The Hyperspectral Bidirectional Reflectance of Snow:
Modeling, Measurement, and Instrumentation

A Thesis submitted in partial satisfaction
of the requirements for the degree of
Doctor of Philosophy
in
Geography
by
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And homage to the mountains of my youth that shaped my path
One afternoon the last week in April
Showing Kai how to throw a hatchet
One-half turn and it sticks in a stump.

He recalls the hatchet-head
Without a handle, in the shop
And go gets it, and wants it for his own.
A broken-off axe handle behind the door
Is long enough for a hatchet,
We cut it to length and take it
With the hatchet head
And working hatchet, to the wood block.
There I begin to shape the old handle
With the hatchet, and the phrase
First learned from Ezra Pound
Rings in my ears!

"When making an axe handle
the pattern is not far off."
And I say this to Kai
"Look: We'll shape the handle
By checking the handle
Of the axe we cut with."

And he sees. And I hear it again:
It's in Lu Ji's Wen Fu, fourth century
A.D. "Essay on Literature"—in the
Preface: "In making the handle Of an axe
By cutting wood with an axe
The model is indeed near at hand.—
My teacher Shih-hsiang Chen
Translated that and taught it years ago
And I see: Pound was an axe,
Chen was an axe, I am an axe
And my son a handle, soon
To be shaping again, model
And tool, craft of culture,
How we go on.

Gary Snyder, 'Axe Handles'
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Painter, T. H., D. A. Roberts, R. O. Green, and J. Dozier, 1996, Sub-pixel snow-covered area and snow grain size from mixture analysis with AVIRIS data, in Summaries of the Sixth Annual JPL Airborne Earth Science Workshop, R. O. Green, ed., preliminary, Jet Propulsion Laboratory, Pasadena, CA.


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ABSTRACT

The Hyperspectral Bidirectional Reflectance of Snow:
Modeling, Measurement, and Instrumentation

by
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Seasonally snow-covered areas in the Earth's mountains are an integral component of the global hydrologic cycle. These regions account for the majority of fresh water resources over much of the mid-latitudes, affect regional climate, and are sensitive indicators of climate change due to the ephemeral nature of snow. Imaging spectroscopy in the solar spectrum allows us to analyze the spectral reflectance of the seasonal snow cover to infer snow properties such as snow covered area, grain size, albedo, and algal concentration. However, the models that retrieve these properties rely on a correct linkage between the spectral and angular distribution of reflectance and the physical properties of the snow surface. The characterization of the bidirectional reflectance of snow has been limited in terms of complete coupling of spectral range and resolution, angular range and resolution, and description of the snow stratigraphy. Moreover, we lack knowledge of the effect
of anisotropic snow reflectance on the inference of snow covered area and albedo from imaging spectroscopy models.

This thesis presents a radiative transfer/spectral mixture model for measuring snow properties from imaging spectrometer data, a spherical robot for the measurement of the bidirectional reflectance factor of snow at fine spectral and angular resolution, comparisons of measurements of the bidirectional reflectance factor of snow for a range of particle sizes and solar zenith angles, and the sensitivity of imaging spectroscopy models to anisotropic snow reflectance. The model had accuracies for subpixel snow-covered area, grain size, and albedo of 4%, 74μm, and 2.5%, respectively. The spherical robot, coupled with a field spectroradiometer, facilitates rapid, repeatable measurements of the spectral bidirectional reflectance from snow into any direction. Measurements with the spherical robot showed significant wavelength-dependent changes in bidirectional reflectance with change in grain size and solar zenith angle. The sensitivity study showed that by assuming that snow reflects isotropically, inference of snow-covered area and albedo with current imaging spectroscopy models may have errors of up to 20% and 11%, respectively, for topographically-realistic solar and view geometries.
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Chapter I

Automated Algorithm for the Retrieval of Sub-Pixel Snow-Covered Area and Grain Size from Imaging Spectrometer Data
ABSTRACT

I describe and validate an automated model for the retrieval of sub-pixel snow-covered area (SCA) and grain size from Airborne Visible Infrared Imaging Spectrometer (AVIRIS) data. The model performs multiple endmember spectral mixture analysis with a spectral library of snow, vegetation, rock, and soil. The snow spectral endmembers of varying grain size are derived from a radiative transfer model. Spectra for vegetation, rock, and soil were collected in the field and laboratory. I analyzed three AVIRIS images of Mammoth Mountain, CA (April 5, 1994; March 29, 1996; April 29, 1998) that span common snow conditions for winter through spring and with which I collected simultaneous high spatial resolution aerial photographs and geographically-located snow samples. I validated the estimates of subpixel SCA with photographic analysis of the aerial photographs for SCA and validated the estimates of grain size with stereological analysis of the snow samples. The root mean squared error for snow-covered area retrieved from AVIRIS for the combined set of three images was 4.0%. The mean and best-case root mean squared errors for snow grain size retrieved from a 3-by-3 window of AVIRIS data for the combined set of three images were 74 and 48 μm, respectively. The root mean squared errors for inferred albedo were 0.025 and 0.018 respectively for the mean and closest grain size.
1 INTRODUCTION

Mid-latitude alpine snow cover and its subsequent melt can dominate local-to-regional climate and hydrology in the world's semi-arid regions. In recent research, snow hydrologists have investigated the spatial distribution of snowpack dynamics and snowmelt through distributed physical snow models (Kimbauer et al., 1994; Cline et al., 1998; Luce et al., 1998; 1999; Colee et al., 2000). Spatially-explicit snow models require field measurements and remotely sensed imagery for initialization, validation, and/or re-initialization. Measurements of snow properties in the field provide direct determination, but at a limited spatial and temporal extent and resolution and frequently under risky conditions. Remote sensing techniques can regularly and safely provide maps of snow cover properties for the entire model domain at a range of resolutions.

Distributed snow models require as inputs and/or re-initialization the following spatially distributed parameters: snow-covered area, grain size, albedo, snow water equivalent, snow temperature profile, meteorological and radiation conditions. Of these parameters, optical remote sensing can deliver snow-covered area, grain size, and albedo, and active microwave remote sensing is close to delivering snow water equivalence (Shi and Dozier, 2000a; Shi and Dozier, 2000b). The remaining parameters come from a combination of in situ measurements and topographic modeling.
Snow-covered area in alpine terrain frequently varies at a spatial scale finer than that of the ground instantaneous field-of-view of remote sensing instruments. This spatial heterogeneity poses a "mixed pixel" problem in that the sensor may measure radiance reflected from snow, rock, soil, and vegetation. To use the snow characteristics in distributed physical models, we must therefore map snow-covered area at subpixel resolution in order to accurately represent its spatial distribution. Grain size is needed to estimate the surface permeability and to infer spectral albedo, and we must estimate these quantities for the fractional snow cover, accounting for the signal from the other surfaces within the pixel. Imaging spectrometers provide the spectral leverage necessary to infer these properties.

In this chapter, I describe and validate an automated model for the retrieval of subpixel snow-covered area and snow grain size from imaging spectrometer data. The algorithm uses a multiple endmember spectral mixture analysis (MESMA) approach (Roberts et al., 1998) to simultaneously solve for subpixel SCA and the grain size of the fractional snow cover. Grain size estimates coupled with an estimate of impurity concentration can then be used to estimate the albedo of the fractional snow cover.

I applied the algorithm to a set of Airborne Visible/Infrared Imaging Spectrometer (AVIRIS) scenes of Mammoth Mountain, CA acquired on April 5, 1994, March 29, 1996, and April 29, 1998. The set of AVIRIS scenes pre-
sents a wide range of snow cover and grain sizes. I validated the SCA and
grain size results with high spatial resolution aerial photographs and stereo-
logical analysis of snow samples, respectively.

2 Remote Sensing of Snow-Covered Area and Snow Grain
Size

Early remote sensing of snow properties focused primarily on mapping
the snow extent with multispectral sensors (Rango and Itten, 1976), such as
the Landsat Multispectral Scanner Subsystem (MSS) and Thematic Mapper
(TM), and the NOAA Advanced Very High Resolution Radiometer (AVHRR),
producing binary maps of snow cover. Based on a two-stream radiative trans-
fer model of snow reflectance, Dozier (1989) proposed a suite of normalized
band differences for mapping snow and qualitative grain size with TM data.
Most current methods of “binary” mapping of snow cover using multispectral
sensors, by which each pixel is classified as either “snow” or “not snow,” are
derived from this method (Hall et al., 1995). Rosenthal and Dozier (1996) ex-
tended this work by developing linear spectral mixture analysis for subpixel
SCA from Landsat TM.

Grain size is the snow parameter that determines its spectral albedo in
the near-infrared wavelengths, while absorbing impurities and, for shallow
snow only, snow water equivalence affect its albedo in the visible spectrum
(Warren, 1982). Dozier et al. (1981) showed that AVHRR data could qualita-
tively retrieve both snow grain size and snow water equivalence. Dozier and
Marks (1987) explored the possibility of mapping the spatial distribution of
snow grain size with TM data, again arriving at qualitative estimates. Bour-
delles and Fily (1993) mapped grain size over Adélie, Antarctica using TM
data and a two-stream snow reflectance model. Their results matched those
found in the literature but they lacked field validation. Fily et al. (1997) esti-
mated grain size from TM data over the Haute Savoie region of the French
Alps using a model for the bi-directional reflectance of snow.

Estimates of grain size from Landsat TM rely on bands 4 and 5, which
span the wavelength ranges 0.76-0.90 μm and 1.55-1.75 μm. Band 4 is only
modestly sensitive to grain size and band 5 effectively saturates near 0% re-
fectance once the snow grain size reaches r~250 μm. Therefore, robust
measures of grain size are more tractable with an instrument that covers at
high spectral resolution the wavelength range 1.0 ≤ λ ≤ 1.3 μm, where the
spectral reflectance of snow is most sensitive to grain size.

The advent of imaging spectrometers, such as the Airborne Visible In-
frared Imaging Spectrometer (AVIRIS), facilitated development of spectros-
copy models that improved retrievals of those properties, of snow and other
surface covers, mapped previously with multispectral instruments and allowed
the retrieval of previously inaccessible properties (Green et al., 1998). AVIRIS
measures reflected radiance in the wavelength range $0.4 \leq \lambda \leq 2.5 \mu m$ with 0.01 $\mu m$ spectral resolution and with a nominal spatial resolution of 20 m.

The spectral range and high spectral resolution of imaging spectrometers provide leverage for mapping a larger set of snow cover properties than coarser resolution instruments (Painter et al., 1998; Nolin and Dozier, 2000; Green et al., 2002). In the coming decades, a suite of airborne and spaceborne imaging spectrometers will provide extensive regular coverage of seasonally snow-covered regions.

Initial retrievals from imaging spectrometers focused on subpixel snow covered area and grain size (Nolin and Dozier, 1993; Nolin et al., 1993). Painter et al. (1998) demonstrated that subpixel SCA mapping improved if the snow endmember was allowed to vary to match the spectral shape of the pixel's snow reflectance. More recent models infer snow liquid water content (Green et al., 2002) and snow algae concentration (Painter et al., 2001).

2.1 Snow-Covered Area

Imaging spectroscopy enables mapping snow cover at the subpixel scale via spectral mixture analysis (SMA), a method of inverting multispectral and hyper-spectral radiance or reflectance data for the subpixel coverage of snow, vegetation, rock, and other surfaces through assumptions relating spectral and spatial fractions (Adams et al., 1993; Mertes et al., 1993; Roberts et al., 1998; Okin et al., 2001).
Nolin et al. (1993) first demonstrated SMA for subpixel snow cover mapping. They modeled two AVIRIS radiance datasets with single endmember suites of (a) snow, rock/soil, water, and vegetation, and (b) snow, shade, and vegetation. The spectral band subset consisted of 18 bands: 3 in visible wavelengths (0.46, 0.55, 0.66 μm), 3 in the near-infrared (0.72, 0.85, 0.94 μm), and 12 spanning the 1.03 μm ice absorption feature.

Painter et al. (1998) used five snow endmembers with grain sizes ranging from 120-500 μm, selected from AVIRIS imagery to produce an optimized map of subpixel SCA. SMA was performed on the entire AVIRIS band set (omitting those bands in water vapor absorption wavelengths) with snow endmembers of each grain size coupled with a rock and a vegetation endmember. Selection of the grain size spectral mixture model with the smallest pixel-by-pixel error produced the optimized map of subpixel SCA. The spectral reflectance of snow decreases with increasing grain size because of an increased path length over which photons may be absorbed. Therefore, varying grain sizes in a scene translate into variability of spectral reflectance, and multiple snow endmembers of varying grain size are necessary to characterize the spectral space of a scene and, in turn, characterize subpixel SCA.

2.2 Snow Grain Size

Nolin and Dozier (1993) described a method for remotely sensing grain size using the band depth of the ice absorption feature at λ = 1.03 μm. The
method was sensitive to sensor noise and required solid knowledge of the solar and viewing geometry. Subsequently, they presented a more robust algorithm (Nolin and Dozier, 2000) that uses the integral across the entire $\lambda = 1.03 \mu m$ absorption feature, which is scaled spectrally by its continuum.

Green et al. (2002) used AVIRIS data to map the solid, liquid, and vapor phases of water by analyzing and distinguishing among the absorption caused by the different phases of water at 0.94 $\mu m$ (vapor), 0.98 $\mu m$ (liquid), and 1.03 $\mu m$ (solid). They quantified the distribution of optical path lengths for ice (related to grain size) and liquid water (related to surface liquid water content).

All previous grain-size algorithms have the constraint that each pixel analyzed must have complete snow cover. The model in this chapter accommodates the mixed-pixel problem by determining the grain size of the fractional snow cover as well as the fractional snow cover itself.

3 Model Description

The multiple endmember snow covered area and grain size (MEMSCAG) model automatically maps subpixel SCA and subpixel grain size simultaneously using spectral mixture analysis coupled with a radiative transfer model. MEMSCAG is an application of the multiple endmember spectral mixture analysis (MESMA) method (Roberts et al., 1998). This method of SMA
allows the number of endmembers and the endmembers themselves to vary pixel-by-pixel and thereby address sub-scene spatial heterogeneity. Okin et al. (2001) used MESMA to map vegetation characteristics in semi-arid regions and Collins et al. (2001) used a variation of MESMA on thermal imaging spectrometer data.

3.1 **Snow Library Endmembers**

An endmember is the spectral reflectance of a pure surface cover. MEMSCAG uses a snow spectral library generated with the discrete-ordinates radiative transfer model DISORT (Stamnes et al., 1988). I modeled snow reflectance spectra for monodispersions of spheres of radii 10-1100 \( \mu m \) at 10 \( \mu m \) resolution. I calculated the single scattering properties at each AVIRIS band for these ice spheres using the Mie scattering theory (Mie, 1908; Nussenzveig and Wiscombe, 1980; Wiscombe, 1980). The scattering properties consist of the single scattering albedo (\( \omega \)), the extinction efficiency (\( Q_{ext} \)), and the moments of the Legendre polynomials used to approximate the scattering phase function. In this work, I used 20 Legendre moments and a semi-infinite snowpack with an optical depth of 4000 at all wavelengths. For each date, I estimated the proportions of direct and diffuse spectral irradiance using atmospheric transmission model *Santa Barbara DISORT Atmospheric Radiative Transfer* (SBDART) model (Ricchiazi et al., 1998) (Figure 2). These properties initiated DISORT to compute the angular distribution of
spectral reflected intensity and the bidirectional reflectance factor for each grain size. A snow spectral library was generated for each AVIRIS acquisition, varying according to the respective solar geometry and diffuse and direct components of irradiance. Each snow spectral library had 110 spectra. This configuration of the model does not use a digital elevation model. Hence, I model only the zenith spectral bidirectional reflectance factor, corresponding to \( BRF(0,0) \). The validity of the use of the nadir BRF is tested in Chapter 5. Figure 3 shows a subset of the snow spectral library.

3.2 Rock, Soil, Vegetation, and Lake Ice Endmembers

With a field spectroradiometer (Analytical Spectral Devices, 2001), I collected 60 spectra for vegetation, rock, soil, and lake ice in the Sierra Nevada. The spectra were collected with solar zenith angles of 20-50° and nadir views. BRF for these spectra were calculated using the method described in Chapter 2. From the 0.003-0.010 \( \mu m \) sampling resolution, I convolved these spectra to the AVIRIS bandpasses using the AVIRIS band center and a Gaussian pass filter with the AVIRIS full-width half maximum. The complete spectral library, snow and non-snow, thus consists of 170 spectral endmembers. Figure 4 shows a subset of the non-snow spectral library.

3.3 Retrieval of Apparent Surface Reflectance

MEMSCAG analyzes apparent surface reflectance spectra (\( \text{ASRF} \)) which is the ratio of the radiance \( L \) measured at the sensor to the radiance
from a completely reflecting Lambertian target given the irradiance, \( E_{\lambda}(\theta_0, \phi_0) \), on an infinite level surface under the atmospheric conditions and solar geometry, \( \theta_0, \phi_0 \), at the time of the acquisition:

\[
(1) \quad ASR_{\lambda} = \frac{L_{\text{AVIRIS},\lambda}}{(E_\lambda(\theta_0, \phi_0)/\pi)} = \frac{\pi L_{\text{AVIRIS},\lambda}}{E_\lambda(\theta_0, \phi_0)}
\]

By definition, the \( ASR \) represents the bidirectional reflectance factor for those surfaces that are level.

I inverted AVIRIS calibrated radiance for apparent surface reflectance using a nonlinear least squares water vapor fitting model (Green et al., 1993) paired with the atmospheric transmission model MODTRAN 3.0 (Berk et al., 1989). This algorithm accounts for atmospheric spatial heterogeneity by solving for the atmospheric conditions pixel-by-pixel from the AVIRIS radiance data themselves and computes \( ASR_{\lambda} \) from AVIRIS data with the following equation:

\[
(2) \quad ASR_{\lambda} = 1/[\left\{ (F_0 T_d T_u / \pi)/(L_{\text{AVL},\lambda} - F_0 r_a / \pi) \right\} + S]
\]

where \( F_0 \) is the exoatmospheric solar irradiance, \( T_d \) is the downward direct and diffuse transmittance of the atmosphere, \( T_u \) is the upward total atmospheric transmittance to the AVIRIS, \( L_{\text{AVL},\lambda} \) is the total upwelling spectral radiance at AVIRIS, \( r_a \) is the atmospheric reflectance, and \( S \) is the albedo of the atmosphere above the surface. This model is run as a series of FORTRAN 77 programs.
3.4 Spectral Mixture Analysis

Linear SMA is based on the assumption that the radiance measured at the sensor is a linear combination of radiances reflected from individual surfaces. The linear assumption is appropriate for spatial scenarios such as snow and rock cover above timberline where the surface is near planar. Nonlinear SMA, which accounts for multiple scattering between surfaces, before is necessary when the surface has a structure, such as vegetation that reflects and transmits radiation to the snow or soil substrate and other vegetation (Roberts et al., 1993).

Spectral mixture analysis with AVIRIS data is based on a set of simultaneous linear equations:

\[ ASR_\lambda = \sum_{i=1}^{N} F_i R_{\lambda,i} + e_\lambda \]

where \( F_i \) is the fraction of endmember \( i \), \( R_{\lambda,i} \) is the reflectance of endmember \( i \) at wavelength \( \lambda \), \( N \) is the number of spectral endmembers, and \( e_\lambda \) is the residual error at \( \lambda \) for the fit of the \( N \) endmembers (Gillespie et al., 1990). I solve the system of equations with the modified Gram-Schmidt orthogonalization (Golub and Van Loan, 1996) method.

The residual error is a rearrangement of the linear mixture model:

\[ e_\lambda = ASR_\lambda - \sum_{i=1}^{N} F_i R_{\lambda,i} \]
Analysis of residuals reveals the spectral regions of poor modeling and can be useful for separating near-degenerate spectra (Roberts et al., 1993).

The root-mean-squared-error provides a spectrum-wide measure of goodness-of-fit for a mixture model:

\[
RMSE = \left( \frac{1}{M} \sum_{n=1}^{M} e_n^2 \right)^{1/2}
\]

\(M\) is the number of imaging spectrometer bands used in the SMA. Painter et al. (1998) and Roberts et al. (1998) used the \(RMSE\) as a fundamental metric for optimizing selection of model results in the multiple end-member spectral mixture analysis approach.

The estimate of subpixel snow-covered area comes from the shade-normalized snow fraction:

\[
f_s = \frac{F_s}{\sum_{p=n,s,v} F_p} = \frac{F_s}{1 - F_{\text{shade}}}
\]

where \(F_s\) is the snow spectral fraction, \(F_p\) are the physical spectral fractions (non-shade), and \(F_{\text{shade}}\) is the spectral fraction of photometric shade. Normalizing by the additive complement of the shade fraction accounts for topographic effects on irradiance (Adams et al., 1993).

3.5 MEMSCAG Model

MEMSCAG analyzes individual linear spectral mixtures for each permutation of two, three, or four endmembers of the spectral library, in which no more than one endmember from a surface cover class is present \(i.e.\) at most
one snow endmember). A model is considered valid if it meets established constraints on spectral fractions, RMSE, and consecutive residuals, as follows: spectral fractions must lie in the range \([-0.01, 1.01]\), RMSE < 2.5\%, and no seven consecutive residuals may exceed 2.5\%. For each \(n\)-endmember suite of models that meet the constraints for a pixel, MEMSCAG selects the SCA and grain size values associated with the model with the least RMS error. MEMSCAG then attributes to the pixel the snow-covered area and snow grain size of the model that has the lowest number of endmembers, according to the MESMA modeling approach (Roberts et al., 1998). Additional endmembers will force modeling error to decrease or not change (Golub and Van Loan, 1996) but the modeling constraints are considered to be sufficient to determine a spatially significant model without the addition of unnecessary or trivial endmembers.

For example, suppose that several 2-endmember models and several 3-endmember models meet the modeling constraints for a given pixel. Among the 2-endmember models, the subpixel SCA and grain size of the model with the least RMSE is attributed to the final 2-endmember images. Among the 3-endmember models, the subpixel SCA and grain size of the model with the least RMSE is likewise attributed to the final 3-endmember images. For the given pixel, however, the final values come from the 2-endmember model (i.e. the model with the least number of endmembers). MESMA chooses the
model with the least number of endmembers because as the number of endmembers increases, models may meet the constraints with decreasing spatial significance. Figure 5 shows the flow of MEMSCAG.

For slopes facing the sun, I loosened the spectral metrics to account for the increase in irradiance, reflected radiance, and in turn, apparent surface reflectance. For example, the apparent surface reflectance of a snow slope facing the sun as viewed from AVIRIS will exceed that of the snow endmember generated with the radiative transfer model. Therefore, the constraints on spectral fractions must be loosened to greater than 1.0. Furthermore, it is necessary to relax the spectral constraints for those surfaces that are not included in the library. Spectral endmembers for analogous surfaces may be sufficient to determine the spatial extent of such surfaces under loosened constraints. Without topographic correction, however, MEMSCAG is likely to be more sensitive to the effects of snow impurities.

MEMSCAG incorporates the following assumptions: (i) the imaged portion of the bidirectional reflectance distribution function (BRDF) of snow is identical to the zenith bidirectional reflectance factor (BRF) for the solar geometry and atmospheric conditions at the time of each acquisition, (ii) the effects of impurities and the effects of thin snow on snow spectral reflectance are not separable and that these effects do not impact retrievals of subpixel SCA and grain size, (iii) linear spectral mixture analysis is valid for hyperspec-
tral scenes of alpine terrain, and (iv) liquid water in the snowpack does not affect the retrievals of subpixel snow-covered area and snow grain size.

I evaluate the first assumption in Chapter 5 of this dissertation. Assumption (iii) cannot be adequately addressed with currently available digital elevation data. Once more accurate digital elevation data are available, robust incorporation of impure/thin snow endmembers will facilitate the discrimination of dirty/shallow snow from less-illuminated slopes which have similar spectral signatures when imaged from AVIRIS. At the time or writing, high quality digital elevation data from the NASA Shuttle Radar Topography Mission (SRTM) have come available for California. Assumption (iii) has been validated in areas where trees are absent or sparse. (Nolin et al., 1993; Rosenthal and Dozier, 1996; Painter et al., 1998) However, we know that vegetated regions exhibit nonlinear mixing. (Roberts et al., 1993; Ray and Murray, 1996) Therefore, nonlinear mixture analysis may become necessary as canopy density increases. Robert O. Green is investigating the sensitivities associated with assumption (iv) in his dissertation. Subsequent to that work, he and I will perform detailed sensitivity studies of the impact of liquid water in the snowpack on retrievals from several other spectroscopy models. However, because MEMSCAG incorporates most AVIRIS bands in the visible and near-infrared spectrum, retrievals should be insensitive to subtle shifts in the shape of the reflectance spectrum.
4 Validation

I used three AVIRIS images of Mammoth Mountain, CA (Figure 6) for validation of the estimates of sub-pixel snow-covered area and snow grain size; April 5, 1994, March 29, 1996, and April 29, 1998. Snow conditions on these dates ranged from fresh, fine-grained to coarse, melting snow. The imaged region of Figure 6 is about 11 by 9 km with a 17-m ground instantaneous field of view. Table 1 shows the solar ephemeris data, atmospheric conditions, and snow conditions for each of the AVIRIS scenes.

Mammoth Mountain lies on the crest of the Sierra Nevada at latitude 37°37' N, longitude 119°02' W, and elevation 3600 m. The snow depths and snow water equivalents at the Mammoth Mountain Cooperative Snow Study Site (http://neige.bren.ucsb.edu/) at 2960 m were large enough to make the snowpack optically semi-infinite on all image dates.

4.1 Snow-Covered Area

High-resolution color-infrared photographs accompany each AVIRIS acquisition. Their spatial resolution is approximately 2 m, compared with a spatial resolution of ~17 m for AVIRIS flying above a surface elevation of 3000 m. I digitized the photographs at 600 dpi in color and co-registered each to its respective AVIRIS scene. From each scene, I randomly picked 20 subregions of size 22 x 22 pixels from the AVIRIS scene and 210 x 210 pixels from the color-infrared photograph. The subregions from the photograph
were manually classified to snow/non-snow using subregion-specific thresholding. Figure 7 shows example classified subregions from the AVIRIS scenes and color-IR photographs. Co-registration errors for photographs to the AVIRIS base scene were approximately 1 AVIRIS pixel, i.e., 17 m.

Table 1. Solar ephemeris and atmospheric conditions for three AVIRIS acquisitions at Mammoth Mountain, CA, with meteorological data from the Mammoth Mountain Cooperative Snow Study Site, located at 2960 m on its north slope.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_0$, solar zenith angle</td>
<td>31°</td>
<td>37°</td>
<td>23°</td>
</tr>
<tr>
<td>$\phi_0$, solar azimuth, clockwise from north</td>
<td>179°</td>
<td>209°</td>
<td>166°</td>
</tr>
<tr>
<td>Air temperature at 2960 m elevation</td>
<td>4.0°C</td>
<td>-7.3°C</td>
<td>7.6°C</td>
</tr>
<tr>
<td>Precipitable water vapor retrieved from AVIRIS data</td>
<td>21 mm</td>
<td>26 mm</td>
<td>16 mm</td>
</tr>
<tr>
<td>Snow condition</td>
<td>Fresh, fine-grained</td>
<td>Fresh, fine-grained</td>
<td>Melting, coarse-grained</td>
</tr>
<tr>
<td>Snow depth</td>
<td>1.6 m</td>
<td>4.1 m</td>
<td>5.0 m</td>
</tr>
<tr>
<td>Snow water equivalence</td>
<td>0.6 m</td>
<td>1.6 m</td>
<td>2.4 m</td>
</tr>
</tbody>
</table>

4.2 Snow Grain Size

Snow samples were collected within one hour of the time of acquisition for each image; three samples on April 5, 1994, seven samples on March 29, 1996, and nine samples on April 29, 1998 (Figure 8). Snow samples came
from all cardinal aspects and included level areas. In 1994 and 1996, we located the sampling locations on topographic maps. In 1998, we used differentially corrected Global Positioning System measurements. I estimated the location accuracy of the 1994 and 1996 samples to be ~40 m and the location accuracy of the 1998 samples was ~2 m. The samples were prepared for stereological analysis in a cold room immediately after collection in order to avoid further metamorphism. I analyzed each sample for snow grain radius using stereological techniques for plane sections. Chapter 4 describes the method in greater detail.

