

Chapter Three: Reading Notes and Assignment

My Intro: Dispersive and non-Dispersive Waves

Waves transfer energy through space, and are characterized by their wavelength (λ , [m]) and frequency (ν , [s^{-1}]). The speed of a wave is just the wavelength times the frequency ($c = \lambda \nu$ [$m s^{-1}$]). There are many kinds of waves. We are all familiar with sound waves, electromagnetic waves, and water waves. Waves are either dispersive or non-dispersive. In non-dispersive waves, the frequency and wavelength are inversely proportional, which results in fixed wave speeds. Electromagnetic radiation is a very good example. The speed of light (c^* , $3 \times 10^8 m s^{-1}$) is fixed and λ and ν have an inverse relationship:

$$\nu = \frac{c^*}{\lambda}, \lambda = \frac{c^*}{\nu}$$

This means that all electromagnetic radiation travels at the same speed, and that the energy does not spread out over time (disperse). This strongly affects how we see the world. Imagine if we received the blue light from a lit match before the red. The universe would rapidly decay into an information jungle.

Most atmospheric waves are dispersive – their frequency and wavelengths are **not** determined by a simple inverse relationship – and the waves tend to travel at different speeds. There are many different atmospheric wave type types:

In the tropics and sub-tropics¹, westward propagating equatorial Rossby and easterly waves and eastward propagating Kelvin waves and Madden-Julian oscillations help create patterns of divergence and convergence – creating rainfall.

In the mid-latitudes eastward propagating Rossby waves² contribute to baroclinic disturbances and storm events.

Mesoscale mountain waves³ (internal gravity/buoyancy): These are created by wind flowing across terrain under stable conditions. The restorative force of the ATM creates oscillations. Models of these waves can be used to help estimate rainfall over mountainous areas.

¹ See http://www.cdc.noaa.gov/map/clim/olr_modes/ for some examples of equatorial waves-Smith linear mountain wave model.

² See http://www.cdc.noaa.gov/map/images/rnl/500z_07a.rnl.anim.html

³ See http://www.nrlmry.navy.mil/~doyle/linear_model/linear.html for examples of the Doyle-Smith linear mountain wave model

3.1 Photons and Minority Constituents

Very small amounts of radiatively active gasses influence strongly the temperature and temperature distribution of the planet. Radiative transfer models can be used to describe this process.

3.2 The nature of Electromagnetic Radiation

At the quantum level, light is quantized in packets, called photons. The energy of a photon (E_v) is proportional to its wavelength ($E_v=h\nu$, h is Planck's constant).

99% of the sun's energy comes from the visible (0.4– 0.75 μm , μm = micrometer= 10^{-6} m) and near infrared (0.75– 5 μm) portions of the electro-magnetic spectrum.

Earth's energy emission is almost all thermal with wavelengths between 4 and 200 μm .

3.3 Description of Radiative Energy

When radiation hits something it can be transmitted, scattered or absorbed and re-emitted.

transmitted	Passes through substance unchanged (i.e. visible light through O_2)
scattered	Direction (but not wavelength) of photons altered (i.e. scattering of pure liquid H_2O)
absorbed & re-emitted	Photons absorbed by molecule or atom, and are re-emitted at different wavelengths. Ex: Absorption of energy by atmospheric water vapor.

3.4 Planck's law of blackbody radiation

Planck's law predicts the emitted energy of a blackbody at a specific temperature

$$B_v(T) = \frac{2\hbar v^3}{c^2} \frac{1}{(e^{h\nu/kT} - 1)}$$

where h is Planck's constant and k is Boltmann's constant. Integrating this equation over all possible frequencies (ν) yields the Stefan-Boltzmann law:

$$\pi \int_0^{\infty} B_v(T) d\nu = \sigma T^4$$

One corollary of Planck's law is **Wien's law of displacement: the wavelength of maximum emission is inversely proportional to temperature.** Higher temperature objects emit more energy at shorter wavelengths.

3.4 Selective Absorption and Emission by Atmospheric Gases

The energy of an atom or molecule only occurs at discrete levels. The total energy of an atom or molecule is the combination of translational, rotational, vibrational and electronic components.

$$E_{\text{total}} = E_{\text{translational}} + E_{\text{rotational}} + E_{\text{vibrational}} + E_{\text{electronic}}$$

A molecule can only absorb the energy of a photon if the energy of the photon corresponds to a valid step in energy in the molecule. Electronic transitions tend to be the biggest and translation transitions the smallest.

Translational or Kinetic Energy (Temperature) Transitions: These relatively small energies are associated with the motions of molecules through space and are **not** quantized. Translational processes tend to broaden the other absorption spectra, and can be used to estimate the temperature of air.

Vibrational Energy Transitions: Atoms bonded together have energy stored in their oscillations about their center of mass. Energy with wavelengths of less than 20 μm can alter these vibrational modes, which are quantized. These absorption bands are typically very sharp, and involve combinations of oxygen, carbon, nitrogen and silicon with maximum absorption in the mid-infrared. They thus play an important role in the greenhouse effect. Triatomic molecules tend to play the most important role because they can vibrate in several modes (symmetric, bending and anti-symmetric). The bending mode of CO_2 is very important because it appears near the peak of the terrestrial emission spectrum. Water vapor and ozone also have strong absorption bands in longwave frequencies.

