The role of fluid residence time and topographic scales in determining chemical fluxes from landscapes

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ABSTRACT

The role of fluid residence time and catchment length scales in controlling the chemical composition of rivers is evaluated by comparing numerical simulations and scaling arguments to concentration–discharge data from small catchments. The analysis suggests that poorly-crystalline aluminosilicates are an important control on the composition of stream waters and therefore chemical equilibrium between the dissolving and precipitating phases determines the maximum concentration and the maximum silicate weathering flux. The modeling results suggest that the residence time of fluid relative to the residence time required to approach chemical equilibrium can be used to assess the controls on solute fluxes in small catchments, and possibly larger rivers. Catchments that show little variability in concentration with discharge (or “chemostatic behavior”) likely have average fluid residence times that exceed the time required to reach chemical equilibrium. Conversely, decreases in concentration with increasing discharge are explained by average residence times shorter than required to approach chemical equilibrium, resulting in dilution. Solute fluxes are also strongly impacted by the distribution of fluid residence times in a basin. The fluid residence time model provides an alternative framework for assessing both the relationship between discharge and concentration observed for individual catchments, and controls on the solute fluxes of rivers. If fluid residence times are a dominant control on weathering fluxes, the chemistry of different rivers could vary entirely as a function of the nature subsurface flow paths and the composition of the system at equilibrium, which is complex to predict and strongly coupled to biological processes, temperature and the composition of the system. As a result of thermodynamic and hydrologic restrictions on the amount of weathering, global solute fluxes may depend more strongly on the geometry, relief, runoff and permeability of basins than on temperature and rates of erosion.

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1. Introduction

The flux of dissolved silicate and carbonate rock is an important control on the composition of the oceans and the atmosphere. In most treatments of global elemental cycles, weathering fluxes in the past are scaled according to semi-empirical formulations that account for runoff, temperature, erosion and biological processes (Arvidson et al., 2006; Berner and Kothavala, 2001; Wallmann, 2001). These scaling parameters are often based on observations from modern rivers of varying sizes and climates, and are thus assumed to apply in the past (Dessert et al., 2003; Gaillardet et al., 1999; Ludwig et al., 1998). An alternative approach to modeling solute fluxes is to develop a mechanistic but simple model for solute production and test it against modern river chemistry. Such a model must incorporate the effects of runoff, erosion, temperature and biological processes, while avoiding the incorporation of empirical parameters that may have varied in the past. Such a model must also consider how fluids travel through the zone of solute generation and the thermodynamic constraints that limit solute generation.

The ability to accurately predict the generation of solutes is integral to understanding the relationships between climate, tectonic uplift/erosion and chemical weathering (West et al., 2005). In many catchments, solute fluxes are linearly correlated with discharge, suggesting that solute concentrations are constant over a broad range of discharge (Godsey et al., 2009; Stefansson and Gislason, 2001). Even for large rivers draining diverse lithologies, a nearly linear relationship is observed between solute flux and discharge (Gaillardet et al., 1999). This behavior has been called “chemostatic” and has been interpreted as evidence for strong climatic or hydrologic control of solute fluxes (Clow and Mast, 2010; Godsey et al., 2009).

Models of the relationship between concentration and discharge are also commonly used in catchment hydrology. Physical models often relate concentration–discharge relationships to mixing of end-member waters from different physical locations in the landscape (Evans and Davies, 1998). Chemical models can be differentiated into several categories: models that assume fluxes depend on reaction rates (Godsey et al., 2009), models that assume the concentration...
varies as a function of the subsurface contact time (Anderson et al., 1997a; Hornberger et al., 2001; Johnson et al., 1969), and models that assume solute concentrations are determined by chemical equilibrium between a primary and secondary phase (Bowser and Jones, 2002; Drever and Zobrist, 1992; Garrels and Mackenzie, 1967; Stefansson and Gislason, 2001; Tardy, 1971). Models that neglect chemical equilibrium would allow for concentrations to surpass equilibrium values, whereas models that assume chemical equilibrium would not predict lower concentrations when fluids are not chemically equilibrated, especially at high discharge or short subsurface contact times. Thus, any model linking solute fluxes to physical or chemical processes must consider both kinetic and equilibrium processes.

The goal of this work is to develop a conceptual framework for solute generation in catchments that can explain the response of rivers to changes in observable parameters such as discharge, temperature and the residence time of fluids in the subsurface. This conceptual framework must take into account the balance between chemical equilibrium, kinetic and hydrologic controls. To accomplish this, a numerical model is used to assess the extent of chemical equilibration along a flow path for different parameters known to impact solute generation (e.g., mineral composition, temperature, soil gas CO$_2$ concentration, water content, flow rate and effective surface area). The numerical model results are used to develop a scaling approach that can describe chemical fluxes as a function of the ratio of the actual fluid residence time to the time required to reach chemical equilibrium. The model is then tested against concentration–discharge data from single rivers. If it can be shown to scale with catchment size, this framework provides an alternative approach for interpreting solute fluxes from catchments that can be used to evaluate rates of chemical denudation or concentration–discharge relationships.

