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Models of hierarchically dominated spatial interaction

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Abstract. Strongly asymmetrical spatial flows are analyzed by using as example the flows of news between major Canadian cities. Conceptually such flows can be modelled by combining a vertical dimension of dominance with two spatial dimensions. The conceptual model is interpreted mathematically by appropriate use of the attraction and emissivity terms in the spatial interaction model. A family of models is identified and calibrated using the Canadian data, allowing the positions of cities on the dominance dimension to be identified. By inverting the model it is possible to scale positions in the spatial dimensions to reflect interaction more correctly. The result is a three-dimensional image which gives a 'fuzzy' interpretation to the conventional topological tree model of a hierarchy. Relationships with Tobler's 'winds' model are examined.

Introduction

The past ten years or so have seen an awakening of interest in what are now known as spatial interaction models, occasioned no doubt by the theoretical work of Wilson (1971), and by the development of improved methods and tools for calibration (see, for example, Batty, 1976).

Relatively little effort has been expended on developing a taxonomy of the phenomena to which such models can be applied, which range from journeys to work and to shop, through migration, to consignments of goods, and such measures of social interaction as telephone calls. This paper is concerned with those cases in which the set of origins is identical to the set of destinations, yielding square data matrices, as for example in most studies of migration flows. In such cases two flows or interaction measures are available between each pair of places; they may be referred to as I_{ij} and I_{ji}. The classic gravity model, which identified mass with population, would predict a symmetrical interaction since

$$I_{ij} = \frac{GP_iP_j}{D_{ij}^b} \qquad (1)$$

where
P_i is the population of place i,
D_{ij} is the distance between i and j,
G, b are constants, and
I_{ij} is the flow from i to j.
Thus $I_{ij} = I_{ji}$.

Although this may be empirically a valid prediction (within experimental error) for certain phenomena, others exhibit varying degrees of asymmetry. If j were the more attractive place to migrants, we might expect $I_{ij} > I_{ji}$, other things being equal. In fact it is possible to visualize a spectrum of increasing asymmetry, leading in the extreme to uni-directional flows between two places, as exemplified by diffusion phenomena.

Asymmetrical flows are usually found to be weakly transitive, that is, if $I_{ij} > I_{ji}$ and $I_{jk} > I_{kj}$, then $I_{ik} > I_{ki}$ for all i, j, k. This being so, it is possible to find a ranking of places such that if i precedes j in the ranking, then $I_{ij} > I_{ji}$. In the case of migration data the ranking would be one of attraction, as in Lycan's (1969) work on the interprovincial Canadian data. For other phenomena the scale can be seen as one of dominance, whereas in the diffusion of innovations it is a temporal ordering of adoption.

Tobler (1976) has extended this ordinal analysis by postulating an interval-scale "forcing function" whose value at each place determines flow asymmetry. The forcing function may be thought of as a pressure; when the pressures at the two ends of a link are different, a 'wind' results, with intensity determined by the difference in pressure and the length of the link. Other analogies are possible; temperature difference generates a heat flow, and voltage difference an electric current. The wind, in turn, affects the symmetry of interaction by increasing downwind flows and decreasing upwind flows.

Interaction can now be interpreted in terms of three dimensions. Distance in the horizontal plane has the conventional effect of attenuating interaction, whereas dominance or relative attraction is visualized as resulting from separation in a third, vertical dimension, and is responsible for interaction asymmetries. Broadly, the horizontal plane corresponds to spatial effects, and the vertical dimension to hierarchical effects.

Although the predictions of the classic gravity model were symmetrical, this is not in fact a property of spatial interaction models in general. Consider the model

$$I_{ij} = E_i A_j \mathrm{f}(D_{ij}) \tag{2}$$

where

E_i is origin effect or 'emissivity',

A_j is a destination effect or 'attraction',

$\mathrm{f}(D_{ij})$ is a function of distance.

In order not to lose generality, E_i and A_j can be assumed to include optional summations required by constraints. When applied to square matrices, such models presume that the properties of a place as origin (the E_i) have no direct relation to those of the same place acting as destination (the A_j). Tobler's model, on the other hand, in effect proposes that both inflow and outflow can be related to the same forcing function.

