that typical of the geodetic model, in which there is only one tree and a highly accurate anchor (see Figure 3).

To link the two databases together, the highest level of the more accurate database is established as the common root. Suppose that its anchor consists of a well-defined point resolvable to better than 1m, such as a survey monument or a photography control point. Suppose also that this location corresponds to that of one of the points in the highest level

of the less accurate tree, although the resolution of this anchor point in the less accurate tree is probably much lower (for example, this anchor point might be described as an intersection between two streets, and the intersection might contain the monument or control point anchor of the less accurate tree, see Figure 4). Figure 3 shows the link that is now built between the new common anchor and the anchor of the less accurate tree, integrating the two trees into one. This link appears as a pseudo-measurement, with a displacement of zero and a standard error equal to 100m. Since the two trees were established and anchored independently, it is reasonable to assume zero covariance between the errors in the measurements in the two subtrees.
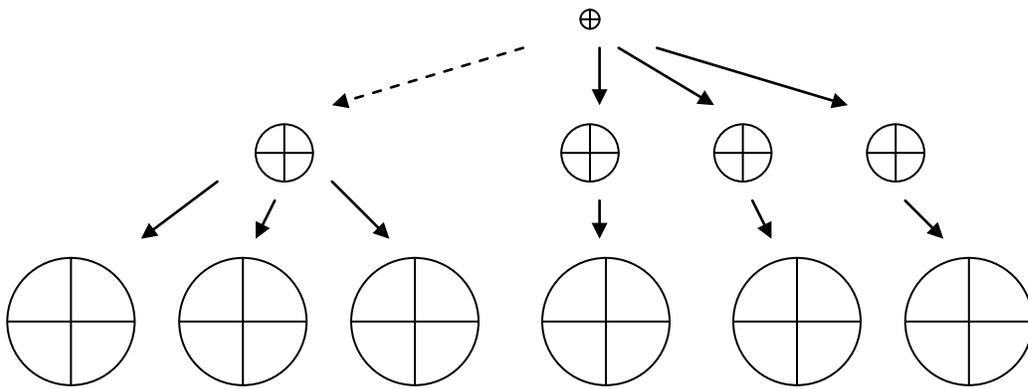


**Figure 3:** Merging of two data sets with distinct inheritance hierarchies. The three least-accurate points on the left depend on a single parent node of moderate accuracy, while the three least-accurate points on the right depend on a more-accurate set, and on a single high-accuracy monument. The dashed arrow represents the pseudo-measurement that is inserted to merge the two trees.

## Discussion and conclusions

If the measurements used to establish position in a spatial database are not retained, but instead all positions are defined only by coordinates with respect to the Earth frame, then it is impossible to correct or update parts of the database without creating geometric and topological distortions that are frequently unacceptable. The almost universal adoption of this design by the GIS software industry is based on the perception that it is possible to know location exactly, and is reflected in the frequent use of precision that greatly exceeds accuracy in the internal representation of coordinates. But in practice exact location is not knowable, and all measurements on which locations are based are subject to some level of error.

By retaining measurements and the functions needed to derive coordinates, it is possible to support incremental update and correction, and to provide much more informed estimates of the impacts of uncertainty in GIS operations. Thus measurement-based GIS designs offer the potential for dramatic reductions in the cost of database

maintenance, support for transaction-based operations, and much greater usefulness. Measurement-based principles can be implemented to support database integration, even when the databases being integrated have very different levels of positional accuracy. But such integration is much more problematic using traditional coordinate-based designs.
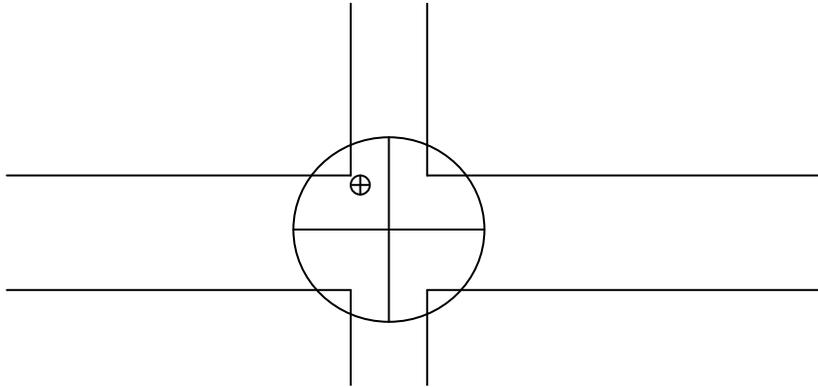


**Figure 4:** Example of two inheritance hierarchies (see Figure 3). One hierarchy is anchored to the high-accuracy monument shown as the small circle, located at the kerb. The other is anchored with low accuracy to the entire intersection.

To return to a point made at the outset, this distinction between coordinate-based and measurement-based GIS strikes at a fundamental issue: are the designs that were developed early in the history of GIS, and remain influential as legacy systems, still optimal given current concepts? Clearly the answer based on this example is *no*.

Two distinct strategies are available to support measurement-based GIS: one can design such a GIS from ground up, or one can adopt a framework that incorporates the necessary elements. Hierarchical databases have fallen out of fashion in the past two decades, but the relational model that has largely replaced them has no inherent concept of hierarchy. On the other hand object-oriented database designs include concepts of inheritance, and provide some of the necessary forms of support. Smallworld's GIS has support for measurement-based designs, through appropriate use of its inheritance features. But full adoption of a measurement-based paradigm is more problematic, since it involves respecification of many functions to include explicit propagation of error characteristics, and explicit storage of error variance–covariance matrices. Thus implementation of a prototype measurement-based GIS, with its associated database and functionality, remains as a challenge to the GIS research community.

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