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Lakes on Fractal Surfaces: A Null Hypothesis for Lake-Rich Landscapes[1]

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A class of stochastic processes known as fractional Brownian motion (fBm) provides strikingly realistic simulations of certain types of terrain, particularly those which appear to be unmodified by geomorphological and geological processes. In addition to their less serious applications in video games and science fiction movies, fractal terrain simulations have proven useful in a number of areas of spatial analysis. For example, they can provide sample data sets for testing the efficiency of data structures and algorithms designed for topographic applications. Previous work has shown that stream networks simulated on fBm surfaces show the same deviations from accepted theories of channel network topology as do real stream networks, implying that such deviations originate in the geometrical constraints of packing channels onto surfaces, rather than from geological or other environmental controls. In effect, this work demonstrates the usefulness of fBm as a null hypothesis for terrain. One difficulty, however, stems from the abundant pits which occur in the simulations, because peaks and pits are equally likely. Flooding of pits on fBm surfaces was simulated to obtain lakes. Lake-rich stream networks were extracted and represented with a suitable integer code. The relative frequencies of various network topologies and groups of topologies were compared to known characteristics of channel networks on real lake-rich landscapes. 'Lake-string' topologies are significantly less abundant than in glaciated landscapes. Lake areas show good fits to hyperbolic distributions, but lake in-degrees do not fit the negative binomial model. fBm surfaces are appropriate null hypotheses of scale-free, lake-rich landscapes.

KEY WORDS: Fractal, fractional Brownian motion, stochastic process, lake, topological randomness.

INTRODUCTION

The notion of noninteger dimension was first associated with the geometry of natural landscape features by Mandelbrot in a paper (Mandelbrot, 1967) on the lengths of coastlines. Since that time terrain has become one of the more important areas of application of the rapidly growing fractal literature, and striking similarities between simulations of fractal surfaces, achieved by realizing a stochastic process known as fractional Brownian motion (fBm), and certain types

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of physical landscape (see the illustrations in Mandelbrot, 1977, 1982), are widely accepted.

In this paper, the usefulness of fBm simulations as the basis for statistical analyses of lake-rich landscapes is investigated. Simulations can be regarded as appropriate null hypotheses for terrain, because the simulations lack any evidence of modification by geomorphic processes. They therefore represent a point of reference or norm against which to compare results of statistical analyses of real landscapes. The application of fractal concepts to terrain are reviewed first. A series of algorithms for extracting lakes and associated hydrologic networks from fBm surfaces, and the results of analyses carried out on these surfaces are described. These results are compared to those reported in the literature for real lake-rich landscapes. The final section discusses the significant differences which are observed, and interprets them in the light of the processes which have shaped real landscapes.

FRACTALS AND TERRAIN SIMULATION

Although fractal simulations may be visually satisfying when used in science fiction movies and video games, geomorphologists tend to have little difficulty in distinguishing between real terrain surfaces and fractal simulations. Mark and Aronson (1984) reported an experiment in which subjects were presented with pairs of computer-generated perspective views, one member of each pair being a real surface and the other a fractal simulation with the same fractional dimension. Subjects with little training in geomorphology rarely reported difficulty in identifying the simulation in each pair.

Several properties of fractal simulations make them unrealistic and therefore readily distinguished from real terrain. First, fractal surfaces are by definition self-affine, being generated by a stochastic process with no inherent scale. This means that a suitably enlarged portion of the surface has the same visual appearance as the surface as a whole, or, more specifically, that it would be impossible to reject a null hypothesis that the part and the whole were generated by the same stochastic process (we use the term self-affine rather than self-similar because the required vertical and horizontal scalings are not necessarily equal). Visually, self-affinity implies that the landscape possesses no cues as to its scale. On the other hand, geomorphic processes tend to operate nonuniformly across scales; for example, a glaciated landscape may have a relatively smooth appearance over distances of a meter or less, but may be rugged over distances of several kilometers. Thus landscapes which have been modified by erosion tend to possess readily identified cues to scale. In contrast, fractal landscape simulations have a raw appearance suggestive of lunar topography.