2. Methods

2.1. Conceptual model

The residence time of water is expected to be an important control on solute concentrations because longer transit times favor the accumulation of more solute (Berner, 1978; Wolock et al., 1997). Tracer studies of small headwater catchments yield mean residence times of water ranging from a few months to several years (Asano et al., 2002; McGuire and McDonnell, 2006, 2010; McGuire et al., 2002; McGuire et al., 2005; McGuire et al., 2007; Soulsby et al., 2000; Stewart et al., 2010; Tetzlaff et al., 2009). For large river basins, mean residence times can approach decades because of longer flow paths (McGuire and McDonnell, 2006). Fig. 1 depicts the possible flow paths through the hillslope to the stream and the chemical evolution along the flow path. The hydrologic setting is simplified to focus on the chemical aspects of solute generation and to highlight the importance of the ratio between the flow path length ($L$) and the flow path length required to reach chemical equilibrium ($L_{eq}$), where the latter can depend on a number of different factors including water flux, water content, mineral surface area, soil gas CO$_2$ and the equilibrium concentration (Maher, 2010). This conceptual model emphasizes the importance of physical length scales relative to chemical length scales as an important factor in solute fluxes.

2.2. Data sources

Runoff generation is clearly a dynamic process so it is surprising that many catchments show minimal variation in solute concentrations across wide ranges of discharge. To assess model predictions
of concentration and discharge, sites from the United States Geological Survey's (USGS) Hydrologic Benchmark Network (HBN) [http://nyef.usgs.gov/hbn/index.cfm] are used (Mast and Clow, 2000). The mean annual discharge–weighted concentrations (calculated following Godsey et al., 2009; Zeman, 1978) show the same general trends as the individual events so the individual events are used for the model fitting and evaluation. The data were not corrected for rainfall inputs or evapotranspiration in order to accurately compute mineral saturation states. Fig. 2A shows Si flux as a function of runoff illustrating the approximate “chemostatic” behavior for individual catchments as noted by Godsey et al. (2009). Fig. 2B shows the concentration–discharge relationships for three catchments indicating the three general patterns observed: (1) decreasing concentration with increasing discharge, (2) constant concentration or “chemostatic” behavior and (3) decreasing concentration only at high discharge.

2.3. Reactive transport simulations

The reactive transport code CrunchFlow (Maher et al., 2006; Maher et al., 2009; Steefel, 2001; Steefel and Maher, 2009) was used to evaluate the increase in solute concentrations as a function of flow rate, temperature, mineral surface areas, water saturation and soil gas CO₂ (PCO₂). The model simulations were run at 5 °C, 15 °C and 25 °C and 1 atm, with 15 °C representing the average global temperature (Berner and Kothavala, 2001). The model domain consisted of a one dimensional soil column with a flux boundary condition at the base of the profile, and a Dirichlet boundary condition for aqueous and gaseous species at the land–atmosphere interface. The simulations infiltrated dilute rainwater into a variably water saturated column of minerals with a porosity of 17.5%. The simulations were run with a starting composition approximating granite (18.2 wt.% plagioclase (20% anorthite (An₀₂₀), 18.2 wt.% K-feldspar, 60.6 wt.% quartz, and 3 wt.% secondary minerals). Mineral surface areas were 0.5 m²/g for feldspars and 10 m²/g for secondary minerals corresponding to the ranges of values for feldspars reported in White and Brantley (2003) and for clay minerals (Maher et al., 2009; Yang and Steefel, 2008).

The majority of the equilibrium constants for minerals and the relevant aqueous complexes used in the modeling and interpretation are from the EQ3/6 thermodynamic database (Wolery et al., 1990). Log K values for anorthite/alkali solid solution series are from Arnorsson and Stefansson (1999). The rate constants and activation energies are from a compilation by Palandri and Kharaka (2004). The kinetic rate law for feldspar dissolution includes a non-linear dependence on Al activity and feldspar saturation state (Gautier et al., 1994; Maher et al., 2006; Maher et al., 2009; Oelkers et al., 1994; Schott et al., 2009). Model profiles shown in Fig. 1B (and subsequently) correspond to 500 years, ample time to reach quasi-steady state such that the fluid compositions are not changing appreciably over time at a given point (Lichtner et al., 1986).