5 SENSITIVITY STUDY

I performed a modeling study to determine the inherent sensitivity of retrievals of subpixel SCA and grain size to the fractional snow cover, to the complementary surface cover type (vegetation or rock), and to the snow grain size.

I generated 6 synthetic hyperspectral scenes, consisting of spectral mixtures of snow and rock for snow grain sizes 50, 250, and 1000 μm, and snow and vegetation for snow grain sizes 50, 250, and 1000 μm, for snow fractions ranging from 0 to 100% and the rock or vegetation fraction ranging as the additive complement to 100%. I used a granite spectrum for the rock endmember and a lodgepole pine spectrum for the vegetation endmember,
both from the spectral library. Each “scene” was a single row of 101 samples and 224 bands. Figure 9 shows a subset of the spectra in the 50 μm rock and vegetation synthetic scenes, incremented by 10% mixtures.

Figure 10 demonstrates that MEMSCAG is robust in mapping subpixel SCA when the non-snow endmember is rock, for all snow cover fractions and grain size. MEMSCAG grain size retrievals are likewise robust for most snow cover fractions. However, the sensitivity of the grain size retrievals to snow cover depends on grain size. Grain size retrievals were accurate down to snow fractions of 15% for the 50 μm grain size, 45% for the 200 μm grain size, and 55% for the 1000 μm grain size. For a snow cover fraction of 10%, retrieval errors are 10 μm for the 50 μm grain size image, 20 μm for the 200 μm grain size image, and −200 μm for the 1000 μm grain size image.

Figure 11 shows that as the vegetation fraction increases past 20% (as the snow fraction decreases below 80%), MEMSCAG maintains stable estimation of subpixel SCA, but the grain size estimate becomes increasingly erroneous, especially for larger grain sizes. The discrepancy between MEMSCAG results for rock and vegetation mixtures most likely derives from the liquid water absorption present in the vegetation spectrum at λ = 0.98 μm and absent from the rock spectrum. As the vegetation fraction increases, the liquid water absorption increasingly draws the 1.03 μm ice absorption feature to-
ward shorter wavelengths. This makes discrimination of the spectral mixture of snow and its associated grain size more difficult.

6 Results

Below, I present the results of MEMSCAG analysis of the three AVIRIS acquisitions for sub-pixel snow fraction and snow grain size on April 5, 1994 (Figure 12), March 29, 1996 (Figure 13), and April 29, 1998 (Figure 14). The April 5, 1994 scene shows high snow cover above timberline and patchy snow cover below timberline. Because vegetation canopies obscure the view of the complete ground cover from AVIRIS, spectral mixture analysis can only produce maps of viewable snow cover. Inference of the snow cover below the canopy will rely on empirical relationships developed from field and model data (*Robert E. Davis*, unpublished data). Snow cover was nearly complete above timberline on March 29, 1996. Snow cover was likewise nearly complete above timberline on April 29, 1998, an El Niño year with heavy snowfall throughout the Sierra Nevada.

*MEMSCAG* maps smaller grain sizes at high elevations with increasing grain size at lower elevations, particularly in the April 5, 1994 scene (Figure 12). Grain sizes were uniformly small in the March 29, 1996 acquisition because of recent snowfall and consistently cold temperatures (Figure 13). Grain sizes were large in the April 29, 1998 acquisition due to rapid metamor-
phism driven by a period of high temperatures and strong irradiance (Figure 14). All scenes exhibit consistent spatial continuity in grain size for those regions free of vegetation. However, MEMSCAG maps vegetated regions as having much larger grain radii than vegetation-free regions, and in a proportional relationship with vegetation density. The errors are consistent in magnitude but opposite in sign with those errors due to increasing vegetation fraction in the sensitivity study. It is unclear why the errors are opposite in sign, but may result from increased shading of the snow substrate with increasing canopy density, an effect that the sensitivity study did not model. A more complete treatment would use a hybrid geometric optics/radiative transfer model (Ni et al., 1999) to determine the relative spectral fractions of directly illuminated snow, directly illuminated vegetation, and vegetation-shaded snow. While grain size variability may indeed contribute to the apparent variability that the images show, the combined effects of absorption by liquid water in the vegetation and vegetation shading of the snow cover are more likely to be confounding the spectroscopic analysis.

6.1 SCA Validation

In Figure 15, I compare the MEMSCAG-derived snow fraction with the photographic derived snow fraction for all years. For all years, the root mean squared error in snow-covered area was 4.0%. The respective RMS errors for
the 1994, 1996, and 1998 acquisitions were 3.5%, 4.5%, and 4.0%. The linear regression line for these data is given by

\[ \text{SCA}_{\text{MEMSCAG}} = 0.97 \text{SCA}_{\text{PHOTO}} + 0.0039. \]

The 95% confidence intervals for slope and y-intercept were [0.94,1.01] and [-0.02,0.03], respectively. Therefore, at a 95% confidence level, the slope and y-intercept were not significantly different from 1.0 and 0.0 respectively and the MEMSCAG SCA retrieval had comparable accuracy to that of high spatial resolution aerial photographs.

6.2 Grain Size Validation

Given the location uncertainty of snow samples discussed above and co-registration errors of ~1 pixel, it was necessary to validate grain size with search windows about the pixels containing the UTM coordinates of the snow sample locations. Below, I present results for mean values and closest values from 3-by-3 search windows. In Figure 16, I compare the mean and closest \textit{MEMSCAG} derived snow grain radii from 3 by 3 windows with those derived from stereological analysis, respectively. The vertical bars in Figure 16 represent the spans of grain radii in the 3 by 3 windows about the apparent points of collection. Because of errors in grain size associated with vegetation density, I have eliminated pixels from each search window if the respective vegetation fraction \( f_v > 20\%. \)
The best-case root mean squared error for snow grain size retrieved from a 3-by-3 window of AVIRIS data for the combined set of three images is 48 μm, with respective errors of 12 μm (April 5, 1994), 45 μm (March 29, 1996), and 57 μm (April 29, 1998). Using the mean grain size from the 3-by-3 window around the apparent pixel of the snow sample, the RMS error for grain size for the combined set of three images was 74 μm, with respective errors of 58 μm (April 5, 1994), 77 μm (March 29, 1996), and 76 μm (April 29, 1998). The scale of spatial variability of snow grain size can be quite small and depends on the scale of topographic variability that, at times, may be of the order of a 17 m pixel or smaller.

Figure 17 shows the grain size errors plotted against the stereology-derived grain sizes for the 3-by-3 windows. The mean value from MEMSCAG usually underestimated the grain size retrieved from stereological analysis. No relationship between residuals and grain size existed but the greatest errors occurred for grain sizes between 250 and 450 μm, reaching Δr = −180 μm. The residuals for the closest values from MEMSCAG were much closer to 0, particularly for stereology grain sizes greater than 200 μm. The closest grain size tended to underestimate the stereology grain size for small grain sizes and overestimate for large grain sizes.
More pertinent is the effect that grain size errors have on inferred albedo. Figure 18 shows the associated errors in albedo ($\Delta \alpha$) calculated using the respective atmospheric conditions and solar ephemeris.

For the mean and closest grain sizes, $\Delta \alpha$ decreases from a range of [0.0, 0.05] at small grain sizes to less than 0.01 for grain sizes greater than 600 $\mu$m. For the closest grain size, only 2 points exceeded $\Delta \alpha$ of 0.03 and these corresponded to grain sizes less than 150 $\mu$m. These results suggest that MEMSCAG maps albedo with a decreasing error as the snow grain size increases. The respective root mean squared errors for albedo for the mean and closest grain sizes were 0.025 and 0.018.

7 CONCLUSIONS

I have presented the multiple endmember snow covered area and grain size (MEMSCAG) model for imaging spectrometer data. The model used MESMA coupled with a radiative transfer model for the spectral reflectance of snow to map subpixel snow covered area and the grain size of the subpixel snow cover simultaneously without topographic data. The model infers a first-order approximation of snow albedo from the grain size estimate and solar geometry. The snow spectral library for mixture analysis consists of spectra generated with the discrete ordinates method for snow with sphere radii ranging from 10 to 1100 $\mu$m.
I validated MEMSCAG with three AVIRIS acquisitions that had snow conditions ranging from fresh, fine-grain snow to coarse, melting snow. For all acquisitions, MEMSCAG had a RMSE for SCA of 4.0% relative to SCA determined with high-resolution photographs. For grain radii ranging from 80 to 750 μm as determined from stereological analysis of snow samples, MEMSCAG had a RMSE for grain size of 74 μm for the mean of 3-by-3 AVIRIS windows. For the closest grain size within the 3-by-3 AVIRIS windows, MEMSCAG had a root mean squared error of 48 μm. The RMSE for inferred albedo were 0.025 and 0.018 respectively for the mean and closest grain sizes in the 3-by-3 windows.

A sensitivity study shows that MEMSCAG grain size estimates for a nadir view should have errors less than 10% for SCA down to 0.2 when the non-snow constituent is rock and down to between 0.5 and 0.75 depending on particle size when the non-snow constituent is vegetation.

Because the scan angle of AVIRIS is only 15° either side of nadir and the solar zenith angles for the acquisitions presented here were relatively small (23 ≤ θ₀ ≤ 37°), the local view and solar zenith angles generally lie within 40° of the surface normals on Mammoth Mountain. As will be shown in Chapter 4, MEMSCAG has little sensitivity to the anisotropic reflectance of snow for these view angles. Therefore, we may apply MEMSCAG to AVIRIS data under these viewing and illumination conditions with small SCA and
grain size errors. However, future spaceborne imaging spectrometers will have larger view zenith angles and will image regions under larger solar zenith angles. For those cases, we will expand MEMSCAG to include ancillary topographic data and a bidirectional reflectance spectral library.

Because MEMSCAG utilizes the entire AVIRIS spectrum rather than a specific absorption feature, it is applicable to data from other spectroradiometers that have bands that are sensitive to grain size, such as the EOS Terra Moderate Resolution Spectroradiometer (MODIS). The MODIS band centered at 1.24 µm provides sensitivity to grain size necessary for MEMSCAG to map subpixel SCA and grain size. However, because AVIRIS has many more bands than MODIS, retrievals with MODIS will likely have greater sensitivity to individual band noise than with AVIRIS. Furthermore, the 1 km footprint increases the likelihood of spatial mixing hurdles such as multiple slopes and aspects, discrete surface grain sizes, or vegetation types within a pixel. I will address these issues in future research.
**Figures**

Figure 1. Directional-hemispherical reflectance of snow for grain radii of 50 to 1000 μm
Figure 2. Spectral diffuse and direct components of irradiance used to model the bidirectional reflectance of snow in the spectral libraries.
Figure 3. Subset of snow spectral library for April 29, 1998 for varying grain size ranging from 10 to 1100 μm.
Figure 4. Subset of non-snow spectral library, for rock/soil (red) and vegetation (green).
Figure 5. Flow of MEMSCAG, with the tightest spectral metrics at right (for slopes with greater irradiance than level surfaces, these constraints are relaxed)
Figure 6. AVIRIS scene (11x9 km, ~17 m IFOV) of Mammoth Mountain, CA acquired on March 29, 1996
Figure 7. Sub-regions of co-registered aerial photographs (left) and AVIRIS snow covered area images (right). The black regions in the top images were exposed rock and in the bottom images were conifer forest.
Figure 8. Locations of snow samples acquired for validation on Mammoth Mountain
Figure 9. Example of spectral mixtures used in sensitivity study, for the 50 μm snow endmember, rock endmember (top), and vegetation endmember (bottom)
Figure 10. Sensitivity modeling results for \( r = 50, 250, \) and 1000 \( \mu \text{m} \) snow endmembers and a granite endmember
Figure 11. Sensitivity modeling results for $r = 50, 250, \text{ and } 1000$ $\mu m$ snow endmembers and a vegetation endmember.
AVIRIS: Mammoth Mountain, CA: April 5, 1994

Figure 12. Snow covered area and grain size results from MEMSCAG for April 5, 1994
Figure 13. Snow covered area and grain size results from MEMSCAG for March 29, 1996
Figure 14. Snow covered area and grain size results from MEMSCAG for April 29, 1998
Figure 15. Validation of sub-pixel snow covered area for 1994, 1996, and 1998 MEMSCAG retrievals
Figure 16. MEMSCAG grain size validations with mean of 3×3 window and closest grain size of 3×3 window. The dashed line represents the 1:1 line.
Figure 17. MEMSCAG grain size residuals versus stereology grain size for the mean grain sizes in 3x3 windows and the closest grain sizes in 3x3 windows.
Figure 18. MEMSCAG albedo errors versus stereology grain size for mean grain sizes in 3×3 windows and closest grain sizes in 3×3 window.
Chapter II

Automated Spectro-Goniometer: A Spherical Robot for the Measurement of the Bidirectional Reflectance of Snow
ABSTRACT

I describe an automated spectro-goniometer (ASG) that rapidly measures the spectral bidirectional reflectance factor (BRF) of snow across the wavelength range $0.4 \leq \lambda \leq 2.5 \mu m$. The ASG has a two-link spherical robot coupled with a field spectroradiometer. Few measurements of snow's BRF exist in the literature, in part caused by a lack of a portable instrument capable of rapid, repeatable sampling. The ASG samples the BRF at arbitrary angular resolution and 0.5 Hz sampling rate. The arm attaches to the fixed-point frame 0.65 m above the surface. With vertical and oblique axes, the ASG places the end of the optical cable of the field spectrometer at any point on the hemisphere above a snow target. The kinematics of the ASG is derived using Rodrigues' formula for the 2 degree of freedom arm. I describe the inverse kinematics for the ASG and solve the inverse problem from a given view angle to the necessary rotation about each axis. Its two-dimensional hemispheric sampling space facilitates the measurement of spectral reflectance from snow into any direction. With a total weight of 41 kg, the ASG is easily portable by an individual via sled.
1 INTRODUCTION

Distributed snowmelt models use snow-covered area, grain size, albedo, and snow water equivalent for input and validation. Estimates of all but snow water equivalent are available from optical (solar) remote sensing. Algorithms for retrieving these snow properties have been based on the simplifying assumption that snow reflects solar radiation isotropically. We know that snow has an anisotropic reflectance distribution that depends on wavelength and grain morphology (Warren, 1982; Leroux et al., 1998), but our understanding is hampered by the lack of measurements that cover the full spectral and angular distribution. Further, our use of the reflectance distribution in remote sensing requires extensive computational power and storage.

Satellites sample only a part of this spectral bidirectional reflectance factor which has been convolved with interactions on heterogeneous terrain. The assumption of isotropic reflectance will frequently produce errors in quantitative retrievals of snow physical properties (Jin and Simpson, 1999).

Recent increases in computational power and data storage have enabled scientists to develop and refine quantitative retrievals of snow properties by incorporating the anisotropic reflectance distribution of snow. Existing models for the BRF of snow (Stamnes et al., 1988; Leroux et al., 1998) have
not been compared with a comprehensive sampling of the \textit{BRF} and its dependence on solar geometry, wavelength, grain size distribution, grain morphology, liquid water content, and impurities. To meet this need, we have developed an automated spherical robot for use with a high spectral resolution field spectroradiometer (Figure 19).

In this chapter, I describe the automated spectro-goniometer (ASG) in terms of its constraints, spectral range and resolution, kinematics, sampling protocols, and specifications. In this section, I define \textit{BRDF} and \textit{BRF}, review previous measurements of snow \textit{BRF}, and review other available goniometers for measurements of angular reflectance. In Section 2, I present system constraints and the spectrometer specifications. Section 3 describes the kinematics of the Automated Spectro-Goniometer and their numerical inversion. In Section 4, I describe the motion control configuration. In Section 5, I describe the angular calibration of the ASG. Section 6 presents the calculation of \textit{BRF} from the ASG. Preliminary results are given in Section 7 and Section 8 presents conclusions and a discussion of future work.

\subsection{Bidirectional Reflectance Factor}

The bidirectional reflectance distribution function (\textit{BRDF}) of a surface describes the magnitude of reflected radiance into a direction relative to the irradiance from a given direction (Figure 20):
(8) \[
BRDF_{\lambda}(\theta_o, \phi_o; \theta_r, \phi_r) = \frac{L_{\lambda}(\theta_r, \phi_r)}{\mu_0 E_{\lambda}(\theta_o, \phi_o)}
\]

where \(L_{\lambda}\) is spectral radiance (W m\(^{-2}\) sr\(^{-1}\) \(\mu m\)^{-1}), \(E_{\lambda}\) is spectral irradiance (W m\(^{-2}\) \(\mu m\)^{-1}) onto a plane normal to the beam, \(\theta_o\) is zenith angle of irradiance, \(\phi_o\) is the azimuth angle of irradiance, \(\theta_r\) is the zenith angle of reflected light, \(\phi_r\) is the azimuth angle of reflected light and \(\mu_0\) is the cosine of \(\theta_o\). \(BRDF\) has units of inverse steradians (sr\(^{-1}\)).

The commonly used, dimensionless form of the distribution is the \textit{bidirectional reflectance factor (BRF)}:

(9) \[
BRF_{\lambda}(\theta_o, \phi_o; \theta_r, \phi_r) = \pi BRDF_{\lambda}(\theta_o, \phi_o; \theta_r, \phi_r)
\]

The \(BRF\) is the ratio of radiance reflected into the direction \(\theta_r, \phi_r\) to the radiance that would be reflected into any direction by a perfectly reflecting Lambertian (i.e. isotropically reflecting) surface for the given irradiance.

For energy balance and snowmelt modeling, the important optical property is the spectral albedo, \(A_{\lambda}\). It is expressed as the integral of the \(BRF\) over the upper hemisphere:

(10) \[
A_{\lambda}(\theta_o, \phi_o) = \frac{1}{\pi} \int_0^{\pi/2} \int_0^{2\pi} BRF_{\lambda}(\theta_o, \phi_o; \theta_r, \phi_r) \sin \theta_r \cos \theta_r \ d\theta_r \ d\phi_r .
\]

\(A_{\lambda}\) depends on \(\phi_o\) if the snow surface has regularly oriented grains or surface features (e.g., sastrugi) that affect the structure of the \(BRF\) which is not symmetric across the solar principal plane. If the \(BRF\) does not depend on \(\phi_o\), the albedo is given by
\[ A_\lambda(\theta_s) = \frac{1}{\pi} \int_0^{2\pi} \int_0^{\pi/2} BRF_{\lambda}(\theta_0; \theta_s, \phi_s) \sin \theta_s \cos \theta_s \, d\theta_s \, d\phi_s. \]

1.2 Previous Snow BRF Measurements

The literature describing the BRF of snow is sparse. Previous efforts to characterize the BRF of snow have been limited with respect to one or more of angular resolution and range, spectral resolution and spectral range, speed of acquisition, and quantitative documentation of snow properties (O'Brien, 1975; Steffen, 1987; Grenfell et al., 1994). Steffen (1987) measured the BRDF of snow for the wavelength range \(0.5 \, \mu m \leq \lambda \leq 0.6 \, \mu m\) for a suite of solar zenith angles and view geometries. Dozier et al. (1988) addressed deficiencies of previous experiments with greater angular sampling and resolution, greater spectral resolution, and more detailed snow characterization. However, the spectrometer they used was sensitive only to \(\lambda = 1.0 \, \mu m\), at the shorter wavelength end of the spectral region in which the reflectance of snow is most sensitive to grain size. Leroux et al. (1998) compared a model with measurements of snow BRF accompanied by detailed quantification of grain size distribution and grain morphology, but they reported measurements only for \(\lambda = 1.68 \, \mu m\).

1.3 Goniometers

Goniometers are instruments that measure angles. The word 'goniometer' is derived from the Greek word 'gonia' meaning angle. Remote
sensing-specific goniometers have been developed primarily for the study of the BRF of vegetative covers. The first goniometric field instrument regularly used for remote sensing and radiative transfer analysis of vegetation was the Portable Apparatus for Rapid Acquisition of Bidirectional Observations of the Land and Atmosphere (Deering and Leone, 1986). The PARABOLA, a dual-axis up- and down-looking 3-band radiometer, rides a tram cableway over a target such as a vegetation canopy. In geometric terms, PARABOLA looks from the center of a hemisphere out to discrete points on the hemisphere. This geometry dictates that PARABOLA views different targets with each pair of zenith and azimuth view angles and so configured, relies on the assumption that the surface cover is spatially homogeneous.

The Remote Sensing Laboratories (RSL) at the University of Zürich developed the field goniometer system (FIGOS) for hyperspectral measurement of the bidirectional reflectance distribution of vegetation and soils (Sandmeier and Itten, 1999). FIGOS has a 2 m radius zenith arc with a tram that carries a GER Corporation GER-3700 spectroradiometer, and an azimuth support track. FIGOS can cover the hemisphere at 15° angular sampling in approximately 18 minutes, making 66 measurements. While the FIGOS weighs 230 kg and requires 2 technicians for assembly and operation, it has the advantages over PARABOLA that it measures the complete optical spectrum at high spectral resolution and views the same target for all spectra.

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Voss et al. (2000) developed an enclosed instrument to measure the in-situ BRDF of surfaces for eight illumination angles from 0 to 65°, three wavelengths (0.475, 0.570, and 0.658 μm), and 100 fixed viewing angles. The instrument has fixed attachment points for view fibers and illuminates the surface with three light-emitting diodes (LEDs) that use separate fibers.

Snyder et al. (1997) used the Spectral Infrared Bidirectional Reflectance and Emissivity (SIBRE) instrument to measure bidirectional thermal emission (3-14 μm) from sand and soil surfaces. SIBRE has a hemispherical positioning system on which a Fourier transform spectrometer rides. The SIBRE has a hemispherical radius of 1.5 m.

Aoki et al. (2000) sampled snow BRF and albedo with a goniostage coupled to a tripod. This instrument is similar to PARABOLA in that it views the surface from the center of the hemisphere and assumes spatial homogeneity. While this instrument is inexpensive to manufacture and deploy, the occultation of many view zeniths and azimuths by the tripod prevents complete BRF sampling.

2 System Description

A comprehensive characterization of snow BRF must address two sets of parameters, one set corresponding to instrumentation and the other corresponding to snow and atmospheric characterization. The instrument parame-
eters consist of spectral range and resolution, acquisition speed, angular range and resolution, angular accuracy, and acquisition speed (if using the sun as an illumination source). Characterization of the snow and atmosphere includes the spectral ratio of diffuse to direct irradiance, grain size distribution, grain morphology and its distribution, snow liquid water content, density, and impurities. This section discusses the instrumentation.

2.1 Field Spectrometer

The core of the Automated Spectro-Goniometer is an Analytical Spectral Devices FieldSpec FR spectroradiometer (Analytical Spectral Devices, 2001). I show the specifications for the ASD-FR in Table 2. Of note here are the complete spectral range (0.35 – 2.5 \( \mu \text{m} \)), high spectral resolution (0.003-0.010 \( \mu \text{m} \)), and the rapid spectrum acquisition (10 spectra / s and an average spectrum automatically recorded at 1 Hz). With the rapid spectrum acquisition, the ASG may access a dense grid of points on the hemisphere above the snow target with little change in solar geometry.

2.2 Configuration

The ASG has two independently controlled motorized arms serially attached to a boom that extends from a circular base (Figure 21). The boom has an attachment point for the first motor and the upper joint housing that encloses an anti-backlash worm wheel, worm-shaft, and ball bearing assembly (Figure 22). Two ball bearings support the worm wheel. The second motor
attaches to the end of the first arm in a lower joint housing (Figure 23). This housing contains the second worm gear, worm-shaft, and a single ball bearing.

The ASD-FR optic cable endpoint attaches to the end of the lower arm of the ASG (Figure 24). The optic cable points at the center of the base circle (discussed in Section 3) from the scanning space of the hemisphere above this center. Brushless servomotors drive rotation about the vertical (-z) and oblique (k) axes, placing the optic cable endpoint at any point on the hemisphere. Hence, the ASG has arbitrary angular range. In design and manufacture, we specified angular accuracy of 2°. The Pittman motor/encoders described below have 2000 counts per revolution which combined with 72 and 95 teeth worm gears provide potential angular resolutions of 0.0025° and 0.0019°.

### Table 2. Specifications for Analytical Spectral Devices FR FieldSpec spectroradiometer

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spectral range</td>
<td>0.35 – 2.5 μm</td>
</tr>
<tr>
<td>Spectral resolution</td>
<td>0.003 μm (0.35 ≤ λ ≤ 1.0 μm)</td>
</tr>
<tr>
<td></td>
<td>0.010 μm (1.0 ≤ λ ≤ 2.5 μm)</td>
</tr>
<tr>
<td>Spectrum sampling rate</td>
<td>10 spectra / s</td>
</tr>
<tr>
<td>Spectrum recording rate</td>
<td>1 spectrum / s (max. rate)</td>
</tr>
<tr>
<td>Weight</td>
<td>7 kg + 2.2 kg (battery) = 9.2 kg</td>
</tr>
<tr>
<td>Optic cable length</td>
<td>2.5 m</td>
</tr>
<tr>
<td>Foreoptics</td>
<td>1°, 4°, 8°</td>
</tr>
</tbody>
</table>
3 Kinematics

In this section, I present the kinematics of the mechanism. The kinematics describe the geometrical properties of the motion of a robotic manipulator (Tsai, 1999). In reference to Figure 2, the kinematics of our 2 degree-of-freedom robot describe the view vector as a function of the joint angles $\theta_1$ and $\theta_2$. The fore-optic mount moves in the hemisphere of constant radius about the target.

The kinematics of the ASG are derived using Rodrigues' formula (Tsai, 1999) for a spherical displacement of a rigid body. The protocol for use of the ASG is determined by a prescribed set, $U$, of viewing angles or view vectors $u$. Hence, we must solve the inverse kinematics problem: given $u$, find $\theta_1$, $\theta_2$ which are the respective rotations about $-z$ and $k$. Except at full reach and full retraction of the ASG arms, there are two solutions to the inverse kinematics problem.

All joint rotations are about axes that pass through the center, $O$, of the hemisphere is coincident with the target. If $k$ is an axis of rotation, the 3 by 3 rotation matrix $R_k$ that represents the rotation about $k$ by an angle $\theta$ is given by

$$R_k = (1-\cos \theta)kk^T + \cos \theta I + \sin \theta [k \times]$$

where
\( k^T \) is the transpose of \( k \), and \( \theta \) is the angle of rotation about \( k \). Equation (12) is Rodrigues’ formula.

Figure 21 depicts the ASG in a general position. However, when both joint angles are zero, the spectrometer foreoptic points in the \(-z\) direction and the second joint axis \( k_0 \) lies in the \( x-z \) plane. From Figure 21,

\[
(14) \quad k_0 = \begin{pmatrix} \sin \beta \\ 0 \\ \cos \beta \end{pmatrix}.
\]

When the ASG elbow rotates \( \theta_2 \), the new orientation of the foreoptic is

\[
(15) \quad R_{k_0}(\theta_2)(-z)
\]

where \( R_{k_0}(\theta_2) \) is the rotation about \( k_0 \) by \( \theta_2 \). When the ASG shoulder rotates \( \theta_1 \) about \(-z\), the orientation becomes

\[
(16) \quad u = R_z(\theta_1)R_{k_0}(\theta_2)(-z)
\]

where the rotation matrices can be computed from equation (12).

We must now solve this equation for \( \theta_1 \) and \( \theta_2 \) given \( u \). Multiply (12) on the left by \( z^T \) to get

\[
(17) \quad z^T u = z^T R_z(\theta_1)R_{k_0}(\theta_2)(-z)
\]

and since \( z^T R_z = z^T \) (\( z \) is a left eigenvector of a rotation about \( z \)), we have
(18) \[ z^T u = z^T R_{k_0}(\theta_2)(-z) \]

which may be solved for \( \theta_2 \).