Second, fractal simulations are invertible, with similar appearance from above and below; peaks and pits occur on the surface with equal likelihood.

Hydrologically, such surfaces would be either lake-rich or karstic, again limiting the acceptability of the simulation.

Goodchild (1982), Mark and Aronson (1984), Brown and Scholz (1985), and Burrough (1981) have reported numerical rather than visual evaluations of the similarity between real topography and the self-affine fractal model at various scales. In general, they conclude that the fractional dimension is a useful parameter of the behavior of measures of terrain features (for example, contour and shoreline lengths) over different scales (Goodchild, 1980). However, dimension remains constant only over limited ranges of scale and may change rapidly (Mark and Aronson, 1984).

Despite these limitations, several uses have been found for fractal terrain simulations. The variogram of fBm is a simple power function, the power being determined by a parameter H, $0 < H < 1$. By varying H, one can create a range of surfaces with different degrees of ruggedness from white noise ($H = 0$) to a plane ($H = 1$) (Fig. 1). Furthermore, contours of surfaces should be fractals with fractional dimension $D = 2 - H$. Suitable terrain simulations have been used to test the efficiency of alternatives for spatial data storage, and the effectiveness of surface interpolation algorithms.

NULL HYPOTHESIS TERRAIN

As already noted, fBm is unlikely to provide an adequate simulation of an eroded terrain because its property of self-affinity is inconsistent with the scale-specific effects of most geomorphic processes. Any real terrain can be viewed as the result of processes operating on some pre-existing form, so that a complete understanding of its present appearance requires models both of the process and of the prior form. Davis (1899) proposed an uplifted block as the initial form at the beginning of each cycle of erosion, whereas Sprunt (1972) and Hugus and Mark (1984, 1985) used tilted planes. Both the block and the tilted plane are regular surfaces, so, in order to simulate an irregular outcome, an element of randomness must be included in the simulated process. Sprunt (1972) introduced a random precipitation input, whereas Hugus and Mark (1984, 1985) included a random factor in the process–response part of the model. The problem is avoided if the initial surface is irregular, and this, together with the raw, unmodified appearance of fBm surfaces makes them particularly attractive as starting points in process simulation. Kirkby (1986) used an initial surface of fBm; Craig (1980) used a surface of independent elevations ($H = 0$) but assumed planar facets between them.

In the hypothesis testing tradition in statistics, a null hypothesis is a stochastic process which is thought to differ from reality only in the absence of the effect of interest; rejection of the null hypothesis then confirms the presence of the effect. A suitable null hypothesis for terrain might represent the expected

