Visual Evidence for Urban Potential fields[1]

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Abstract

A satellite picture of adjacent cities clearly illustrates the analytical dynamics of urban growth.

Keywords: Dallas, Dynamics, Fort Worth, Satellite Picture, Urban Growth

For over a hundred years attempts have been made to apply to the social sciences concepts expropriated from physics. The danger inherent in reasoning from analogy has always caused these efforts to be regarded with some suspicion. The gravity model of human interaction, however, is one model that seems to have gained widespread acceptance, mainly because it describes aggregate human behavior quite well (1).

Most of the evidence used to calibrate the social gravity model has come from census enumerations, the time and space scales of human interactions being such that direct sensory observations are difficult. The satellite photograph (fig. 1), showing two terrestrial cities, it is suggested, provides direct pictorial confirmation of the existence of potential functions in the field of human geography.

In terms of potential theory, if a source of strength k_1 is located at $z = a$ of the complex plane, and another strength of k_2, is located at $z = -a$, then the complex potential is:

$$\Omega(z) = k_1 \ln (z + a) + k_2 \ln (z - a)$$

and this is readily separated into the equipotential and stream functions (2). The general configuration shown in the satellite photograph appears to conform to these equations. In particular, the developing suburbanization between the two cities appears to outline the stream function, a = 25 km., $k_1 = 679{,}684$, (= 12.9 km. radius) the Dallas population in 1960; and the Fort Worth population in 1960 $k_2 = 356{,}268$, (= 9.7 km. radius) . The second figure shows this somewhat more schematically, with the city areas approximated by circles of appropriate size (3). This diagram has been sized to fit over the satellite photograph.

There remains the possibility that the example is spurious or fortuitous; time sequences or examples from twin cities in other parts of the world should illuminate this possibility. In particular, it is suggested that further high resolution photographic exploration of this unusual planet may reveal other phenomena of equally great interest, including further instances similar to that reported here.

[1]*Mappemonde* 1/91, 46-47.

(1) J. Stewart and W. Warntz, *Journal of Regional Science,* vol. 1, 1968, pp. 99-123;
Gunnar Olsson, *Distance and Human Interaction,* Philadelphia, Regional Science Research Institute, 1965*;*
Robert E. Nunley, *Living Maps of the field Plotter: Analog Simulation of Selected Geographic Phenomena,* Washington, D.C.: Association of American Geographers, Commission on College Geography, Technical Paper n° 4, 1971.
(2) M. Spiegel, *Theory and Problems of Complex Variables,* Schaum. 1968, p. 248.
(3) W. Tobler, "Satellite Confirmation of Settlement Size Coefficients", *Area,* vol. 1, 1969, pp. 30-34.

Figure 1: Apollo VI photograph: AS6-2-1462, showing the built up area of Fort Worth (32°45' N, 97°20' W) and Dallas (32°47' N, 96°48' W).