In a multi-component system characterized by irreversible reactions, the reaction endpoint is highly dependent on the mineral kinetics and the components in the system (Lasaga et al., 1994; Steefel and Van Cappellen, 1990). The goal of this study is to understand how departures from chemical equilibrium affect solute fluxes, and not to determine the exact phases that control the equilibrium concentrations as these may vary with climate, vegetation and lithology. Fig. 3 shows the water compositions for the majority of the HBN catchments are also consistent with either kaolinite or a metastable aluminosilicate (likely halloysite). Saturation with respect to halloysite, allophane or imogolite is common for both river waters (Stefansson and Gislason, 2001) and soil waters (Maher et al., 2009; White et al., 2009). As a result, the model mineral system that produced the most consistent values for pH, Si, K, Na compared to the HBN river data was composed of quartz, K-feldspar, plagioclase (An₀₂₀ to An₂₀) and halloysite (Fig. 3). The dashed lines in Fig. 3 show that the phase boundaries will shift depending on the solubility of the secondary aluminosilicate, and in general the identity and solubility of the secondary minerals are poorly known (Violette et al., 2010; Yang and Steefel, 2008). However, clay minerals composed of the more soluble elements (such as smectite) tend to form in dry environments, while kaolinite group minerals tend to form in wetter climates (Chadwick and Chorover, 2001; Curtis, 1990; Folkoff and Meentemeyer, 1985). Most of the basins considered have rainfall amounts typical of halloysite, consistent with the model predictions. Overall the chemical endpoints in the model are highly uncertain, yet the overall progression towards chemical equilibrium is not.

3. Results

3.1. Reactive transport model sensitivity analysis

Reactive transport simulations were used to evaluate the sensitivity of solute profiles to key variables (Fig. 4). The goal is to understand: (1) the parameters that impact the final equilibrium concentration and (2) the parameters that control the time required for fluids to equilibrate. The equilibrium time and concentration relative to the system length scale determine the maximum solute flux and are thus important
for assessing restrictions on solute generation imposed by either chemical equilibrium or flow path lengths. Variations in water content and the boundary condition for $P_{CO_2}$ impact both the final equilibrium concentrations and the equilibration length scale or equilibration time (Fig. 4A, B). In both cases, the evolution to different chemical endpoints is a result of differences in the final pH, as suggested by the activity diagram shown in Fig. 3. For unsaturated flow, the model $P_{CO_2}$ is effectively constant throughout the profile, creating a competition between the dissolution of CO$_2$ into the pore water, which lowers the pH, and the feldspar hydrolysis reactions. The net result is an increase in equilibration length and equilibrium concentration with increasing $P_{CO_2}$. This difference in chemical evolution emphasizes the strong coupling between hydrologic and biogeochemical processes—abiotic and biotic factors that influence soil water pH, including the presence of organic acids, are thus likely to strongly impact the extent of weathering. In contrast, changes in mineral surface area and porosity do not impact the final equilibrium concentration, just the length scale (Fig. 4C). Increases in mineral surface area decrease the equilibration length scale because reaction rates increase. The reactive transport model also predicts longer times to reach equilibrium at lower temperatures due to slower reaction kinetics. This highlights the expansive set of variables that factor into the generation of solutes from weathering processes (Beaulieu et al., 2010; Berner, 1992; Bluth and Kump, 1994; Maher et al., 2009; Richards and Kump, 2003; West et al., 2005).

In general, parameters that slow the approach to equilibrium (e.g. reduced mineral surface area/reaction rate or increased flow rate) result in longer equilibration length scales. Parameters that change the composition of the system tend to change both the equilibrium length scale and the concentration (soil $P_{CO_2}$ or pH, unsaturated flow, mineral solubilities and to some extent temperature). The reaction time in the unsaturated zone relative to the saturated zone will also be an important control on the equilibration length and concentration. The sensitivity analysis demonstrates that weathering evolution can still be effectively described by a theoretical equilibration length and equilibrium or endpoint concentration. The theoretical model equilibration lengths (0.5 to 3 m) are similar to those of soils and hillslopes. However, detailed field studies that assess equilibration...
lengths and endpoints in tandem with the lithology, temperature and hydrology are critical for evaluating this approach.

### 3.2. Concentration as a function of temperature and fluid residence time

Fig. 5 shows the theoretical evolution of solute concentrations as a function of fluid residence time for different temperatures and saturated flow. The evolution of concentrations are similar across 2 orders of magnitude in flow rates (e.g., 0.1 to 10 m/yr) for a given temperature, suggesting that fluid residence time is a useful variable for understanding chemical evolution. The decreasing final concentration with increasing temperature predicted for fully saturated flow conditions is the reverse of the temperature dependence predicted for unsaturated flow in Fig. 4D. The difference is due to the presence of CO$_2$(g) throughout the profile for unsaturated flow, compared to saturated flow where the amount of CO$_2$ dissolved in the water entering the profile is determined by the temperature and boundary condition for PCO$_2$ (which was held constant at 0.5% in both cases). As a result, for unsaturated flow both water flux and temperature impact the final equilibrium concentration. In reality, soil PCO$_2$ levels likely decrease with temperature due to lower biological productivity. In addition to the pH effects due to CO$_2$(g) solubility and transport, temperature also impacts the reaction kinetics. For example, at short fluid residence times the concentration is higher at temperatures because of elevated reaction kinetics.

Between or within the six HBN catchments considered here, water composition shows no consistent relationship to water temperature—some catchments show no correlation between concentration and temperature (SHCA, KMHI, HCID), while others show a weak positive correlation (ECCA) or a weak negative correlation (ACWA, WCMI)—likely because the effects of temperature are complicated by the boundary conditions. Detailed field studies examining the dynamics of CO$_2$(g) in the soil zone and the resulting impacts on solute generation would help to evaluate the actual relationship between temperature, equilibrium composition, extent of weathering and hydrologic conditions.