Newspaper datelines

Kariel and Welling (1977) have described and analyzed a matrix showing the flows of news between seventeen major Canadian cities. Each entry I_{ij} is a count of the number of news stories originating in city i and carried by the sampled newspaper in city j during the sample period. The reader is referred to their paper for the details of data collection. Only 21 of the 680 triads ijk in the matrix are intransitive against the weak criterion, and all of these can be ascribed to sampling error (Kwan, 1977), indicating a very strong pattern of dominance.

The data are clearly well-suited to analysis using asymmetric models in general, and the three-dimensional model in particular. However, for reasons which will be detailed below, Tobler's approach to calibration and interpretation is not suitable for this data set. The purposes of this paper are to develop more appropriate versions of the same general class of model, to explore methods of calibration and to make a direct empirical test.

Models

Let there be a forcing function, more simply referred to as a height, at each place, denoted by H_i. The models investigated are of the general form

$$I_{ij} = P_i \mathrm{f}(D_{ij}) \mathrm{g}(H_i, H_j) \, . \tag{3}$$

The functions f and g are measures of horizontal and vertical separation respectively.

Two forms for g are investigated in the following section: first a difference, $g(H_i, H_j) = [1+(H_i-H_j)]$, and then a ratio, $g(H_i, H_j) = H_i/H_j$. Their validity can be tested without prior assumptions about the nature of f by the following transformations.

For the difference model let

$$I_{ij} = aP_i[1+(H_i-H_j)]f(D_{ij})$$

and let

$$x_{ij} = \left(\frac{I_{ij}}{P_i}-\frac{I_{ji}}{P_j}\right) \bigg/ \left(\frac{I_{ij}}{P_i}+\frac{I_{ji}}{P_j}\right).$$

Then

$$x_{ij} = H_i - H_j$$

given that $f(D_{ij}) = f(D_{ji})$. It is therefore hypothesized that a vector of constants H can be found such that each observed value x^*_{ij} is equal to the difference between the appropriate pair of constants.

For the ratio model write

$$x_{ij} = \frac{I_{ij}}{I_{ji}} ,$$

which leads to the hypothesis

$$x_{ij} = \frac{P_i H_i^2}{P_j H_j^2} .$$

Such hypotheses are not of course limited to this particular context. Equation (2), the more general model, when transformed using $x_{ij} = I_{ij}/I_{ji}$, also leads to the hypothesis $x_{ij} = C_i/C_j$, where C is a vector of constants, but without, of course, the direct interpretation in terms of height.

A test of these hypotheses requires some assumption about the sampling distribution of I_{ij}. Most authors have taken the multinomial distribution, but this seems inappropriate in this case in view of the absence of constraints, particularly on $\sum_i \sum_j I_{ij}$. Rather, each observation I^*_{ij} is regarded as a sample count of random, independent events, leading to the assumption of a Poisson sampling distribution, in accordance for example with Bexelius et al (1969) and Kirby (1974). Each I^*_{ij} was taken as a Poisson mean, and the sampling distribution of x_{ij} simulated by 50 independent generations of Poisson deviates.

Both models can now be calibrated and tested by means of weighted least squares. Consider the difference model first, and let x^*_{ij} represent the observed transformed value, and $x_{ij} = H_i - H_j$. The simulated sampling distribution of x_{ij} is approximately normal, with standard deviation σ_{ij}. Then the logarithm of the likelihood of a matrix $\mathbf{X}$ is given by

$$\log L = \sum_i \sum_j \log(2\pi)^{\frac{1}{2}} \exp(-\tfrac{1}{2} z_{ij}^2) ,$$

where

$$z_{ij} = \frac{x^*_{ij} - x_{ij}}{\sigma_{ij}} .$$

Thus the maximization of $\log L$ is equivalent to the minimization of $\sum_i \sum_j (x^*_{ij} - x_{ij})^2/\sigma^2_{ij}$, which is a weighted least-squares criterion. This can be achieved by linear regression if x^*_{ij}/σ_{ij} is regressed against a series of dummy variates δ_{ijk}/σ_{ij}, and δ_{ijk} is given the value 1 if $k = i$, -1 if $k = j$ and 0 otherwise, with the diagonal elements $i = j$ treated as missing. The regression coefficients are the required values of H, and the residual sum of squares is directly related to the likelihood function.

