Satellite confirmation of Settlement Size Coefficients †

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Several investigators 'have demonstrated that the built-up area and equivalent radius of settlements can be estimated with considerable accuracy from a knowledge of the number of inhabitants. A recent circumstance, somewhat fortuitous, has led to an interesting, independent confirmation of this result. Specifically, on the basis of theoretical and empirical deductions it has been concluded that the built-up area of a settlement should be proportional to the population raised to some exponent (<1). From this it follows that the radius of a circle of the same area should be given by a formula of the form.

$$r = aP^b$$

where P is the population. The population density is also readily deduced from this formula. For the United States empirical estimates yield the values a = 0.035, b = 0.44, when the radius is measured in kilometres. In general the built-up area differs from the area of the legal jurisdiction, and from the physical area used for enumerations.

Using these coefficients, with census data plus latitude and longitude coordinates, it is possible to use a digital plotter to produce population maps, as illustrated in Fig. 1 for a portion of Michigan. Comparable maps prepared for sections of southern Germany, central Spain, and central Turkey yielded similarly reasonable results, but for a portion of the Nile Delta they suggested village overlapping to an outrageously absurd degree. There is no evidence that the Egyptian census data are seriously in error. A somewhat casual comparison of the Egyptian map plot with a Gemini photograph[2] of the same area suggested that dividing the radii of the villages by four would yield estimates of the correct order of magnitude (Fig. 2). The implication is that Egyptian towns are 16 times more compact than United States towns. This suggested a re-examination of the previous studies[1], even though the actual project for which the maps were being prepared is relatively unrelated in objectives to these works. Table 1 summarizes the relevant observations. The phenomenon discussed for the Nile Delta is clearly consistent with Nordbeck's data for Japan. On this basis it is suggested that the coefficient a, (or a^2/π, provides a useful characterization of settlement packing, with clear value in distinguishing between various strategies employed by society for the organization of spatial activity. Stewart and Warntz suggest that it is related to their notion of population potential. Extrapolation of the formulae to one individual yields a reasonable measure of space consumption, in principle capable of independent estimation. World-wide samples of actual measurements on the built-up area of settlements would appear to be possible if high resolution satellite photographs were available.

The analysis can, of course, also be inverted, e.g., using the Apollo VI photograph (AS6-2-1462) of Dallas (Fig. 3). With $1/10^{th}$ inch coordinate paper to measure the built-up area and inferred consequences of the Public Land Survey System as a surrogate for an actual scale, fifteen minutes suffice to estimate the population at 668,000 people, which is within 2% of the 1960 census figure. To sharpen the efficacy of such estimates requires theoretical studies of the concept of built-up area, especially along the lines suggested by Nystuen[3]. The general utility and applicability of the settlement size relations, however, seems well established.

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References

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[2]Particular benefit was obtained from J. F. Kolar's study of this photograph, as reported in:
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[3]Nystuen, J., 1967. Boundary shapes and boundary problems. *Papers Peace Research Society,* 7, 107-28.