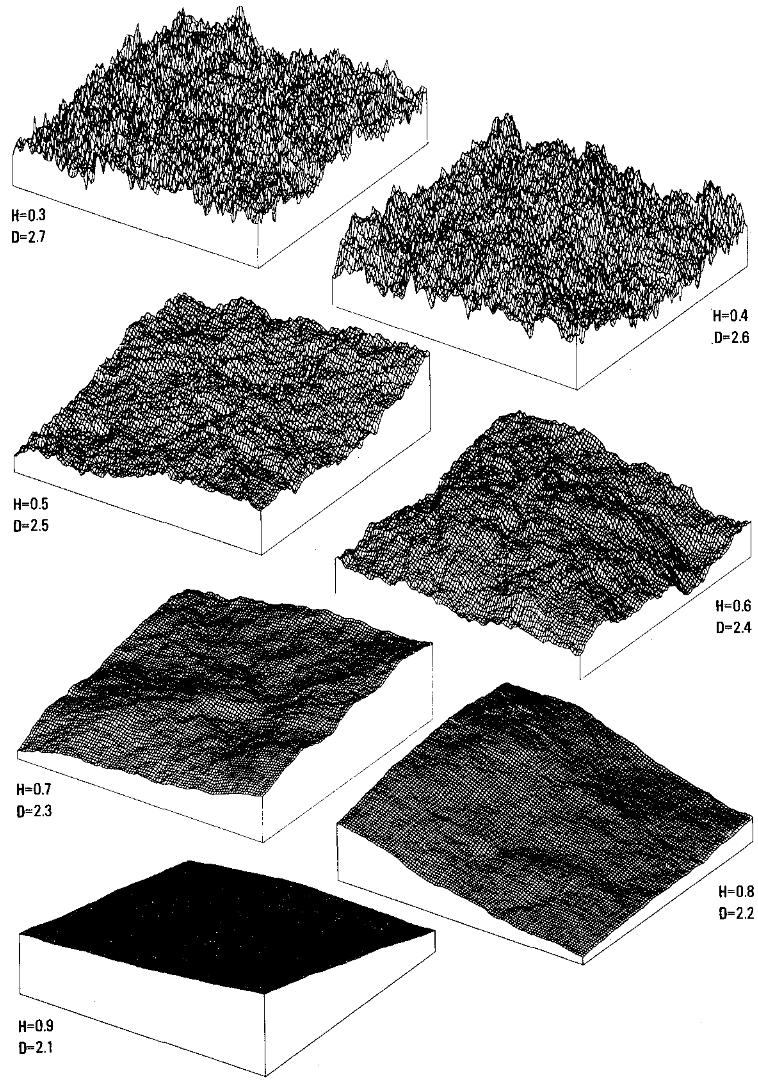

Fig. 1. Self-affine surfaces generated by the fractional Brownian process with variable parameter H.

appearance of terrain in the absence of a particular erosion process; rejection could then be taken to confirm the action of the process.

Goodchild et al. (1985) have used this approach in connection with channel networks. Shreve (1966, 1967) proposed that the observed frequencies of different channel network topologies could be explained by assuming that all possible, distinct topologies were equally likely, and early tests showed good fits to the model. However, now abundant evidence of significant departures from the model exists (for review, Abrahams, 1984), particularly in greater than expected probabilities of "fishbone" topologies, having one major central channel and numerous short feeders.

By simulating channel networks on fBm surfaces, Goodchild et al. (1985) were able to show that similar overabundances of fishbone topologies occur on unmodified terrain. In effect, topological randomness is not an appropriate null hypothesis for channel networks because it removes not only the effects of geomorphic processes, but also the geometric constraints imposed when basins must be packed together on a surface. On the other hand, fBm networks differ from real ones only in the absence of geomorphic processes, and acceptance of an fBm null hypothesis therefore implies that no significant constraints of a geologic or geomorphic nature have affected channel development.

Unfortunately the abundance of pits on fBm surfaces casts doubt on the generality of these results, and on the suitability of fBm as a general null hypothesis for terrain. However, if pits are assumed to fill as lakes, fBm might offer a useful model for lake-rich landscapes and their associated hydrologic networks. Mark and Goodchild (1982), Mark (1983), and Mark and Averack (1984) have analyzed various aspects of lake-rich channel networks, and have interpreted the differences observed between networks in different regions in terms of the respective regional geology and geomorphology. However what statistical properties would have been expected in the absence of geological and geomorphic constraints, and what differences were significant is not clear, because no suitable null hypothesis is available. The remainder of the paper examines the use of fBm as a suitable null hypothesis.

EXTRACTION OF LAKE-RICH CHANNEL NETWORKS

Realizations of fBm were generated as digital elevation models for values of H between 0.3 and 0.7, using the methods described in Mandelbrot (1975). Square arrays of 256×256 were used, one realization being generated for each of 0.3, 0.4, 0.5, and 0.6, and three at 0.7, because this value of H has the greatest similarity to real terrain.

Pits were then flooded to form lakes. The terrain was assumed to be lower everywhere outside the array, so lakes were not allowed to form in the first and last rows and columns. Lakes were initiated at pits, defined as cells having no strictly lower neighbors (the four Rook's case neighbors), and were allowed to flood and expand until an outlet was reached. A cell was defined as a lake outlet if it lay in a lake, and if one of its four neighbors was not in the lake and was lower in elevation. Note that this neighbor might be in another lake if it was not also the outlet of that other lake; if it was the outlet, the two lakes would coalesce and continue to expand until a valid outlet was found.