Note: Tropospheric and stratospheric ozone have different influences. Ozone is made up of three oxygens combined with a single and double bond: $\text{O}-\text{O}=\text{O}$.

Stratospheric Ozone: An electronic transition causes the double bond to jump - a high energy transition associated with absorption in the visible and ultraviolet.

Tropospheric Ozone: bending mode causes ozone to absorb in the mid-infrared

3.6 The Lambert-Bouget-Beer law of extinction

The change in the radiation flux (dF) along a path of length ds is a function of the density of the absorber (ρ_a) and an absorption coefficient (k_{abs}).

$$dF = -k_{abs}\rho_a F ds$$

If we assume the sun is straight overhead the $ds = dz$ and we can estimate the vertical change in the radiation flux as

$$\frac{dF}{dz} = -k_{abs}\rho_a F$$

If τ is the optical depth along the path, defined as

$$\tau = \int_z^{\infty} k_{abs} \rho_a dz$$

then $d\tau = -k_{abs}\rho_a dz$ and

$$\frac{dF}{d\tau} = -F, F = F_{\infty} e^{-\tau}$$

The flux F thus decays exponentially with increasing optical depth.

3.6.1 Absorption Rate

The hydrostatic balance gives us an exponential decay of density with increasing heights:

$$\rho_a = \rho_{as} e^{(-z/H)}$$

where H is the scale height of the atmosphere ($H=RT/g$). R is the gas constant, g is the acceleration due to gravity and ρ_{as} is the density of the ATM at the surface. Plugging this into our definition of optical depth and assuming that k_{abs} is constant gives us

$$\tau = \frac{P_s}{g} M_a k_{abs} \left[e^{-z/H} \right]$$

Where M_a is the mass mixing ratio (ρ_a/ρ) of the absorber. From this relationship we can derive an expression for the change in the optical depth with height

$$\frac{d\tau}{dz} = -\frac{\tau}{H}$$

Now the absorption rate per unit volume is = (Flux)(density)(absorption coeff). If we assume that our solar angle is 0, then we can estimate the absorption rate as:

$$AbsRate = \frac{dF}{dz} = k_{abs} \rho_a F = -\frac{d\tau}{dz} F = F_{\infty} e^{-\tau} \frac{\tau}{H}$$

3.7 Net Radiation (p. 59)

The net flux of terrestrial radiation is the difference between the upward and downward fluxes

$$F(z) = F^{\uparrow}(z) - F^{\downarrow}(z)$$

The heating rate associated with the divergence of the terrestrial flux density is

$$\frac{\partial T}{\partial t} = \frac{1}{\rho c_p} \frac{\partial F}{\partial z}$$

c_p is the specific heat of air at a constant pressure

3.8 Heuristic Model of Radiative Equilibrium

Consider an atmosphere with two layers. Further assume that the ATM is transparent to shortwave but not longwave radiation. The energy balance at the top of the atmosphere must balance with the incoming solar radiation

$$\frac{S_o}{4} (1 - \alpha_p) = \sigma T_e^4 = \sigma T_1^4, \text{balance_at_top_of_atm}$$

At layer 1 the energy balance must be

$$\sigma T_2^4 = 2\sigma T_1^4, \text{balance_at_layer1}$$

This is because:

- layer 1 must emit energy equivalent to the incoming radiation upward to balance the incoming radiation
- layer 1 must also emit the same radiation downward, and
- layer 2 must emit enough energy to balance the up & down flux of layer 1

Repeating this logic gives us a balance for the layer 2.

$$\sigma T_1^4 + \sigma T_s^4 = 2\sigma T_2^4, \text{balance_at_layer2}$$

$$\frac{S_o}{4} (1 - \alpha_p) + \sigma T_2^4 = \sigma T_s^4, \text{balance_at_surface}$$

These 4 equations can be used to solve for the surface temperature

$$T_s^4 = 3 \frac{\left(\frac{S_o}{4}\right)(1 - \alpha_p)}{\sigma} = 3T_e^4$$

Which can be generalized

$$T_s = \sqrt[n+1]{n+1} T_e$$

The radiative equilibrium surface temperature for a 2 layer model is 335K – way too hot – conduction and convection also transport heat away from the surface.

The surface temperature and lapse rate of a ‘purely radiative’ atmosphere are both higher than the observed values.

In radiative-convective equilibrium models, ‘convective adjustment’ can be used to force the temperature profile to some maximum value (such as $6.5^\circ\text{C km}^{-1}$).

Homework: Answer question 5 in the back of chapter three. You will also answer 2 short essay (each answer ~ 1 page of writing) based on the Science Article reading handout. Chapter questions and essay answers will receive equal weights when the assignment is graded.

Essay Q1: Based on the Science articles, summarize the expected changes in population, biodiversity, tropical soils and food security, energy demands and freshwater resources that will likely occur over the next 50 years. How might these changes affect the radiative balance of the earth atmosphere – i.e. the earth’s surface albedo, top of atmosphere albedo, greenhouse balance, and net radiation patterns.

Essay Q2: Based on the Science articles, discuss how the climate is likely to change. Will these changes tend to affect adversely people in the tropics or mid-latitudes more? If the economy is a feedback system, what might this imply for political support for reducing greenhouse gas emissions?