5. OHLIN, B. *Interregional and International Trade.* Cambridge: Harvard University Press, 1933.
6. OLSSON, G. *Distance and Human Interaction.* Philadelphia, Pa.: Regional Science Research Institute, 1965, p. 45.
7. PRAIS, S. J. "Econometric Research in International Trade: A Review," *Kyklos*, 15 (1962), 560–79.
8. SUITS, D. B. "The Use of Dummy Variables in Regression Equations," *Journal of the American Statistical Association*, 52 (1957), 548–51.

Local Structure in the Town Populations of Iowa

M. F. Goodchild

Attempts to identify a central place hierarchy in the observed town populations of an area have been largely unsuccessful. Studies which have claimed a measure of success have either assumed the population levels of the hierarchy, or at least assumed the number of levels. Berry et al. [1] identified a hierarchy in functions and functional units, and demonstrated a close relationship between population and number of functions, but reported no study of a population hierarchy.

The line of search for a population hierarchy must be to look for nodality in the town population frequency distribution. The number of nodes is unknown: any fitted model must therefore include it as a variable, together with the node populations. For such a search to have any degree of statistical significance, the study would require a large number of towns, and consequently would cover a large geographical area. Over such an area, social, economic and technological factors might vary considerably, changing the absolute levels, if not the form, of any central place town population hierarchy. Berry et al. [1] have suggested that this is the reason for the apparent lack of success of such studies.

This study considers the hypothesis that, although the absolute levels of the hierarchy may vary spatially, an analytic relationship is maintained between them. Specifically, two tests concerning the differences and ratios of population are discussed.

Tests

The tests define local scale by considering relationships only between nearest neighbors. The first test involves taking population differences between neighbors, the second, taking the ratios of the populations.

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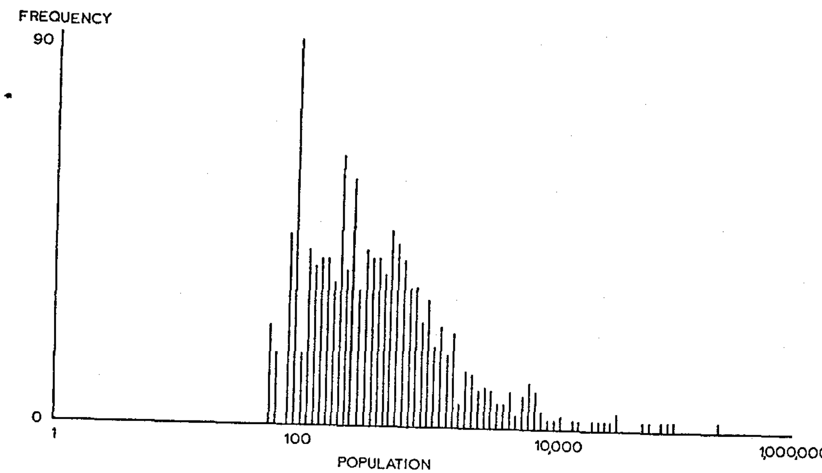

FIG. 1. Frequencies of Town Populations.

Figure 1 is a frequency plot of town size for the study area, the State of Iowa, with 1170 towns of population 60 or over. The x-axis is defined by the transformation

$$i = 20.0 \log_{10}\text{abs}(P + 1)$$

where P is the population and the integer values of i form the 130 group boundaries.

Figure 2 shows the frequency plot using the same x-axis, of the absolute population difference between each town and its nearest neighbor. In the ideal Central Place environment, this histogram for an n-level hierarchy would consist of a discrete distribution of $n(n-1)/2$ peaks. Subjective attempts might be made to identify these peaks.

There are, however, more objective possibilities. Figure 3 is a histogram of population differences between all possible pairs of towns in the state. The hypothesis that nearest neighbors do not have random population differences was tested using this curve by regarding the observed differences as a sample.

Since Figure 3 is not a smooth, analytic function, the statistics of a sample drawn from it are not easy to derive. A numerical method, therefore, was used. Each town was allocated a neighbor on a random basis from the set of 1170 towns, the entire procedure being repeated 40 times. The actual set of population differences was then compared with the 40 generated sets.

The preferable method of comparison would involve a statistic

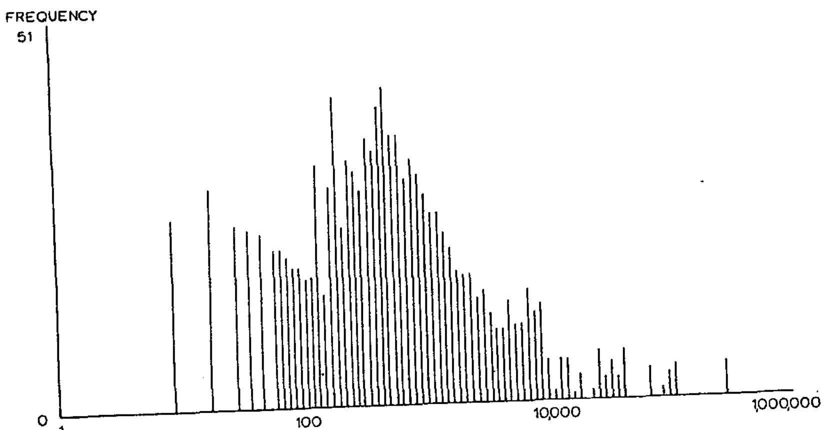

FIG. 2. Frequencies of All Possible Population Differences.