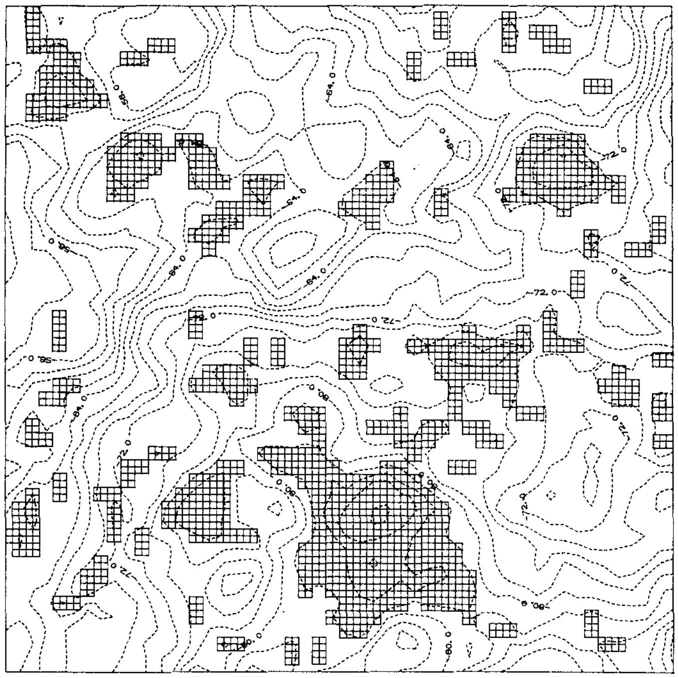

Fig. 2. A 50 × 50 subset of the H = 0.6 surface flooded to show lakes.

Results for a 50 × 50 area of the H = 0.6 realization are illustrated (Fig. 2). As expected, lakes are more abundant on more rugged surfaces of small H.

To complete hydrologic networks, flows were simulated in channels and into and out of lakes, and the results were represented using the coding scheme for lake-rich networks developed by Mark and Goodchild (1982). A cell was assumed to flow to the lowest of its four neighbors provided at least one neighbor was strictly lower; if not, the cell would already have been flooded and be part of a lake. Cells were coded 1, 2, 3, or 4 depending on the direction of flow and coded by sequential lake number if part of a lake. A search routine was then used to find each simple channel network tree by working upstream from every lake, or from the edge in the case of channels which flowed off the array. Finally, every simple network was merged to form a set of lake-rich network trees rooted at the array edge.

This method of network simulation using four neighbors may produce junctions formed by three inflowing streams and one outflowing stream, whereas the coding scheme and the bulk of the literature on channel networks assume that all junctions have only two inflowing streams. The search routine deals with this problem by breaking each such four-valent junction into two three-valent junctions (Goodchild et al., 1985). In order to avoid directional bias, the search generates the two alternative topologies with equal probability; in approximately half of the cases the streams incident from the left and center will join upstream of the one incident from the right, and in the remaining cases the one incident from the right will join upstream of the one from the left.

Other simulations of drainage networks on terrain models have used somewhat different approaches to network extraction. Yuan and Vanderpool (1986) used eight rather than four neighbors for each cell, as did O'Callaghan and Mark (1984). This method allows more accurate representation of flow direction, but has the disadvantage of allowing up to seven inflowing streams to join at each junction, and is therefore less suitable for studies of network topology. O'Callaghan and Mark (1984) also introduced a constraint on stream formation, requiring a minimum number T of upslope cells with overland flow before a channel was allowed to develop. The method used here allows streams to be initiated in cells immediately adjacent to each watershed, i.e., $T = 0$. In further research, the effects of nonzero T on the relative frequencies of different network topologies will be investigated.

The coding scheme uses a string of integers to represent each complete network. The network is scanned from the root, working first up the left side of the first channel link, turning left at each junction or lake, and then around the end of each source. A junction is coded as a 0 when first encountered, a terminal branch or source as a 1, and a lake as $2 + n$ where n is the number of inlets. This coding gives a unique representation to every possible topological arrangement of stream links, junctions, and lakes, provided the network is a simple tree containing no circuits. An example coding is shown in (Fig. 3).