Many studies have assumed that silicate weathering reactions are slow relative to transport such that rates are kinetically- or surface-controlled. Based on this assumption, equations have been developed that relate weathering to fluid flux and temperature impact the chemical weathering rates across different environments. The concentration of a dissolving solute, $c$, along a flow path $x$ as a function of the flow rate $[q(m/yr)]$, the volumetric water content $\theta$ and the net of the dissolution $(\sum R_{di} [mol/m^3-fluid/yr])$ and precipitation $(\sum R_{pi})$ rates summed over each mineral $(i)$, can be described in simplified form as:

$$\frac{dc}{dt} = -\frac{q}{\theta} \frac{dc}{dx} + \sum_i \mu R_{di} \left(1 - \frac{c}{c_{eq}}\right)^n - \sum_i \mu R_{pi} \left(1 - \frac{c}{c_{eq}}\right)^m.$$  \(1\)

The exponents $n$ and $m$ reflect the observation that the dissolution and precipitation behavior of many minerals exhibits a non-linear dependence on the departure from equilibrium (Hellmann and Tisserand, 2006) and $\mu$ is the stoichiometric coefficient. The value of $c_{eq}$ reflects the equilibrium concentration where rates of dissolution and precipitation are zero, and reflects the decrease in rates as fluids approach equilibrium. The terms $R_{di}$ and $R_{pi}$ are the product of the dissolution rate constant $k_i$ (mol/m$^3$-mineral), the surface area $A_i$ (m$^2$/mineral), the term describing inhibition or activation, and the mineral to fluid ratio $M$ (m$^3$-mineral/m$^3$-fluid). The dispersion term in Eq. (1) (not shown) is neglected for simplicity. After an initial relaxation time which is on the order of a year or less, the shape of the solute profile $(dc/dx)$ should reach a quasi-steady state as long as mineral volumes are

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Fig. 5. (A) SiO$_2$(aq) and Na and (B) calcium and alkalinity (K and Al not shown) concentrations as a function of fluid residence times at different temperatures for a system of K-feldspar, plagioclase $(An_{30})$ and halloysite; The individual lines correspond to a range of flow rates from 0.1 m/yr to 10 m/yr showing the correspondence between fluid residence time and the evolution of concentration across flow rates. Simulations assume 100% liquid saturation. Minerals included in the simulation are plagioclase $(An_{30})$ (18.2 wt.%), K-feldspar (18.2 wt.%), quartz (60.6 wt.%), and halloysite (3.0 wt.%). Note the linear concentration scale on the y-axis and logarithmic scale on the x-axis.
changing slowly (ca. 10,000s of years) (Lichtner et al., 1986) so that the
solute profile can be represented by:

\[
dc \frac{dt}{dx} = \frac{\omega}{q} \sum_{i} \mu R_{di} \left(1 - \left(\frac{c}{c_{eq}}\right)^{\omega}\right) - \frac{\omega}{q} \sum_{i} \mu R_{pi,1} \left(1 - \left(\frac{c}{c_{eq}}\right)^{\omega}\right) \tag{2}
\]

Based on Eq. (2), the concentration gradient \(dc/dx\) depends on the balance between the flow rate and the reaction rates. Once the fluid has reached equilibrium \(c_{eq}\) with the dissolved and precipitating phases, \(dc/dx\) is zero and the maximum possible solute flux \(F_{\text{max}}\) is achieved \(F_{\text{max}} = q c_{eq}\). Until equilibrium is reached, the solute flux varies as a function of the flow rate and the concentration \((\text{Fig. 1B})\). Conceptually, this idea can be represented by considering the change in concentration as a function of the residence time of the fluid \((\tau = L/\omega)\) at distance \(L\) as shown in Fig. 5. If the fluid residence times are long enough that fluids reach equilibrium with the solids, the primary control on the solute flux is discharge.

In order to understand how solute fluxes vary with catchment fluid residence time, it is helpful to have an equation that predicts the solute curve shown in Figs. 4 and 5. This can be done by assuming that \(\sum \mu R_{di} - \sum \mu R_{pi,1}\) is equal to \(R_{\text{net}}\), the net increase in solute, and solving for the concentration as a function of distance based on Eq. (2):

\[
c(x) = c_{eq} \exp \left(-\frac{R_{\text{fl}}}{\omega c_{eq}} x\right) + c_{eq} \left(1 - \exp \left(-\frac{R_{\text{fl}}}{\omega c_{eq}} x\right)\right) \tag{3}
\]

where \(c_{eq}\) is the initial solute concentration at the inlet (nominally zero) and \(x\), the distance along the flow path \((c_{eq} \text{ and } R_{\text{fl}}\) are a function of temperature, composition \((\text{including mineral and gasses})\) and liquid saturation). This approximation assumes no net addition of solute from other sources along the flow path. Eqs. (2) and (3) could be amended if this assumption is violated. However, if all fluids are either equilibrated with the solids before they mix, or isolated from mixing in the subsurface, this assumption has a negligible effect on the composition of waters.