A similar approach can be taken for the ratio model. Here x_{ij} has an approximately log-normal distribution, and likelihood can be maximized using the weighted least-squares criterion $\sum_i \sum_j (\log x_{ij}^* - \log x_{ij})^2 / s_{ij}^2$, where $x_{ij} = P_i H_i^2 / P_j H_j^2$ and s_{ij} is the simulated-sample standard deviation of $\log x_{ij}$. Ordinary least squares (OLS) is applied with $(\log x_{ij}^*)/s_{ij}$ as the dependent and δ_{ijk}/s_{ij} as the independent variables. The resulting regression coefficients are values of $\log P_i H_i^2$, and again the residual sum of squares is related to likelihood.

Table 1 shows the results for both models. It is clear that the ratio model yields a much better fit. With the exception of eight anomalous interactions, the values of z_{ij} are within experimental error. The standard errors shown in table 2 were computed by repeated recalibration of the model with values of I_{ij} distorted by Poisson simulation. The values of $\log P_i H_i^2$ are shown normalized to an arithmetic mean of zero, and the H_i to a geometric mean of 1. Values of H are also shown in figure 1.

The eight anomalous interactions, defined as $|z_{ij}| > 3$, occur in the following ways: Victoria dominates Vancouver to a greater degree than expected given their interactions with the other cities; Ottawa dominates Toronto to a smaller degree than expected; and Montreal dominates both Toronto and Quebec to a greater degree than expected.

Table 1. Results of calibration for the difference and ratio models after transformation.

$\|z_{ij}\|$	Difference model		Ratio model	
	observed	expected	observed	expected
0-1	100	180	148	177
1-2	81	71	77	70
2-3	41	11	26	11
>3	41	1	8	1

Table 2. Calibrated values of $\log P_i H_i^2$ for the ratio model.

Place i	$\log P_i H_i^2$	Standard error	H_i	H_i [model (6)]
Victoria	0·802	0·061	1·89	1·64
Vancouver	0·664	0·029	0·75	0·68
Calgary	−0·221	0·057	0·79	0·76
Edmonton	0·083	0·029	0·83	0·84
Regina	−0·947	0·050	0·93	0·85
Winnipeg	0·334	0·051	0·91	0·83
Windsor	−1·374	0·107	0·56	0·46
London	−1·419	0·077	0·52	0·54
Hamilton	−1·224	0·052	0·43	0·43
Toronto	1·899	0·021	0·90	0·87
Ottawa	2·471	0·051	2·49	3·51
Montreal	2·146	0·051	1·00	0·93
Quebec	0·737	0·031	1·17	1·08
Fredericton	−1·082	0·106	2·14	2·55
Halifax	−0·104	0·053	1·13	1·22
Charlottetown	−1·324	0·131	2·10	2·04
St. John's	−1·444	0·051	0·75	0·99

Although the ratio model fits the data remarkably well, the identification of the results in terms of heights or forcing functions is of course only one possible interpretation, since any model of the form given in equation (2) yields the same hypothesis and the same results. Modelling of the x_{ij} has merely indicated that the ratio model is much more acceptable than the difference model. The next section is concerned with the modelling of I_{ij}, and the form of the function f.