Substituting (14) into (12) gives:

\[ R_{k_0}(\theta_2)(-z) = -[(1 - \cos \theta_2)k_0^T z + \cos \theta_2 z + \sin \theta_2 [k_0 \times](-z)] \]

(19)

\[ = - \begin{bmatrix} (1 - \cos \theta_2) & 0 & \cos \beta \\ \sin \beta & 0 & \cos \theta_2 \\ \cos \beta & \cos \theta_2 & 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \cos \theta_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \sin \theta_2 & -\sin \beta \end{bmatrix} \]

\[ = (\cos \theta_2 - 1) \begin{bmatrix} \sin \beta \cos \beta \\ 0 \\ \cos^2 \beta \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ \cos \theta_2 \end{bmatrix} - \begin{bmatrix} 0 \\ \sin \theta_2 \sin \beta \\ 0 \end{bmatrix} \]

Combining (16) and (19) gives

(20) \[ u_z = \cos \theta_2 (\cos^2 \beta - 1) - \cos^2 \beta \]

and, in turn

(21) \[ \theta_2 = \cos^{-1}\left(\frac{u_z + \cos^2 \beta}{(\cos^2 \beta - 1)}\right) \]

where \( u_z = Z^T u \). Since there exist two branches to the \( \cos^{-1} \), there exist generally two solutions to choose from at this point in the calculation. A single branch is used throughout a scan except when occlusion by the arm forces a switch to a different branch.

Given \( \theta_2 \), we now solve for \( \theta_1 \). We define \( w \) as

(22) \[ w = R_{k_0}(\theta_2)(-z) \]

and solve
\[ u = R_z(\theta_i)w \]

\[
\begin{bmatrix}
  u_x \\
  u_y \\
  u_z
\end{bmatrix} =
\begin{bmatrix}
  \cos \theta_i & -\sin \theta_i & 0 \\
  \sin \theta_i & \cos \theta_i & 0 \\
  0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
  w_x \\
  w_y \\
  w_z
\end{bmatrix}.
\]

\[ \Rightarrow \begin{bmatrix}
  u_x \\
  u_y \\
  u_z
\end{bmatrix} = \begin{bmatrix}
  \cos \theta_i & -\sin \theta_i \\
  \sin \theta_i & \cos \theta_i
\end{bmatrix}
\begin{bmatrix}
  w_x \\
  w_y
\end{bmatrix}.\]

We define \( \alpha \) and \( r \) as follows:

\[ \alpha = \arctan_2\left( \frac{w_y}{w_x} \right) \]

\[ r = \sqrt{w_x^2 + w_y^2} \]

so that

\[ w_x = r \cos \alpha \]
\[ w_y = r \sin \alpha \]

Combining these definitions with (23) yields

\[
\begin{bmatrix}
  u_x \\
  u_y
\end{bmatrix} =
\begin{bmatrix}
  \cos \theta_i & -\sin \theta_i \\
  \sin \theta_i & \cos \theta_i
\end{bmatrix}
\begin{bmatrix}
  r \cos \alpha \\
  r \sin \alpha
\end{bmatrix}
\]

\[
= \begin{bmatrix}
  r \cos \theta_i \cos \alpha - r \cos \theta_i \sin \alpha \\
  r \sin \theta_i \cos \alpha + r \cos \theta_i \sin \alpha
\end{bmatrix}
\]

\[ = r \begin{bmatrix}
  \cos (\theta_i + \alpha) \\
  \sin (\theta_i + \alpha)
\end{bmatrix} \]

\[ \Rightarrow \theta_i + \alpha = \arctan_2\left( \frac{u_x}{u_y} \right) \]
Therefore,

\[(27) \quad \theta_i = \arctan_2 \left( \frac{u_y}{u_x} \right) - \alpha.\]

For each solution for \(\theta_i\), there exists a corresponding solution to (27). Hence, there are two solutions overall for the pair \(\theta_1, \theta_2\).

The above parameters for the ASG are given in Table 3. For a given set \(U\) of view angles, I then calculated the necessary rotations \(\theta_1, \theta_2\) (Figure 26). I chose \(\beta = 46.5^\circ\) so that the arm would be bent when at the largest view zenith angle. This prevents occultation of the sun at the maximum extension of the arm.

At the scale of field instrumentation, occultation of the sun by the instrument must occur when the view zenith and azimuth angles match those of the sun. Occultation occurs with the ASG when the foreoptic mount (the end of the robotic arm) aligns with the sun and the target center.

### 3.1 Numerical Inversion

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\beta)</td>
<td>46.5°</td>
</tr>
<tr>
<td>(k_0)</td>
<td>([\sin 46.5^\circ, 0, \cos 46.5^\circ] \Rightarrow [0.725, 0, 0.688])</td>
</tr>
<tr>
<td>Hemisphere radius</td>
<td>0.65 m</td>
</tr>
</tbody>
</table>
For a prescribed set of view zenith and view azimuth angles, I numerically solved for the necessary relative \( \theta_1 \) and \( \theta_2 \) in terms of motor counts. Rotations about \( \theta_1 \) and \( \theta_2 \) are translated to digital motor counts \( \theta_{1,MC} \) and \( \theta_{2,MC} \), respectively, by the following relationships:

\[
\begin{align*}
\theta_{1,MC} &= \frac{\theta_1 (N_{wg1}) (C_{rev})}{360} \\
\theta_{2,MC} &= \frac{\theta_2 (N_{wg2}) (C_{rev})}{360}
\end{align*}
\]

where \( N_{wg1} \) and \( N_{wg2} \) are the respective numbers of teeth on the worm gears and \( C_{rev} \) is the number of motor counts per motor axle revolution. For the ASG, \( N_{wg1} \) and \( N_{wg2} \) are 95 and 72 respectively and \( C_{rev} \) is 2000.

4 Control Hardware

The summary of the specifications of the ASG appears in Table 4. Pittman Series 3400 brushless servomotors drive rotation about the axes of the ASG. A Galil Motion Control DMC-2400 motor controller and two Advanced Motion Control B12A6 amplifiers control the motors. The integrated system draws 110V AC power. An autonomous laptop drives the motor controller via Universal Serial Bus (USB) and interfaces via a 9-pin RS-232 interface with a laptop integrated with the ASD-FR spectroradiometer. The ASD-FR runs in automatic regular acquisition and sends a pulse to the serial port upon completion of spectrum sampling. This pulse initiates a discrete motion of the ASG arm to the next point in the sampling grid.
The ASD-FR can sample automatically at a 1 s interval. However, running the ASD-FR at a 2 s (0.5 Hz) interval allows the time needed for ASG arm movement (~600 ms for 15° rotation) and vibration settling. The ASG has three fundamental sampling protocols: hemisphere at 10° zenith and azimuth sampling, hemisphere at 15° zenith and azimuth sampling, and 1° zenith sampling in the solar principal plane. The respective sampling times for these protocols are 5:40 (mm:ss), 2:36, and 5:42.

Table 4. ASG specifications.

<table>
<thead>
<tr>
<th>Automated Spectro-Goniometer</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Angular Resolution</td>
<td>0.002° (azimuth)</td>
</tr>
<tr>
<td></td>
<td>0.0025° (zenith)</td>
</tr>
<tr>
<td>Angular Range</td>
<td>Full Hemisphere</td>
</tr>
<tr>
<td>Angular Accuracy</td>
<td>2°</td>
</tr>
<tr>
<td>Degrees of Freedom</td>
<td>2</td>
</tr>
<tr>
<td>Motors</td>
<td>Pittman 3400 Series Brushless Servo Motors</td>
</tr>
<tr>
<td>Amplifiers</td>
<td>Advanced Motion Control B12A6 Trapezoidal velocity profile</td>
</tr>
<tr>
<td>Motor Controller</td>
<td>Galli DMC-2400</td>
</tr>
<tr>
<td>ASG Hemisphere Radius</td>
<td>0.65 m</td>
</tr>
<tr>
<td>Total Weight</td>
<td>41 kg</td>
</tr>
</tbody>
</table>
Figure 27 shows that in the winter and spring months, a 6-minute sampling interval results in solar geometry change comparable to the 2° angular accuracy specification for the ASG. Hence, changes in solar geometry should have negligible impact on the BRF results. During the summer months the changes in solar geometry are more pronounced at mid-day. However, the small solar zenith at these times results in a less pronounced BRF. Nonetheless, I change to the 15° sampling grid to decrease the sampling time and in turn decrease the change in solar geometry.

The ASG base lies fixed in a north-south orientation for each sample. The plane that bisects the frame symmetrically lies coincident with the north-south plane. Sampling begins with a real-time determination of the solar azimuth angle. A front-end for the motion control software on the autonomous laptop calls subroutines for solar ephemeris (given input date, time, and geographic location) and for the axis rotations (Figure 28). The solar ephemeris subroutines (FORTRAN77) come from the Naval Research Laboratory ftp site and I modified them with FORTRAN90 calling functions. I coded the inversion algorithms (Section 3) for motor rotations in FORTRAN90. The ASG collects spectra from a near-perfect reflecting white panel. The ASG arm then unfolds to full extension and rotates about −z until  is in the solar principal plane. A complete sampling then follows. The sampling path fixes view zenith angle
and passes through all view azimuth angles, steps to the next smaller view zenith and repeats through to the nadir view.

The threaded foreoptic mount at the end of the lower arm can accommodate any of the standard ASD foreoptics (1°, 4°, and 8° field-of view). Figure 29 shows the theoretical sampling footprint of the ASG with the 4° field-of-view foreoptic relative to view zenith $\theta_v$.

5 Angular Calibration

The angular accuracy of the ASG was evaluated through a laser pointing experiment. I attached the fiber optic cable of a collimated light source to the optic mount of the ASG. In this configuration, the laser could point with each view vector $\hat{u}$. I centered two grids with cell spacing of 0.375 cm under the vertical axis $-z$, at the base plane of the ASG and height 1.15 cm above the base plane of the ASG. The ASG was then run through a complete 10° protocol, noting the Cartesian coordinates of the interception points of the laser with both grids. I then solved for the view zenith and azimuth of each view vector by solving the following relationships:

$$\theta_r = \cos^{-1}\left(\frac{\sqrt{x^2 + y^2}}{\sqrt{x^2 + y^2 + h^2}}\right)$$

$$\phi_r = \tan^{-1}\left(\frac{x}{y}\right)$$

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The results of this experiment are shown in Figure 30. The root mean squared error for zenith and azimuth were 1.3° and 2.2° respectively and the mean errors were 0.7° and −0.1° respectively.

6 BRF Computation and Data Formats

6.1 Calculation of BRF

Measuring BRF with the ASD-FR consists of sampling spectra from a calibrated, near-Lambertian Spectralon panel and sampling spectra from the snow target. The BRF for the geometry \((\theta_0, \phi_0; \theta_\perp, \phi_\perp)\) is given by:

\[
BRF_\lambda(\theta_0, \phi_0; \theta_\perp, \phi_\perp) = \left( \frac{DN_\lambda(\theta_\perp, \phi_\perp) - DC_\lambda}{DN_{\lambda, \text{Spectralon}}(\text{nadir}) - DC_\lambda} \right) \frac{C_{\lambda, \text{Spectralon}}}{C_{\lambda, \text{BRF-Spectralon}}} \]

\(DN_\lambda(\theta_\perp, \phi_\perp)\) is the spectral digital number recorded by the ASD-FR for the view zenith \(\theta_\perp\) and view azimuth \(\phi_\perp\), \(DC_\lambda\) is the spectral dark current measurement, \(DN_{\lambda, \text{Spectralon}}(0,0)\) is the spectral digital number recorded by the ASD-FR for the nadir view \(\theta_\perp = 0^\circ, \phi_\perp = 0^\circ\), \(C_{\lambda, \text{BRF-Spectralon}}\) is the spectral correction factor for the bidirectional reflectance of the Spectralon panel provided by the manufacturer, and \(C_{\lambda, \text{Spectralon}}\) is the spectral calibration coefficient that accounts for sub-unity reflectance of the Spectralon panel. The BRDF is given by scaling equation (30) by \(1/\pi\):
\[ BRDF_{\lambda}(\theta_0, \phi_0; \theta_r, \phi_r) = \frac{1}{\pi} \frac{DN_{\lambda}(\theta_r, \phi_r) - DC_{\lambda}}{\frac{DN_{\lambda, \text{Spectraon}}(\text{nadir})}{C_{\lambda, \text{Spectraon}}} - DC_{\lambda}} \]

\(DN_{\lambda}(\theta_r, \phi_r)\) and \(DN_{\lambda, \text{Spectraon}}(0,0)\) for an ASG acquisition in February, 2001 are shown in Figure 31. Figure 32 shows \(C_{\lambda, \text{BRF, Spectraon}}\) for varying solar zenith angles (Robert O. Green, unpublished data). The Spectraon calibration spectrum, \(C_{\lambda, \text{Spectraon}}\), is shown in Figure 33.

### 6.2 Correction for VNIR Dark Current Drift

The ASD-FR records total digital numbers in the visible and near-infrared wavelengths \((0.35 \leq \lambda \leq 0.975 \, \mu m)\) but digital numbers corrected for the digital number equivalent of the dark current in the shortwave infrared wavelengths \((0.976 \leq \lambda \leq 2.5 \, \mu m)\). Dark current is the digital number recorded by the ASD-FR when the foreoptic is completely occulted. As such, the dark current represents the inherent instrument noise. Because the digital numbers recorded in the SWIR1 and SWIR2 detectors are dark-current corrected, we may use the reflectance calculation at the shortest wavelength of this range \((\lambda = 0.976 \, \mu m)\) to determine the dark current in the visible and near-infrared spectrometer. I assume that the \(BRF\) calculated in the shortest wavelength band of the SWIR1 spectrometer is valid. I then set the \(BRF_{\lambda=0.975\mu m} = BRF_{\lambda=0.976\mu m}\), and solve for the \(DC_{\text{VNIR}}\) as follows:
\[ DC_{VNR} = \frac{1}{1 - BRF_{0.976\mu m}} \left( DN_{\lambda} - BRF_{0.976\mu m} \left( \frac{DN_{\lambda, \text{Spectral (nadir)}}}{C_{\lambda, \text{BRF - Spectral}} C_{\lambda, \text{Spectral}}} \right) \right) \]

6.3 Data Formats

ASD-FR files are stored as 2151-band floating-point binary files with a 484 byte data header that describes instrument dynamic range, time of acquisition, and instrument configuration. With a suite of Interactive Data Language (IDL) programs, I calculate the BRF for the complete spectral \((0.35 \leq \lambda \leq 2.5 \mu m)\) and geometric space \((0^\circ \leq \theta \leq 80^\circ, 0^\circ \leq \phi \leq 180^\circ)\) for each ASG acquisition. Each ASG BRF set is stored as an ENVI spectral library, a format used in the ENVI image processing software (Research Systems, Inc., Boulder, CO). The ENVI spectral library format is a floating-point image with the number of samples equal to the number of spectral bands and the number of lines equal to the number of angular spectra. Each ENVI spectral library file has an associated header file that contains information such as data type, instrument configuration, wavelengths, and viewing geometry (Figure 34).

7 Preliminary Results

Figure 35 shows a complete ASG BRF acquisition set. The angular and spectral structure of the BRF is readily apparent by this figure. Snow BRF spectra and angular distributions of spectral BRF are shown in Figure 36 and Figure 37, respectively.
8 CONCLUSIONS

Continued development of quantitative optical remote sensing algorithms for snow properties requires knowledge of the angular distribution of reflected solar radiation from snow surfaces. The Automated Spectro-Goniometer (ASG) is a 2 degree of freedom, 2-axis, spherical robotic arm for the characterization of the bidirectional reflectance distribution function \((BRDF)\) of snow. The ASG uses an Analytical Spectral Devices FR field spectroradiometer to sample reflectance spectra covering the solar spectrum at \(\leq 10\) nm spectral resolution. The geometry of the robotic arm places the spectroradiometer foreoptic at any point on the hemisphere of radius 0.65 m in order to sample the distribution of reflected solar spectra. The angular displacement of the robotic arm is derived using Rodrigues' formula. Rodrigues' formula is inverted for the necessary rotations about both axes to describe the positions of the view vectors. A laptop coupled with a Galil DMC-2400 motor controller drives motion control. The ASG has three standard sampling protocols: \(10^\circ\) angular resolution over the hemisphere, \(15^\circ\) angular resolution over the hemisphere, and \(1^\circ\) angular resolution in the solar principal plane. Sampling at 0.5 Hz (1 point / 2 s), these protocols translate to total sampling times of 5:40 (mm:ss), 2:36, and 5:42. Over these periods, solar geometry change is minimal.
The scale of the ASG permits application to other smooth surfaces such as desert, soil, tundra, and pavement. These high angular and spectral resolution data will improve our capacity to model the angular reflectance of snow and other smooth surfaces, and in turn improve inference of surface properties from remotely sensed multi-spectral and hyper-spectral data.
Figures
Figure 19. Automated Spectro-Goniometer on snow
Figure 20. Representation of bidirectional reflectance
Figure 21. Geometry for the Automated Spectro-Goniometer. The elbow axis $k_0$ lies in the $x$-$z$ plane, with a zenith angle of $\beta = 46.5^\circ$ off of the $z$ axis. The three vectors $z$ (vertical axis), $k_0$ (the oblique axis), and $u$ (the view vector) intersect at the hemisphere origin. $\theta_1$ and $\theta_2$ are the rotations about $z$ and $k_0$, respectively. The elbow axis $k_0$ lies in the $x$-$z$ plane when $\theta_1 = 0$. 

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Figure 22. ASG upper joint assembly. The joint housing and upper motor housing (right) are suspended from the ASG boom. A worm shaft (threaded) extension to the motor shaft drives the anti-backlash worm gear (center). The upper arm extends off to the left from the upper shaft.
Figure 23. ASG lower joint housing. The lower joint assembly is similar to that of the upper motor but uses fixed mounts rather than the suspended mounts found on the upper joint assembly. The far housing shown contains the lower motor and the near housing contains the lower anti-backlash worm gear and lower shaft. The lower motor, to the end of which the foreoptic is mounted, extends out of the photograph to the right.
Figure 24. ASG foreoptic mount. The mount is threaded to accept all Analytical Spectral Devices foreoptics.
Figure 25. ASG system signal flow
Figure 26. Motor counts for complete ASG acquisition, relative to the $\theta_r = 80^\circ$ in the backscattering half of the solar principal plane
Figure 27. Changes in solar zenith and azimuth angles in a 6 minute period for different seasons and times of day at Mammoth Mountain, CA (37° 37’ N, 119° 02’ W)
Figure 28. Graphical user interface (GUI) for ASG
Figure 29. ASG ground instantaneous fields-of-view (GiFOV) for the range of view zenith angles. The dashed ellipses represent the GiFOVs for $\theta_v = 60^\circ$ for other view azimuth angles.
Figure 30. Results from angular calibration experiment. The ASG angles represent the zenithally averaged and azimuthally averaged results for the azimuth and zenith accuracy respectively.
Figure 31. Spectral digital numbers (DN) for ASG acquisition in February, 2001 at Sherwin Meadows, CA. The solid spectrum shows the DN for the Spectralon reflectance panel and the dashed spectrum shows the DN for the snow target for a the view geometry (0°, 0°).
Figure 32. Bidirectional reflectance calibration spectra for Spectralon reflectance calibration panel relative to the spectrum for a view zenith of 50° (data provided by Robert O. Green, Jet Propulsion Laboratory). These measurements were made under nadir irradiance. Reciprocity is assumed in order to use these data to correct nadir viewing measurements of the Spectralon panel for bidirectional reflectance according to the respective illumination zenith angle. Under nadir illumination, Spectralon has a BRF near 1.0 across the spectrum near a view zenith of 50°.
Figure 33. Spectralon calibration spectrum for directional-hemispherical reflectance given illumination zenith angle of 8°
ENVI

description = {
samples = 2151
lines = 289
bands = 1
header offset = 0
file type = ENVI Spectral Library
data type = 4
interleave = bsq
sensor type = Automated Spectro-Goniometer coupled with ASD-FR
byte order = 0
x start = 0.00000
y start = 0.00000
z plot titles = { Wavelength [171133m], BRF }
band names = { ASG ENVI Spectral Library }
wavelength = {
    0.350000,
    0.351000,
    0.352000,
    0.353000,
    0.354000,
    0.355000,
}

Figure 34. ENVI spectral library header used for ASG acquisitions
Figure 35. Image of a complete ASG acquisition for snow made on February 23, 2001 in ENVI spectral library format. The image shows 8 distinct zenithal bands that contain 36 spectra associated with azimuth angle. In each zenithal band, the lines represent view azimuths of 0 - 350° at 10° intervals.
Figure 36. Snow BRF spectra (left) collected with θ₀ = 47° at 10° angular resolution and the anisotropy factor (right) which gives the ratio between respective BRF spectra and the nadir spectrum. The top plots correspond to a view azimuth of φ_r = 0°, the middle plots correspond to a view azimuth of 90°, and the bottom plots correspond to a view azimuth of 180°. The surface grain size, estimated with stereological analysis, was ~70 μm.
Figure 37. Polar plots of the snow BRF shown in Figure 36 for all view angles for 6 wavelengths across the solar spectrum. The radial distance from the center of each plot represents the view zenith angle. Rotation about the center represents a change in azimuth. The azimuth angle of 0° is the forward reflectance half of the solar principal plane.
Chapter III

Measurements of the Bidirectional Reflectance of Snow at Fine Spectral and Angular Resolution
ABSTRACT

This chapter presents 2 days measurements of the bidirectional reflectance factor (BRF) of snow made at fine spectral and angular resolution with the Automated Spectro-Goniometer (ASG) for a range of solar zenith angles ($\theta_0 = 40\text{-}50^\circ$) and snow textures (surface grain size = 80 - 240 $\mu$m). Measurements of the stratigraphy of snow texture and density accompanied each day’s suite of measurements. The BRF for fine grain, faceted snow exhibited a local backscattering peak at the view zenith near the solar zenith angle, whereas those for medium grain, clustered snow did not have a local backscattering peak. The BRF decreased at all wavelengths for an increase in measured grain radius from 80 $\mu$m to 240 $\mu$m. However, the decrease in BRF in the visible wavelengths was strongest at $\theta_r = 80^\circ$ in the forward direction and strongest for $\lambda > 1.8\ \mu$m near $\theta_r = 30^\circ$ in the backward direction. As solar zenith angle decreased from 47° to 41° the BRF increased near nadir for $\lambda \leq 1.03\ \mu$m but decreased with coherent angular structure for $\lambda > 1.03\ \mu$m. I compared forward radiative transfer modeling results of the BRF with the BRF measurements. The forward model used single-scattering parameters for ice spheres with radii that matched the surface-area-to-volume ratio derived from stereological analysis of snow samples and a stratigraphic distribution of optical depths from measured density and modeled extinction efficiency. All BRF
models underestimated reflectance for $\lambda > 1.30 \ \mu m$ and had large errors in the perpendicular plane. Mean RMS errors in reflectance for the fine grain, faceted snow case were 0.09 at $\lambda = 1.3 \ \mu m$ and 0.14 at $\lambda = 1.85 \ \mu m$. Mean RMS errors for the medium grain, clustered snow were 0.04-0.06 at $\lambda = 1.3 \ \mu m$ and 0.04-0.06 at $\lambda = 1.85 \ \mu m$. The models for the more spherical medium grain snow had better overall spectral and angular fits than those for the non-spherical fine grain snow. The spherical radii inferred from the surface-area-to-volume ratio from stereological analysis of snow with non-spherical particles have a greater effective path length than the actual snow particles, resulting in underestimates of bidirectional reflectance.
1 INTRODUCTION

Improvement of quantitative retrievals of snow physical properties from remote sensing will require knowledge of the spectral and angular structure of snow's bidirectional reflectance distribution function. Models for subpixel snow covered area, grain size, albedo, and liquid water content invert directional reflectance data from multi-spectral and hyperspectral remote sensing platforms and field measurements (Davis et al., 1993; Painter et al., 1998; Nolin and Dozier, 2000; Jin and Simpson, 2001; Green et al., 2002). A comprehensive knowledge of the bidirectional reflectance of snow allows us to evaluate the range of validity for those models that do not incorporate directional reflectance information or topographic correction and will improve efforts to incorporate directional reflectance into retrievals.

Several investigators have documented portions of the bidirectional reflectance of snow (O'Brien, 1975; Kuhn, 1985; Steffen, 1987; Dozier et al., 1988; Leroux et al., 1998; Aoki et al., 2000), giving us a coarse understanding of its structure and dependencies. Table 5 shows the complete spectral and angular sampling spaces for these experiments. Dozier et al (1988) gave the most comprehensive characterization of the BRDF of snow, but the spectrometer they used was sensitive only for $\lambda < 1.0 \, \mu m$. From other work, we know that the BRF of snow is most anisotropic at $\lambda > 1.0 \, \mu m$. In this work, I
present spectroscopic measurements of the bidirectional reflectance factor of snow at 10° angular resolution in the view zenith and azimuth. These measurements address the sensitivity of the BRF of snow to wavelength, snow texture, and solar zenith angle.

**Table 5. Spectral and geometric parameters of previous measurements of the snow bidirectional reflectance distribution function**

<table>
<thead>
<tr>
<th></th>
<th>Spectral Range (μm)</th>
<th>Spectral Resolution (nm)</th>
<th>View Geometry</th>
<th>Solar Geometry</th>
</tr>
</thead>
<tbody>
<tr>
<td>O'Brien and Munis, 1975</td>
<td>0.6–2.5</td>
<td>0.3-3.0</td>
<td>$\theta_r = 5^\circ, 30^\circ$</td>
<td>$\theta_0 = 0^\circ, 5^\circ$</td>
</tr>
<tr>
<td>Kuhn, 1985</td>
<td>0.45, 0.75, 1.0</td>
<td>15-25</td>
<td>$\theta_r = 60-87^\circ$ by 10°</td>
<td>$\theta_0 = 60^\circ, 67^\circ$</td>
</tr>
<tr>
<td>Steffen, 1987</td>
<td>0.5-0.6</td>
<td>Single band</td>
<td>$\theta_r = 45, 60, 75^\circ$</td>
<td>$\theta_0 = 28-76^\circ$</td>
</tr>
<tr>
<td>Dozier et al., 1988</td>
<td>0.37-1.1</td>
<td>5-10</td>
<td>$\theta_r = 0-75^\circ$ by 15°</td>
<td>$\theta_0 = 30-50^\circ$</td>
</tr>
<tr>
<td>Leroux et al., 1998</td>
<td>1.65</td>
<td>100</td>
<td>$\theta_r = 0-80^\circ$</td>
<td>$\theta_0 = 48-63^\circ$</td>
</tr>
<tr>
<td>Aoki et al., 1999</td>
<td>0.35 – 2.5</td>
<td>3 for λ &lt; 1.0 μm</td>
<td>$\theta_r = 0 – 80^\circ$ by 10°</td>
<td>$\theta_0 = -56^\circ$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>10 for λ &gt; 1.0 μm</td>
<td>$\phi_r = 0 – 337.5^\circ$ by 22.5°</td>
<td></td>
</tr>
</tbody>
</table>
2 METHODS

2.1 Site

I measured snow BRF with the Automated Spectro-Goniometer (ASG) in Sherwin Meadows, immediately south of Mammoth Lakes, California at an elevation of 2450 m. Mammoth Lakes lies on the east side of the Sierra Nevada near Mammoth Mountain Ski Area at latitude 37°37' N and longitude 119°02' W.

2.2 Bidirectional Reflectance Factor

The previous chapter describes the Automated Spectro-Goniometer in detail. The ASG collected data with the 10° protocol during the winter 2001 acquisitions.

I present the data in terms of the BRF and the anisotropy factor (ANIF) that facilitates the analysis of the spectral variability in BRF data (Sandmeier and Itten, 1999). The ANIF is given by normalizing the BRF by the nadir BRF. Appendix A contains more details.

\[ BRF(\theta_0, \phi_0; \theta_r, \phi_r) = \frac{\pi l(\theta_r, \phi_r)}{\cos \theta_0 E(\theta_0, \phi_0)} \]

\[ ANIF(\theta_0, \phi_0; \theta_r, \phi_r) = \frac{BRF(\theta_0, \phi_0; \theta_r, \phi_r)}{BRF(\theta_0, \phi_0; 0, 0)} \]

\( l \) is the radiant intensity reflected from the surface into reflectance angle and azimuth \( \theta_r, \phi_r \) given irradiance \( E \) at incident angle and azimuth \( \theta_0, \phi_0 \).
2.3 Spectral Atmospheric Optical Depth

I used a Reagan Sun Photometer (Optical Sciences Center) to characterize the atmospheric optical depth for ASG acquisitions. The sun photometer has a 10-channel parallel co-aligned field-of-view tube with a 3.2° field-of-view. The sun photometer collected voltage measurements once per minute at the following wavelengths: 0.382, 0.41, 0.501, 0.611, 0.669, 0.721, 0.78, 0.872, 0.94, and 1.03 μm with full-width half-maxima of 0.008-0.012 μm. I used the Langley method to calculate spectral atmospheric optical depth (Stephens, 1994).