Eq. (3) can also be expressed in terms of fluid residence time \((\tau = L/\omega)\), if \(x\) is equal to the length scale of the system \((L)\):

\[
c(\tau) = c_{eq} \exp \left(-\frac{R_{\text{fl}}}{c_{eq} \omega} \tau\right) + c_{eq} \left(1 - \exp \left(-\frac{R_{\text{fl}}}{c_{eq} \omega} \tau\right)\right) \tag{4}
\]

Determining the effective net reaction rate for a natural system is challenging. However, a measure of the net reaction rate is the distance \((\text{or fluid residence time})\) required for the system to approach equilibrium. The ratio of \(c_{eq}/R_{\text{fl}}\) is effectively a measure of the time required for the fluid to reach equilibrium, or the “equilibrium fluid residence time”, \(\tau_{eq}\). As demonstrated in Figs. 4 and 5, \(\tau_{eq}\) can vary depending on a number of factors, although timescales of months to years are within the range of median residence times for many catchments (McGuire and McDonnell, 2006; Stewart et al., 2010). Eq. (4), while simple, is consistent with the theoretical evolution of the system: as actual fluid residence times become large relative to the ratio of \(c_{eq}/R_{\text{fl}}\), the concentration approaches \(c_{eq}\) (Aagard and Helgeson, 1982; Lasaga and Rye, 1993; Steefel and Lasaga, 1994).

An alternative way to estimate the net reaction rate is to consider the ratio of the dissolution rate to the precipitation rate. Secondary minerals cannot precipitate more quickly than minerals dissolve so \(R_{d}/R_{p}\) must be close to 1. In the reactive transport simulations, \(R_{d}/R_{p}\) was between 1.66 and 1.76. Experimental batch studies also demonstrated a ratio of oligoclase dissolution to kaolinite precipitation of 1.62 (Zhu et al., 2010). Using this relationship, the equilibrium fluid residence time \(\tau_{eq}\) is:

\[
\tau_{eq} \approx \frac{c_{eq}}{R_{\text{fl}}} \approx c_{eq} \frac{1}{F_{\text{fl}}}
\]

where \(\epsilon\) describes proportionality between net precipitation and dissolution rates and is likely on the order of 0.56 for Si when the stoichiometry \((\text{e.g., albite to halloysite})\) is accounted for. Using the data from the reactive transport model, \(c_{eq}\) at 15 °C is 348 μmol/L, \(R_{d}\) is 1.7 × 10^{-3} mol L/yr, and \(\tau_{eq}\) is equal to 0.3 yr, in agreement with the model profiles. If \(c_{eq}/R_{d}\) is assumed to be proportional to the fluid residence time where the system reaches equilibrium \((\tau_{eq})\), Eq. (4) can be recast to describe the change in concentration along a flow path as a function of the ratio of the actual fluid residence time to the fluid residence time required for the system to approach equilibrium:

\[
c(\tau) = c_{eq} \exp \left(-\frac{f \tau}{\tau_{eq}}\right) + c_{eq} \left(1 - \exp \left(-\frac{f \tau}{\tau_{eq}}\right)\right)
\]

A scaling factor \((f\omega)\) is introduced to control where the concentration approaches equilibrium relative to the equilibrium fluid residence time such that \(f = -\ln \left(1 - f\right)\) where \(f\) is the fraction of the equilibrium concentration achieved. Values for \(f\) and \(\omega\) are assumed to be 0.9 and 2.3 respectively. In Fig. 6, Eq. (6) is compared to the results from the reactive transport simulations for saturated flow assuming the same equilibrium concentrations and fluid residence times.

The equilibrium fluid residence time is a measureable parameter that reflects the combined influence of multiple factors, including mineral surface areas, temperature, mineral solubility, biological productivity and hydrologic properties. The \(\tau_{eq}\) is thus likely to vary widely between lithologies, climates and ecosystems and may be difficult to estimate \(a\ priori\), but could be assessed through geochemical measurements. This equation predicts a similar behavior to that of Scanlon et al. (2001), although it is applicable over a broader range of fluid residence times and predicts a maximum concentration that is dependent on the composition of the system. The reactive transport simulations included multiple phases and reaction kinetics with a non-linear dependence on reaction affinity. As a result, Eq. (6) reproduces the general profiles but departs from the numerical model near equilibrium. Overall, this approach captures the evolution of the solute profile determined by the numerical model without relying on strictly empirical parameters. Eq. (6) is not a perfect representation of a multi-component reactive transport system, but it is simple and relies on measureable parameters and may thus offer a useful interpretive tool that could be strengthened by further comparisons to well-studied catchments.

4. Discussion

The reactive transport model approach provides a framework for understanding how different variables may influence the generation
of solutes and control chemical fluxes from landscapes. This framework was used to generate a simple model based on scaling arguments that captures the primary variables. An additional utility of a simple model is that variables can be determined for individual catchments without relying on empirical parameters.