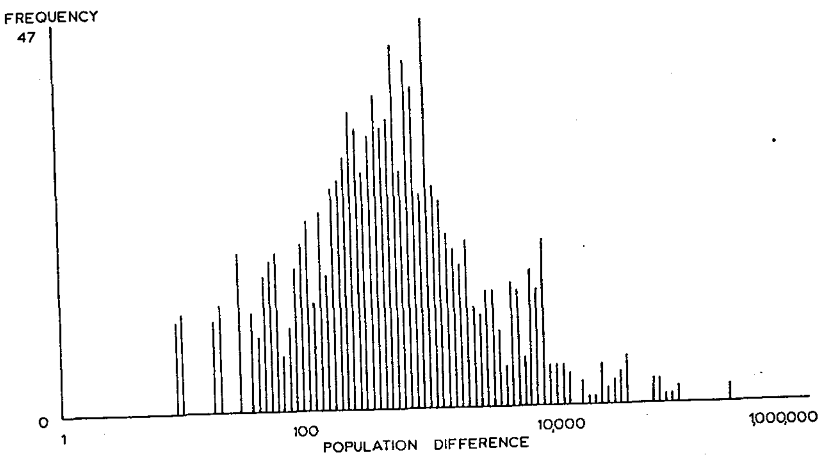

FIG. 3. Frequencies of Observed Population Differences between Nearest Neighbors.

of goodness of fit to the ideal, $n(n - 1)/2$ node distribution. The positions of the nodes, and n, are all unknown but could be derived by regression: however, the necessary regression analysis is extremely complex and insensitive. Statistics were used therefore, with the aim of demonstrating an indefinite local structure without regard to its actual form. Two statistics were used: the mean linear, and mean square deviation between Figures 2 and 3. These were compared with the same statistics derived after random allocations of nearest neighbors. The use of the two statistics gives a crude test both of

TABLE 1

SIMULATED VALUES AND FREQUENCIES

Constant Difference Test				Constant Ratio Test			
Means		Variances		Means		Variances	
13.0	0	13.0	2	5.0	2	1.0	14
14.0	31	14.0	10	6.0	8	2.0	21
15.0	9	15.0	16	7.0	25	3.0	4
16.0	0	16.0	12	8.0	5	4.0	1
17.0		17.0	0	9.0		5.0	
		18.0					
Observed Values:							
18.8		18.0		16.3		10.6	

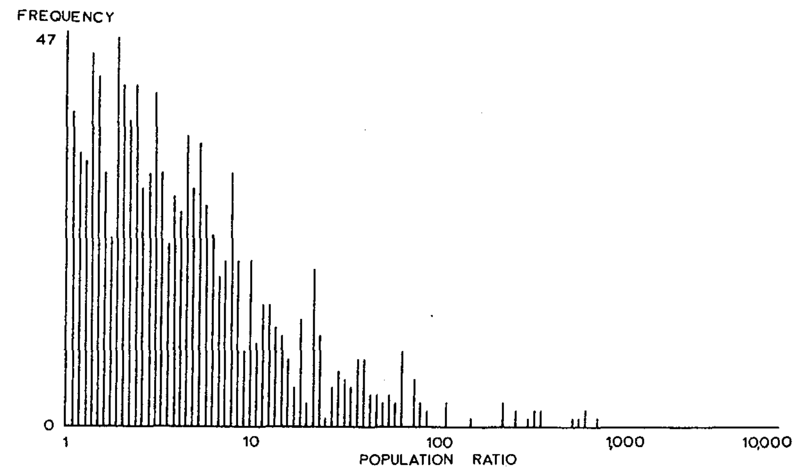

FIG. 4. Frequencies of Observed Population Ratios.

divergence and of nodality. In Table 1 there are also the results for the second, constant ratio test based on Figure 4 in which the x-axis is defined as

$$\mathrm{abs}(30.0 \log_{10} P_1/P_2).$$

In every case (Table 1) the statistics for the real town structure are highly significant, and prompt the conclusion that a local structure exists which is not apparent at the state-wide scale, and which is compatible with some degree of nodal, hierarchical structure. Further, the second test is the more successful.

The influence of local structure may be expected to decrease with distance between pairs of towns. To investigate this effect, the process was repeated by using a scanning radius about each centre rather than by the selection of only the nearest neighbor. As the scanning radius increased, the size of the sample increased, so that at a 10 mile scanning radius, the local structure had fallen to a point where mean linear differences were no longer significantly greater than those randomly generated. The other statistics however, were still highly significant. Scanning radii were not increased further since computer time for random generation was becoming prohibitive.

LITERATURE CITED

1. BERRY, B. J. L., H. G. BARNUM, and R. J. TENNANT, "Retail Location and Consumer Behavior," *Papers and Proc.*, Regional Science Assoc., 9 (1962), 65–106.