Blended Map Projections are Splendid Projections.

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The homolosine projection, popular in atlases, was first described by Professor John Paul Goode in 1925. This projection uses a combination of the homolographic (Mollweide 1805) and the sinusoidal (variously attributed to Mercator, Sanson, or Flamsteed) projections. Goode's procedure was to use the sinusoidal projection, because of its supposedly superior properties, in the latitude interval from 40.7366 degrees south to 40.7366 degrees north, and to use Mollweide's projection, from there as far as the poles, for a comparable reason. The particular latitude chosen is where the two projections coincide if the maps are laid on top of each other, or as Professor Goode states "have the same scale". The resulting pseudocylindrical projection has a discontinuity in the first derivatives at the join, and this can be seen by the user as a small kink in the curvature of the meridians. The kink can be avoided by blending the projections instead of just joining them. This blending is achieved by using a tapered weighted average of the two projections causing them to grade smoothly into each other.

Goode's homolosine projection can also be considered a weighted average of the two projections, but with the weight assigned to the sinusoidal taking on a value of one between the bounding latitudes, and a value of zero from there to the poles. Thus W_s is one if -40.7366 < φ < +40.7366 (φ denotes latitude), and zero otherwise. The weight of the homolographic (Mollweide) projection is then given by 1 - W_s.

Simple arithmetrical averaging of two projections uses constant weights of one half for each part of the entire map using two equal area projections, as has been done by Foucaut (1862), Hammer (1890), Boggs (1929), Tobler (1973), and several others. Snyder (1993) references twenty six authors who have attempted to improve the pseudocylindric class in this manner, including several who have provided multiple variants. Blending, in contrast, is accomplished by use of a continuously variable weighting, given as a function of latitude $W = W(\varphi) \geq 0$, the major requirement being only that the derivative of this function with respect to latitude be negative. That is, the weights should decrease from one at the equator to zero at the poles. The weight for the second projection is always 1 - W, i.e., decreasing from the poles to the equator.

Possible blending functions are numerous, and any equal area map projection with horizontal parallels - the equal area pseudocylindrical class of projections, - can be combined in this manner. These will continue to be equal area. Two easy choices for weighting functions are

$$W = 1 - 2|\varphi|/\pi \quad \text{and} \quad W = \cos^n(\varphi),\ n > 0,$$

the first, with n equal to two, and the simplest of these being the linear decline from the equator to the pole. The illustration shows this using the interrupted version of the (blended) homolosine. Naturally there are many other possibilities. The equation for the latitude spacing on the combined projection then becomes

$$y = W\ F_s + (1-W)\ F_m\ ,$$

where the F_s and F_m refer to the ordinate values on the conventional sinusoidal and Mollweide projections. In order to retain the equal area property we must ensure that $x = \lambda\ \cos(\varphi)/(\partial y/\partial\varphi)$, assuming a sphere of radius one. In the present instance

$$\partial y/\partial\varphi = W\ \partial F_s/\partial\varphi + (1 - W)\ \partial F_m/\partial\varphi + (F_s - F_m)\ \partial W/\partial\varphi\ ,$$

and the combination retains the equal area property. We also require that $\partial y/\partial\varphi > 0$ and also that $\partial x/\partial\varphi < 0$ for $\varphi, \lambda > 0$ to avoid meridians with reversing curvature. Combining two other equal area projections simply requires replacing the subscripts s and m in the example by the appropriate values. Using Lambert's cylindrical area projection for the region near the equator and Mollweide's for the polar regions yields an easily computed and attractive alternative map.

References:

S. Boggs, 1929, "A new equal area projections for world maps", Geographical Journal, 73:241-245.

P. Faucaut, 1862, Notice sur la construction de nouvelle mappemondes et de nouveau atlas, Arras, 5-10.

E. Hammer, 1890, "Unechtcylindrische and unechtkonische flaechentreue Abbildungen, Petermann's Geographische Mitteilungen, 46:42-46.

J. Goode, 1925, "The Homolosine Projection: A New Device for Displaying the Earth's Surface Entire", *Annals*, AAG, 15, 3: 119-125

C. Mollweide, 1805, "Ueber die von Prof. Schimdt *in* Giessen in der zweyten Abteilung seines Handbuchs der Naturlehre S 595 angegebene Projection der Halbkugelflaeche" , *Zach's Monatliche Correspondenz*, 12 (Aug): 152-163.

J. Snyder, 1993, Flattening the Earth, University of Chicago Press.

W. Tobler, 1973 "The Hyperelliptical and other new Pseudo-Cylindrical Equal Area Map Projections", *Journal of Geophysical Research*, 78, 11, (1973), pp. 1753-1759.