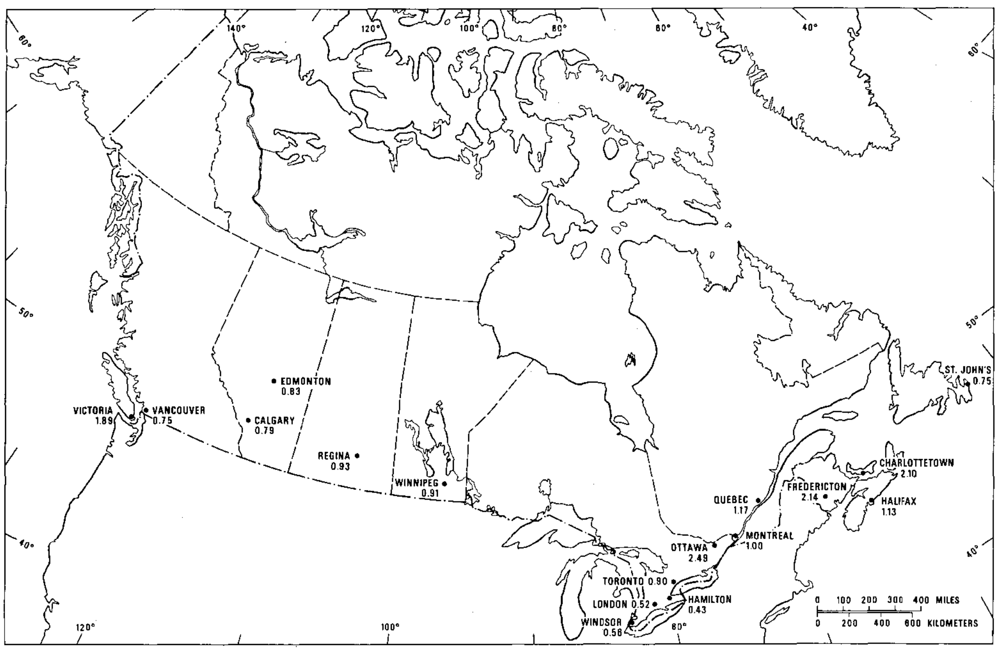

Figure 1. Solution space, transformed ratio model.

The impedance function

Adopting the ratio model, we have

$$f(D_{ij}) = \frac{I_{ij}H_j}{P_i H_i} ,$$

where I_{ij} is the predicted value of interaction. The impedance $f(D_{ij})$ can be estimated by taking the observed flow together with the constants determined in the previous section. Assume that f is a monotonically decreasing function. Then it is possible to ask whether a set of locations exists such that the distances between them, measured according to some metric, are in the appropriate rank order given the estimates of impedance. Figure 2 shows the best solution for the locations, using the straight-line Pythagorean metric, and the nonmetric multidimensional scaling routine TORSCA-9 (Young and Torgerson, 1968). The stress statistic, which in this case measures the degree to which the distances between locations violate the observed rank order of impedances, is 0·220, compared to an optimum of 0·0. It represents the combination of sampling errors in the determination of impedance, and structural error in the specification of the metric and the model.

The locations in figure 2 show several systematic distortions when compared with figure 1. The nationally dominant cities, Ottawa and Toronto, occupy the centre of the configuration, as they do in reality. Large, regionally dominant centres, the second rank in the national hierarchy, are drawn in towards the centre from their true positions. Thus Vancouver and Calgary, which interact far more with the central cities than their distances from them would indicate, appear near the centre. Other Western and Atlantic cities show the same influence to a lesser degree. The lowest centres, the consistent news sinks rather than news sources, are pushed out to the periphery. London, Hamilton, and Windsor interact with each other much less than their mutual proximity would suggest and, although they receive from the centre, they contribute very little to it. St. John's and Halifax show similar effects. Finally, Montreal and Quebec City are forced to the perimeter because of their linguistic isolation from the rest of the system, which here is equated with effective distance.

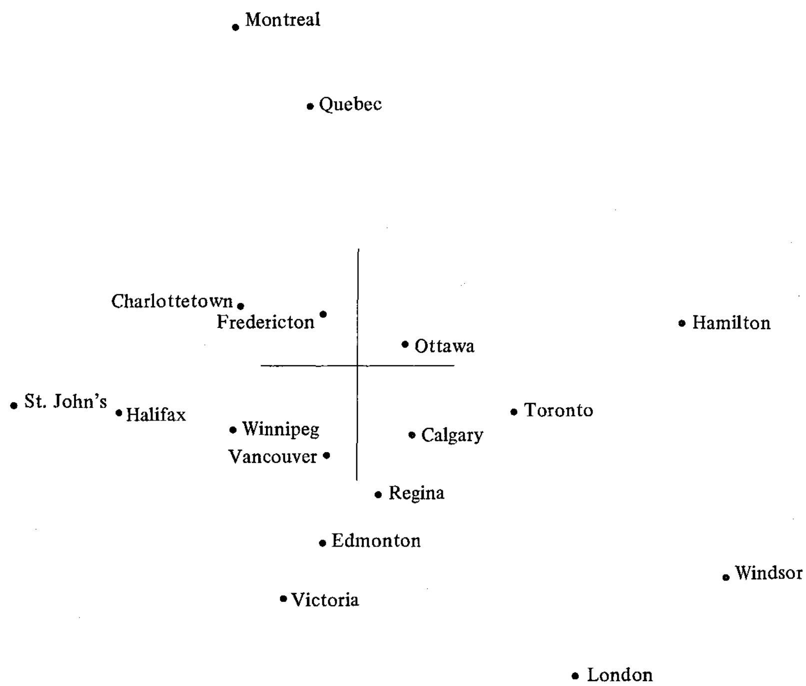

Figure 2. Solution found by scaling the impedance function of the ratio model.