The model analysis suggests that the primary controls on weathering fluxes are the residence time of water in the saturated and unsaturated zones, mineral surface areas and solubilities and the dynamics of CO₂ production and transport. These factors, along with temperature, control the rate of approach to chemical equilibrium and the final chemical equilibrium concentration, which in turn determine the maximum solute flux. The fact that individual catchments show less variability in concentration than discharge may result from equilibrium controls (or long fluid residence times such that the maximum solute concentration is reached before the fluids reach the channel). This is consistent with a number of studies that have proposed that Si concentrations are comprised of a distribution of travel times for water between the point of input and outflow. The sine-wave, piston approach for catchment transit time distributions have included the mean transit time, but as of yet no direct scaling relationship has emerged from basin to basin (Tetzlaff et al., 2002, 2009).

As a result of the distribution of fluid residence times some waters in a catchment will have spent enough time in the subsurface to equilibrate, while others may not have approached equilibrium. The distribution of fluid residence times is thus important when considering the application of Eq. (6) to catchments. The distribution of fluid transit times h(τ) within small catchments (first or second order) is most commonly described as an exponential function (McGuire and McDonnell, 2006):

\[ h(\tau) = \frac{1}{\tau_f} \exp\left(-\frac{\tau_f}{\tau_m}\right) \]  

(7)

However, recent studies have suggested that a gamma distribution may be a better empirical representation of residence times (Kirchner et al., 2000; 2001; McGuire et al., 2005):

\[ h(\tau) = \frac{\tau^{\alpha-1}}{\beta^\alpha \Gamma(\alpha)} \exp\left(-\frac{\tau}{\beta}\right) \]  

(8)

In Eqs. (7) and (8) \( \tau_m \) is the mean residence time and \( \alpha \) is the shape factor, which influences the shape of the distribution and extent of tailing. The other factors in the gamma distribution are the scale factor \( \beta = \tau_m/\alpha \) and the gamma function \( \Gamma(\alpha) \). Other approaches for catchment transit time distributions have included the sine-wave, piston flow and binomial models. Recent studies have found that the gamma distribution (with \( \alpha \approx 0.5 \)) reproduced the power spectral scaling observed in some catchments better than the exponential model (Godsey et al., 2010). In addition, a recent comparison between transit times measured with either stable isotopes (\( ^{18}\text{O}, \delta^{13}\text{C} \)) and/or tritium (\( ^{3}\text{H} \)) suggests that stable isotope approaches are biased towards shorter mean residence time estimates because waters older than about 4 years no longer show detectable variation in \( ^{18}\text{O}, \delta^{13}\text{C} \) (Godsey et al., 2010). Old ground water components reflective of deep storage can be a substantial fraction of the discharge in headwater catchments, while mean residence times and ground water components of large rivers are potentially even greater (Rice and Hornberger, 1998; Scanlon et al., 2001). For example, 60% of the flow in the Ohio River is groundwater with a mean residence time of 10 years, while 90% of the flow in the Missouri River is groundwater with \( \tau_m \) of 4 years (Michel, 2004). Most importantly, the large contributions of groundwater and mean residence times on the orders of months to years suggests that for many catchments surface contact time is sufficient for chemical equilibration, in contrast to other studies which assume chemical equilibration is unlikely.

The effect of a distribution of fluid residence times on river solute concentrations is shown in Fig. 7 for both the exponential and gamma distributions assuming Eq. (6). The fluid residence time required to reach equilibrium is assumed to be 1 year and the equilibrium concentration is 350 μmol/L. Compared to a homogenous flow path, the concentrations are lower for all distributions because the packets of fluid with residence times less than 1 year do not equilibrate with the solids. The lowest concentration occurs for the gamma distribution with \( \alpha = 0.25 \) because the distribution is highly skewed towards shorter residence times and less reaction. The exponential and gamma distributions with a higher \( \alpha \) value move more closely approach the case for a single flow path. Seasonal or event-driven shifts in fluid residence time may not appreciably affect the concentration of discharge as long as \( \tau_m \) remains greater than \( \tau_{eq} \). Conversely, if \( \tau_m \) is shorter than \( \tau_{eq} \), waters may show considerable variability in Si and other weathering derived solutes—heterogeneity in flow paths can have a substantial impact on concentrations of a reacting solute. This analysis suggests that the average dimensions and flow routing of catchments will strongly determine the solute flux from weathering.

The effect of basin size was also observed by Wolock et al. (1997), where base cation concentrations, acid neutralizing capacity and pH varied sharply as basin size increased to 3 km², but showed small to no variations with additional increases in size. An increase in solute flux with increasing basin size and/or the transition from mountainous regions to alluvial plains was also observed for both the Ganges Basin (West et al., 2002) and the Mackenzie River (Lemarchand and Gaillardet, 2006; Millot et al., 2003). These observations, which are supported by the model, suggest that the factors controlling solute fluxes may change with the scale of observation.