2.4 Snowpack Characterization

For each suite of acquisitions, I sampled a snow pit immediately adjacent to the snow target and collected snow samples from the target for characterization of snow grain size and snow grain morphology.

Snow Pit Analysis

I sampled each snow pit for density and temperature at the end of a day's suite of ASG acquisitions. Density was measured with a 250 ml snow cutter for depths 0-1 and 1-2 cm, a 500 ml snow cutter for depths 2-5, 5-8, and 8-11 cm, and a 1000 ml snow cutter for depths 11-21, 21-31, and 31-41 cm. Snow temperature was measured with a dial stem thermometer.
Stereological Analysis for Snow Grain Size

Snow samples were collected from the target for depths 0-10 cm, 10-20 cm, and 20-30 cm. I then transported these samples to a cold room at the nearby Sierra Nevada Aquatic Research Laboratory. The samples were stored for transit in an electric cooler that had been prepared with snow stored at −10°C and kept insulated in a snow pit near the target site.

I filled the pore space of these samples with liquid dimethyl phthalate, supercooled to −10°C. Once frozen, the samples were then halved with a saw, cutting in a cross-sectional plane. One half was returned to storage in a freezer while the other half was mounted on a steel plate that was thread-mounted to a sledge microtome. I then shaved each sample with the microtome to a smooth surface. The sample sublimated over a 48-hour period, leaving a dimpled surface where the dimples represented the ice volume. Dimples were filled with powder copier toner and the sample was subsequently shaved with the microtome, leaving a surface of white (pores) and black (ice). With a digital camera in macro-mode and a 1x magnification, I photographed a 2mm grid and each sample in 2 cm intervals. I then analyzed the photographs for the sphere radius with the surface-area-to-volume ratio determined with the stereological relationships (Davis et al., 1987b).
2.5 **BRF Modeling**

Several authors have suggested that a snow layer or cloud of ice spheres with the same surface-area-to-volume ratio (SVR) as the non-spherical particles in the natural snow or clouds would have the same reflected and transmitted flux (Dozier, 1989; Grenfell and Warren, 1999). However, most suggest that the equivalent SVR may not provide accurate results in the case of intensity or bidirectional reflectance. With the data collected by the ASG and the snow texture stratigraphy, I compared SVR model results with measurements.

I modeled each ASG acquisition with a multi-layer discrete-ordinates forward calculation. I calculated the single scattering properties (Appendix I) for each 2 cm layer using spheres of the radii derived from stereological analysis of the snow samples. The optical depth for each 2 cm layer came from equation 23 in Appendix I using snow density measured in the field and the extinction efficiency $Q_{ext}$ modeled in the single scattering calculations.

3 **Results**

3.1 **Acquisition Set**

The acquisition set from the ASG consists of 1 scan for fine-grain snow on February 23, 2001, and 2 scans for medium-grain snow on March 13, 2001. Herein, I will refer to the February 23 acquisition as F1 and the two
March 13 acquisitions as M1 and M2. The respective solar ephemerides for these acquisitions are shown in Table 6. Figure 38 and Figure 39 show the respective acquisition sets as images.

<table>
<thead>
<tr>
<th></th>
<th>February 23, 2001</th>
<th>March 13, 2001</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>F1</td>
<td>M1, M2</td>
</tr>
<tr>
<td>$\theta_0$, $\phi_0$</td>
<td>47.3°, 176.9°</td>
<td>46.5°, 144.3°</td>
</tr>
<tr>
<td>$\theta_0$, $\phi_0$</td>
<td></td>
<td>41.9°, 161.0°</td>
</tr>
</tbody>
</table>

*Snowpack Characterization*

Figure 40 shows the density and temperature profiles for the snow pits sampled on February 23 and March 13. In Figure 41, I present plane section images for the snow surfaces on February 23 and March 13. Figure 42 shows the grain size profiles as determined with stereological analysis of the snow samples. Snow grain shapes on February 23 consisted primarily of slightly rounded remnants of the original plates and dendritic forms, on March 13, surface grains were primarily grain clusters formed through melt-freeze cycling (Figure 42). On both dates, the snow temperature was less than 0°C at the time of acquisition.

On February 23, density decreased from 200 kg/m³ near the surface to 130-150 kg/m³ at 2-11 cm depth and increased to greater than 300 kg/m³ be-
between 31 and 41 cm depth (Figure 40). This pack had increased in density over 2 days since a large snowfall with density ~80-100 kg/m³. On March 13, density decreased from 380 kg/m³ in the top 2 cm to 180 kg/m³ between 8 and 11 cm and then increased to ~ 250 kg/m³ in the 21-31 cm layer. Liquid water from surface melting in the top 2 cm had increased the density by forming grain clusters and accelerating settling. March 13 came a few days into a period of above normal air temperatures and clear skies that lasted through the remainder of March. Therefore, surface layers exhibited greater densification while the deeper layers had not yet densified appreciably.

The stratigraphy of grain size exhibited a similar time series (Figure 40). Grain size on February 23 was relatively constant with depth, increasing slightly from ~70 µm at depths 0–16 cm to ~125 µm at 18-20 cm. The entire snowpack (52 cm) came from consecutive storms separated by cold temperatures and slow grain growth. Grain size on March 13 showed greater scatter and values up to 284 µm in the top 5 cm. Between depths of 10 and 18 cm, grain size ranged from 100 to 210 µm and between depths of 20 and 30 cm, the grain size decreased to a range of 65 to 95 µm. The stratigraphy on March 13 exhibits the same response to warm temperatures and full insolation as that by the stratigraphy of density.
Atmospheric Optical Depth

Figure 44 shows the Langley plots and instantaneous optical depths for all wavelengths for the March 13 acquisitions. The sun photometer was not available for the February 23 acquisition. Figure 45 shows the Langley derived spectral optical depths for the March 13, 2001 acquisitions. The stability of instantaneous total optical depths during the March 13 acquisition shows that atmospheric conditions remained constant.

3.2 BRF Relative to View Azimuth

Figure 46, Figure 47, and Figure 48 show the BRF spectra and ANIF spectra for all view zenith angles for view azimuths 0°, 90°, and 180° for the F1, M1, and M2 ASG acquisitions (Table 6), respectively. Bad data existed at \((\theta_r = 40^\circ, 70^\circ; \phi_r = 0^\circ)\), at the occultation geometry (Chapter 2) in the F1 data, and at the occultation geometry in the M2 data. All BRF spectra exhibit noise near \(\lambda = 1.85 \mu m\) and 2.5 \(\mu m\) that comes from the low signal to noise in these water vapor absorption features.

The F1 BRF spectra show a small local backscattering reflectance peak at \(\theta_r \equiv 40^\circ\) for \(\lambda > 0.9 \mu m\) (Figure 46). The M1 and M2 BRF spectra do not show this peak. Steffen (1987) observed this backscattering peak near the solar zenith in the shorter wavelength band 0.5 \(\mu m \leq \lambda \leq 0.6 \mu m\). While it has similar bidirectional structure as the retro-solar reflectance peak for vegetation (the ‘hot spot’), the mechanism is different. The vegetation hot spot is
due to the absence of shadowed surface in the field-of-view whereas for snow, the retro-solar peak, if present, is primarily due to local peaks in the single-scattering phase function (Appendix I) associated with greater faceting on the snow grains.

The F1 data show high reflectance that is consistent with the fine grain size (Figure 46). The nadir BRF was 0.98 at $\lambda = 0.55 \, \mu m$, 0.75 at $\lambda = 1.03 \, \mu m$, 0.55 at $\lambda = 1.3 \, \mu m$, and 0.2 at $\lambda = 2.25 \, \mu m$. In the forward half of the solar principal plane ($\phi_r = 0^\circ$), BRF exhibited the largest zenithal increase. Anisotropic factors ranged from 1.5 at $\lambda = 0.55 \, \mu m$ to 4.0 at $\lambda = 2.25$. The BRF in the perpendicular plane ($\phi_r = 90^\circ$) exhibited a small, monotonic zenithal decrease at wavelengths $\lambda \leq 0.98 \, \mu m$ and a small monotonic zenithal increase that increased with wavelength at wavelengths $\lambda \geq 0.98 \, \mu m$. In the backscattering half of the solar principal plane ($\phi_r = 180^\circ$), the BRF exhibited a larger monotonic zenithal decrease at $\lambda \leq 0.98 \mu m$ and little change at other wavelengths. However, the slight backscattering peak at $\theta_r = 40^\circ$ occurred at all wavelengths $> 0.8 \, \mu m$, increasing in ANIF with increasing wavelength.

M1 and M2 show lower reflectance than that for F1, consistent with the measured increase in grain size (Figure 47). In the M1 acquisition ($\theta_b = 47^\circ$), the nadir BRF was 0.99 at $\lambda = 0.55 \, \mu m$, 0.5 at $\lambda = 1.03 \, \mu m$, 0.25 at $\lambda = 1.3 \, \mu m$, and 0.07 at $\lambda = 2.25 \, \mu m$ (Figure 47). The BRF for $\phi_r = 0^\circ$ increased 

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monotonically at all wavelengths to lower reflectance than in the F1 data but with larger ANIF. BRF in the perpendicular plane decreased slightly for \( \lambda \leq 0.86 \ \mu m \) and increased for \( \lambda \geq 0.86 \ \mu m \) with larger ANIF in the perpendicular plane than exhibited in the F1 data. For \( \phi_r = 180^\circ \), the BRF decreased with zenith angle for \( \lambda \leq 0.97 \ \mu m \) and increased with zenith angle at longer wavelengths. The magnitude of ANIF change indicates that the M1 data had a greater relative change with view zenith than the F1 data. As with the F1 data, the M1 data exhibited a backscattering peak at \( \theta_r = 50^\circ \).

The M1 and M2 data (\( \theta_0 = 41^\circ \)) had only subtle differences (Figure 48), as one would expect given that the difference in solar zenith angle between the two was \( 6^\circ \). M2 had a smaller increase in BRF with increasing view zenith across the spectrum for \( \phi_r = 0^\circ \). In the perpendicular plane, M2 had a larger zenithal decrease for \( \lambda \leq 0.95 \ \mu m \) and a smaller zenithal increase for \( \lambda \geq 0.95 \ \mu m \). For \( \phi_r = 180^\circ \), M2 had a larger zenithal decrease for \( \lambda \leq 1.17 \ \mu m \) and remained nearly unchanged for \( \lambda \geq 1.17 \ \mu m \). The outlier spectrum in the backscattering half of the principal plane at \( \theta_r = 40^\circ \) was an artifact of occultation of the sun by the goniometer. In section 3.5, I will show that forward reflectance increased with the \( 6^\circ \) increase in solar zenith angle.
3.3 Angular Structure of the BRF Relative to Wavelength

Figure 49, Figure 50, and Figure 51 show the polar plots of the BRF for six wavelengths across the spectrum for the respective ASG acquisitions. These figures show the angular structure of the bidirectional reflectance factor.

In all acquisitions, the BRF structure drifted from convex about the forward half of the solar principal plane (ϕ = 0°) at shorter wavelengths to concave with respect to the forward direction. The concave structure develops because at longer wavelengths, the single-scattering co-albedo (absorption) is greater and single scattering dominates the intensity of reflected radiation. Obviously then, the angular structure of the BRF at longer wavelengths where single scattering dominates should mimic the mapping of the phase function, at an oblique incidence, into the view hemisphere. This produced the concave angular structure. Warren (1982) describes the intersection of the scattering phase function with the surface in detail.

The magnitudes of BRF for the M1 and M2 acquisitions were lower than those for the F1 acquisition, consistent with the larger grain sizes on March 13. The M1 data (θϕ = 47°) exhibited higher BRF values and greater forward reflectance anisotropy than the M2 data (θϕ = 41°), consistent with the larger solar zenith angle.
The structure of the F1 data remained slightly convex at wavelengths up through 1.03 \( \mu \text{m} \) whereas the structure of the M1 and M2 data became concave at 1.03 \( \mu \text{m} \).

3.4 \textit{BRF Relative to Snow Texture}

The F1 and M1 data both had solar zenith angles of 47° but surface grain sizes from stereological analysis of 80 \( \mu \text{m} \) and 284 \( \mu \text{m} \), respectively. A comparison of the two datasets demonstrates the sensitivity of the BRF to grain size (Figure 52). The BRF for F1 at all wavelengths was generally higher than that for M1 because F1 had the finer grain size. As wavelength increased, \( \text{BRF}_{\text{M1}}/\text{BRF}_{\text{F1}} \) (the spectral ratio of the BRF for M1 with the BRF for F1) decreased in magnitude. This decrease is due to the increase in single-scattering co-albedo with grain size (Appendix I).

Figure 52 shows that the angular distribution of \( \text{BRF}_{\text{M1}}/\text{BRF}_{\text{F1}} \) transitions with increasing wavelength from higher values in backscattering angles than forward angles to higher values in forward angles than at backscattering angles. At \( \lambda = 0.55 \mu \text{m} \), \( \text{BRF}_{\text{M1}}/\text{BRF}_{\text{F1}} \) decreased from \( \sim 1.05 \) in the backscattering angles to \( \sim 0.9 \) in the forward angles. At \( \lambda = 0.85 \mu \text{m} \), \( \text{BRF}_{\text{M1}}/\text{BRF}_{\text{F1}} \) was uniform across all angles with a slight decrease in the forward angles. At \( \lambda = 1.03 \mu \text{m} \), \( \text{BRF}_{\text{M1}}/\text{BRF}_{\text{F1}} \) had little small azimuthal dependence but significant zenithal dependence, ranging from 0.67 at \( \theta_r = 0^\circ \) to 0.8 at \( \theta_r = 80^\circ \). However, backscattering \( \text{BRF}_{\text{M1}}/\text{BRF}_{\text{F1}} \) was \( \sim 0.03 \) less than the BRF for analogous angles in the forward direction. \( \text{BRF}_{\text{M1}}/\text{BRF}_{\text{F1}} \) at \( \lambda > 1.30 \mu \text{m} \) had increasing azimuthal dependence with an increasing trend with higher values in the forward scattering angles than in the backscattering angles. For these data, the
wavelength for which $\text{BRF}_{M1}/\text{BRF}_{F1}$ was isotropic lay in the range $0.85 \mu m \leq \lambda \leq 1.03 \mu m$.

The dynamics of the single-scattering co-albedo and scattering phase function together explain this trend. As wavelength increases, the single-scattering co-albedo increases and the asymmetry parameter increases. Likewise, both increase with increasing particle size. At shorter wavelengths, multiple scattering dominates and at longer wavelengths, single scattering dominates.

Therefore, at shorter wavelengths, multiple scattering with near-forward events has a strong contribution to the forward reflectance peak of the BRF. With an increase in particle size, the single-scattering co-albedo increases (absorption increases) slightly and the asymmetry parameter increases, driving scattered photons deeper into the snowpack. The increase in particle size then decreases the forward reflectance peak while leaving the remainder of the reflectance distribution unchanged. At longer wavelengths ($\lambda > 1.2 \mu m$), single scattering begins to dominate the contribution to reflectance and thus the angular distribution becomes dominated by forward reflectance. More scattering events are required to redirect photons to near-complete backscattering ($\theta_\|^\approx 47^\circ \pm 10^\circ$, $\phi \|^\approx 180^\circ$) when the particle is a strong forward scatterer. Therefore, the increase in the single-scattering co-albedo with increasing particle size decreases the probability that a photon would survive the scattering events required for redirection to backscattering. This process explains the minimum in the $\text{BRF}_{M1}/\text{BRF}_{F1}$ centered about the view zenith angle that matches the solar zenith angle in the backscattering direction. In Figure 53, the profiles of $\text{BRF}_{F1}$, $\text{BRF}_{M1}$, and $\text{BRF}_{M1}/\text{BRF}_{F1}$ in the solar principal plane indicate the trend seen in Figure 52.

3.5 BRF Relative to Solar Zenith Angle

The M1 and M2 data had solar zenith angles of $47^\circ$ and $41^\circ$ respectively for the same snowpack. A comparison of the two datasets demon-
strates the sensitivity of the BRF to solar zenith angle (Figure 54). Most of the variance in $\text{BRF}_{\text{M}_2}/\text{BRF}_{\text{M}_1}$ comes with the change in wavelength. Ranges of $\text{BRF}_{\text{M}_2}/\text{BRF}_{\text{M}_1}$ increase in breadth while the values themselves decrease from $[0.98, 1.07]$ at $\lambda = 0.55 \, \mu\text{m}$ to $[0.47, 0.76]$ at $\lambda = 2.25 \, \mu\text{m}$.

The angular distribution of $\text{BRF}_{\text{M}_2}/\text{BRF}_{\text{M}_1}$ changed from azimuthally independent at $\lambda = 0.55 \, \mu\text{m}$ to higher in the forward direction with a local maximum about the nadir point. At $\lambda = 0.55 \, \mu\text{m}$, most of the angular domain lay above 1.0. The distribution was slightly skewed with more of the large view zenith angles in the forward direction lying just below 1.0. At $\lambda = 0.85 \, \mu\text{m}$, the cap of $\text{BRF}_{\text{M}_2}/\text{BRF}_{\text{M}_1}$ lying above 1.0 spanned to $\theta_r = 60^\circ$ in the backward half of the principal plane and to $\theta_r = 40^\circ$ in the forward half of the principal plane. For $\lambda \geq 1.30 \, \mu\text{m}$, the angular distribution of $\text{BRF}_{\text{M}_2}/\text{BRF}_{\text{M}_1}$ had distinct local maxima at $\theta_r > 45^\circ$ in the forward half of the principal plane and in a complex structure centered at $\theta_r = 10^\circ$ in the backward half of the principal plane. The range of $\text{BRF}_{\text{M}_2}/\text{BRF}_{\text{M}_1}$ increased with wavelength.

Whereas the change in BRF with grain size derived largely from the change in the single-scattering co-albedo, the change in BRF with solar zenith comes from the change in the orientation of the phase function. The phase function of non-spherical particles frequently has a predominant narrow forward scattering peak ($\Theta \equiv 0^\circ$) and a broad backscattering peak ($\Theta \equiv 180^\circ$) that is several orders of magnitude smaller. In the visible wavelengths, the
decrease in solar zenith angle orients the backscattering peak closer to nadir. Therefore, the near-nadir BRF will increase due to enhanced single scattering into the near-nadir directions. As wavelength increases, the backscattering peak remains in the phase function but at one order of magnitude smaller relative contribution. At the longer wavelengths, single scattering dominates and the overlapping regions in the phase functions that have significant single-scattering intensities will contribute to the BRF. In this case, the overlaps in the backscattering peaks functions and in the tails of the forward scattering peaks of the two differently oriented phase functions correspond to the peaks in $\text{BRF}_{M2}/\text{BRF}_{M1}$ in the backscattering angles and the large forward angles, respectively.

Figure 55 shows the profiles of the $\text{BRF}_{M1}$, $\text{BRF}_{M2}$, and $\text{BRF}_{M2}/\text{BRF}_{M1}$ in the solar principal plane. These profiles show the trend discussed above.

3.6 Comparison with BRF from Forward Modeling

Spectral Comparison

Figure 56 shows the February 23 ASG measurements and associated forward modeling results for $\theta_r = 30^\circ$, $60^\circ$ and $\phi_r = 0^\circ$, $90^\circ$, and $180^\circ$. Figure 57 and Figure 58 show the measurements and modeling results for the two March 13 acquisitions.

The forward model consistently underestimated the snow bidirectional reflectance factor in the F1 dataset, particularly for $\lambda > 1.2 \ \mu m$ (Figure 56).
The RMS errors averaged over the angular domain were 0.066, 0.086, 0.091, and 0.137 at wavelengths 0.55 μm, 1.03 μm, 1.30 μm, and 1.85 μm, respectively. The nadir BRF spectrum showed better agreement between measurement and model. Among the view geometries, the forward model most closely matched the ASG measurements near the λ = 1.03 μm absorption feature. At nadir and in the backscattering half of the principal plane, the model and measurements differed by less than 0.01 across the 1.03 μm absorption feature.

Model results more closely matched the M1 and M2 BRF data (Figure 57 and Figure 58). As with the F1 data, the forward model consistently underestimated the measured BRF at all view angles for λ > 1.2 μm. However, the magnitudes of the average error over the angular domain in this wavelength range were smaller for M1 and M2. The RMS errors averaged over the angular domain of the M1 data were 0.052, 0.097, 0.056, and 0.064 at wavelengths 0.55 μm, 1.03 μm, 1.30 μm, and 1.85 μm, respectively. The RMS errors for the M2 data were 0.055, 0.109, 0.041, and 0.037 respectively. The forward model for M1 and M2 overestimated the BRF for λ < 1.2 μm over much of the angular domain.

Figure 59 shows the spectral residuals for the F1, M1, and M2 model comparisons. These data show that the forward model for F1 underestimates
the measured BRF at all wavelengths. For the M1 and M2 data, the forward models were within ± 0.05 at most view geometries.

The underestimate by the forward model for λ > 1.2 μm at all view geometries suggests that either the estimate of snow particle size from stereological analysis was too large and/or that the phase function for spheres is sufficiently different from the non-spherical particles found in the snowpack.

Angular Comparison

Figure 60 shows the angular distribution of the ratio of February 23 ASG measurements with forward modeling results for wavelengths 0.55, 0.85, 1.03, 1.30, 1.80, and 2.25 μm. Figure 61 and Figure 62 show the angular distributions of the ratios for the two March 13 acquisitions.

The angular distribution of the error between the forward model and the F1 ASG measurements had a broad maximum in the backscattering angles, a narrow maximum at the largest view zeniths in the forward direction, and a broad minimum at the largest view zeniths for φ = 30-150°. The range of errors changed from [-0.22, 0.08] at λ = 0.55 μm to complete underestimates of [-0.2, 0.07] at λ = 2.25 μm. As described in Chapter 5, some imaging spectroscopy models analyze the ice absorption feature at λ = 1.03 μm. For the view geometries sampled by an imaging spectrometer over rough terrain, say θ ≤ 60°, the errors at λ = 1.03 μm had an RMS of 0.046.
The errors in the M1 modeling exhibited a similar angular structure to those in the F1 modeling but with smaller magnitude errors. The range of errors changed from [-0.23, 0.08] at \( \lambda = 0.55 \, \mu m \) to [-0.15, -0.02] at \( \lambda = 2.25 \, \mu m \). However, most of the angular domain was closer to 0.0 for the M1 data than the F1 data at all wavelengths. For \( \theta_r \leq 60^\circ \), the modeling errors at \( \lambda = 1.03 \, \mu m \) had an RMS of 0.045.

The angular distribution of errors in the M2 modeling had the maximum in the backscattering angles and the minimum in the perpendicular plane but lacked the narrow maximum in the forward direction. For all wavelengths, the M2 modeling had the lower magnitude errors than for F1 and M1. The range of errors changed from [-0.22, 0.02] at \( \lambda = 0.55 \, \mu m \) to [-0.11, 0.02] at \( \lambda = 2.25 \, \mu m \). For \( \theta_r \leq 60^\circ \), the modeling errors at \( \lambda = 1.03 \, \mu m \) had an RMS of 0.052.

Therefore, the forward model generally underestimated measured BRF at \( \lambda > 1.2 \, \mu m \) and strongly underestimated measured BRF for \( \theta_r > 60^\circ \) in the perpendicular plane for \( \lambda \leq 1.3 \, \mu m \). The angular structure of errors suggests that the phase functions of spheres used in the forward models do not match those of the irregularly shaped particles found in the snowpack. The lack of the forward maximum in the modeling errors for M2 in which the solar zenith angle was closer to nadir reinforces the hypothesis that the angular structure was due to the phase function mismatch.
The increase in magnitude of underestimate of BRF with wavelength shows that the spheres with the same surface-area-to-volume ratio as that determined from stereology had a greater absorbing path length. It is possible that the stratigraphic resolution was too coarse to resolve finer grain sizes near the surface. However, the estimate of sphere radius from the surface-area-to-volume ratio derived from stereological analysis necessarily overestimates the volume if the sample particles are non-spherical. A sphere has the minimum surface area for a given volume, whereas non-spherical particles will have a larger surface area for the same volume. The non-spherical particles will intersect a plane with a surface density that, when assumed to be that of spheres, overestimates volume.

Additional error in using the sphere determined from stereological analysis may come from fractal sensitivity to magnification (Davis et al., 1987a). Davis et al. demonstrated that retrieved surface density increased with increasing imaging magnification while the retrieved volume did not change. Because the images used in the stereological analysis in this work had a magnification of 1x, the retrieved surface density was relatively low compared with the volume giving a relatively large sphere radius. Therefore, an optimal magnification may exist that would give a larger surface density and in turn the smaller sphere radius necessary to better model bidirectional reflectance.
While decreasing the sphere radius would improve the magnitude of the modeled BRF, the angular distribution for snow of non-spherical particles would still be in error because the scattering phase functions of spheres are significantly different from those of non-spherical particles (Leroux et al., 1998). The closer modeling results for the M1 and M2 data, which had more spherical grains, with respect to magnitude and angular distribution are consistent with these arguments.

4 Conclusions

This chapter presented measurements at high spectral and angular resolution of the bidirectional reflectance factor of snow. Measurements were made on two dates for fine grain and medium grain snow at solar zeniths 41°-47°. The BRF spectra for fine grain snow had a small backscattering peak at $\theta_r = 50^\circ$, whereas the BRF distributions for medium grain snow did not. The fine grain snow had remnants of the original dendritic morphology that can have a single-scattering phase function with a strong backscattering peak. The backscattering peak in the phase function then produces a retro-solar reflectance peak. The medium grain snow consisted mostly of grain clusters that most likely lack the backscattering peak in the phase function.

Analysis of the sensitivity of the BRF to snow texture showed that the BRF decreased at all view angles with increasing grain size. However, the
decrease in BRF was accompanied by a monotonic trend with wavelength, decreasing more at large forward reflectance angles in the visible wavelengths to decreasing more near $\theta_r = 20-30^\circ$ in the backscattering direction for $\lambda \geq 1.30 \mu m$.

Analysis of the sensitivity of the BRF to solar zenith angle showed that for a change of solar zenith from 47° to 41° and $\lambda < 1.03 \mu m$, the BRF increased in a zenithal range around nadir that decreased with increasing wavelength. For $\lambda > 1.03 \mu m$, the BRF decreased more with increasing wavelength with the greatest decreases at increasing view zenith for all $\phi_r = 90-$180° and at $\theta_r = 40-50^\circ$ for $\phi_r = 0^\circ$.

I compared the results from a radiative transfer model of snow BRF with the measurements. The model study used the stratigraphy of sphere radii inferred from stereology to drive calculations of single-scattering parameters that in turn drove the discrete-ordinates calculation of the BRF. In each case, the forward model underestimated the BRF for $\lambda \geq 1.3 \mu m$ and had large errors in the perpendicular plane. However, the models for the medium grain snow had smaller errors in the spectral and angular domain than the fine grain snow.

The errors in the spectral and angular domains were most likely due to the effect of snow particle morphology on the inference of particle size from stereological analysis and on the shape of the single-scattering phase func-
tion. Stereological analysis infers a larger absorbing volume for snow particles by assuming a spherical shape when the true particle shapes have large aspect ratios, such as those found in the fine grain snow. The larger particle volume gives a greater path length for absorption leading to an underestimate of the snow BRF.

Other investigators (Liou, 1980; Leroux et al., 1998) have demonstrated the effect of morphology on the single-scattering phase function and multiple scattering results. The angular distribution of modeling errors indicates a mismatch of phase functions that is consistent with the difference between spheres and non-spherical particles. Likewise, the smaller errors in the angular distributions for the medium grain size snow are consistent with the finding of more spherical grain clusters.