Models based on true location

The general class of models introduced in equation (3) describes interactions between places located in a three-dimensional space. The vertical, or H dimension, describes the position of a place in the national hierarchy, and its propensity either to generate or to accept interaction. The two horizontal dimensions, x and y, describe spatial relationships and the effects of distance in attenuating interaction.

Table 3. Calibration statistics: true location models.

	Model (4)	Model (5)	Model (6)
Constants	19	20	53
Criterion value	5659	4639	2121
Mean absolute percentage error	60·1	53·5	43·7
Root mean-square error	50·3	46·7	27·5
R^2	0·674	0·720	0·903
Residuals			
>3 standard errors	134	102	63
<3 standard errors	138	170	209

In the vertical dimension there are no exogenously determined locations, and so it is possible to choose a scale of H such that the function g takes a simple form. But in the horizontal, two approaches are possible. Cities can be located in their exogenous, real-world locations, and the function f defined to give the best possible predictions. On the other hand, f can be defined in some a priori manner, and locations selected to give the best possible predictions. The alternatives may be mathematically equivalent, but are conceptually different. The former of the two approaches is adopted in the following section.

Two models are investigated below. The first is a simple gravity-type model with negative exponential impedance function

$$I_{ij} = aP_i\frac{H_i}{H_j}\exp(-b_0D_{ij}) . \tag{4}$$

The second model reflects the distortion noted in figure 1. Cities at the upper levels of the hierarchy appear to interact more, given the distances between them, and cities at the lower levels less. The constant b_0 in equation (4) should therefore be combined with a function of height, as it is in the second model:

$$I_{ij} = aP_i\frac{H_i}{H_j}\exp[-b_0\exp(-b_1H_i^{1/2}H_j^{1/2})D_{ij}] . \tag{5}$$

The additional term increases the effect of distance when both H_i and H_j are small, and decreases it when they are large.

To calibrate models (4) and (5), the sampling distribution of I_{ij} was again taken as Poisson. Maximum likelihood estimation would be inappropriate in view of the probability of model specification errors, so both models were calibrated using a weighted least-squares criterion. Each observation was weighted by the inverse of the standard error of the Poisson distribution, which led to the following criterion: minimize $\sum_i\sum_j(I_{ij}^* - I_{ij})^2/I_{ij}$. A quasi-Newton method (Fletcher, 1972) was used for optimization and gave satisfactory results [for a comparison of various alternatives see Batty and Mackie (1972)]. Distances were measured along shortest great circles.

Calibration statistics for both models are shown in table 3. The second is a substantial improvement over the first, given that only one constant has been added. Residuals have been tabulated in terms of the standard error of the Poisson sampling distribution of each observation, so that 138 observations out of 272 can be said to have been correctly predicted within experimental error in the case of model (4), and 170 for model (5).

We have already seen that distance affects interaction in different ways depending on the level in the hierarchy at which interaction occurs. In addition the distance measured along a great circle may have little relationship to impedance, which may in fact be better explained by some sort of perceptual distance. To some degree these problems can be overcome by a careful choice of the impedance function f. But a dramatic improvement in the fit of the model can only be achieved by a redefinition of the locations of the places in the system to reflect their effective, rather than true, positions.

A model of perceptual distance

The approach adopted in this section is to define a standard impedance function, but to allow places to find locations that best predict interactions. This can be expressed as follows:

$$I_{ij} = aP_i\frac{H_i}{H_j}\exp\{-b_0[(x_i-x_j)^2+(y_i-y_j)^2]^{1/2}\} . \tag{6}$$

The vectors x and y are treated as unknowns, together with the vector H and scalars a and b_0. This gives a total of 53 unknowns in the model, against 272 valid observations. For H, scale is arbitrary, whereas for x and y both scale and origin are arbitrary.

The model was again calibrated using the weighted least-squares criterion, a Poisson sampling distribution, and the quasi-Newton algorithm. In this, as in previous cases, convergence to three decimal places was achieved in less than thirty complete iterations. To minimize the possibility of encountering local minima, the initial estimates of H were taken from the OLS estimates of the transformed calibration, and x and y from the TORSCA solution configuration referred to earlier. The value of a was taken from model (4). Calibration statistics are given in table 3 for comparison, and figure 3 shows the three-dimensional configuration given by x, y, and H.