In addition, the data set extracted from each fBm surface included the area and perimeter of each lake and the length of each stream link. Although lakes were extracted from the full 256 × 256 arrays, networks were obtained from 100 × 100 subsets because of limited central memory.

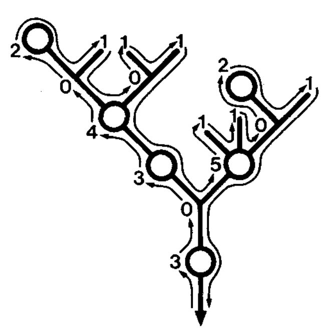

Fig. 3. Representation of lake-rich networks by integer strings: circles denote lakes.

RESULTS

Results obtained from these simulations will be discussed in two sections, the first dealing with populations of lakes and the second with lake-rich drainage networks. In each case, the objective is to compare populations derived by simulation with results already reported in the literature for real terrains. Simulations will play the role of a null hypothesis, representing what would be expected by chance on a landscape free of constraints. Significant differences then can be ascribed to the action of geologic or geomorphic constraints.

Lake Populations

Korčák (1940) reported that lake areas tend to follow a hyperbolic or Pareto distribution, with a cumulative probability distribution in the form of a simple power law

$$\Pr(A > a) = k\, a^{-b} \tag{1}$$

where k and b are constants and $\Pr(A > a)$ is the probability that a randomly chosen lake has an area A greater than some area a. Fréchet (1941) regressed Korčák's data and obtained 0.48 for b. Hyperbolic distributions are typical of features of self-affine, fractal landscapes because they lack scale-specific parameters. Mandelbrot (1982) argued that areas of islands formed by flooding fractal terrains should have hyperbolic distributions, and that parameter b should be equal to $D/2$ where D is the fractional dimension of contour lines on the surface and is constrained to lie between 1 and 2. However, the definitions of lakes and islands are not symmetric. Islands result from the flooding of the entire surface to a fixed level, whereas lakes result from flooding of pits to the point of overflow. It follows that inversion of the surface does not convert lakes to islands, and vice versa, in any simple fashion. Thus, despite his conclusions for islands, Mandelbrot was unable to show that the fractal model leads to hyperbolic lake area distributions, even though they are observed widely.

Pareto fits (Table 1), in the form of correlations between log(area) and log(rank), where rank is defined as the position of each lake when sorted from largest to smallest, have been calculated and the goodness of fit (Fig. 4) plotted.

Clearly (Table 1 and Fig. 4), lake areas on fBm surfaces fit the Pareto model well. On the other hand, estimates of b do not behave consistently with the expectation that $b = D/2$ and $D = 2 - H$, and all estimates of D are outside the feasible range. Instead, support is seen for the proposition that b is constant and independent of H, especially if large values for $H = 0.7$ are discounted because of small sample sizes; also, r^2 is generally smaller for $H = 0.7$.

The effect of a large b is to shorten the tail of the Pareto distribution, making large lakes less likely; possibly the method of simulation has had this

Table 1. The Pareto Distribution Fitted to Lake Area Populations

H	Sample size	Mean area	Standard deviation	r^2	b	$D = 2b$
0.3	481	8.9	29.0	0.974	1.02	2.04
0.4	485	6.9	24.9	0.976	1.18	2.36
0.5	358	8.3	36.6	0.970	1.11	2.22
0.6	283	5.8	11.6	0.964	1.18	2.36
0.7*a*	35	4.0	3.4	0.955	1.37	2.74
0.7*b*	120	6.5	16.4	0.939	1.09	2.18
0.7*c*	109	4.5	7.8	0.932	1.31	2.62

effect. Consider the 100 × 100 array used here as if it were embedded in the center of a much larger array. If the larger array were flooded, almost certainly some of the lakes which would develop from pits outside the 100 × 100 area would flood onto it, increasing the proportion of cells covered by lakes. In effect, use of a limited area for simulation has caused an underrepresentation, particularly of large lakes, and thus inflated the estimate of b.