Further work will investigate the BRF under a larger range of solar zenith angles and for a larger range of particle sizes and morphologies. A thorough investigation of the relationship between the variables retrieved with stereological analysis (surface density, volume density, etc.) and the true particle shape, size, and size distribution is also needed.
Figure 38. Complete ASG acquisition for February 23, 2001 at Sherwin Meadows, CA. Each view zenith angle has 36 associated view azimuths increasing from 0° (top) to 360° (bottom), which gives the symmetry within each view azimuth band.
Figure 39. Complete ASG acquisitions for March 13, 2001 at Sherwin Meadows, CA: (a) BRF for θ₀ = 47°; (b) BRF for θ₀ = 41°
40. Snow density and temperature stratigraphy for (a) February 23 and (b) March 13 ASG acquisitions (instrument failure prevented measuring temperature on February 23)
Figure 41. Snow sample photographs for stereological analysis. The upper two photographs show snow of equivalent sphere radii of 80 µm and 284 µm. The bottom photograph shows the 2mm grid used for particle size calibration.
Figure 42. Grain size stratigraphy for (a) February 23 and (b) March 13
Digital photomicroscope images of snow particles collected on (a) February 23 and (b) March 13. The characteristic morphology on February 23 was the rounded remnant of the original dendritic forms with a high surface-area-to-volume ratio. The characteristic morphology on March 13 was the grain cluster formed through melt-freeze cycling. Grain clusters have a lower surface-area-to-volume ratio.
Figure 44. Langley plots and instantaneous optical depth measured with a Reagan Sun Photometer at Sherwin Meadows, CA during the March 13 acquisitions (legends show wavelengths in nanometers)
Figure 45. Optical depth spectrum from Langley calculations for March 13 measurements at Sherwin Meadows, CA
Figure 46. BRF spectra (left) and the ANIF (right) from the F1 ASG acquisition ($\theta_0 = 47^\circ$, surface grain size = 80 $\mu$m) for all view zenith angles at view azimuth angles 0$^\circ$, 90$^\circ$, and 180$^\circ$. The spectrum for $\theta_r = 50^\circ$, $\phi_r = 180^\circ$ is much lower due to occlusion of the sun by the ASG foreoptic.
Figure 47. BRF spectra (left) and the ANIF (right) from the M1 ASG acquisition ($\theta_0 = 47^\circ$, surface grain size = 284µm).
Figure 48. BRF spectra (left) and the ANIF (right) from the M2 ASG acquisition ($\theta_0 = 41^\circ$, surface grain size = 284 $\mu$m)
Figure 49. Polar plots of the BRF for all view angles for the February 23 ASG acquisition for 6 wavelengths across the spectrum. The radial distance from the center of each plot represents the view zenith angle. Rotation about the center represents a change in azimuth. The azimuth angle of 0° is the forward reflectance half of the solar principal plane.
Figure 50. Polar plots of the BRF for all view angles for the first March 13 ASG acquisition (θ₀ = 47°).
Figure 51. Polar plots of the BRF for the second March 13 ASG acquisition (acquisition ($\theta_0 = 41^\circ$))
Figure 52. Polar plots of the ratio $BRF_{M1}/BRF_{F1}$ at the wavelengths 0.55, 0.85, 1.03, 1.30, 1.80, and 2.25 μm. These data show the change in BRF with an increase of particle size from 80 μm (F1) to 284 μm (M1).
Figure 53. Cross-sections of solar principal plane from backscattering ($\theta_r = -80$) to forward scattering ($\theta_r = 80$) for the F1 data, M1 data, and the ratio $BRF_{M1}/BRF_{F1}$
Figure 54. Polar plots of the spectral ratios $\text{BRF}_{\lambda_2}/\text{BRF}_{\lambda_1}$ at wavelengths 0.55, 0.85, 1.03, 1.30, 1.80, and 2.25 $\mu$m.
Figure 55. Cross-sections of solar principal plane from backscattering ($\theta_r = -80$) to forward scattering ($\theta_r = 80$) for the M1 data, M2 data, and the ratio $\text{BRF}_{M2}/\text{BRF}_{M1}$.
Figure 56. Comparison of ASG measurements (solid) and DISORT forward modeling results (dashed) for February 23 ($\theta_0 = 47^\circ$). The nadir BRF spectra ($\theta_r = 0^\circ, \phi_r = 0^\circ$) for the ASG and DISORT forward modeling are shown in bold in the upper right plot ($\theta_r = 60^\circ, \phi_r = 0^\circ$) for comparison.
Figure 57. Comparison of ASG measurements (solid) and DISORT forward modeling results (dashed) for the first March 13 (M1) acquisition ($\theta_0 = 47^\circ$). The nadir BRF spectra ($\theta_r = 0^\circ$, $\phi_r = 0^\circ$) for the ASG and DISORT forward modeling are shown in bold in the upper right plot ($\theta_r = 60^\circ$, $\phi_r = 0^\circ$) for comparison.
Figure 58. Comparison of ASG measurements (solid) and DISORT forward modeling results (dashed) for the second March 13 acquisition ($\theta_0 = 41^\circ$). The nadir BRF spectra ($\theta_r = 0^\circ$, $\phi_r = 0^\circ$) for the ASG and DISORT forward modeling are shown in bold in the upper right plot ($\theta_r = 60^\circ$, $\phi_r = 0^\circ$) for comparison.
Figure 59. Spectral residuals between DISORT and ASG measurements for the F1 (solid), M1 (dotted), and M2 (dashed) acquisitions at the view geometries in Figure 56.
Figure 60. Polar plots of residuals between DISORT forward modeling results and ASG measurements for the February 23 acquisition (F1) at wavelengths 0.55, 0.85, 1.03, 1.3, 1.85, and 2.25 μm
Figure 61. Polar plots of residuals between DISORT forward modeling results and ASG measurements for the first March 13 acquisition (M1)
Figure 62. Polar plots of residuals between DISORT forward modeling results and ASG measurements for the second March 13 acquisition (M2)
Chapter IV

The Effect of Anisotropic Bidirectional Reflectance on Imaging Spectroscopy Models for Retrieving Snow Properties
ABSTRACT

This chapter describes a sensitivity study to determine the effect of snow bidirectional reflectance on models that map snow properties from imaging spectrometer data. The study applies two spectroscopy models to synthetic spectral reflectance images with prescribed snow-covered area and snow grain size. The first model (MEMSCAG) performs multiple endmember spectral mixture analysis to determine sub-pixel snow-covered area and the grain size of the fractional snow cover. The second model (Nolin/Dozier) analyzes the ice absorption feature at wavelength $\lambda \approx 1.03 \, \mu m$ for an estimate of snow grain size under the assumption of complete snow cover. The synthetic images span a range of grain sizes (spheroids with a- and c-axes of 41 $\mu m/104 \, \mu m$, 208 $\mu m/520 \, \mu m$, and 624 $\mu m/1560 \, \mu m$), fractional snow cover (100%, 75%, 50%, and 25%), view geometries ($0^\circ \leq \theta_v \leq 60^\circ$, $0^\circ \leq \phi_r \leq 180^\circ$), and solar zenith angles ($\theta_b = 30^\circ$, $60^\circ$). Retrievals of subpixel snow covered area (SCA) with MEMSCAG had increasing sensitivity to the BRF with decreasing particle size, increasing solar zenith angle, and increasing snow-covered area. SCA overestimates generally occurred in the forward reflectance angles ($0^\circ \leq \phi_r \leq 90^\circ$) and underestimates generally occurred in the backward reflectance angles ($90^\circ \leq \phi_r \leq 180^\circ$). Maximum SCA errors ranged from $-0.15$ to 0.2. Grain size retrievals from MEMSCAG had increasing sensi-
tivity to BRF with increasing particle size and increasing solar zenith angle but were insensitive to snow-covered area. The largest inferred grain sizes occurred around a peak in the backward reflectance angles and the smallest generally occurred at the largest view zenith angles in the forward reflectance angles. Grain size retrievals from the *Nolin/Dozier* model had the same general angular distribution as those from *MEMSCAG*. Retrievals of albedo from *MEMSCAG* and *Nolin/Dozier* had similar sensitivities to the BRF, with albedo errors up to 5% for $\theta_0 = 30^\circ$ and up to 11% for $\theta_0 = 60^\circ$. The magnitudes of relative albedo differences between the two models were less than 0.015 for all particle sizes and solar zenith angles.
1 INTRODUCTION

Several models have been developed for mapping snow properties from imaging spectrometer data (Painter et al., 1998; Nolin and Dozier, 2000; Green et al., 2002). These models remain untested with respect to their sensitivities to the anisotropic reflectance of snow. Many of these models focus on the wavelength range $1.0 \leq \lambda \leq 1.5 \mu m$ where snow reflectance is most sensitive to grain size.

However, for a given solar geometry, the bidirectional reflectance factor (BRF) of snow (Appendix A) becomes more forwardsly anisotropic as wavelength increases in this range (Warren, 1982). Moreover, the BRF of snow becomes more forwardsly anisotropic as the solar zenith angle increases and as the grain size increases.

Imaging spectrometers and multi-spectral imagers sample much of the distribution of bidirectional reflectance of snow because of instrument view geometry and topography. Remote sensing models frequently ignore bidirectional reflectance because of the number of dimensions of the variable space associated with the variance in the reflectance spectrum shape. These dimensions correspond to solar zenith angle, the ratio of diffuse and direct irradiance, snow grain size, and snow grain morphology. However, in alpine terrain, topography dominates the variance in remotely sensed bidirectional
reflectance. Variation in slope and aspect translates to variation in local solar illumination angle and local view zenith and azimuth angles.

Jin and Simpson (1999) presented a radiative transfer model to correct biases associated with bidirectional reflectance in retrieving albedo from Advanced Very High Resolution Radiometer (AVHRR) data over the Arctic Ocean. Stroeve et al. (1997) likewise used DISORT (Stamnes et al., 1988) to correct AVHRR data over Greenland to improve retrievals of albedo. Both of these models assume a level surface and complete snow cover.

In alpine terrain, any correction of bidirectional reflectance requires knowledge of the topography at a scale that controls the variation in irradiance and view geometry. Presently, we lack accurate topographic data for all but a few basins in the Earth’s mountain ranges at the scale necessary for bidirectional reflectance correction of remotely sensed data. Scientists at the Jet Propulsion Laboratory are currently processing interferometric data collected during the Shuttle Topography Radar Mission (SRTM). These data will provide accurate digital elevation data for most of the latitude range ±60° at 30 m spatial resolution. However, further work will be required to determine the quality of these elevation data for modeling the radiation field in alpine terrain.

In the absence of topographic-correction, model development has proceeded by accounting for changes in the magnitude of irradiance (Painter et
al., 1998; Nolin and Dozier, 2000; Green et al., 2001). However, these models do not account for bidirectional reflectance effects. In this chapter, I describe a sensitivity study of bidirectional reflectance effects on retrievals of snow covered area, grain size, and in turn, albedo. The study also evaluates the impact on retrievals due to the spectral structure of the non-snow reflectance in mixed pixels of varying fractional coverage with anisotropic snow reflectance.

2 METHODS

2.1 Imaging Spectroscopy Models

I analyzed the sensitivity to bidirectional reflectance of two models that retrieve snow physical properties from imaging spectrometer data. The first is the MEMSCAG model, described in Chapter 1 of this dissertation. The second is the scaled integral method of Nolin and Dozier (2000). Neither model has been analyzed for its sensitivity to changes in bidirectional reflectance with changes in view and illumination geometry.

MEMSCAG

MEMSCAG is described in detail in Chapter 1. MEMSCAG simultaneously retrieves sub-pixel snow-covered area (SCA) and grain size of that fractional snow cover. It does not require that the radiance from each pixel's in-
stantaneous field-of-view comes from 100% snow cover, and can thus retrieve the grain size and in turn the albedo of the sub-pixel snow cover.

*Nolin-Dozier Scaled Integral Model*

The Nolin and Dozier (2000) model inverts the integral of the continuum-scaled ice absorption band centered at $\lambda \equiv 1.03 \ \mu m$ for snow grain size alone. The model does not retrieve snow-covered area. The breadth and depth of this absorption feature increase with increasing grain size. The model relates snow grain size to the following integral:

$$
(33) \quad \int_{0.95 \mu m}^{1.09 \mu m} \frac{R_{\text{cont}, \lambda} - R_{\text{snow}, \lambda}}{R_{\text{cont}, \lambda}} d\lambda
$$

where $R_{\text{cont}, \lambda}$ is the continuum reflectance at wavelength $\lambda$ and $R_{\text{snow}, \lambda}$ is the snow reflectance at wavelength $\lambda$. The continuum reflectance is the reflectance interpolated between the shoulders of the absorption feature and represents the reflectance in the absence of the ice absorption (Figure 63). Scaling by the continuum reflectance accounts for changes in irradiance. I will refer to this model as $ND$ herein.

2.2 *Synthetic Imaging Spectrometer Data*
I generated synthetic imaging spectrometer images using the discrete-ordinates model DISORT described in Appendix I. I generated 24 images for varying grain size, solar zenith angle, and snow fraction. While synthetic images generated using a digital elevation model might present a spatially realistic image, the results would be convoluted by the topography and difficult to understand. Therefore, I generated images that consisted of the bidirectional reflectance factor at view zenithal and azimuthal resolutions of 10° for level surfaces of varying grain sizes and illuminated from two solar zenith angles. These images present monotonic gradients in BRF from which we may more easily understand the sensitivity of the imaging spectroscopy models to anisotropic reflectance.

I modeled the phase functions and single-scattering albedos for spheroids of respective a-axis/c-axis radii of 41 μm/104 μm, 208 μm/520 μm, and 624 μm/1560 μm (Figure 64). I used spheroids to create the BRF images because the BRF of spheroids have less angular structure than those of

Table 7 Model parameters for BRF test images.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Snow spheroid a-/c-axes</td>
<td>41 μm/104 μm, 208μm/520 μm, 624μm/1560μm</td>
</tr>
<tr>
<td>Solar zenith angle</td>
<td>30°, 60°</td>
</tr>
<tr>
<td>View zenith angles</td>
<td>0° to 60° in 10° increments</td>
</tr>
<tr>
<td>View azimuth angles</td>
<td>0° to 180° in 10° increments</td>
</tr>
<tr>
<td>Snow fractional coverage</td>
<td>25% to 100% in 25% increments</td>
</tr>
</tbody>
</table>
spheres and are more like those measured in the field. These spheroids have the same surface area to volume ratios as spheres of radii 50, 250, and 750 \( \mu m \), respectively, while maintaining an aspect ratio of 0.4. The phase functions were generated using ray tracing code made available via ftp by Andreas Macke (2001).

Figure 65 shows the respective phase functions for the 41 \( \mu m/104 \mu m \), 208 \( \mu m/520 \mu m \), and 624 \( \mu m/1560 \mu m \) spheroids at several wavelengths spanning the visible and near-infrared range. Figure 66 shows the respective spectral single-scattering co-albedos for these spheroids. Single scattering co-albedo, \( 1 - \omega \), is the additive complement of the single scattering albedo (Appendix I) and represents the likelihood of absorption in an interaction of a photon with a particle. I solved for 20 Legendre moments, \( \omega_i \), from each phase function using the following relationship

\[
(34) \quad \omega_i = \frac{\int_{-1}^{1} P(\cos \Theta) P_i(\cos \Theta) (d \cos \Theta)}{\int_{-1}^{1} P(\cos \Theta) (d \cos \Theta)}.
\]

\( P(\cos \Theta) \) is the scattering phase function, \( P_i(\cos \Theta) \) is the \( i \)-th Legendre polynomial (Liou, 1980), and \( \Theta \) is the single scattering angle.

Each snow grain size layer had a prescribed semi-infinite optical depth of 4000. Using the Santa Barbara DISORT Atmospheric Radiative Transfer (SBDART) model (Ricchiazzi et al., 1998), I modeled the spectral direct and diffuse contributions to total irradiance for solar zenith angles 30 and 60° for a
mid-latitude winter atmosphere with 23 km visibility, and a surface elevation of 3.0 km. With the optical properties of the snow layer and irradiance fields, I modeled the spectral bidirectional reflectance factor for each grain size and solar zenith angle using the discrete-ordinates model DISORT. Figure 67, Figure 68, and Figure 69 show the BRF spectra for the respective particle sizes for reflected azimuth directions 0°, 90°, and 180° given θ0 = 30°. Figure 70, Figure 71, and Figure 72 show the BRF spectra for reflected azimuth directions 0°, 90°, and 180° given θ0 = 60°. Figure 73, Figure 74 and Figure 75 show the spectral polar representations of the BRF for θ0 = 30°. Figure 76, Figure 77, and Figure 78 show the spectral polar representations of the BRF for θ0 = 60°.

The snow fractions and solar zenith angles are given in Table 7. Figure 79 shows the granite reflectance spectrum used in the spectral mixtures. I measured this spectrum in the field with an Analytical Spectral Devices FR field spectroradiometer (Analytical Spectral Devices, 2001) and, in the spectral mixture images, assumed that the rock surface had Lambertian reflectance. While rock reflectance is not isotropic, the main focus of this research is the impact due to the spectral structure of the complementary reflectance on retrievals with anisotropic snow reflectance rather than the impact due to the magnitude of the complementary reflectance.

2.3 Snow Albedo Inference

I calculated snow albedo for the three model spheroid layers and all snow sphere layers used in the spectral libraries. Snow albedo in the wave-
length range $0.4 \leq \lambda \leq 2.5 \mu m$, $\alpha$, comes from the convolution of the spectral albedo, $\alpha_\lambda$, determined with DISORT (Stamnes et al., 1988) and the spectral irradiance, $E_\lambda$, as modeled with SBDART (Ricchiazzi et al., 1998):

$$
\alpha = \frac{\int_{0.4\mu m}^{2.5\mu m} \alpha_\lambda E_\lambda d\lambda}{\int_{0.4\mu m}^{2.5\mu m} E_\lambda d\lambda}
$$

(35)

3 RESULTS

I applied MEMSCAG and ND to all 24 BRF images. Below, I present the SCA and grain size results from MEMSCAG and the grain size results from ND. Subsequently I compare the retrievals from the two models. Finally, I present inferred albedo results from MEMSCAG and ND, along with a comparison of the albedo results for the two models.

3.1 MEMSCAG SCA and Grain Size

Figure 80, Figure 82, Figure 84, and Figure 86 show the snow covered area and grain size retrievals from MEMSCAG for BRF images of all grain sizes for the snow fractions 100%, 75%, 50%, and 25% respectively for $\theta_0 = 30^\circ$. Figure 81, Figure 83, Figure 85, and Figure 87 show the SCA and grain size retrievals for BRF images of all grain sizes for the same snow fractions and $\theta_0 = 60^\circ$. The use of spheres by both models for calculations of model
spectra and spheroids for the bidirectional reflectance images inhibits a direct grain size comparison.

100% Snow Cover BRF

For the 41/104 BRF, retrieved SCA was 0.98-1.0 in much of the angular domain (Figure 80). However, retrieved SCA decreased to 0.85-0.9 near \(\theta_r = 45^\circ, \phi_r = 120-180^\circ\) as well as near \(\theta_r = 60^\circ, \phi_r = 30^\circ\), and increased to 1.03 near \(\theta_r = 30^\circ, \phi_r = 0-90^\circ\). The retrieved grain size ranged from 50 \(\mu m\) in the forward reflectance angles to 90 \(\mu m\) in the backscattered and nadir angles.

For the 208/520 BRF, retrieved SCA ranged from 0.95 to 1.0 with the greatest errors in lobes in the backscattering half of the BRF. Retrieved grain sizes ranged from 200 to 440 \(\mu m\), with the smallest grain sizes in the forward reflectance angles and the largest grain sizes near \(\theta_r = 10^\circ\) in the backscattering half of the principal plane \(\phi_r = 180^\circ\).

For the 624/1560 BRF, retrieved SCA was essentially uniform at 1.0 for all view directions, except near \(\theta_r = 10^\circ, \phi_r = 180^\circ\) where retrieved SCA was up to 1.15. The retrieved grain size ranged from 500 \(\mu m\) in the forward principal plane to 1100 \(\mu m\) in much of the backward principal plane and into the forward principal plane up to \(\theta_r = 20^\circ\). The grain size retrieval saturated at 1100 \(\mu m\), which was the maximum snow grain size in the spectral library. The
extensive nature of the grain size saturation and the error in SCA in the same angular domain as the maximum grain sizes in the distributions for 41/104 and 208/520 show that the MEMSCAG would provide more accurate retrievals of grain size and SCA with larger grain size snow spectra in the spectral library.

Figure 81 shows the MEMSCAG results for 100% snow cover and $\theta_0 = 60^\circ$. For 41/104, retrieved SCA ranged from 0.85 at ($\theta_r \equiv 50-60^\circ$, $\phi_r \equiv 90-120^\circ$) to 0.9 scattered through the backscattering angles and to 1.05-1.2 in a broad band at ($\theta_r \equiv 45-60^\circ$, $\phi_r \equiv 0-70^\circ$). Retrieved grain size ranged from 10 $\mu$m in the most forward reflectance angles continuously through to a lobe of 70 $\mu$m at ($\theta_r \equiv 0-45^\circ$, $\phi_r \equiv 150-180^\circ$).

For the 208/520 BRF, retrieved SCA was primarily in the range 1.0-1.03 but for a band of SCA exceeding 1.10 at ($\theta_r \equiv 45-60^\circ$, $\phi_r \equiv 0-50^\circ$). Retrieved grain size ranged from 50 to 350 $\mu$m with the largest grain sizes near ($\theta_r = 25^\circ$, $\phi_r = 180^\circ$), decreasing continuously to the smallest grain sizes near ($\theta_r = 60^\circ$, $\phi_r = 0^\circ$).

For the 624/1560 BRF, retrieved SCA was uniformly between 1.0 and 1.02 but for an increase to 1.05 near $\theta_r \equiv 58^\circ$, $\phi_r \equiv 0-15^\circ$. Retrieved grain size ranged from 200 $\mu$m at ($\theta_r = 60^\circ$, $\phi_r \equiv 20^\circ$) continuously to a peak of 1100 $\mu$m at ($\theta_r = 20-30^\circ$, $\phi_r \equiv 180^\circ$).
In summary, for $\theta_0 = 30^\circ$ and 100% snow cover, errors in retrieved SCA decreased from [-0.15,0.03] to [0.0,0.17] with increasing particle size. Underestimates generally occurred in the backscattering angles and overestimates occurred in the forward scattering angles. However, the large overestimate for the 624/1560 BRF appears to be due to limited grain size range in the spectral library and not the technique. The retrieved grain size range increased from [50 µm, 90 µm] to [550 µm, 1100 µm] as particle size increased, with larger grain sizes in the backscattering angles and smaller grain sizes in the forward scattering angles.

For $\theta_0 = 60^\circ$ and 100% snow cover, errors in retrieved SCA decreased from [-0.15,0.2] to [0.0,0.05] as the particle size increased. Again, negative SCA errors generally occurred for backscattering angles and positive SCA errors occurred in the forward scattering angles. At the largest particle size, the remaining small SCA errors were in the forward scattering angles. The ranges of retrieved grain sizes increased from [10 µm, 70 µm] to [100 µm, 1100 µm] as particle size increased. Peak retrieved grain size appears near $\theta_f \equiv 25^\circ$ in the backscattering half of the solar principal plane ($\phi_f = 180^\circ$) and decreases slowly as $\theta_f$ increases in $\phi_f = 180^\circ$ and decreases more rapidly to the minimum at $\theta_f = 60^\circ$ and $\phi_f = 0^\circ$. 

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75% Snow Cover BRF

Figure 82 shows MEMSCAG results for the 75%/25% snow/rock cover mixture for \( \theta_0 = 30^\circ \). For the 41/104 BRF, retrieved SCA ranged from 0.71 to 0.84. MEMSCAG generally underestimated SCA in the backscattering range \((\theta_r \equiv 45-60^\circ, \phi_r \equiv 90-180^\circ)\), overestimated SCA by up to 0.08 in the near-nadir and forward scattering range \((\theta_r \equiv 45-60^\circ, \phi_r \equiv 0-60^\circ)\), and had values 0.75±0.1 in an almucantar band defined by \((\theta_r \equiv 20-40^\circ, \phi_r \equiv 0-180^\circ)\). The retrieved grain size distribution was very similar to that of the 100% snow cover case (Section 0). The only significant difference was the more abrupt decrease in grain size with increasing view zenith in the backscattering domain. Whereas MEMSCAG mapped grain size of 90 \( \mu \text{m} \) for \( \theta_r \equiv 0-45^\circ \) in the 100% snow cover case, grain size dropped from 90 \( \mu \text{m} \) to 70-80 \( \mu \text{m} \) abruptly at \( \theta_r \equiv 30^\circ \) for the 75% snow cover case.

For the 208/520 BRF, retrieved SCA ranged from 0.71 in the range \((\theta_r \equiv 40-60^\circ, \phi_r \equiv 90-180^\circ)\) to 0.76 in the range \((\theta_r \equiv 0-60^\circ, \phi_r \equiv 0-30^\circ)\). The grain size distribution remained essentially unchanged from that for 100% snow cover.

For the 624/1560 BRF, retrieved SCA also ranged from 0.71 to 0.76 with nearly the same distribution as that for the 208/520 BRF. Likewise, the grain size distribution was unchanged from that for the 100% snow cover case.

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Figure 83 shows the MEMSCAG results for the 75%/25% snow/rock mixture for $\theta_0 = 60^\circ$. For the 41/104 BRF, retrieved SCA ranged from 0.70-0.72 in the backscattering angles to 0.8-0.85 in the forward scattering angles. Only near nadir and in a band spanning ($\theta_r \equiv 15-25^\circ$, $\phi_r \equiv 0-60^\circ$) was the retrieved SCA 0.75±0.01. As in the case of $\theta_0 = 30^\circ$, the grain size distribution mimicked that of the 100% snow cover grain size distribution but with a narrower angular range for the peak of 90 $\mu$m.

For the 208/520 BRF, retrieved SCA ranged from 0.7 in the backscattering angles through 0.75 just forward of nadir to $> 0.8$ for $\theta_r > 45^\circ$ in the forward half of the principal plane. Retrieved SCA in the perpendicular plane ($\theta_r = 0-60^\circ$, $\phi_r \equiv 90^\circ$) was nearly constant at 0.72-0.73. The grain size range was 50-350 $\mu$m with the same distribution as for 100% snow cover with $\theta_0 = 60^\circ$.

For the 624/1560 BRF, retrieved SCA had a similar distribution to that of the 208/520 results but with a broader range of angles exhibiting retrieved SCA in the range 0.71-0.79. Retrieved SCA was $> 80\%$ only for $\theta_r > 55^\circ$ and $\phi_r \equiv 0-30^\circ$. The grain size range was 100-1100 $\mu$m with the same distribution as for 100% snow cover with $\theta_0 = 60^\circ$.

In summary, for $\theta_0 = 30^\circ$ and 75% snow cover, errors in retrieved SCA decreased from [-0.04,0.1] to [-0.04,0.0] as particle size increased. Underestimates generally occurred in the backscattering angles and overestimates
occurred in the forward scattering angles. The retrieved grain size range increased from [40 $\mu$m, 90 $\mu$m] to [600 $\mu$m, 1100 $\mu$m] as particle size increased, with larger grain sizes in the backscattering angles and smaller grain sizes in the forward scattering angles.

For $\theta_0 = 60^\circ$ and 75% snow cover, the span of errors in retrieved SCA was the same for all particle size BRF images [-0.05,0.10], however the angular span of large errors decreased with increasing particle size. For 41/104, most of the angular domain was dominated by errors of $\sim$0.02 in the backscattering domain or $> 0.05$ in the forward scattering domain. As particle size increased to 208/520 and 624/1560, the angular domains of these errors decreased. For 624/1560, nearly all retrieved SCA at angles $\phi_r$ for $\theta_r < 45^\circ$ were within 0.03 of 0.75. The angular distribution of retrieved particle size was nearly unchanged from that for 100% snow cover. The sole significant difference was the narrower angular peak at 70 $\mu$m for the 41/104 BRF relative to that of the 41/104 BRF case for 100% snow cover and $\theta_0 = 60^\circ$.

50% Snow Cover BRF

Figure 84 shows the MEMSCAG results for the 50%/50% snow/rock cover mixture and $\theta_0 = 30^\circ$. For the 41/104 BRF, retrieved SCA ranged from 0.46 to 0.52. The underestimates occurred primarily in a near-almucantar band defined by ($\theta_r > 40^\circ$, $60^\circ \leq \phi_r \leq 180^\circ$). The grain size distribution was unchanged from that for 75% snow cover and $\theta_0 = 30^\circ$.  

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For the 208/520 BRF, retrieved SCA ranged from 0.45 to 0.5. The underestimates occurred in oblique iso-lines primarily in the backscattering angles where $\theta_r > 30^\circ$. SCA was $0.5 \pm 0.01$ for an angular lobe defined by $(\theta_r < 30^\circ, \phi_r = 180^\circ)$ to $(\theta_r = 60^\circ, \phi_r = 0^\circ)$. The grain size distribution was unchanged from that for 75% snow cover and $\theta_0 = 30^\circ$.