Several points can be made about these results. First, the improvement in fit of model (6) is substantial, but not surprising in view of the considerable increase in the number of calibrated constants. Only 63 out of 272 observations remain unexplained to within sampling error, which compares very favourably with the goodness-of-fit record for spatial interaction models reviewed by Openshaw (1976).

Several cities share the same locations in the horizontal plane in figure 3. This is to a degree attributable to the use of the negative exponential function, which reaches a finite maximum at zero distance. Since a negative power function tends to infinity as distance tends to zero, we would not expect coincident locations if such a function had been selected for f.

Seen as a map, figure 3 is a striking portrayal of news dominance and interaction between Canadian cities. Ottawa and Toronto occupy the centre, dominating the English-language news pattern. The three Western Ontario sinks, Hamilton, London, and Windsor, have been forced away from the centre because their level of interaction with the centre does not reflect their true geographic proximity to it; and rotated because Hamilton does not interact as much as its greater proximity would suggest.

The western cities are placed in their true locations with respect to each other, but again rotated, for the same reasons. Cities with a strong hierarchical relationship at

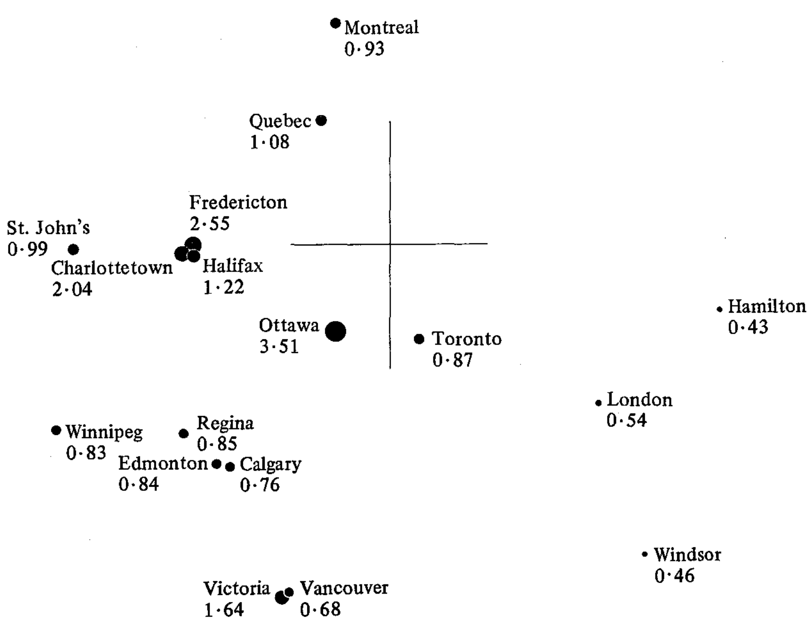

Figure 3. Solution space, model (6).

the regional level, as for example between Victoria and Vancouver, and Calgary and Edmonton, appear closer together than in reality. Montreal's isolation, except with respect to Quebec, is reflected in its peripheral location. And finally, the Maritime cities appear as a compact group, with St. John's in isolation.

Figure 3 has been constructed by calibration of the model under two major assumptions: that locations can be represented in a space of two dimensions, and distances measured using the Pythagorean metric. If the number of dimensions were decreased, locations would be forced to collapse onto a line, and the fit of the model would deteriorate. Conversely, a better fit could be achieved if three dimensions were allocated. In either case we suspect that the general preservation of regional relationships shown in figure 3 would continue. In three dimensions the Maritime and British Columbia regions would no longer be forced into coincidences.

Similarly, it may be that a better fit could be achieved by the use of some other metric. Figure 3 suggests that a better choice might be one which exaggerates short distances, and reduces long ones, relative to the Pythagorean lengths. But although this might give a better fit, it would reduce the usefulness of visual display, since distances would be less directly related to interaction.

The vertical, H dimension shows the position of a city in the competitive hierarchical system. It shows the propensity of a city to generate news, after the removal of the effects of population, and also to absorb it. Values are shown in figure 3 and table 2. They are consistent with the first calibration by transformation of model (3).

The most dominant cities, relative to their size, are the strictly political capitals: Ottawa, of course, together with Fredericton, Charlottetown, and Victoria. The large commercial centres, Toronto, Montreal, Vancouver, etc, are not dominant once population effects have been removed. London, Hamilton, and Windsor, each large in their own right, are nevertheless net absorbers rather than net generators of news.