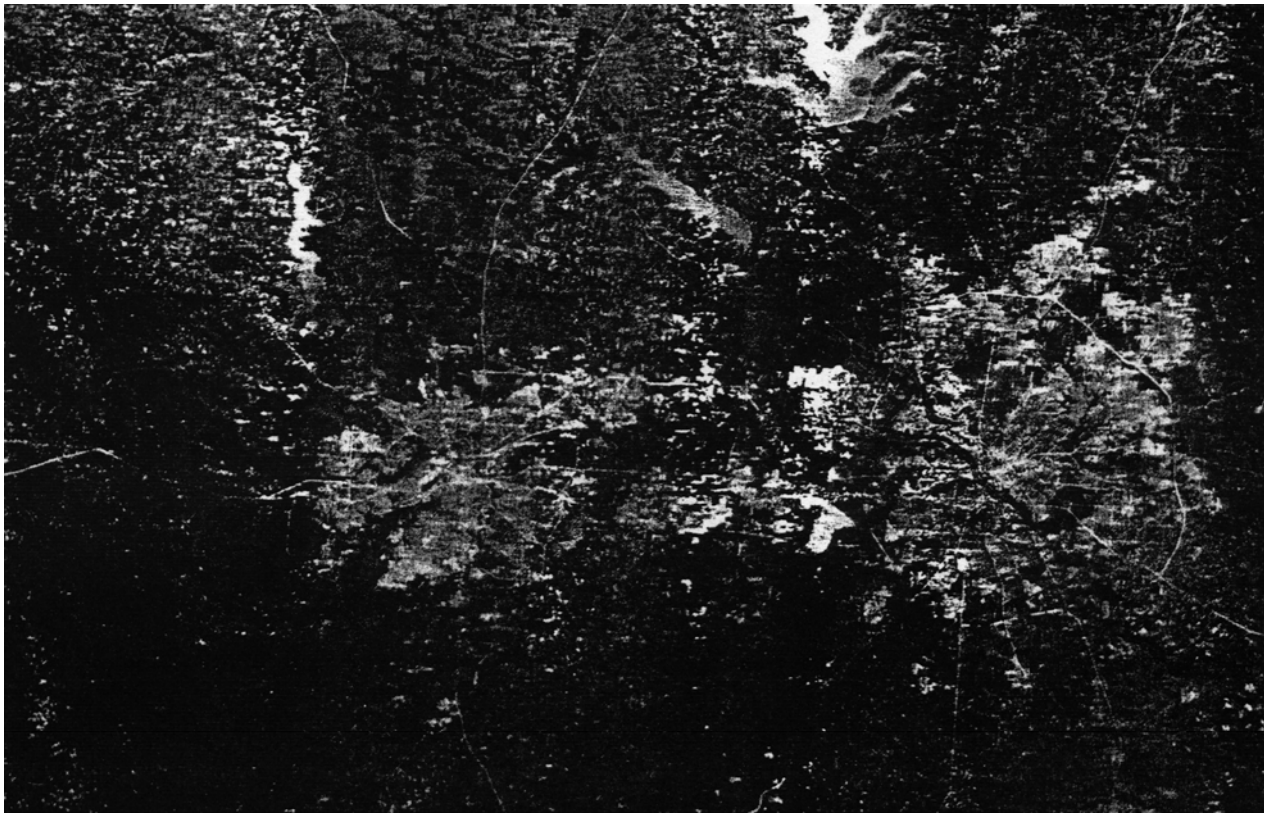
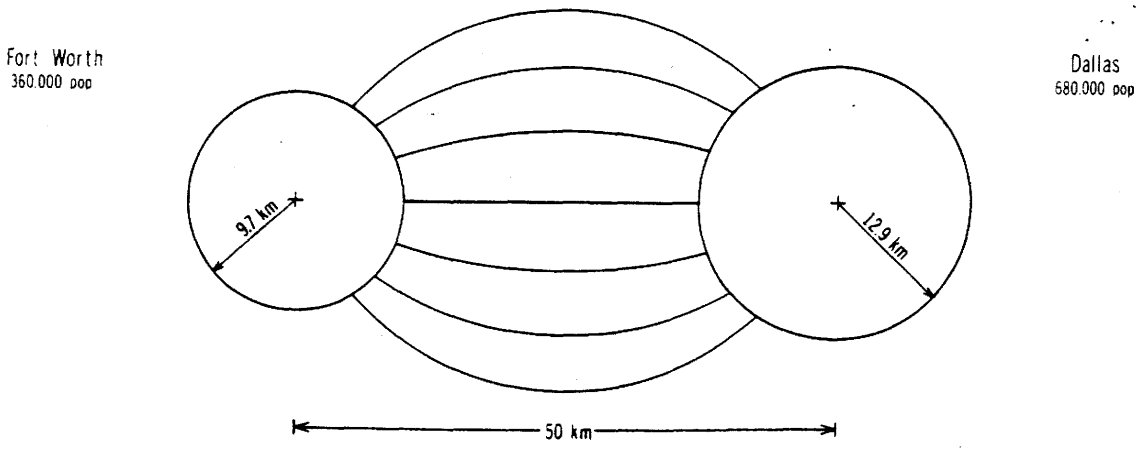
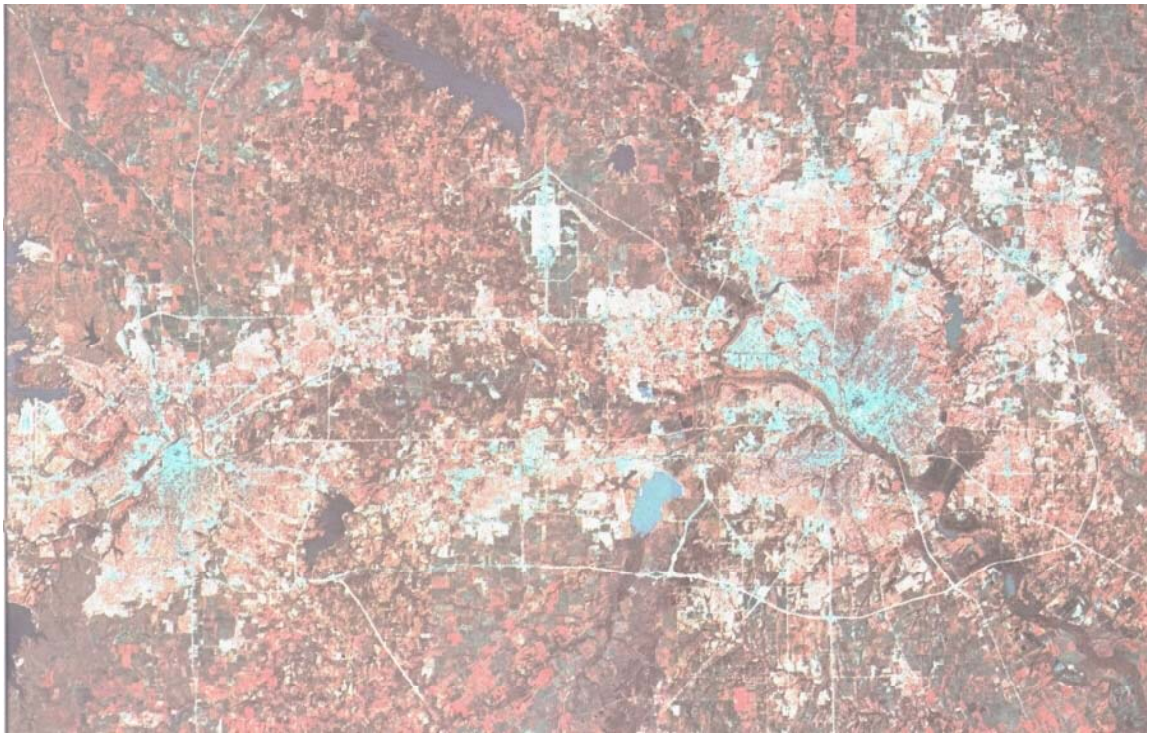


Figure 2. Schematic diagram of a portion of the expected stream function if Fort Worth and Dallas induce a potential field. For comparison with the satellite photograph in Figure 1. (Scaled to fit Figure 1; you should be able to superimpose the two figures)



A more recent photograph of the same area.



And a map of the area