However, although this process may account for infeasible values of D, the same argument can be made of any analysis of lake areas because the results will always be affected by the choice of study area. Thus, although lakes on fBm surfaces appear to have Pareto distributions consistently with observations of real lakes, the associated Pareto parameters are unlikely to be shown to be

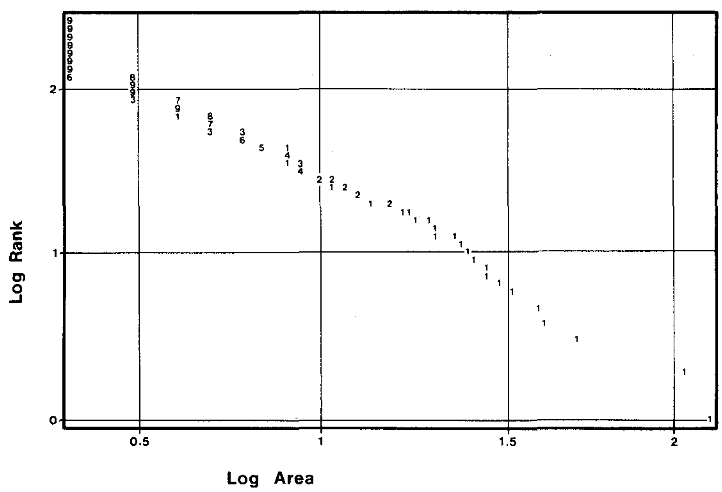

Fig. 4. Scattergram of rank against area for lakes on the $H = 0.6$ surface, showing goodness of fit to the Pareto or hyperbolic distribution.

Table 2. Regressions of Log(Perimeter) Against Log(Area) for fBm Lakes

H	n	r^2	Slope	D	$2 - H$
0.3	3054	0.988	0.72	1.44*	1.7
0.4	2896	0.988	0.69	1.38*	1.6
0.5	2396	0.986	0.67	1.34*	1.5
0.6	1616	0.986	0.64	1.28*	1.4
0.7*a*	378	0.983	0.64	1.28*	1.3
0.7*b*	723	0.986	0.64	1.28*	1.3
0.7*c*	680	0.988	0.64	1.28*	1.3

NOTE: * Different from $2 - H$ at .05 level of significance.

consistent with either Mandelbrot's expectations or Korčák's results, and, because of sampling problems, estimates of the Pareto parameter will likely be biased.

A population of lakes with smooth boundaries, for example circular lakes, would show a relationship between perimeter and area where perimeter varies as area to the one-half power. For lakes on fractal surfaces, a lake of four times the area would have a perimeter rather longer than twice the length, because with a constant spatial resolution the larger lake would tend to show more boundary detail. More precisely, the perimeter should vary as area to the power $D/2$.

Results of regressing log(perimeter) against log(area) for populations of lakes on each of the full 256×256 fBm realizations (Table 2) show good fits to the simple power law, and slopes decrease monotonically with H as expected. But the results conflict with the expected relationship between slope and H.

Network Topologies

Networks found on simulated terrain are compared here with results which have been reported for real lake-rich networks. The number of possible topologies of channel networks with a given number of streams is large for lake-free basins and much larger for lake-rich basins. Consider, for example, a simple lake-free network with five first-order (source) streams and four three-valent junctions. Lake-free basins are represented by binary strings of 0s and 1s, and a basin with n ones must have exactly $n - 1$ zeros. One possible topology would be represented by the string 000011111, and the total number of topologically distinct basins is given by

$$P = (2n - 1)!/[(n - 1)!\, n!\, (2n - 1)] \quad (2)$$

or 14. By comparison, 15 topologically distinct ways exist in which two lakes and one source stream can be combined (Mark and Goodchild, 1982).

The commonly used model of topological randomness of lake-free basins proposes that all possible and topologically distinct arrangements of a given number of source streams are equally likely to occur. Thus a 0 or 1 appears at any specified position in the binary string representing an infinitely large network with probability $\frac{1}{2}$ independent of the pattern anywhere else in the sequence. Mark and Goodchild (1982) suggest that one suitable interpretation of topological randomness in the case of lake-rich basins would be to propose that the various integer elements similarly occur in a string with probability independent of position. The probability of any given string or basin then could be computed from the product of the probabilities of occurrence of each of its elements.