![Fig. 7. Model predicted Si concentrations for different fluid residence time distributions (see text for description of models). The exponential and gamma models are equivalent when \( \alpha = 1 \). Range of mean transit times measured for small basins is summarized from McGuire and McDonnell (2006) and Stewart et al. (2010) and references therein. Also shown are mean transit time for the Missouri and Ohio Rivers (Michel, 2004), Sacramento and Mississippi Rivers (Michel, 1992) and the Danube River (Yurtsever, 1999). Numbers in parenthesis correspond to the measured mean transit time in years.](image-url)
4.2. Concentration–discharge relationships of individual catchments

Small changes in the concentration of waters over large changes in discharge as observed in many catchments (cf. Godsey et al., 2009) could occur in systems where fluid residence times exceed the time required to approach equilibrium. “Chemostatic” behavior is thus attributed here to chemical equilibrium between primary and secondary phases, and most likely a poorly crystalline precursor phase. Fig. 8 shows the model fit to the data for six HBN catchments with different discharge–concentration patterns using Eq. (6). Each catchment was fit by varying the ratio of the product of the path length and the porosity, \( L\theta = (\text{m}) \), to the equilibrium fluid residence time \( \tau_{eq} \) so that \( L\theta /\tau_{eq} = (\text{m/yr}) \) is the actual adjustable parameter because true equilibrium length scales are not known. Thus, the ratio of the fluid residence time to the equilibrium fluid residence time varies with discharge.

As the general relationship between concentration and fluid residence time does not change appreciably when the distribution of fluid residence time is considered, a mean flow path length is assumed. For the Type 1 catchments, Sagehen Creek, in the Sierra Nevada Mountains (SHCA) and Kahakuloa Stream, Hawaii (KMHI), no plateau is apparent so the Si concentration at low discharge is assumed to represent the equilibrium concentration (Fig. 8A, B). To fit Sagehen Creek also required that the initial concentration was elevated (260 \( \mu \text{mol/L} \)). Sagehen Creek is dominated by spring snow melt and fracture-controlled perennial springs with residence times on the order of decades (Rademacher et al., 2005), so the value for the initial concentration may reflect spring input which would not vary with discharge. For all of the other catchments the initial concentration was zero.

For Type 2 or “chemostatic” catchments, the \( L\theta /\tau_{eq} \) values are minimum values (Fig. 8C, D). The Type 3 catchments, which show a chemostatic behavior only at low discharge (Andrews Creek in the...
Cascade Mountains of Washington (ACWA) and Washington Creek in Michigan (WCMI)), the concentration plateau is assumed to represent the equilibrium concentration (Fig. 8E, F). In general, the $L/	au_q$ values are greatest for the chemostatic catchments, as expected. The value of $\tau_q$ (0.08 years) is likely a minimum value — $\tau_q$ values for the Santa Cruz marine chronosequence are closer to 1 to 2 years (Maher et al., 2009). $L/	au_q$ also cannot be directly estimated for HBV catchments because it reflects the combined soil thickness and hillslope length. Given these uncertainties and the possibility that $\tau_q$ varies between catchments, it is difficult to assess whether the predicted $L/	au_q$ values are reasonable. Assuming $\tau_q$ is between 0.08 and 1 year, $L/	au_q$ values would range from 0.5 to 12 m to 0.63 to 150 m respectively, values that are generally consistent with hillslope length scales. A potentially useful outcome of the model is that if $\tau_{eq}$ is known, concentration–discharge data can be used to determine average flow path lengths and porosity. The ability to describe different types of concentration–discharge patterns also emphasizes the importance of a mixed-equilibrium kinetic framework for interpreting chemical weathering processes. However, although the simple model provides a useful approach for describing different concentration–discharge relationships, it still requires testing and refinement at sites with well-defined hydrologic systems and detailed compositional data (e.g. knowledge of soil and groundwater flow paths and water fluxes, primary and secondary mineral distributions, solute and reaction affinity gradients along flow paths, and discharge–concentration or discharge–reaction affinity relationships for stream waters). Comparisons between the simple model framework and detailed catchment-scale reactive transport models (e.g. Godderis et al., 2009; Violette et al., 2010) may also help to refine the scaling relationships defined here so that they can be applied with more confidence to less monitored catchments, or potentially up-scaled to describe large rivers.

4.3. Topographic controls on solute fluxes

Topography is known to be an important control on the movement of water through a basin (Beven, 1987; Tetzlaff et al., 2009) and on solute fluxes (Millet et al., 2003; Stallard and Edmond, 1983; West et al., 2002; West et al., 2005; Wolock et al., 1997). The primary role of topography as considered here is to determine the mean fluid residence time relative to the equilibrium fluid residence time. As basin scale increases, the evolution of catchment geomorphology may lead to changes in the dynamics of hydrologic processes. Larger basins favor longer flow paths, longer fluid residence times and thus potentially the highest specific solute fluxes. Erosional processes may influence both the thickness of soils and the supply of fresh mineral (Waldbauer and Chamberlain, 2005), which would influence the fluid residence time and the equilibration time, respectively. For smaller basins with the same runoff and soil thickness, a larger fraction of the flow paths may not allow for equilibrium to be reached, resulting in lower fluxes (e.g. Fig. 7). It may not necessarily be the overall size of the basin, but rather the geometry or “wavelength” of the drainage network that determines solute fluxes. In any case, the physical and chemical compositions and length scales of sub-basins within a catchment are an important control on solute fluxes.