For the 624/1560 BRF, the angular distribution of retrieved SCA was nearly identical to that for the 208/520 BRF. Two differences were (a) the subtle decrease in SCA at $\theta_r \equiv 55-60^\circ$ for $\phi_r = 0^\circ$ and (b) the ring of overestimate up to 0.53 centered on $\theta_r \equiv 10^\circ$ for $\phi_r = 180^\circ$. The latter was due to the limited range of grain sizes in the snow spectral library. Again the grain size distribution was essentially unchanged from that for 75% snow cover and $\theta_0 = 30^\circ$.

Figure 85 shows the MEMSCAG results for the 50%/50% snow/rock cover mixture and $\theta_0 = 60^\circ$. For all particle size BRF images, retrieved SCA ranged from 0.44 to 0.53 in similar angular distributions. The retrieved SCA for the 41/104 BRF had more high angular frequency structure than the retrieved SCAs for the 208/520 and 624/1560 BRF images, which were nearly identical. The isolines of retrieved SCA lay in planes slightly oblique in their intersection of the BRF hemisphere. Underestimates occurred throughout the backscattering half of the distribution and into much of the forward scattering half of the distribution. The grain size distribution was unchanged from that for 75% snow cover and $\theta_0 = 60^\circ$. 

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In summary, for $\theta_0 = 30^\circ$ and 50% snow cover, errors in retrieved SCA decreased slightly in range from [-0.04,0.02] to [-0.05,0.0] as particle size increased. Underestimates generally occurred in the backscattering angles and overestimates occurred in the forward scattering angles. The retrieved grain size range increased from [40 $\mu$m, 90 $\mu$m] to [600 $\mu$m, 1100 $\mu$m] as particle size increased, with larger grain sizes in the backscattering angles and smaller grain sizes in the forward scattering angles.

For $\theta_0 = 60^\circ$ and 75% snow cover, the span of errors in retrieved SCA was the same for all particle size BRF images [-0.06,0.03] and the angular distributions of SCA were essentially unchanged with increasing particle size. Retrieved SCA for the 41/104 BRF had slightly more angular structure than the retrieved SCA for either 208/520 or 624/1560 BRF image. The angular distribution of retrieved particle size was unchanged from that for 75% snow cover and $\theta_0 = 60^\circ$, ranging from [20 $\mu$m, 70 $\mu$m] to [100 $\mu$m, 1100 $\mu$m] as particle size increased.

25% Snow Cover BRF

Figure 86 shows the MEMSCAG results for the 25%/75% snow/rock cover mixture and $\theta_0 = 30^\circ$. Retrieved SCA distributions for all BRF images were nearly identical, ranging from 0.22 to 0.255. Underestimates lay in angular bands defined by ($30^\circ \leq \theta_r \leq 60^\circ$, $\phi_r = 180^\circ$) to ($\theta_r = 60^\circ$, $\phi_r = 30^\circ$). Retrieved SCA of 0.25 ±0.01 lay in an angular lobe defined by up to $\theta_r = 30^\circ$ in the
backscattering directions to nearly all the forward scattering half of the BRF domain. The grain size distributions remained unchanged from those of 50% snow cover and $\theta_0 = 30^\circ$.

Figure 87 shows the MEMSCAG results for the 25%/75% snow/rock cover mixture and $\theta_0 = 60^\circ$. Retrieved SCA distributions for all particle size BRF images were nearly identical, ranging from 0.20 to 0.26. Isolines of retrieved SCA lay nearly orthogonal to the projection of the solar principal plane into the plane of the snow surface. This distribution may be thought of as a 'sliced-bread' distribution. The SCA band of $0.25\pm0.01$ had its intersection with the solar principal plane near ($\theta_r = 25^\circ$, $\phi_r = 0^\circ$) and forward. All retrieved grain size distributions remained unchanged from those of 50% snow cover and $\theta_0 = 60^\circ$.

3.2 ND Grain Size

Figure 88 and Figure 89 show the ND grain size retrievals from all particle size BRF images with 100% snow cover for $\theta_0 = 30^\circ$ and 60$^\circ$, respectively. The ND model assumes that snow cover is complete. The analysis here evaluates the 100% snow cover case. However, it is likely that small fractions of non-snow cover would have negligible to small effects on grain size retrievals. Therefore, further research should analyze the snow fraction range of validity of this model.
For $\theta_0 = 30^\circ$ (Figure 88), grain sizes for the 41/104 BRF ranged from 50 $\mu$m in the forward scattering direction ($\phi_r \equiv 0^\circ$, $\theta_r \equiv 60^\circ$) continuously to a peak of $\sim$90 $\mu$m in the backscattering direction near-nadir ($\phi_r \equiv 180^\circ$, $\theta_r \equiv 10^\circ$). Grain sizes for the 208/520 and 624/1560 particle BRF images ranged from 200 to 450 $\mu$m and 600 to 1200 $\mu$m, respectively, with similar angular distributions. However, the peak increased in breadth with increasing particle size, reaching to larger $\theta_r$ in the backscattering direction.

For $\theta_0 = 60^\circ$ (Figure 89), grain sizes ranged from 15 to 75 $\mu$m, 70 to 370 $\mu$m, and 200 to 1150 $\mu$m for the 41/104, 208/520, and 624/1560 particle BRF respectively. The grain size distributions were similar for all particle size BRF images with the smallest retrieved grain sizes at the forward scattering angles and a peak grain size near $\theta_r \equiv 25^\circ$. Unlike the retrievals for $\theta_0 = 30^\circ$, the breadth of the peak grain size decreased with increasing grain size.

### 3.3 Comparison of MEMSCAG and ND Grain Size

Figure 90 and Figure 91 show $\Delta r = r_{ND} - r_{MEMSCAG}$ for 100% snow cover for $\theta_0 = 30^\circ$ and 60$^\circ$, respectively.

For $\theta_0 = 30^\circ$ (Figure 90), $\Delta r$ ranged from $-20$ to $15 \mu$m, $-3$ to $25 \mu$m, and $15$ to $105 \mu$m for the 41/104, 208/520, and 624/1560 particle BRF, respectively. The distribution of $\Delta r$ for the 41/104 BRF shows smaller grain sizes for ND than MEMSCAG in the backscattering angles and larger grain
sizes for ND in the forward scattering angles. The distribution of $\Delta r$ for the 208/520 BRF shows the greatest $\Delta r$ at near-nadir angles and decreasing to ~0 as $\theta_r$ increases for all $\phi_r$. The distribution of $\Delta r$ for the 624/1560 BRF was wavelike about a maximum of 105 $\mu$m at ($\theta_r = 10^\circ$, $\phi_r = 180^\circ$) but generally larger in the backscattering angles than in the forward scattering angles. As particle size increased, the distributions migrated from largest $\Delta r$ in the forward scattering angles (41/104) to quasi-symmetric about nadir (208/520) to largest $\Delta r$ in the backscattering angles (624/1560).

For $\theta_0 = 60^\circ$ (Figure 91), $\Delta r$ ranged from $-5$ to $25 \mu$m, 3 to $45 \mu$m, and $-20$ to $115 \mu$m for the 41/104, 208/520, and 624/1560 particle BRF, respectively. The distributions of $\Delta r$ for $\theta_0 = 60^\circ$ were more anisotropic than for $\theta_0 = 30^\circ$. The distribution of $\Delta r$ for the 41/104 BRF had a range of [5 $\mu$m, 25 $\mu$m] in the forward half of the BRF and [-5 $\mu$m, 10 $\mu$m] in the backward half of the BRF. $\Delta r$ for 208/520 was near-symmetric about ($\theta_r = 5^\circ$, $\phi_r = 180^\circ$) with the highest values near-nadir. The distribution of $\Delta r$ for the 624/1560 BRF had a range of [-20 $\mu$m, 70 $\mu$m] in the forward half of the BRF and [65$\mu$m, 115 $\mu$m] in the backward half of the BRF. As with the $\Delta r$ distributions for $\theta_0 = 30^\circ$, the $\Delta r$ were largest in the forward scattering angles (41/104), symmetric about near-nadir (208/520), and largest in the backscattering angles (624/1560).
The use of spheres by both models and spheroids for the bidirectional reflectance images brings to mind the question that has puzzled snow remote sensing scientists for years, “What is grain size?” While the analysis of the model sensitivities to anisotropic reflectance in retrieving grain size is enlightening from a snow optics standpoint, the more pertinent question that extends from grain size retrievals is “How does anisotropic reflectance affect inference of snow albedo with these models?”

3.4 MEMSCAG Albedo

I inverted the $\text{n}_{\text{MEMSCAG}}$ for albedo as described in section Chapter IV2.3 above. Figure 92 and Figure 93 show albedo inferred from the MEMSCAG model for the BRF images for the all three particle sizes for $\theta_0 = 30^\circ$ and $\theta_0 = 60^\circ$. The snow layers for particle sizes 41/104, 208/520, and 624/1560 had respective albedos of 0.80, 0.71, and 0.64 for $\theta_0 = 30^\circ$ and 0.83, 0.75, and 0.68 for $\theta_0 = 60^\circ$. Inferred albedos for $\theta_0 = 30^\circ$ had ranges of [0.79, 0.82] for the 41/104 BRF, [0.71, 0.75] for the 208/520 BRF, and [0.645, 0.687] for the 624/1560 BRF, with corresponding spans 0.03, 0.04, and 0.042. Inferred albedos for $\theta_0 = 60^\circ$ had ranges of [0.83, 0.92] for the 41/104 BRF, [0.75, 0.84] for the 208/520 BRF, and [0.69, 0.79] for the 624/1560 BRF, with corresponding spans 0.09, 0.09, and 0.10. The range of inferred albedos increased slightly with increasing grain size and by more than a factor of 2 with increasing solar zenith.
I then computed the error in inferred albedo. Figure 94 and Figure 95 show angular images of the errors in inferred albedo with respect to the prescribed albedo (e.g. $\alpha_{MEMSCAG} - \alpha_{41/104}$) for $\theta_0 = 30^\circ$ and $\theta_0 = 60^\circ$. In Figure 94 ($\theta_0 = 30^\circ$), albedo errors ranges increased in span from [-0.005, 0.025] for the 41/104 particles, to [-0.005, 0.037] for 208/520 particles, and to [0.005, 0.05] for 624/1560 particles. For the 41/104 BRF, albedo errors were less than 0.01 in all of the backscattering angles and the range $0^\circ \leq \theta_r \leq 40^\circ$ in the forward half of the BRF, and approached 0.025 at ($\theta_r \equiv 60^\circ$, $\phi_r = 0^\circ$). For the 208/520 BRF, albedo errors were less than 0.01 in most of the backscattering angles and to $\theta_r \equiv 30^\circ$ in the forward scattering angles. Errors increased continuously in the forward direction to 0.037 at ($\theta_r \equiv 60^\circ$, $\phi_r = 0^\circ$). The distribution of albedo errors for the 624/1560 BRF was similar to that of the 208/520 BRF. The only differences were (a) near-nadir, the error was ~ 0.005 rather than ~ 0.005 and (b) the increase in error was steeper, ending with an error of 0.05 at ($\theta_r \equiv 60^\circ$, $\phi_r = 0^\circ$).

Albedo errors for $\theta_0 = 60^\circ$ were greater than for $\theta_0 = 30^\circ$ (Figure 95). Error ranges were [0.005, 0.09] for the 41/104 particles, [0.01, 0.09] for 208/520 particles, and [0.01, 0.11] for 624/1560 particles. Therefore, errors in inferred albedo increased with increasing grain size and increasing solar zenith angle. This results from increasing anisotropy with increasing particle size and increasing solar zenith angle.
3.5 ND Albedo

Figure 96 and Figure 97 show albedo inferred from the ND model for the BRF images for the all three particle sizes for $\theta_0 = 30^\circ$ and $\theta_0 = 60^\circ$, respectively.

For $\theta_0 = 30^\circ$, the angular distributions of ND-retrieved albedo were essentially the same as those from MEMSCAG. However, the albedo gradient in the ND 41/104 albedo image was more continuous than that in the MEMSCAG 41/104 albedo image. The ranges were [0.79, 0.82], [0.71, 0.75], and [0.64, 0.69] for the 41/104, 208/520, and 624/1560 BRF images respectively.

Figure 98 shows the albedo errors for $\theta_0 = 30^\circ$. As with the errors for the MEMSCAG albedo retrievals, error ranges increased in breadth from [-0.005, 0.018] to [-0.008, 0.038] and to [0.003, 0.048] for the 41/104, 208/520, and 624/1560 BRF images, respectively.

In summary, for all $\phi_r$ with $\theta_r < 30^\circ$, albedo errors were less than 0.01. For all $\phi_r$ with $\theta_r < 45^\circ$, errors in inferred albedo increased with particle size from ~0.02 to ~0.045 at the largest view zenith angles and $\phi_r = 0-30^\circ$.

Figure 97 shows the albedo retrievals for $\theta_0 = 60^\circ$. The retrieved albedo ranges were [0.825, 0.9], [0.755, 0.835], and [0.685, 0.79] for the 41/104, 208/520, and 624/1560 BRF images, respectively. These spans increased from 0.075 to 0.08 and 0.105 with increasing particle size. The angular distributions were similar to those from MEMSCAG. This similarity follows
from the similarity of retrieved grain size distributions. Figure 99 shows the errors in retrieved albedo for $\theta_0 = 60^\circ$. Error ranges were [0.005, 0.07], [0.005, 0.09], and [0.005, 0.11], respectively.

In summary, for all $\phi$, with $\theta_r < 30^\circ$, ranges of albedo errors increased monotonically from [0.005, 0.015] to [0.005, 0.04] for 41/104 to 624/1560. For all $\phi$, with $\theta_r < 45^\circ$, ranges of albedo errors increased monotonically from [0.005, 0.03] to [0.005, 0.07] for 41/104 to 624/1560. The greatest errors increased with particle size from $-0.07$ to $-0.11$ at the $\theta_r \equiv 60^\circ$, $\phi_r = 0^\circ$.

The range of inferred albedo increased with increasing grain size and with increasing solar zenith as did the errors in inferred albedo. This results from increasing anisotropic reflectance with increasing particle size and increasing solar zenith angle.

3.6 Comparison of MEMSCAG and ND Albedo

Figure 100 and Figure 101 show the differences in inferred albedo ($\Delta \alpha = \alpha_{ND} - \alpha_{MEMSCAG}$) between the ND and MEMSCAG models for all particle sizes for the 100% snow cover images for $\theta_0 = 30^\circ$ and $60^\circ$, respectively.

For $\theta_0 = 30^\circ$, differences ranged from $[-0.008, 0.012]$ for the 41/104 BRF image, $[-0.004, 0.0]$ for the 208/520 BRF image, and $[-0.006, 0.0]$ for the 624/1560 BRF image. The only significant differences ($|\Delta \alpha| > 1.0 \%$) lay in the angular range ($45^\circ \leq \theta_r \leq 52^\circ$, $150^\circ \leq \phi_r \leq 180^\circ$) for the 41/104 BRF image.
For $\theta_0 = 60^\circ$, differences ranged from [-0.022, 0.0025] for the 41/104 BRF image, [-0.0075, 0.0] for the 208/520 BRF image, and [-0.005, 0.0] for the 624/1560 BRF image. The only significant differences between the two models occurred in the 41/104 BRF image with $\Delta \alpha \equiv [-0.02, -0.01]$ through much of the forward reflectance angles.

Therefore, MEMSCAG and ND have nearly identical sensitivities to bidirectional reflectance in retrieving albedo for a threshold of $|\Delta \alpha| < 0.01$. Only for small particle snow do the two models diverge up to $|\Delta \alpha| \equiv 0.02$.

Table 8 Summary of MEMSCAG and ND results for 100% snow cover with $\theta_0 = 30^\circ$ and 60°. Error ranges are presented for SCA and $\alpha$ and the range of retrieved particle sizes. This error would be mitigated by larger particle size snow endmember in the spectral library of MEMSCAG.

<table>
<thead>
<tr>
<th>100% SCA, $\theta_0 = 30^\circ$</th>
<th>100% SCA, $\theta_0 = 60^\circ$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ellipse size a.c ((\mu m))</td>
<td>Ellipse size a.c ((\mu m))</td>
</tr>
<tr>
<td>41/104</td>
<td>208/520</td>
</tr>
<tr>
<td>$\Delta$SCA</td>
<td>$\Delta \alpha$</td>
</tr>
<tr>
<td>50.90</td>
<td>-0.01</td>
</tr>
<tr>
<td>200.450</td>
<td>0.01</td>
</tr>
<tr>
<td>600.1100</td>
<td>0.0</td>
</tr>
<tr>
<td>10.80</td>
<td>0.01</td>
</tr>
<tr>
<td>70.370</td>
<td>0.01</td>
</tr>
<tr>
<td>200.1150</td>
<td>0.01</td>
</tr>
</tbody>
</table>
4 CONCLUSIONS

The two models for inverting imaging spectrometer data for snow physical properties, MEMSCAG and Nolin/Dozier, are comparably sensitive to the anisotropic reflectance of snow in angular domains accessed by current airborne and spaceborne imaging spectrometers and multi-spectral imagers.

Table 9 Summary of MEMSCAG $\Delta$SCA results for $\theta_0 = 30^\circ$ and varying SCA from 75 to 25%.

<table>
<thead>
<tr>
<th>$\Delta$SCA, $\theta_0 = 30^\circ$</th>
<th>75% SCA</th>
<th>50% SCA</th>
<th>25% SCA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ellipse size a:c (µm)</td>
<td>min, max</td>
<td>min, max</td>
<td>min, max</td>
</tr>
<tr>
<td>41/104</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>208/520</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>624/1560</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 10 Summary of MEMSCAG $\Delta$SCA results for $\theta_0 = 60^\circ$ and varying SCA from 75 to 25%.

<table>
<thead>
<tr>
<th>$\Delta$SCA, $\theta_0 = 60^\circ$</th>
<th>75% SCA</th>
<th>50% SCA</th>
<th>25% SCA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ellipse size a:c (µm)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>41/104</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>208/520</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>624/1560</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

SCA retrievals from MEMSCAG exhibit increasing sensitivity to BRF with decreasing grain size, increasing solar zenith angle, and increasing snow cover. Errors in retrieved SCA ranged from overestimates of 0.2 in the for-
ward reflectance angles to underestimates of $-0.15$ in the backward reflectance angles. Grain size retrievals with MEMSCAG had increasing sensitivity with increasing particle size and increasing solar zenith angle, but were insensitive to the snow fraction. The largest retrieved grain sizes generally occurred around a peak in the backward half of the solar principal plane and the smallest grain sizes occurred at the largest view zenith angle in the forward half of the principal plane. The grain size distributions retrieved from the ND model in the 100% snow cover case closely matched those from MEMSCAG, independent of the image particle size or solar zenith angle.

Both models had albedo errors up to 5% for $\theta_v = 30^\circ$ and 11% for $\theta_v = 60^\circ$. Albedo overestimates occurred at the largest view zenith angles in the forward half of the solar principal plane and underestimates occurred about a peak in the backward half.

The Airborne Visible Infrared Imaging Spectrometer (AVIRIS) has an off-nadir scan of $\pm 14^\circ$. In this angular range, over a level surface, anisotropic reflectance would have little effect on the accuracy of inferred subpixel SCA and albedo. However, over rough terrain, large local view zenith angles in the forward half of the BRF are more prevalent, increasing the likelihood of erroneous snow-covered area and albedo retrievals. The same arguments hold for the current spaceborne imaging spectrometer Hyperion (EO-1 satellite),
which has an angular range of ±6° on either side of nadir and has pointing capability to place the view zenith near 22° at the end of a scan.

The Moderate Resolution Imaging Spectroradiometer (MODIS), the flagship instrument aboard the NASA Terra satellite, has an angular range of ±55° on either side of nadir. Therefore, level snow surfaces observed at the edge of the swath from the forward half of the BRF may have inferred SCA errors of 15% and inferred albedo errors up to 10% when using MEMSCAG. MODIS does not have sufficient spectral resolution in the λ = 1.03 μm ice band to allow analysis with the ND model. The ground instantaneous field of view of MODIS ranges from 250 to 1000 m, relative to the 30 m GIFOV of Hyperion and the Landsat Thematic Mapper. In rough terrain, the instantaneous field of view of MODIS is more likely to view radiance from various local view geometries, further complicating retrievals. An analysis of this issue should be performed in conjunction with the use of MEMSCAG or other physically based models on MODIS data.

Spatially distributed energy balance models require accurate parameterization of snow spatial and physical properties. This study demonstrated the sensitivity of quantitative retrievals of subpixel SCA, grain size, and albedo to anisotropic reflectance. Even for imaging spectrometers with relatively narrow scan ranges, retrievals over rough alpine terrain demand accurate digital elevation data in order to meet SCA and albedo requirements of
10% and 2%, respectively (National Research Council Polar Research Board, 1989).
Figure 63. Integral of ice absorption feature at $\lambda = 1.03 \, \mu m$. The red region represents the integral. The line connecting the shoulders of the ice absorption feature gives the continuum reflectance.
Figure 64. Cross sections of spheroids used for synthetic BRF images. The particles have a-axis to c-axis ratios of 41 μm/104 μm, 208 μm/520 μm, and 624 μm/1560 μm. These particles have the same surface area to volume ratios as spheres with radii of 50 μm, 250 μm, and 750 μm, respectively, while maintaining an aspect ratio of 0.4.
Figure 65. Single scattering phase functions, $p_{11}$, at wavelengths 0.55 $\mu$m, 1.03 $\mu$m, and 1.75 $\mu$m for the 41/104, 208/520, and 624/1560 spheroids. The scattering angle is denoted as $\Theta$. 

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Figure 66. Spectral single-scattering co-albedo for the 41/104, 208/520, and 624/1560 spheroids. The co-albedo \((1-\omega)\) gives the probability of absorption during an interaction.
Figure 67. Bidirectional reflectance factor spectra for the 41/104 spheroid for a range of view zenith angles at view azimuths of $0^\circ$, $90^\circ$, and $180^\circ$. The solar zenith angle was $30^\circ$. 

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Figure 68. Bidirectional reflectance factor spectra for the 208/520 spheroid for a range of view zenith angles at view azimuths of 0°, 90°, and 180°. The solar zenith angle was 30°.
Figure 69. Bidirectional reflectance factor spectra for the 624/1560 spheroid for a range of view zenith angles at view azimuths of $0^\circ$, $90^\circ$, and $180^\circ$. The solar zenith angle was $30^\circ$. 
Figure 70. Bidirectional reflectance factor spectra for the 41/104 spheroid for a range of view zenith angles at view azimuths of 0°, 90°, and 180°. The solar zenith angle was 60°.
Figure 71. Bidirectional reflectance factor spectra for the 208/520 spheroid for a range of view zenith angles at view azimuths of 0°, 90°, and 180°. The solar zenith angle was 60°.
Figure 72. Bidirectional reflectance factor spectra for the 624/1560 spheroid for a range of view zenith angles at view azimuths of $0^\circ$, $90^\circ$, and $180^\circ$. The solar zenith angle was $60^\circ$. 
Figure 73. Polar representation of the BRF for the 41/104 spheroid at wavelengths of 0.55 μm, 0.85 μm, 1.03 μm, 1.30 μm, 1.80 μm, and 2.25 μm. The solar zenith angle for these data was 30°.
Figure 74. Polar representation of the BRF for the 208/520 spheroid at wavelengths of 0.55 μm, 0.85 μm, 1.03 μm, 1.30 μm, 1.80 μm, and 2.25 μm. The solar zenith angle for these data was 30°.
Figure 75. Polar representation of the BRF for the 624/1560 spheroid at wavelengths of 0.55 μm, 0.85 μm, 1.03 μm, 1.30 μm, 1.80 μm, and 2.25 μm. The solar zenith angle for these data was 30°.
Figure 76. Polar representation of the BRF for the 41/104 spheroid at wavelengths of 0.55 μm, 0.85 μm, 1.03 μm, 1.30 μm, 1.80 μm, and 2.25 μm. The solar zenith angle for these data was 60°.
Figure 77. Polar representation of the BRF for the 208/520 spheroid at wavelengths of 0.55 μm, 0.85 μm, 1.03 μm, 1.30 μm, 1.80 μm, and 2.25 μm. The solar zenith angle for these data was 60°.
Figure 78. Polar representation of the BRF for the 624/1560 spheroid at wavelengths of 0.55 μm, 0.85 μm, 1.03 μm, 1.30 μm, 1.80 μm, and 2.25 μm. The solar zenith angle for these data was 60°.
Figure 79. Reflectance spectrum for granite used in spectral mixtures with BRF images.
Figure 80. MEMSCAG results for $\theta_0 = 30^\circ$ and 100% snow cover.
Figure 81. MEMSCAG results for $\theta_0 = 60^\circ$ and 100% snow cover.
Figure 82. MEMSCAG results for $\theta_0 = 30^\circ$ and 75% snow cover.
Figure 83. MEMSCAG results for $\theta_0 = 60^\circ$ and 75% snow cover.
Figure 84. MEMSCAG results for $\theta_0 = 30^\circ$ and 50% snow cover.
Figure 85. MEMSCAG results for $\theta_0 = 60^\circ$ and 50% snow cover.
Figure 86. MEMSCAG results for θ₀ = 30° and 25% snow cover.
Figure 87. MEMSCAG results for $\theta_0 = 60^\circ$ and 25% snow cover.
Figure 88. ND grain size retrievals for $\theta_0 = 30^\circ$. 
Figure 89. ND grain size results for $\theta_0 = 60^\circ$. 

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Figure 90. Angular distributions of $\Delta r = r_{\text{ND}} - r_{\text{MEMSCAG}}$ for $\theta_0 = 30^\circ$. 

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Figure 91. Angular distributions of $\Delta r = r_{ND} - r_{MEMSCAG}$ for $\theta_0 = 60^\circ$.
Figure 92. Albedo retrievals using $r_{MEMSCAG}$ for $\theta_0 = 30^\circ$. 

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Figure 93. Albedo retrievals with \( r_{\text{MEMSCAG}} \) for \( \theta_0 = 60^\circ \).
Figure 94. Error in MEMSCAG albedo retrieval for $\theta_0 = 30^\circ$. 
Figure 95. Error in MEMSCAG albedo retrieval for $\theta_0 = 60^\circ$. 

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Figure 96. Albedo retrieved with $r_{ND}$ for $\theta_0 = 30^\circ$. 
Figure 97. Albedo retrieved with $r_{ND}$ for $\theta_0 = 60^\circ$. 
Figure 98. Error in ND albedo retrieval for $\theta_0 = 30^\circ$. 

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Figure 99. Error in ND albedo retrieval for $\theta_0 = 60^\circ$. 

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Figure 100. Difference between ND and MEMSCAG albedo retrievals for $\theta_0 = 30^\circ$. 
Figure 101. Difference between ND and MEMSCAG albedo retrievals for $\theta_0 = 60^\circ$. 
EN.REFLIST
Appendix A

Snow Optical Properties in the 0.4 – 2.5 μm
wavelength range: Measurements and Modeling
5 Optical Properties of Ice and Snow

Snow is a multiple scattering medium composed of ice grains and air and, when the snow temperature reaches 0°C, liquid water at volume fractions ranging from 0 to ~15%. Snow may also contain dust, soot, algae, and other impurities. The optical properties of snow are therefore controlled by the bulk optical properties of ice, the size and morphology of the ice grains, the concentration of liquid water, and the bulk optical properties and size and morphology of absorbing and scattering impurities.