Consider, for example, the subbasin represented by the string 021 consisting of one three-valent junction, one lake with no inlets, and one source stream. According to this definition of topological randomness, this subbasin should occur with the same frequency as 012. Furthermore, the expected frequency of occurrence of all other feasible three-element strings may be computed given the proportions of 0s, 1s, 2s, and 4s observed in the entire network.

The frequencies of all five-element subbasins found on the $H = 0.3$ surface are shown (Table 3). The set of strings observed is a small subset of all possible five-element strings. In particular, all strings which would result by replacing a source stream by a lake with no inlets (a 1 by a 2) are missing. In these simulations with $T = 0$, 2s were rarely found but would likely become much more common if T were increased.

Although the frequencies are small, the proposed definition of topological randomness appears to be consistent with the subbasins observed on the fBm

Table 3. Observed and Expected Frequencies of All Five-Element Subbasins on the $H = 0.3$ Surface

String	Observed	Expected
00111	31	33.22
01011	39	33.22
01411	9	3.85
04111	3	3.85
41011	4	3.85
40111	1	3.85
04311	1	0.05
35111	2	0.49
44111	1	0.44
61111	21	30.02
62111	1	0.14

surfaces. Mark and Goodchild (1982) concluded that the frequencies of sub-basins in a large lake-rich terrain in northern Ontario also were consistent.

A more testable, related proposition is that lake elements (2s and larger) occur independently of nonlake elements (0s and 1s) in integer strings. Mark and Goodchild (1982) rejected this proposition in their test basin, concluding that geologic controls had created lake-rich and lake-poor regions. For fBm surfaces, the null hypothesis of independence is rejected at the .05 level for H = 0.7a, 0.6, 0.5, 0.4, and 0.3. In all cases, the tendency is for positive correlation; lakes are more likely followed by lakes, and nonlake elements by nonlake elements. This is the same effect observed for the test basin in northern Ontario, and so suggests that the conclusion regarding lake-poor and lake-rich regions may be less valid than previously thought. The effect of packing lake-rich basins onto a surface appears to constrain basin development geometrically such that lakes are more likely to occur immediately adjacent to other lakes. In effect, positive correlations are expected under the null hypothesis of a random, unmodified surface and therefore are not due necessarily to the presence of geologic constraints.

Mark and Averack (1984) classified each link in lake-rich basins according to the nature of upstream and downstream nodes. The upstream node could be a stream source (EP), a lake with no inlets (EL), a stream junction (IP), or a lake with inlets (IL). The downstream node could be a lake (L) or a junction (P). Topological randomness could be tested then in terms of the independence of upstream and downstream node classifications in a 4×2 contingency table. Links which joined lake with lake and nonlake with nonlake were found to be more frequent than expected under topological randomness.

Independence was rejected for all fBm simulations. The link types which occur more frequently than expected are IPP (5 of 7 surfaces), ILL (7 of 7), EPP (4 of 7), EPL (5 of 7), and ELL (4 of 7). Again, this suggests that results obtained by Mark and Averack (1984) may be more consistent with the model of topological randomness than previously thought.

Mark (1983) analyzed the distribution of the number of inlet streams to lakes (the in-degree). The number of inlets would be expected to respond to lake size, being greater for lakes with long perimeters. Perimeter length is not well-defined because of the fractal nature of the shoreline, but can be estimated using the square root of area as a surrogate. Mark found that Canada's largest lakes show a power law relationship between number of inlets and area, with a slope of $\frac{1}{2}$, suggesting that the number of inlets would respond to perimeter length and that perimeter length would vary with the square root of area. Similar analysis of the fBm simulations (Table 4) showed strong correlations when log(in-degree) was regressed against log(area). However, slopes were in all cases greater than $\frac{1}{2}$, which is consistent with a dependence of number of inlets on perimeter, but a perimeter/area relationship with a power greater than $\frac{1}{2}$, as

Table 4. Regressions of Log(in-Degree) with Log(Area) for fBm Simulations

H	r^2	Slope
0.3	.787	.862
0.4	.760	.835
0.5	.771	.781
0.6	.792	.812
0.7*a*	.711	.773
0.7*b*	.751	.830
0.7*c*	.734	.784

discussed above. Mark (1983, Fig. 1) would also support a slope of greater than $\frac{1}{2}$.