Discussion

The hierarchical dimension H was introduced through a similarity with the "forcing function" explored by Tobler (1976). As noted earlier, the function could be regarded as generating a pattern of flows, or winds, in order to account for interaction asymmetries. Tobler suggested that such a pattern could be deduced from the equation,

$$C_i = \frac{1}{n-1} \sum_{j \neq i} \frac{I_{ij} - I_{ji}}{I_{ij} + I_{ji}} \frac{W_j}{D_{ij}} [x_j - x_i, y_j - y_i] , \qquad (7)$$

where

C_i is a vector describing the wind at i,

n is the number of places, and

W_j is an inverse function of D_{ij} acting as a weight,

which he showed to be consistent with interpretations of a number of spatial interaction models. Having found estimates of the wind field at each location, the entire pattern was found by spatial interpolation. Finally, integration of the wind pattern allows one to find the forcing function generating it.

The models proposed in this paper in effect model the forcing function directly, as a property of each place. A surface could then be interpolated, and an analogy to winds found by differentiation. However, for this particular data set it would be entirely inappropriate to assume that the height dimension has any degree of spatial continuity, or that it is in any sense differentiable. The dominance of Ottawa for example is lost within a few miles of its city limits. In summary, Tobler's approach is appropriate to a spatially continuous phenomenon, whereas the models developed in this paper correspond to a discrete view of space. The choice of viewpoint is as much determined by the pattern of zones inherent in the data as by the nature of the phenomena under study. If the zones are contiguous and exhaustive, a continuous view is appropriate, as it would have been had this study considered flows of news between provinces, and the domination of one province by another.

A second difficulty arises when the forcing function reaches a sharp local peak, as it does for example at Ottawa. A 'wind' map should show strong divergence around Ottawa, but no net flow at the centre. Equation (7) would give a correct value of zero for $\boldsymbol{C}$. But the interpolated map would be quite wrong, because it would also show values close to zero around Ottawa, instead of a strong divergent pattern. A more correct pattern could be found by modelling the forcing function directly and then differentiating.

Concluding remarks

The spatial interaction literature has paid remarkably little attention to the empirical meaning of the origin and destination terms which are invariably present. It is common to adopt surrogates, such as population, store-floor area, total income, employment, etc, or to incorporate marginal interaction totals, coupled with balancing factors. Yet the identification of the factors governing 'attraction' and 'emissivity' is potentially the most valuable contribution which interaction modelling can make.

No constraints have been placed on the models in this paper. A production constraint would have no meaning. But there is certainly a real constraint on the number of stories any newspaper can consume. Let K_j^* denote the observed marginal total $\sum_i I_{ij}^*$, the number of Canadian stories used during the study period. There seems no reason to require that the predicted interactions in this paper sum to the same precise total, since K_j^* is subject to sampling error. But if the model is to be used for prediction of situations in which origins have been added, deleted, or otherwise changed, then it would be appropriate to make a substitution in the model to obtain, in the case of equation (4),

$$I_{ij} = \frac{P_i H_i K_j^* \exp[-b_0 D_{ij}]}{\sum_i P_i H_i \exp[-b_0 D_{ij}]} .$$

Openshaw (1976) has argued that the goodness of fit of spatial interaction models has been comparatively neglected. The value of various measures of goodness of fit, notably R^2, has not been adequately explored; and perhaps a case can be made that the emphasis on the finer points of calibration found in many theoretical papers is not justified in the light of the poor fits actually obtained in many instances, or the strong structural errors of many models. The method used to compare observations and predictions in this paper against the assumed sampling distribution gives an entirely different perspective to that obtained from the usual error statistics.

Hierarchies have commonly been visualized as trees, with dominance as a binary property; a node dominates all those nodes in the branches which emanate from it, and no others. A number of methods have been devised for reducing interaction or flow matrices to binary data from which simple representations can be drawn, with attendant loss of information (Rouget, 1972; and see the review by Holmes and Haggett, 1977). But in most cases the set of places dominated by a node is fuzzy (Gale, 1976), and interaction is indirectly a measure of set membership. In this paper we have attempted to construct models which allow the analysis of hierarchical relationships without loss of information, and which preserve a strong visual component. The models have considerable predictive power, and many extensions and refinements are possible, particularly in the choice of metrics.

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