The dominant optical property of ice that causes the spectral variation in snow reflectance is the spectral variation in the imaginary part, $k$, of the complex refractive index ($n=n+i\kappa$). For both ice and water, $k$ varies six orders of magnitude across the wavelength range 0.4-2.5 μm whereas the real part $n$ is nearly constant across this spectrum (Figure 102). Ice and water are transparent across the visible wavelengths ($10^{-9} \leq k \leq 10^{-6}$). In the near-infrared spectrum, $k_i$ (ice) and $k_w$ (water) increase nearly monotonically to ~$10^{-5}$ at $\lambda=1.22$ μm (moderate absorption) and to ~$10^{-3}$ at $\lambda=2.09$ μm (strong absorption). Transmission across distance $s$ attenuates in a pure medium as $e^{-4\pi k s/\lambda}$. Thus, the absorption coefficient ($4\pi k/\lambda$) varies from $7.11 \times 10^{-1}$ m$^{-1}$ at $\lambda=0.55$ μm to $1.05 \times 10^{3}$ m$^{-1}$ at $\lambda=1.22$ μm and $6.04 \times 10^{4}$ m$^{-1}$ at $\lambda=2.09$ μm.
$k_w$ and $k_i$ are consistent in magnitude across the spectrum but $k_w$ is shifted to shorter wavelengths, particularly in the absorption features in the range 1.0-2.0 $\mu$m. Hence, the general shape of the spectral signature of snow containing liquid water is unchanged from that of dry snow, but the water causes the snow grains to cluster [Colbeck, 1986 #209], thus decreasing the near-infrared reflectance (O'Brien, 1975; Dozier, 1989). Spectral overlap of bulk ice and water absorption features broadens the composite absorption features in the reflectance spectrum, lowering the local reflectance in the regions of the absorption features. It is this broadening of the wet snow absorption features that Green et al. (2002) analyze to infer liquid water content at the snow surface.

6 Spectral Reflectance of Snow

Figure 103 shows the spectral directional-hemispherical reflectance of pure, optically thick snow for visible and near-infrared wavelengths and snow grain radii from 50-1000 $\mu$m. The spectral reflectance of snow is inversely related to grain size (Warren and Wiscombe, 1980; Wiscombe and Warren, 1980). At visible wavelengths, where ice is relatively transparent ($k \approx 10^{-6}$), the spectral albedo of pure snow is near 100% and is insensitive to a change in grain size. However, absorbing impurities may be present in sufficient volume to significantly decrease the visible reflectance. In the spectral range 0.9
\( \leq \lambda \leq 1.3 \ \mu m \), the reflectance of snow decreases due to moderate absorption by ice \((k \approx 10^{-5})\). It is in this wavelength range that the spectral reflectance of snow is most sensitive to a change in grain size. However, at these wavelengths and longer, the spectral reflectance loses sensitivity to impurity concentration because of the decreasing contrast in \( k \) between ice and impurities coupled with the much larger volume fraction of ice compared to the impurities. At longer wavelengths, \( k \) converges to \( \approx 10^{-2} \) and the spectral albedo drops to less than 5\% when the grain radius exceeds 250 \( \mu m \). As grain size increases beyond 250 \( \mu m \), spectral reflectance becomes progressively less sensitive to grain size. In the context of vector spaces, the spectrum vectors become increasingly degenerate as grain size increases.

The characteristics described above are inherent to the snowpack and independent of the irradiance. However, wavelength-integrated albedo depends on the spectral and angular distribution of the irradiance on the surface.

Snow albedo increases as the solar zenith angle \( \theta_0 \) increases with other variables fixed. The spectral quality of the irradiance \( E_\lambda \) depends on the atmospheric characteristics such as clouds, water vapor concentration, aerosol concentration, and aerosol optical properties, and it also depends on topography. For example, illumination in the shade is predominantly in the wavelengths where snow’s spectral albedo is high \((\lambda < 0.7 \ \mu m)\).
Most snow exhibits diffuse angular reflectance with a forward reflectance peak that grows at longer wavelengths, for larger particles, and for larger solar zenith angles (Warren, 1982). For faceted particles such as dendrites and plates, a retro-solar reflectance peak exists at the view angle (measured from zenith) matching the illumination angle. Leroux et al. (1998) demonstrated through measurements and a doubling-adding radiative transfer model that the bidirectional reflectance of snow is sensitive to particle shape at $\lambda = 1.65 \mu m$. Warren et al. (1998) showed through measurements in Antarctica that the bidirectional reflectance of snow can be more sensitive to surface roughness, such as dunes and sastrugi, than to the snow grains, depending on the orientation of the roughness elements relative to the sun. However, for view angles ranging from $0^\circ$ to $15^\circ$ from zenith, surface roughness has negligible effect.

7 Physical Basis of Reflectance Model

7.1 Single Scattering

Calculations of scattering in snow are usually based on the assumption that scattering by irregularly shaped grains in a snowpack can be matched by that of ice spheres in each other's far field (Wiscombe and Warren, 1980). Arguments for assuming sphericity are presented in Mugnai and Wiscombe (1980), however the discussion focuses on size parameters $x$ (defined below)
much smaller than those commonly found in snow. Nonetheless, the random orientation and strong forward scattering of grains allows us to make such an assumption without large errors in flux calculations (Grenfell et al., 1994). The tremendous availability of code with which to calculate the parameters of single scattering by spheres makes the assumption doubly attractive. The greatest error from the sphericity assumption lies in its effect on the structure of the (single) scattering phase function and in turn, on the structure of the bidirectional reflectance distribution function (Leroux et al., 1998).

The Mie theory (or Mie-Debye) describes the interaction of a plane wave with a sphere that is large with respect to the wavelength of incident radiation (Mie, 1908; Thomas and Stamnes, 1999). Its derivation requires solutions to the vector wave equations, expansion of a plane wave in vector spherical harmonics, and development of the internal and scattered fields (Bohren and Huffman, 1988). These details will be left to the reader to enjoy elsewhere and I will proceed with a description of the solution products, using the notation of Thomas and Stamnes (1999).

The scattering phase function, which describes the angular distribution of light scattered by a given sphere, is given by

$$\rho(\Theta) = \frac{1}{2} \left| S_1 \right|^2 + \left| S_2 \right|^2,$$

where the scattering amplitude functions $S_1(\Theta)$ and $S_2(\Theta)$ are defined as
\[ S_1(\Theta) = \sum_{n=1}^{2n+1} \frac{2n+1}{n(n+1)} [a_n \tau_n + b_n \pi_n] \]

(37)

\[ S_2(\Theta) = \sum_{n=1}^{2n+1} \frac{2n+1}{n(n+1)} [b_n \pi_n + a_n \tau_n] \]

and where

\[ \pi_n(\Theta) = \frac{1}{\sin \Theta} P_n^1(\Theta) \]

(38)

\[ \tau_n(\Theta) = \frac{d}{d\Theta} P_n^1(\Theta) \]

\( P_n^1(\Theta) \) is the associated Legendre polynomial of degree \( n \) and order 1, and

the coefficients \( a_n \) and \( b_n \) are defined as

\[ a_n = \frac{m \psi_n(mx) \psi'_n(x) - \psi_n(x) \psi'_n(mx)}{m \psi_n(mx) \xi_n(x) - \xi_n(x) \psi'_n(mx)} \]

(39)

\[ b_n = \frac{m \psi_n(mx) \psi'_n(x) - m \psi_n(x) \psi'_n(mx)}{m \psi_n(mx) \xi'_n(x) - m \xi_n(x) \psi'_n(mx)} \]

where the size parameter, \( x \), is given by

\[ x = \frac{2 \pi r}{\lambda} \]

(40)

the relative refractive index, \( m \), is

\[ m = \frac{N_p}{N} \]

(41)

\( N_p \) and \( N \) are the refractive indices of the particle and medium, respectively, and the functions \( \psi_n \) and \( \xi_n \) are the Ricatti-Bessel functions. Differentiation is denoted with primes ('). Figure 104 shows the phase functions for an ice sphere of radius 100 \( \mu m \) at \( \lambda = 0.55 \mu m \) and \( \lambda = 1.85 \mu m \).
The mean cosine of the scattering angle is known as the asymmetry parameter, $g$. It is given by

$$g = \frac{1}{2} \int_{-1}^{1} d(\cos \Theta)(\cos \Theta)p(\cos \Theta).$$

For isotropic scattering or scattering that is symmetric about a scattering angle of 90°, $g = 0$. As the scattering approaches complete forward scattering, $g$ approaches 1.0. As scattering approaches complete backward scattering, $g$ approaches −1.0. Figure 105 shows the asymmetry parameter across the wavelength range $0.4 \leq \lambda \leq 2.5 \, \mu m$ for spheres of radii 50 \, \mu m and larger. Because of the strong forward scattering of ice spheres, $g$ lies between 0.88 and 1.0 in this wavelength range. The asymmetry parameter increases with wavelength across the range with local minima at 1.9 and 2.3 \, \mu m, due to local minima in the imaginary part of the complex refractive index. Also, $g$ increases as particle size increases because the diffracted component of scattered amplitude becomes more important when absorption increases. The ripples in the smaller grain sizes are due to surface waves that graze and travel the sphere (Bohren and Huffman, 1998). Effects of ripples are not likely to be seen in nature because they are quickly damped by even a narrow size distribution of particles.

The scattering cross section, $\sigma_n$, and extinction cross section, $k_n$, are calculated as

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(43) \[ \sigma_n = \frac{2\pi}{k^2} \sum_{m=1}^\infty (2n+1)(|a_n|^2 + |b_n|^2), \]

(44) \[ k_n = \frac{2\pi}{k^2} \sum_{m=1}^\infty (2n+1) \Re(a_n + b_n), \]

where \( k = 2\pi/\lambda \) is the angular wavenumber and \( \Re \) indicates the expression's real part. From \( \sigma_n \) and \( k_n \) come the scattering efficiency, \( Q_{\text{sca}} \), and the extinction efficiency, \( Q_{\text{ext}} \), as follows

(45) \[ Q_{\text{sca}} = \frac{\sigma_n}{\pi r^2} \]

(46) \[ Q_{\text{ext}} = \frac{k_n}{\pi r^2} \]

where \( r \) is the radius of the ice sphere. For completeness, the absorption cross section \( \alpha_n \) is given by

(47) \[ k_n = \sigma_n + \alpha_n. \]

The single-scattering albedo, \( \omega \), describes the probability that a photon will survive an extinction event and is defined as

(48) \[ \omega \equiv \frac{Q_{\text{sca}}}{Q_{\text{ext}}} = \frac{\sigma_n}{k_n} \]

and the single-scattering co-albedo, \( 1 - \omega \), describes the probability that a photon will be absorbed during an extinction event. Figure 106 shows the single-scattering co-albedo for the wavelength range \( 0.4 \leq \lambda \leq 2.5 \, \mu m \) for spheres of radii \( 50 \, \mu m \) and larger. The co-albedo increases dramatically across the spectrum. The co-albedo also increases with grain radius, particularly at the longer wavelengths (note the logarithmic scale). These attributes
of the co-albedo almost exclusively determine the shape of the spectral reflectance of snow and its sensitivity to grain size (Wiscombe and Warren, 1980), discussed below.

In order to model the spectral reflectance of wet snow, one must address the inclusion of liquid water in the snow matrix. Green et al. (2002) used the coated sphere approach in which they modeled the single scattering properties of a particle with an ice core and a liquid shell. This approach provides the same suite of single scattering properties that I discussed above for homogeneous ice spheres.


With the single scattering properties \( p(\Theta), Q_{\text{ext}}, \omega \), one may then model the spectral reflectance of snow with the discrete-ordinate solution to the radiative transfer equation.
7.2 Multiple Scattering

The radiative transfer equation describes the radiation field in a scattering, absorbing, and emitting medium [Chandrasekhar, 1960 #213]. Again, the reader is referred to other references for a development of the radiative transfer equation (Thomas and Stamnes, 1999). We begin with the integro-differential form of the radiative transfer equation that describes the intensity field for a multiple scattering and emitting medium:

\[
\frac{\mu}{d\tau} \frac{dI(\tau, \mu, \phi)}{d\tau} = I(\tau, \mu, \phi) - [1 - \omega(\tau)]B(T(\tau)) - \frac{\omega(\tau)}{4\pi} \int_0^{2\pi} d\phi' \int_{-1}^{1} d\mu' p(\tau, \mu, \phi; \mu', \phi') I(\tau, \mu', \phi') d\mu' \\
- \frac{\omega(\tau)I^{inc}}{4\pi} p(\tau, \mu, \phi; -\mu_o, \phi_o) e^{-\eta/\mu_o}
\]  

(49)

where \( I \) is radiance (W m\(^{-2}\) sr\(^{-1}\) m\(^{-1}\)), \( \mu = \cos \theta \), \( \tau \) is optical depth, \( \phi \) is the azimuth angle, \( \theta \) is the zenith angle, \( B(T(\tau)) \) is the Planck function at temperature \( T \), \( I^{inc} \) is the incident radiation at \( \tau = 0 \), and \( \mu_o \) and \( \phi_o \) are the solar zenith and solar azimuth angles, respectively.

Several accurate methods exist to solve the radiative transfer equation (Stamnes, 1987; Thomas and Stamnes, 1999). These include the discrete-ordinates, spherical-harmonic, and adding-doubling methods. A comparison of these methods is beyond the scope of this appendix. Instead, the focus will be on a description of the discrete-ordinates method using the notation of Stamnes (1987), followed by snow spectral reflectance results.

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The method allows analytic solutions at discrete ordinates. Initially, the phase function must be expanded into a series of $2n - 1$ Legendre polynomials

\begin{equation}
\rho(\tau, \cos \Theta) = \sum_{i=0}^{2n-1} (2i + 1) \chi_i(\tau) P_i(\cos \Theta),
\end{equation}

where $P_i$ is the $i$th Legendre polynomial and the $i$th expansion coefficient is

\begin{equation}
\chi_i = \frac{1}{2} \int_{-1}^{1} d(\cos \Theta) P_i(\cos \Theta) \rho(\tau, \cos \Theta).
\end{equation}

The intensity is expanded into a Fourier cosine series

\begin{equation}
I(\tau, \mu, \phi) = \sum_{m=0}^{2n-1} I_m(\tau, \mu) \cos m(\phi_0 - \phi).
\end{equation}

In turn, equation (52) yields $2n$ independent equations of the form

\begin{equation}
\mu \frac{dI_m(\tau, \mu)}{d\tau} = I_m(\tau, \mu) - J_m(\tau, \mu),
\end{equation}

where the source function, $J_m(\tau, \mu)$, is given by

\begin{equation}
J_m(\tau, \mu) = \frac{a}{2} \sum_{i=0}^{2n-1} (2i + 1) g_i^m P_i(\mu) \left[ \int_{-1}^{1} P_i(\mu') m(\tau, \mu') d\mu' + \frac{1}{2\pi} \int_{2\pi}^{2\pi} (2 - \delta_{0,m})(-1)^{+m} P_i(\mu_0) e^{-i\mu_0} \right] + \delta_{0,m}(1 - \omega) \Theta(T(\tau))
\end{equation}

and

\begin{align*}
\delta_{ij} &= 1 & i = j \\
\delta_{ij} &= 0 & i \neq j \\
g_i^m &= g_i \frac{(l - m)}{(l + m)} \\
g_i &= \frac{1}{2} \int_{-1}^{1} d(\cos \Theta) P_i(\cos \Theta) d(\cos \Theta). 
\end{align*}
We then have the discrete ordinate approximation to \( \frac{dI^m(\tau, \mu_s)}{d\tau} \) with

\[
(55) \quad \mu_s \frac{dI^m(\tau, \mu_s)}{d\tau} = I^m(\tau, \mu_s) - J^m(\tau, \mu_s)
\]

where \( \mu_s \) are the cosines of the quadrature angles. For a nonemitting atmosphere, \( B(T(\tau)) = 0 \), and the general discrete ordinate solution for each Fourier component of the diffuse intensity for a parallel beam incident from direction \( \mu_0 \) is then given by

\[
(56) \quad I^m(\tau, \mu_s) = \sum_{j=-n}^{n} G^m_j(\mu_s) e^{-k^m_\tau}
\]

where

\[
G^m_j(\mu_s) = L^m_j g^m_j(\mu_s) \quad \text{for } j \neq 0
\]

\( k^m_j \) and \( g^m_j(\mu_s) \) are the eigenvalues and eigenvectors, and \( L^m_j \) are constants of integration determined from boundary conditions. If \( \tau_N \) is the total optical thickness of the atmosphere (or in the case of snow, the total optical thickness of the snowpack), the intensities for all angles come from the following equations

\[
(57) \quad I^m(\tau, \mu) = \sum_{j=-n}^{n} G^m_j(\mu) \left[ e^{-k^m_\tau} - e^{[k^m_\tau + (\tau_N - \tau)/\mu_j]} \right]
\]

With a homogeneous layer of snow as the plane parallel atmosphere, we must specify the boundary and initial conditions specific to the snowpack. The optical thickness of a homogeneous layer of snow is given by
\[
\tau = \frac{3(SWE)Q_{\text{ext}}}{4\rho_r r} \quad \text{(dimensionless)}
\]

where \(Q_{\text{ext}}\) is the extinction efficiency, \(r\) is the mean snow grain radius (m), and the \textit{snow water equivalent} is defined as

\[(59) \quad SWE \equiv \rho_s d_s \quad \text{(kg m}^{-2}\text{)}\)

We must also establish the direct and diffuse irradiance at the top of the snowpack and specify the substrate reflectance. The irradiance components can be modeled using a standard atmospheric model in an atmospheric transmission model such as the \textit{Santa Barbara DISORT Atmospheric Radiative Transfer} (SBDART) model (Ricchiazzi et al., 1998) or the \textit{MODerate resolution TRANssmittance} (MODTRAN) model (Berk et al., 1989).

With these boundary conditions and the single scattering properties described in Section 7, we can model the radiance field over a homogeneous snowpack. The radiance field allows us to calculate the bidirectional reflectance distribution function (\textit{BRDF}) and the bidirectional reflectance factor (\textit{BRF}), defined as

\[(60) \quad \text{BRDF}_1(\theta, \phi; \theta_r, \phi_r) = \frac{l_1(\theta_r, \phi_r)}{\cos \theta_o E_\lambda(\theta_o, \phi_o)} \quad (\text{sr}^{-1})
\]

\[(61) \quad \text{BRF}_1(\theta, \phi) = \pi \text{BRDF}_1(\theta, \phi; \theta, \phi) = \frac{\pi l_1(\theta_r, \phi_r)}{\cos \theta_o E_\lambda(\theta_o, \phi_o)} \quad \text{(dimensionless)}
\]

where \(\theta, \phi, \theta_o,\) and \(\phi_o\) are the reflected zenith, reflected azimuth, incident zenith, and incident azimuth angles, respectively. The \textit{BRDF} is the ratio of reflected radiance into direction \(\theta, \phi\) to the irradiance from direction \(\theta_o, \phi_o\). The

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BRF gives a more physically intuitive measure of the angular distribution of reflected radiation. It represents the ratio of reflected radiance into direction \( \theta_r, \phi_r \) to that which a Lambertian surface would reflect into direction \( \theta_n, \phi_n \).

Spectral albedo is related to the BRF through the following integral:

\[
A_\lambda(\theta_o) = \int_0^{\pi/2} \int_0^{2\pi} BRF(\theta_o, \phi_o; \theta_r, \phi_r) \sin\theta_r \cos\theta_r d\theta_r d\phi_r.
\]

The relationship between spectral albedo and BRF is analogous to this equation, with the scalar \( 1/\pi \):

\[
A_\lambda(\theta_o) = \frac{1}{\pi} \int_0^{\pi/2} \int_0^{2\pi} BRF(\theta_o, \phi_o; \theta_r, \phi_r) \sin\theta_r \cos\theta_r d\theta_r d\phi_r.
\]

To this point, I have ignored surface roughness. Warren et al. (1998) showed with measurements in Antarctica that the structure of the bidirectional reflectance of snow can be sensitive to surface roughness, such as dunes and sastrugi, depending on the orientation of the roughness elements relative to the sun. However, they demonstrated that for near-nadir view angles (\( \theta_r < 50^\circ \)), the spectral reflectance in visible wavelengths is relatively insensitive to the orientation of the surface roughness and the solar elevation. They consider these results valid up to \( \lambda = 0.9 \, \mu m \). Further work is needed to investigate the sensitivity of snow reflectance to surface roughness for \( \lambda > 0.9 \, \mu m \).

The characteristics described above are inherent to the snowpack and independent of the irradiance. However, albedo depends on the spectral and angular nature of the irradiance on the surface. Snow albedo increases as the
solar zenith angle increases with other variables fixed. The spectral quality of the irradiance for a clear sky depends on the atmospheric characteristics such as water vapor concentration, aerosol concentration, and aerosol optical properties. Whether a change in the spectral quality of the irradiance increases or decreases the albedo depends on the zenith angle of the direct beam irradiance (Warren, 1982).

The spectral reflectance data presented in this dissertation were calculated using the discrete-ordinates FORTRAN subroutine DISORT.f, available via ftp from Warren J. Wiscombe's website ftp://climate.gsfc.nasa.gov/pub/wiscombe/.

Calculations of the diffuse and direct irradiance components were made with SBDART which is available via ftp from the Earth Space Research Group (University of California, Santa Barbara) website http://www.crseo.ucsb.edu/esrg/pauls_dir/.
Figure 102. Real (left) and imaginary (right) parts of complex refractive index of water and ice. Data are available via ftp (Wiscombe, 2001) that include improvements by Kou et al. (1993) and Perovich and Govoni (1991) to Warren's (1984) summary.
Figure 103. Directional-hemispherical reflectance of snow for grain radii $r = 50 - 1000 \, \mu m$. 
Figure 104. Single scattering phase functions for ice spheres of radius 100 μm at $\lambda = 0.55 \mu m$ and $1.80 \mu m$. Note that the forward scattering peak ($\Theta = 0^\circ$) increases by nearly an order of magnitude from $\lambda = 0.55 \mu m$ to $1.8 \mu m$. 

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Figure 105. Asymmetry parameter spectra for ice spheres with radii ranging from 50 to 1000 µm. The ripple structure in the smaller particle spectra comes from the excitation of surface waves on ice spheres of a given radius. We do not observe the ripple structure in nature because the range of particle sizes and shapes smoothes the asymmetry parameter spectrum.
Figure 106. Single scattering co-albedo for ice spheres of radii from 50 to 1000 μm. The co-albedo is the additive complement to the single scattering albedo.
Appendix B

Computer Code for Automated Spectro-Goniometer
1 KINEMATICS CODE

1.1 ASGAngles.f90

PROGRAM ASGAngles
!
! ASGAngles
!
! Purpose: For the Automated Spectro-Goniometer (ASG), compute the rotation
! angles of the azimuth (Theta1) and zenith (Theta2) axes to access
! a given view vector from an initial view vector (default: nadir).
! Author: Thomas H. Painter
! Institution: Institute for Computational Earth System Science
! University of California, Santa Barbara
! Date: 05081998
! Update: 09082000 = Documentation
! 11142000 = Fully functional and validated.
!
! Internal Variables:
!
! real:
! kVec(3,1) = ASG elbow axis vector
! kAng(2,1) = ASG elbow axis polar vector
! z(3,1) = ASG shoulder axis (vertical) vector
! u(3,1) = Input view vectors
! R2(3,3) = Rodrigues' rotation matrix about k (elbow)
! R1(3,3) = Rodrigues' rotation matrix about z (shoulder)
! Beta = Arm arc angles
! Beta2 = Elbow start location (longitude)
!
! integer:
! OpenStatus = Status of opening input file
! AngleCount = Number of view angle pairs for which to determine
! rotations
! AllocateStatus = Status of memory allocation
! i = Counter through view angle pairs
! j = Counter through vectors for writes
!
! character:
! InFileName = Filename of input file containing view angle pairs
! OutFileName = Filename of output file for rotation angles
! Descriptor = String for the text lines in input file
!
! Output Variables:
!
! real:
! Angs(:,;) = View angle pairs - zenith, azimuth
! Vecs(:,;) = Cartesian vectors representing the view angle pairs
! Theta1(:,;) = Rotation about vertical axis
! Theta2(:,;) = Rotation about oblique axis
!
! Author: Thomas H. Painter

IMPLICIT NONE
!
! PARAMETERS
real, parameter :: PI = 3.141593
real, parameter :: Beta = 46.5 * PI / 180.0 ! Beta in radians
real, parameter :: Beta2 = 0.0 * PI / 180.0 ! Beta2 in radians

! VARIABLES
real :: kVect(3,1), kAng(2,1), z(3,1), u(3,1)
real :: R2(3,3), R1(3,3)
real, DIMENSION(:, :), ALLOCATABLE :: Angs ! view angles (degrees)
real, DIMENSION(:, :), ALLOCATABLE :: Vets ! view vectors from view angles
real, DIMENSION(:,), ALLOCATABLE :: Theta1,Theta2 ! rotations (azimuth,zenith)
integer :: dV1, dV2, dV3

character(30) :: InFileName, OutFileName
character(80) :: Descriptor

ingteger :: OpenStatus, AngleCount, AllocateStatus
integer :: i, j

! BEGIN
write(*,*) 'ASG_angles'

! ESTABLISH AXIS OF ROTATION FOR Theta2 - kVect
kAng = reshape((/Beta,Beta2/),/(2,1/))
z = reshape((/0.0, 0.0, 1.0/),/(3,1/))
kAng(1,1) = kAng(1,1) * PI / 180.0 ! degrees to radians
kAng(2,1) = kAng(2,1) * PI / 180.0 ! degrees to radians

call AnglesToVector(kAng,kVect)
write(*,*) 'k vector'
write(*,*) kAng(1,1), kAng(2,1)
write(*,*) kVect(1,1), kVect(2,1), kVect(3,1)
write(*,*)
kVect(1,1) = -1.0 * kVect(1,1)
kVect(2,1) = -1.0 * kVect(2,1)
kVect(3,1) = -1.0 * kVect(3,1) ! positive k
write(*,*) 'kVects : ',kVect(1,1), kVect(2,1), kVect(3,1)

! OPEN CONTROL FILE (NUMBER OF VIEW ANGLES AND ANGLES THEMSELVES)
write(*,*) 'INPUT CONTROL FILENAME?'
read(*,*) InFileName
open(unit=10, file=InFileName, status='old', iostat=OpenStatus)
if (OpenStatus > 0) STOP **** Error Opening Input Control File ****

! OPEN OUTPUT FILE
read(10,*) Descriptor
read(10,*) OutFileName
write(*,*) 'Output File : ', OutFileName
open(unit=11, file=OutFileName, status='unknown')

! OPEN DIAGNOSTICS FILE
open(unit=12, file='Diagnostics.txt', status='unknown')

! INPUT VIEW ANGLES
read(10,*) Descriptor
read(10,*) AngleCount
write(*,*) 'Number of view angles = ', AngleCount

! ALLOCATE MEMORY for ARRAYS Angs, Angs, Vets
allocate(Angs(1:2, 1:AngleCount), stat = AllocateStatus)
if (AllocateStatus > 0) STOP **** Error Allocating Memory for <Angs> ****
allocate(Vets(1:3, 1:AngleCount), stat = AllocateStatus)
if (AllocateStatus > 0) STOP **** Error Allocating Memory for <Vets> ****
allocate(Theta1(1:AngleCount), stat = AllocateStatus)
if (AllocateStatus > 0) STOP **** Error Allocating Memory for <Theta1> ****
allocate(Theta2(1:AngleCount), stat = AllocateStatus)
if (AllocateStatus > 0) STOP **** Error Allocating Memory for <Theta2> ****

! READ VIEW ANGLES, CONVERT TO RADIANS
read(10, *) Descriptor
write( *, *) 'View Zenith View Azimuth'
do I = 1, AngleCount
   read(10, *) (Angs(I, J), J = 1, 2)
   write( *, *) (Angs(J, I), J = 1, 2)
   Angs(1, i) = Angs(1, i) * Pi / 180.0 ! degrees to radians
   Angs(2, i) = Angs(2, i) * Pi / 180.0
end do

! CLOSE CONTROL FILE
close(10)

! COMPUTE VIEW VECTORS FROM VIEW ANGLES
do I = 1, AngleCount
call AnglesToVector(Angs(1:2, i), Vecs(1:3, i))
end do

! COMPUTE Theta2, Theta1 - rotation about zenith axis, azimuth axis
do I = 1, AngleCount
call RotationTheta2(Vecs(1:3, i), Beta, Theta2(i))
call Rodriguez(kVecs, Theta2(i), R2)
call RotationTheta1(Vecs(1:3, i), R2, Theta1(i))
call Rodriguez(kVecs, Theta1(i), R1)
call InvertRotations(R1, R2, u)

! write to diagnostics file
dv1 = 1000.*Vecs(1, i)-u(1, i))
dv2 = 1000.*Vecs(2, i)-u(2, i))
dv3 = 1000.*Vecs(3, i)-u(3, i))
write(12, *) 'View Angles ': Angs(1, i)*180.0/Pi, Angs(2, i)*180.0/Pi
write(12, *) 'Diff in vs. Out':, dv1, dv2, dv3

! OUTPUT THETA1 and THETA2 along with Zenith and Azimuth angles
do I = 1, AngleCount
write(11, *) Theta1(i)*180.0/Pi, Theta2(i)*180.0/Pi, Angs(1, i)*180.0/Pi, Angs(2, i)*180.0/Pi
write(11, *) 360.0+Theta1(i)*180.0/Pi, -1.0* Theta2(i)*180.0/Pi, Angs(1, i)*180.0/Pi, Angs(2, i)*180.0/Pi
end do

! DEALLOCATE MEMORY FROM ARRAYS Angs, Vecs
dealocate(Angs)
deallocate(Vecs)
deallocate(Theta1)
deallocate(Theta2)
close(11)

END PROGRAM ASGAngles
SUBROUTINE AnglesToVector(AngsVect, Vect)
!
! AnglesToVector
!
! Purpose: Compute view vector from view angles (zenith and azimuth).
! It is assumed that the view vectors are always negative in Z,
! that is, from hemisphere toward center of sphere.
!
! Internal Variables:

! Output Variables:

! Author: Thomas H. Painter
! Institution: Institute for Computational Earth System Science
! Date: 05091999

IMPLICIT NONE

! INPUT VARIABLES
real :: AngsVect(2,1)

! OUTPUT VARIABLES
real :: Vect(3,1)

! DETERMINE VECTOR COMPONENTS FROM ZENITH AND AZIMUTH ANGLES IN ANGS
Vect(1,1) = -1.0 * sin(AngsVect(1,1)) * cos(AngsVect(2,1)) ! x-component
Vect(2,1) = -1.0 * sin(AngsVect(1,1)) * sin(AngsVect(2,1)) ! y-component
Vect(1,1) = sin(AngsVect(1,1)) * cos(AngsVect(2,1)) ! x-component
Vect(2,1) = sin(AngsVect(1,1)) * sin(AngsVect(2,1)) ! y-component
Vect(3,1) = -1.0 * cos(AngsVect(1,1)) ! z-component
write(".*") 'Vects: ', Vect(1,1), Vect(2,1), Vect(3,1)
write(".*") sin(AngsVect(1,1)), 0, Vect(3,1)

END SUBROUTINE AnglesToVector
SUBROUTINE RotationTheta2(ViewVect, Beta, Theta2)

! RotationTheta2

! Purpose: Determine rotation about zenith axis of Automated Spectro-Goniometer.