Basin relief will also be an important parameter. As relief increases, the hydraulic gradient also becomes steeper resulting in shorter fluid residence times (Beven, 1987; Tetzlaff et al., 2009). If the transit time through the unsaturated zone is small relative to the overall transit time of water, the hydraulic conductivity $[K_{ave} \text{ (m/day)}]$), effective porosity $[\phi_{\text{effective}}]$ and the hydraulic gradient determine the flow of water. If the hydraulic gradient approximates the topography, then the residence time along a flow path can be expressed as a balance between the hydraulic conductivity, the relief and the length scales:

$$\tau_c = \frac{\phi_{\text{effective}}(\Delta L)^2}{K_{ave} \Delta h}$$

where $\Delta L$ is the distance from recharge to discharge (m) and $\Delta h$ is the change in hydraulic head (m) along the flow path which should be proportional to the relief. As the ratio of $\phi_{\text{effective}}/K_{ave}$ is unlikely to vary substantially, the length scale and hydraulic gradient should be an important determinant of fluid residence times, along with soil structure. Topographic control of solute fluxes is probably weakest in areas where less permeable soils give rise to overland flow because residence times are effectively shorter. These areas include steep bedrock-dominated regions where soil production cannot keep pace with erosion, and more subdued deeply weathered lowlands with poor drainage. Deeply weathered areas may also have very long equilibrium fluid residence times, also weakening topographic effects.

High relief also tends to increase the length of valleys, the spacing between valleys and potentially the area of basins, extending the length of flow paths (Perron et al., 2008). Longer flow paths would increase solute fluxes for a given runoff as long as overland flow is not substantial. If longer flow paths are associated with high physical erosion rates, this may in part explain the correlation observed between weathering fluxes and physical erosion rates (Gaillardet et al., 1999; Hren et al., 2007; Waldbauer and Chamberlain, 2005). However, if increasing relief steepens the hydraulic gradient and shortens fluid residence time, then shorter residence times may result in lower concentrations. In any case, the characteristic length scale of the topography is likely to exert a strong control on the solute flux of a river.

5. Conclusions

The framework presented here suggests that both thermodynamic and hydrologic properties limit the solute fluxes carried by rivers. The thermodynamic limit is set by chemical equilibrium, where the maximum solute flux occurs when the system reaches equilibrium between the dominant primary and secondary phases. The hydrologic limit determines the time available for the fluid to react: the maximum solute flux occurs when fluid residence times allow for the equilibrium limit to be maintained. The thermodynamic and hydrologic scales are connected by the relationship between the equilibrium concentration and the time required for a system to reach the equilibrium concentration.

Evaluating the concentration–discharge patterns of rivers can also suggest how solute fluxes might respond to changes in climate or topography. In Fig. 9, the model concentrations are converted to fluxes and compared to the data for two catchments, ECCA (Type 2) and ACWA (Type 3). Schematic arrows depict how the weathering flux varies with discharge according to the ratio of fluid residence time ($\tau_f$) to equilibrium fluid residence time ($\tau_{eq}$) resulting in the different types of concentration–discharge behavior. Curves corresponding to variations in $L/	au_q$ for ACWA are shown for reference.
ACWA (Type 3). The inset diagram suggests how solute fluxes could change with the evolution of climate and topography. "Chemostratic" or Type 2 catchments reflect the maximum concentration and thus solute fluxes depend on discharge. Unless the equilibrium endpoint changes, Type 2 catchments can be considered as optimized systems. In contrast, for Type 1 and 3 systems where residence times are short (τ < τeq), increasing flow path length would allow waters more time to react and thus become Type 2 systems. However, as runoff increases, if the length scales of the drainage are not sufficient to maintain equilibrium, then concentrations will decrease (e.g. Type 3 behavior). Changes in discharge of 10 to 40% in response to climate change (IPCC, 2007) may not result in proportional increases in solute and alkalinity fluxes if the fluid residence times of catchments become too short or the equilibrium endpoints and length scales change due to temperature/PCO2 effects.

Over geologic timescales, if major rivers of the world all contain waters reacted to the thermodynamic maximum then the primary determinant of global solute fluxes is runoff. Alternatively, increasing the thermodynamic maximum by reducing the water saturation of soils and increasing subsurface CO2(g) will elevate solute fluxes in basins with fluid residence times long enough to equilibrate to the new potential maximum. In contrast, if the waters of major rivers are not reacted to the full extent, then increasing flow path length will drive solute fluxes higher as the fluids have more opportunity to react. Assuming the equilibrium concentrations are fixed, shifts in global solute fluxes may require that both the length scales and the runoff of basins increase in tandem, changes that commonly occur in response to mountain building.

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