Mark (1983) also analyzed the frequency distribution of lake in-degrees, proposing a modified negative binomial model which was found to fit test populations well. The model combined two populations of lakes, those with one inlet corresponding to the strings of small lakes or stream widenings commonly found on glaciated landscapes, and larger lakes whose numbers of inlets might be expected to depend on some measure of lake size. Mark proposed a negative binomial model for the second class, arguing that the first type would increase the probability of lakes with one inlet above the negative binomial expectation.

Table 5. Fit Between Lake in-Degrees and Negative Binomial Model for $H = 0.7b$

In-degree	Observed	Expected
0	1	26
1	11	16
2	13	12
3	36	9
4	12	8
5	14	7
6	4	6
7	5	5
8	4	4
9	3	4
10	4	3
—	—	—
—	—	—

Note: Negative binomial parameters: $p = .104$, $x = .684$, and $n = 120$.

The result when the two classes are combined is a three-parameter discrete distribution.

As might be expected, fBm simulations show no evidence for added 1s. This is consistent with the geomorphologic interpretation, which ascribes added 1s to the process of glaciation. Even the two-parameter negative binomial model fits poorly; fBm distributions typically show too few lakes with few inlets, and too many lakes with large numbers of inlets (Table 5).

Inflowing streams occur in a substantial proportion of all of the cells forming the perimeter of each lake in fBm simulations. The spatial resolution of the entire simulated drainage system is determined by cell size, whereas on topographic maps relatively complex rules are used to identify and generalize lake and stream features. Reasonably, a model developed for good spatial resolution of topographic maps would fit poorly when applied to a discrete cellular system. On the other hand, one might expect simulations with nonzero T, that is, where a fixed number of upslope cells are required before a channel develops, to behave differently.

CONCLUSIONS

A geomorphologist wishing to ascribe a particular aspect of the physical landscape to some geologic or geomorphic cause must have a well-conceived notion of how the landscape would have appeared had the cause been absent; otherwise it will be impossible to ascribe cause with any degree of certainty. This paper has proposed that fBm simulations can provide appropriate null hypotheses for landscape, because of their lack of scale dependence and their visual appearance, which suggests rawness and lack of modification. Because fBm surfaces contain equal numbers of pits and peaks, they are most suitable as null hypotheses for lake-rich landscapes.

Several conclusions can be drawn from a comparison of analyses of simulated surfaces and results in the literature obtained from observations of real lake distributions. The simulations confirm that lake areas can be expected to have Pareto distributions in the absence of geologic control, but that the Pareto parameter is not consistent with expectations drawn from either Korčák (1940) or Mandelbrot (1982). The relationship between lake area and perimeter is consistent with the fractal model, but the expected relationship between regression slopes and the fBm parameter H is not confirmed. Kent and Wong (1982) examined the frequency distribution of area and the relationship between area and perimeter for a sample of Canadian Shield lakes, and found similar inconsistency between the fractal dimension implied by the Pareto distribution and that obtained from the area/perimeter relationship.

Lake-rich channel networks extracted from fBm simulations support the model of topological randomness proposed by Mark and Goodchild (1982), but packing of subbasins onto the terrain surface introduces biases which are consistent with those observed in real networks, suggesting that previous empirical conclusions may not be as strong as previously thought. In particular, lake and nonlake elements both show positive correlations, although the degree of correlation is much less than observed for real networks. Further work is needed to determine the effects of requiring a fixed number of upslope cells before channels are allowed to develop.

In conclusion, fBm surfaces offer a useful null hypothesis for lake-rich terrains. The method of drainage basin extraction used in this paper is not an accurate reflection of real processes, but it generates networks whose topological properties closely resemble those observed on real terrains.

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