! Internal Variables:

! Output Variables:

! Author: Thomas H. Painter
! Institution: Institute for Computational Earth System Science
! University of California, Santa Barbara

! Date: 05091999

IMPLICIT NONE

! INPUT VARIABLES
real :: ViewVect(3,1)
real :: Beta

! OUTPUT VARIABLES
real :: Theta2

! INTERNAL VARIABLES
real :: ZComponent, Numerator, Denominator, Quotient

! ESTABLISH ZComponent
ZComponent = ViewVect(3,1)

! COMPUTE NUMERATOR
Numerator = ZComponent + cos(Beta)**2.0

! COMPUTE DENOMINATOR
Denominator = cos(Beta)**2.0 - 1.0

! COMPUTE Theta2
Quotient = Numerator / Denominator
if (Quotient == 1.0) Quotient = 1.0
Theta2 = acos(Quotient)

END SUBROUTINE RotationTheta2
SUBROUTINE Rodriguez(k Vect, Theta, R)

! Rodriguez
!
! Purpose: Determine Rodriguez rotation matrix for given rotation (Theta)
! and axis vector (kVect).
!
! Internal Variables:
!
! Output Variables:
!
! Author: Thomas H. Painter
! Institution: Institute for Computational Earth System Science
! University of California, Santa Barbara
!
! Date: 05101999

IMPLICIT NONE

! INPUT VARIABLES
real :: kVect(3,1)
real :: Theta
!
! OUTPUT VARIABLES
real, DIMENSION(3,3) :: R
!
! INTERNAL VARIABLES
real, DIMENSION(3,3) :: lx, kkT, Identity
real :: kT(1,3), z(3,1)
!
! ESTABLISH lx MATRIX
lx(1,1) = 0.0
lx(1,2) = -1.0 * kVect(3,1)
lx(1,3) = kVect(2,1)
lx(2,1) = kVect(3,1)
lx(2,2) = 0.0
lx(2,3) = -1.0 * kVect(1,1)
lx(3,1) = -1.0 * kVect(2,1)
lx(3,2) = kVect(1,1)
lx(3,3) = 0.0

! ESTABLISH IDENTITY MATRIX AND z-vector (0.0,1)
Identity = reshape((/1.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,1.0/),(/3,3/))
z = reshape((/0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,1.0/),(/3,1/))
!z = reshape((/0.0,0.0,0.0,1.0/),(/3,1/))
!
! COMPUTE k'k Transpose
kT = transpose(k Vect)
kkT = matmul(k Vect, kT)
!
! COMPUTE R - Rodriguez matrix
R = (1.0-cos(Theta))*kkT + cos(Theta)*Identity + sin(Theta)*lx

END SUBROUTINE Rodriguez
SUBROUTINE RotationTheta1(ViewVect, R, Theta1)

! RotationTheta1
!
!! Purpose: Determine rotation about azimuth axis of Automated Spectro-Goniometer.
!!
!! Internal Variables:
!!
!! Output Variables:
!!
!! Author: Thomas H. Painter
!! Institution: Institute for Computational Earth System Science
!! University of California, Santa Barbara
!!
!! Date: 05121999

IMPLICIT NONE

! INPUT VARIABLES
real :: ViewVect(3,1)
real :: z(3,1)
real :: R(3,3)

! OUTPUT VARIABLES
real :: Theta1

! INTERNAL VARIABLES
real :: w(3,1)
real :: alpha

! ESTABLISH -z - vector to be multiplied by R
z = reshape((/0.0, 0.0, -1.0/),(3,1))

! COMPUTE w = R(Theta2) * (-z)
w = matmul(R,z)

! COMPUTE alpha
alpha = atan2(w(2,1), w(1,1))
write ("(4.d3)", alpha = ", alpha
!!! (alpha < 0) alpha = -1.0 * alpha ! Debugging
write ("(4.d3)", alpha = ", alpha

! COMPUTE Theta1
Theta1 = atan2(ViewVect(2,1), ViewVect(1,1)) - alpha
write("(3,1.16)", (2,1), (1.1), atan2:ViewVect(2,1),View Vect(1,1), Theta1 + alpha

END SUBROUTINE RotationTheta1
SUBROUTINE InvertRotations(R1, R2, u)
!
InvertRotations
!
Purpose: Using rotation matrices for Theta1 and Theta2, backcalculate view vectors
for code validation.
!
Internal Variables:
!
Output Variables:
!
Author: Thomas H. Painter
Institution: Institute for Computational Earth System Science
University of California, Santa Barbara
!
Date: 05131999
!
IMPLICIT NONE
!
INPUT VARIABLES
real, DIMENSION(3,3) :: R1, R2
!
OUTPUT VARIABLES
real, DIMENSION(3,1) :: u
!
INTERNAL VARIABLES
real, DIMENSION(3,1) :: z, w
!
DEFINE z
z = reshape((/0.0, 0.0, -1.0/),(3,1))
!
CALCULATIONS

! COMPUTE w
w = matmul(R2, z)
!
! COMPUTE u
u = matmul(R1, w)
!
END SUBROUTINE InvertRotations

1.2 ASGAngMarg.f90

PROGRAM ASGAngMarg
!
ASGAngMarg.f90
!
Purpose: Program to sort through output files from ASGAngles.f90
and organize into marginal rotations Theta1 and Theta2.
This produces the text strings which the ASG Control GUI
reads and submits to the Galil Motor Controller for the ASG.
!
Internal Variables:
!
External Variables:
Input:

infile Input filename containing the rotation angles for the specific
view angles

Output:

**OutFile** Output filename containing the command strings to be submitted
to the Galli Motor Controller (DMC-2000).

**OutString** Array of command strings written to OutFile

IMPLICIT NONE

! Parameters
integer, parameter :: RC1=2000 ! rotation counts for single rev. on motor 1
integer, parameter :: RC2=2000 ! rotation counts for single rev. on motor 2
integer, parameter :: Teeth1 = 96 ! number of teeth on worm gear, motor1
integer, parameter :: Teeth2 = 72 ! number of teeth on worm gear, motor2

! Declarations
integer :: Count, OpenStatus, NAngles
real :: FullTheta1, FullTheta2
real :: PrevTheta1=0.0, PrevTheta2=0.0
real :: Zenith, Azimuth
real :: LocalTheta1, LocalTheta2
real :: MotorCounts1, MotorCounts2
character(30) :: InFile, OutFile
!character(50) :: OutString

! Open Input File
write(*,*) 'Insert Input File:'
read(*,*) InFile
write(*,*) InFile
open(unit=10, file=InFile, status='old', iostat=OpenStatus)
if (OpenStatus > 0) STOP **** Error Opening Input Angle File ****

! Read Number of Angle Pairs from Input Angle File
read(10,*), NAngles
write(*,*) 'Number of Angle Pairs: ', NAngles

! Open Output File
write(*,*) 'Insert Output File:'
read(*,*) OutFile
write(*,*) OutFile
open(unit=11, file=OutFile, status='unknown', iostat=OpenStatus)
if (OpenStatus > 0) STOP **** Error Opening Output Angle File ****

! Read Input Angles - Determine Local Theta1 and Theta2 for Each
do Count = 1, NAngles
read(10,*) FullTheta1, FullTheta2, Zenith, Azimuth
write(*,*) Zenith, Azimuth, FullTheta1, FullTheta2'
write(*,*) Zenith, Azimuth, FullTheta1, FullTheta2
call CalcLocal(PrevTheta1, PrevTheta2, FullTheta1, FullTheta2, LocalTheta1, LocalTheta2)
call CalcMotorCounts(RC1, RC2, Teeth1, Teeth2, LocalTheta1, LocalTheta2, MotorCounts2)
write(11,*) "PR.,nint(MotorCounts1),": BGX; PR.,nint(MotorCounts2),": BGY;"
write(*,*) "PR.,nint(MotorCounts1),": BGX; PR.,nint(MotorCounts2),": "
PrevTheta1 = FullTheta1
PrevTheta2 = FullTheta2
end do

! Close input and output files
close(unit=10)
close(unit=11)

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! stop
end
SUBROUTINE CalcLocal(PrevTheta1, PrevTheta2, FullTheta1, FullTheta2, LocalTheta1, LocalTheta2)

CalcLocal
Purpose: Calculate local rotation angles Theta1 and Theta2 from the full rotations from the zenith position and the previous rotations associated with the previous view angle. Determines if rotation angles are defined in negative direction and corrects to positive direction.

Internal Variables:

External Variables:
Input:
real PrevTheta1 = previous Theta1 rotation
real PrevTheta2 = previous Theta2 rotation
real FullTheta1 = current full Theta1 rotation
real FullTheta2 = current full Theta2 rotation

Output:
real LocalTheta1 = marginal Theta1 rotation
real LocalTheta2 = marginal Theta2 rotation

Author: Thomas H. Painter
Institution: Institute for Computational Earth System Science
University of California, Santa Barbara
Date: 11.18.2000

IMPLICIT NONE

! Declarations
real :: PrevTheta1, PrevTheta2
real :: FullTheta1, FullTheta2
real :: LocalTheta1, LocalTheta2

! If Angles < 0, Change Quadrant
if (PrevTheta1 < 0) PrevTheta1 = 360.0 + PrevTheta1
if (PrevTheta2 < 0) PrevTheta2 = 360.0 + PrevTheta2
if (FullTheta1 < 0) FullTheta1 = 360.0 + FullTheta1
if (FullTheta2 < 0) FullTheta2 = 360.0 + FullTheta2

! Compute Local Theta1 and Local Theta2
LocalTheta1 = FullTheta1 - PrevTheta1
LocalTheta2 = FullTheta2 - PrevTheta2

END SUBROUTINE CalcLocal
SUBROUTINE CalcMotorCounts(RC1, RC2, Teeth1, Teeth2, LocalTheta1, LocalTheta2, MotorCounts1, MotorCounts2)
!!
!!    CalcMotorCounts
!!
!! Purpose: Calculate the number of motor counts required to affect rotations LocalTheta1 and LocalTheta2 respectively, using the number of counts per revolution and number of teeth per worm gear.
!!
!!
!! IMPLICIT NONE
!!
!! Declarations
integer :: RC1, RC2
integer :: Teeth1, Teeth2
real :: LocalTheta1, LocalTheta2
real :: MotorCounts1, MotorCounts2
real :: Scalar1, Scalar2
!!
!! Compute Angular Count Scalars for Axis 1 and Axis 2
Scalar1 = RC1 * Teeth1 / 360.0
Scalar2 = RC2 * Teeth2 / 360.0
!!
!! Compute Number of Motor Counts for Theta1 and Theta2
MotorCounts1 = LocalTheta1 * Scalar1
MotorCounts2 = LocalTheta2 * Scalar2

END SUBROUTINE CalcMotorCounts

2  IDL ROUTINES FOR ASG POST-PROCESSING

2.1  asg_openws.pro

function asg_openws, prefix, specvec

: asg_openws: program to open, view, and average white standard digital numbers as measured by the ASD in a run of the Automated Spectro-Goniometer.

    input: prefix prefix of ASD spectrum files
          specvec vector of valid white standard spectra indices

    output: mean spectrum of white standard DN

    author: thomas.h.painter
    date: 2.19.2001

    revisions: thomas.h.painter
              6.11.2001
              - changed yrange for plot of white standard ratio
              - added toggle plots for ws ratio and total DN
              7.13.2001

    thomas.h.painter

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- added specvec and refurbished code to accommodate specvec

examples:
ws = asg_openws('010215a.1')
(this opens the first 10 spectra 010215a.000-009,
plots them for quality check, averages them, and
assigns them to ws.)

: size of specvec
svlength = (size(specvec))(1)

: open and read ws DN files
for i=0,svlength-1 do begin
  openr,i=1.string(prefix,string(specvec(i),format='(3.3)'))
endfor
ws=fitarr(svlength,2151)
header=bytarr(484)
for i=0,svlength-1 do begin
  readu,i=1.header
endfor

sws=fitarr(2151)
for i=0,svlength-1 do begin
  readu,i=1,sws & ws(i,*)=sws
  close,i=1
endfor

: open and read wavelength file -> convert to microns from nm
openr,1,'sae1999.wv'
vwl=fitarr(2152)
readf,1,vwl
vwl=vwl(1:2151)
vwl=vwl*0.001
close,1

: create new prefix to put in PostScript filenames
spechold = str_ssep(prefix,'.'
specname = spechold(0)

: calculate mean white standard spectrum
mws=fitarr(2151)
for i=0,svlength-1 do begin
  mws=mws + (1.0/avlength) * ws(i,*)
endfor

: calculate standard deviation of white standard spectra
sdws=fitarr(2151)
sumwvar=fitarr(2151)
for i=0,svlength-1 do begin
  sumwvar=sumwvar + (1.0/avlength) * ((ws(i,*) - mws)^2.0
endfor
sdws = sqrt(sumwvar)
mwdr = mean(sdws)

: calculate coefficient of variation of white standard spectra
cvws=sdws/mws

: character size
lpcharsize=1.25

: plot ratio of ws spectra to mean ws spectrum
window,0.xs=500,ys=500
plot,wvl/ws(0,*)/mws,yrange=[0.9,1.1],xstyle=1,xrange=[0.35,2.5],xtitle='wavelength (1713m)',ytitle='ratio with mean'
for i=1,svlength-1 do begin
plot wvl,ws(1:4)/mws,line=1
endfor

; plot ws DN spectra on top of the mean spectrum
window,1,xa=500,ya=500
plot,wvl,mwsl,xtitle='wavelength (\mu m)',ytitle='digital number',xstyle=1,xrange=[0.35,2.5]
for i=0,svlength-1 do begin
  opplot,wvl,ws(i,:),line=i
endfor

; plot standard deviation spectrum for all ws DN spectra
window,2,xa=500,ya=500
plot,wvl,mdws,xtitle='wavelength (\mu m)',ytitle='spectral standard deviation',xstyle=1,xrange=[0.35,2.5]

; plot coefficient of variation for ws DN spectra
window,3,xa=500,ya=500
plot,wvl,cmws,xtitle='wavelength (\mu m)',ytitle='coefficient of variation',xstyle=1,xrange=[0.35,2.5]

; plot the ratio of ws DN spectra to mean ws DN spectrum
toggle,fil='string(\'WSRATIO\',specname,\',ps\'),xsize=6,ysize=5
plot,wvl,ws(0,:),mws,range=[0.3,1.1],xtitle='wavelength (\mu m)',ytitle='RATIO',xstyle=1,xrange=[0.35,2.5]
for i=1,svlength-1 do begin
  opplot,wvl,ws(i,:),mws,line=1
endfor

toggle

; plot the mean digital number spectrum for white standard and all spectra
toggle,fil='string(\'DIN\',specname,\',ps\'),xsize=6,ysize=5
plot,wvl,mwsl,xtitle='wavelength (\mu m)',ytitle='digital number',xstyle=1,xrange=[0.35,2.5]
for i=0,svlength-1 do begin
  opplot,wvl,ws(i,:),line=i
endfor

toggle

; plot the DN standard deviation spectrum for white standard spectra
toggle,fil='string(\'StD\',specname,\',ps\'),xsize=6,ysize=5
plot,wvl,mdws,xtitle='wavelength (\mu m)',ytitle='spectral standard deviation (DN)',xstyle=1,xrange=[0.35,2.5]
toggle

; plot the coefficient of variation
toggle,fil='string(\'CoefVar\',specname,\',ps\'),xsize=6,ysize=5
plot,wvl,cmws,xtitle='wavelength (\mu m)',ytitle='coefficient of variation',xstyle=1,xrange=[0.35,2.5]
toggle

openw,10,string(specname,\',.ws\')
writeu,10,mws
close,10

return,mws
end

2.2  asg_refl.pro

function asg_refl,prefix,ws1,ws2,nspec,azelen

  asg_refl.pro  Program to process Automated Spectro-Goniometer  
  BRDF DN spectra to bidirectional reflectance  
  function (BRF). It reads the spectra prefix, the  
  mean white standard DN spectrum, and the number  
  of spectra to process. The number of spectra to  
  process varies with the acquisition protocol (10,  
  15, or 1 degree angular resolution).  

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input:

prefix Spectra filenames prefix
wsaspec1 Spectrum of early white reflectance standard DN
wsaspec2 Spectrum of late white reflectance standard DN
nspec Number of reflectance spectra to process
solzen Solar zenith angle for given acquisition (degrees)

internal:
spectra1n Spectrum of calibration values for spectra1n panel. Contained in 'spectra1n1989.spl'. Must be present in the spectrum directory.

snow Array of digital number spectra measured by asg.

brf Bidirectional reflectance factor spectrum for 153 spectra covering view zeniths of 0-80 and view azimuths of 0 - 180

dc Dark current value determined by using the reflectance at the shortest wavelength of the SWIR1 spectrum as true, assigning this refl. to that at the longest wavelength of the VNIR spectrum and then solving for the necessary dark current value.

spectbrf Correction spectrum for bidirectional reflectance of Spectra1n panels (from Kurt Thome/Stu Biggar at University of Arizona)

output:

asgbrf 2-D array of BRDF spectra
format: fitarr(nspec,nbands) nbands=2151
example: asg010215a=asg_br(‘010215a..’,’010215a.ws1’,’010215a.ws2’,158,47.5)
This example opens 158 spectra files (010215a.010 - 157), reads them, divides them by ws, corrects for Spectra1n BRF, and divides them by the spectra1n calibration values, then writes them to the array asg010215a.

author: thomas.h.painter
date: 2.19.2001
revisions: 1. added Spectra1n BRF correction
7.13.2001 t.h.painter
2. added second (late) white standard spectrum interpolation
7.19.2001 t.h.painter

define internal variables
nbands = 2151 ; number of ASD bands
ASDhdrsz= 484 ; size of ASD file header
asg = fitarr(nspec,nbands) ; array of BRDF digital numbers
header = bytarr(ASDhdrsz) ; buffer in which to read ASD header
subasg = fitarr(nbands) ; intermediate variable for reading
subbrf = fitarr(nspec,nbands) ; array of intermediate BRF values
asgbrf = fitarr(nspec,nbands) ; array of BRF values (output)
subBRF = fitarr(nbands) ; intermediate variable for computing BRF
wespec = fitan(nbands) ; white standard spectrum for time step

; open and read spectraion file
openr,1,'spectranion.1999.spt'
spectranion = fitan(nbands)
readf,1,spectranion
mve,spectranion
close,1

; calculate BRF correction for Spectranion panel from solar zenith angle
spectBRF = fitan(nbands)
spectBRF = asg_brfcor(solzen)

; open and read white standard DN files
; starting white standard spectrum
openr,1,ws1
wespec1=fitan(nbands)
readu,1,wespec1
close,1

; ending white standard spectrum
openr,1,ws2
wespec2=fitan(nbands)
readu,1,wespec2
close,1

; calculate spectral slope between the starting and ending ws spectra
wslope = (wespec2-wespec1)/152.

; open input files and read from input files into asg
for i=0,nspec-1 do begin
    openr,1,string(prefix,string(i+10,format='(I3.3))
    readu,1,header
    readu,1,subasg
    asg(i,*') = subasg
    close,1
endfor

; loop through all spectra
for i=0,nspec-1 do begin
    ; calculate white standard spectrum for current time step
    wespec=wespec1 + i*wslope

    ; compute initial bidirectional reflectance factor from asg
    subBRF = asg(i,*')/(wespec*(spectBRF*spectranion))
    subrefl(i,*') = subBRF

    ; calculate VNIR dark current and calculate VNIR BRF
    rfl=subrefl(i,*')
    snow=asg(i,*')

    ; calculate dark current
    dc = (1./rfl(601)-1.))*(rfl(601)*wespec(600)*(spectBRF(600)*spectranion(600))-snow(600))

    ; calculate overall BRF
    bref = (snow - dc)/(wespec*(spectBRF*spectranion))-dc
    subrefl(i,0:500)=bref(i,0:500)
endfor

; assign asgrefl
asgrefl=subrefl
asgrefl=asgrefl(5:157,*) ; purge first spectra

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2.3 **asg_brfcor.pro**

```
function asg_brfcor,

asg_brfcor = Calculate correction spectrum for a Spectralon panel irradiate with incident angle sol_zan.

The measured BRF correction spectra are linear with solar zenith. This function uses linear spectral y-intercept and slope contained in linfitspec.txt to directly calculate the Spectralon panel BRF correction spectrum for the incident angle sol_zan.

The output spectrum represents the ratio of the spectral BRF for the given solar zenith to the spectral BRF for a solar zenith of 50 degrees.

This is based on measurements by Robert O. Green and the AVIRIS team (Jet Propulsion Laboratory).

The primary assumptions are that the BRF at 50 degrees matches the albedo of the Spectralon panel and that the BRF at 50 degrees in constant across wavelength.

These assumptions are supported by S. Sandmeier et al. (1998, RSE).

input

sol_zan  Solar zenith angle for correction spectrum

internal

linfit  Array to hold the spectral y-intercepts and slopes

output

Cbrf  Spectralon BRF correction spectrum

: : : author  thomas.h.painter
: : : date  6.27.2001

:: Read correction spectra database
openr.1,linfitspec.txt
linfit=fitarr(2,2151)
readf,linfit
close,1

:: Compute Spectralon BRF correction spectrum
Cbrf = linfit(0,*) + linfit(1,*)*sol_zan

:: Return Spectralon BRF correction spectrum
return, Cbrf
stop
end
```
2.4 asg_order_10deg.pro

function asg_order_10deg, filename, sol_zn, sol_az

  : asg_order_10deg: function to collate an acquisition-ordered cube of asg
  data and orders in an ENVI spectral library file
  according to the following format:

  input: filename
  filename of the input cube of asg BRF
  that is output from asg_refl.pro
  sol_zn  solar zenith for acquisition
  sol_az  solar azimuth for acquisition

  output: ENVI Spectral Library of asg BRF data
  Number of samples = number of bands
  Number of lines = number of spectra
  In this case, the image is given by fitarn(2151,153)

  author: thomas.h.painter
  date: 6.13.2001

  revisions:

  examples: asg=asg_order_10deg(’010215a.asg’, ’47.5,123.0’)

:: declarations
Nepec = 153 ; Number of spectra
NepecS = 289 ; Number of spectra in symmetric set
; 36(azx)*8(azx) + 1(nadir) = 289
Nbands = 2151 ; Number of bands

:: open and read wavelength file -> convert to microns from nm
openr,1,filename
asgold = fitarn(Nbands, Nepec)
readu,1,asgold
close,1

asgold = fitarn(Nbands, Nepec)
asg = fitarn(Nbands, NepecS)

:: order spectra
for i=0,18 do begin
  asgold(*) = asgold(i)
  asgold(i+19) = asgold(i+37)
  asgold(i+38) = asgold(i+56)
  asgold(i+57) = asgold(i+75)
  asgold(i+76) = asgold(i+94)
  asgold(i+95) = asgold(i+113)
  asgold(i+114) = asgold(i+132)
  asgold(i+133) = asgold(i+151)
endfor
asgold(152) = asgold(152)

:: produce symmetric spectral library (symmetry across solar principal plane)
for i=7 do begin
  asg(36:38*)=reverse(asgold(19:19*:18),2)
endfor
asg(288) = asgold(152)

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:: output ordered spectral file
prefix = str_seps(filename,'.')
prefix = prefix(0)
outputfile = string(prefix,'.sfl')
openw,10,ouputfile
writeu,10,asg
close,10
print,'output file written'

:: create spectra names
asgnames=strarr(NaspecS)
for i=0.7 do begin
  for j=0.36 do begin
    asgnames('36+i') = string('z',string(10*(8-i),format='(i2.2)'),',a','$
    string('10,format='(i3.3)'),'.')
  endfor
endfor
asgnames(288)=20s0'

:: import wavelengths
openr,1,'asd1999.wvl'
wav=fitarr(2152)
readf,1,wavl
wav=wav(1:2151)
close,1

:: determine date and time
datespi = systime(0)

datahdr=string(prefix,'.asg.hdr')
openw,10,datahdr

print,10,'ENVI'
print,10,'description = '
print,10,' New asg Spectral Library [datespi, ]'

print,10,'samples = '.Nbands
print,10,'lines = '.NaspecS
print,10,'bands = 1'
print,10,'header offset = 0'
print,10,'file type = ENVI Spectral Library'
print,10,'data type = 4'
print,10,'interleave = baeq'
print,10,'sensor type = Automated Spectro-Goniometer coupled with ASD-FR1'
print,10,'byte order = 0'
print,10,'x start = 0.00000'
print,10,'y start = 0.00000'
print,10,'z plot titles = (Wavelength [7113m], BRF )'
print,10,'band names = ( ASG ENVI Spectral Library )'

print,10,'wavelength = '
for i=0,2149 do begin
  print,10,wavl(i)*0.001,' : wavelength in microns
endfor
print,10,wavl(2150)*0.001
print,10,'

print,10,'spectra names = '
for i=0,7 do begin
  for j=0,36 do begin

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print,10,'Single(8) : ',asnames(38'!+)].' [000000AZ],"10',"ZN","(8-
)"10',"SA'
sol_az,'SZ',sol_zn',].",format=(a12,a7,a10,i3,3,a3,i3,0,a2,i5,1,a2,i4,1,a2)
endfor
endfor

print,10,'Single(8) : ',asnames(288),'
[000000AZ],"0',"ZN",0,0,"SA'
sol_az,'SZ',sol_zn',].",format=(a12,a7,a10,i3,3,a3,i3,0,a2,i5,1,a2,i4,1,a2)

print,10,')'
close,10

:: write protocol file used by RSL GONIO package *.lst
outfille = string(prehold,'.sii.lst')
openw,10,outfille
print,10,' SOL_AZ SOL_ZN AZIMUTH ZENITH TIME FILE_NAME'
for i=0,7 do begin
for j=0,35 do begin
print,10,sol_az,sol_zn,"i",10.,(8 - i)*10.,' NA ",str_sep(asnames(19'!+)'],")
endfor
endfor
print,10,sol_az,sol_zn,0,0,' NA ", str_sep(asnames(288),")
close,10

return,datespi
end
References


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Optical Sciences Center, U. o. A. (2001), Optical Sciences Center, University of Arizona, [http://www.optics.arizona.edu/rsc/](http://www.optics.arizona.edu/rsc/).