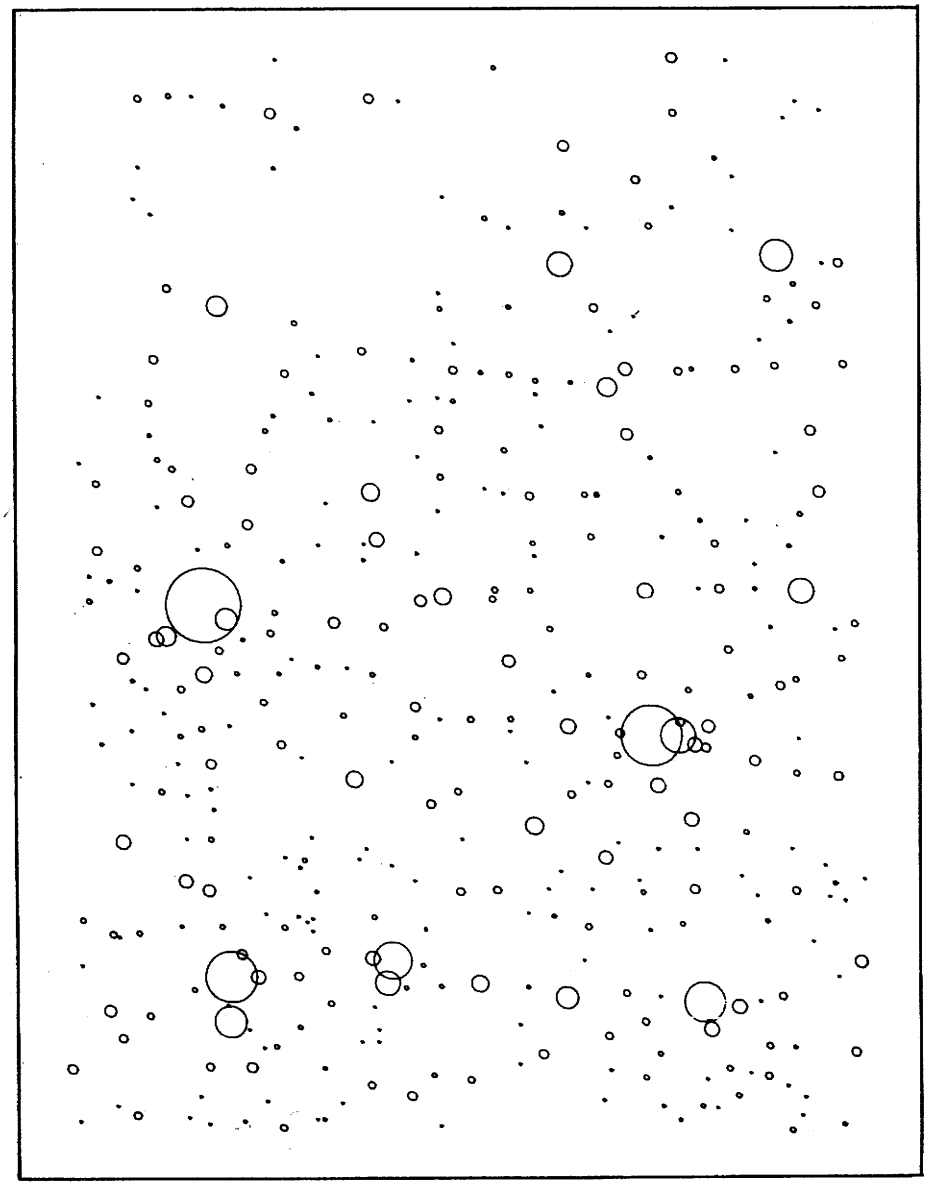

Fig. 1. Built-up areas in a portion of Michigan (latitude 42° N to 44°N, longitude 86°W to 84°W) drawn as described in the text. The overlap, e.g., East Lansing–Lansing at right centre, is due to the definition of equivalent radius, and to the distinction between the legal city and the functional built-up area.

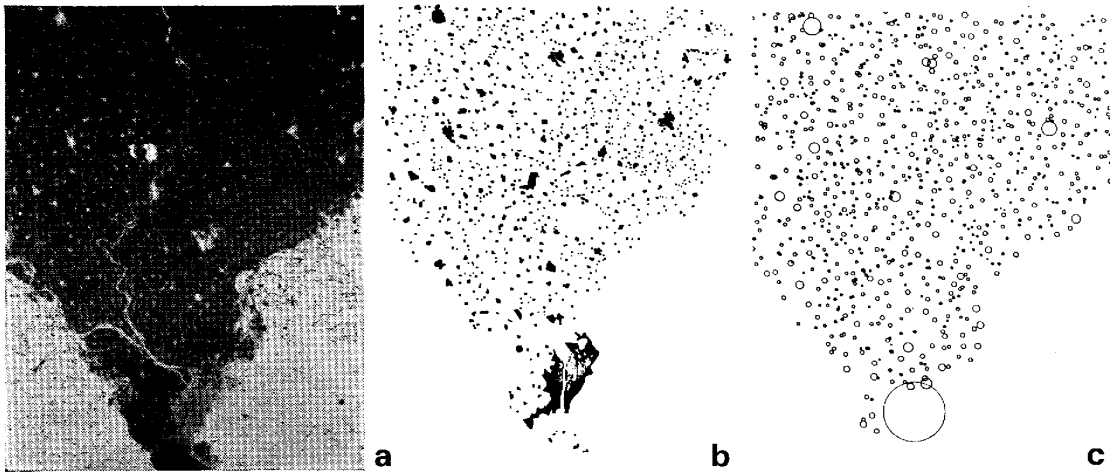

Fig. 2. Comparison of actual towns and theoretically calculated towns, (a) High oblique Gemini photograph (G5-Ri-44) of the southern third of the Nile Delta. The area shown contains 7×10^6 people in towns (1960 census). (b) Tracing of towns from (a). Modified from Kolars (Ref. 2). (c) Computer plot of towns using radii calculated from the 1960 Census of Egypt populations as described in the text and latitudes and longitudes from 1/50 000 topographic maps. Comparison with (b) illustrates the agreement between theory and fact. Also note the overlap of towns actually separated by the Nile River (centre, upper centre) suggesting functional unity. The distance from Tanta (top left) to Cairo (lower centre) is approximately 95 kilometres.

Fig. 3. A portion of Apollo VI photograph AS6-2-1462 including the built-up areas of Dallas and Forth Worth, Texas.

Table 1

Region and data	*Sample size**	a *Coefficient* (proportionality)	b *Coefficient* (exponent)	*Correlation Coefficient*
English and Welsh cities, 1951 (Stewart and Warntz)	157	$4{\cdot}14\times10^{-2}$	0·375	0·87
USA 1950 Census, Urbanized Areas (Nordbeck; Boyce)	155	$3{\cdot}0\times10^{-2}$	0·43	0·928
USA 1960 Census, Urbanized Areas (Nordbeck; Boyce)	213	$3{\cdot}5\times10^{-2}$	0·44	0·922
Swedish Tätorter, 1960 built-up areas (Nordbeck)	70	$5{\cdot}3\times10^{-2}$	0·332	0·976
Japan 1960 Census, Densely Inhabited Districts (Nordbeck)	518	$9{\cdot}46\times10^{-3}$	0·46	0·967
Ontario Cities, developed area, 1960 (Maher and Bourne)	51	$2{\cdot}92\times10^{-2}$	0·437	0·987

*The range of settlements studied is $1{\cdot}75\times10^{2}$ to $1{\cdot}2\times10^{7}$ people.