Beyond Ptolemy
Mercator and other Distorted Maps

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This talk is about transformations

The subject of transformations has a long history.

Of interest here are geometrical transformations.

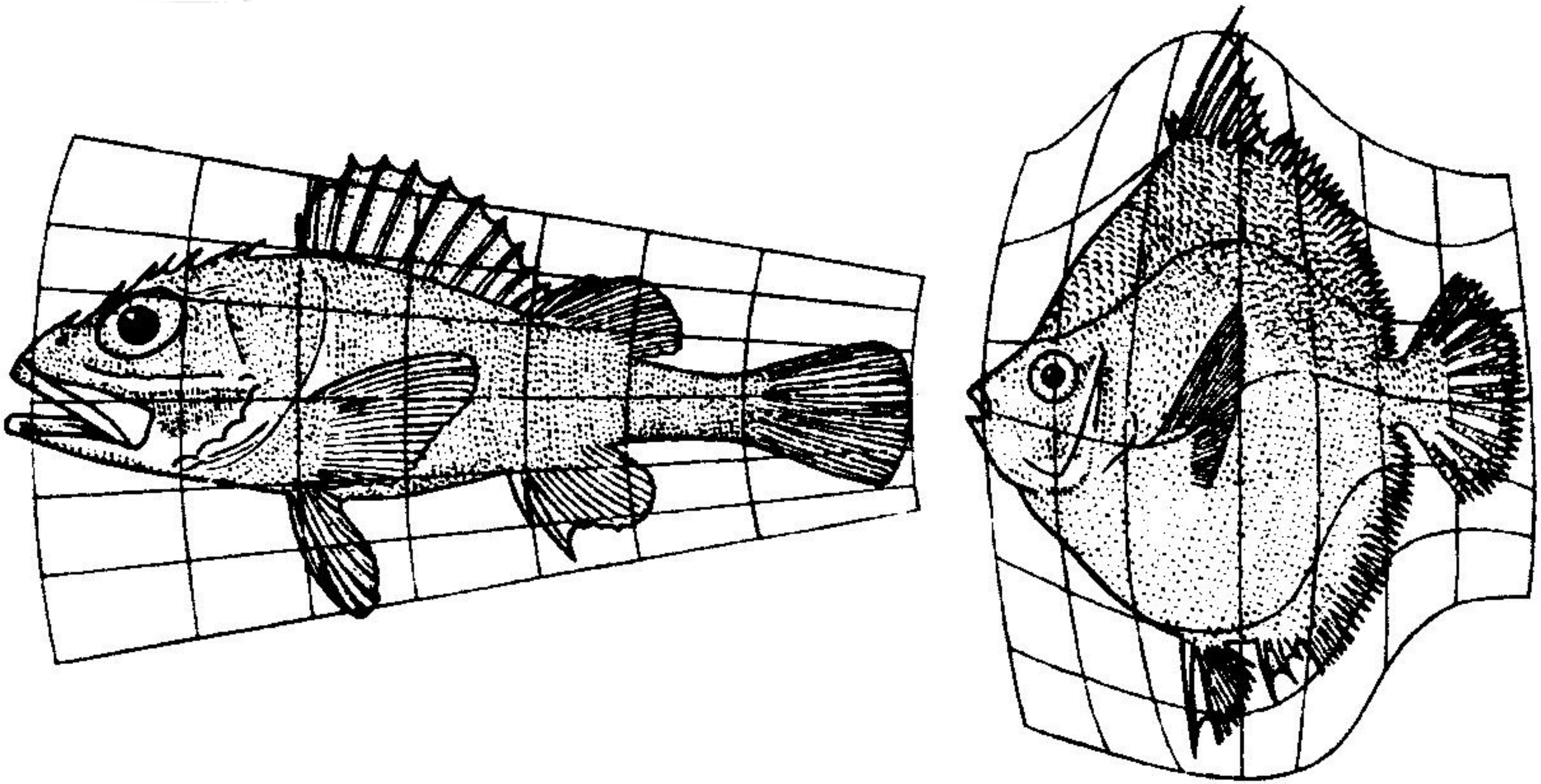
In this talk I describe geographical transformations
as represented by map projections.

First some other views:

In 1919 D' Arcy Wentworth Tompson wrote about biological transformations in the last chapter of his book "On Growth and Form"

Example: different fish are nearly homeomorphisms of each other.

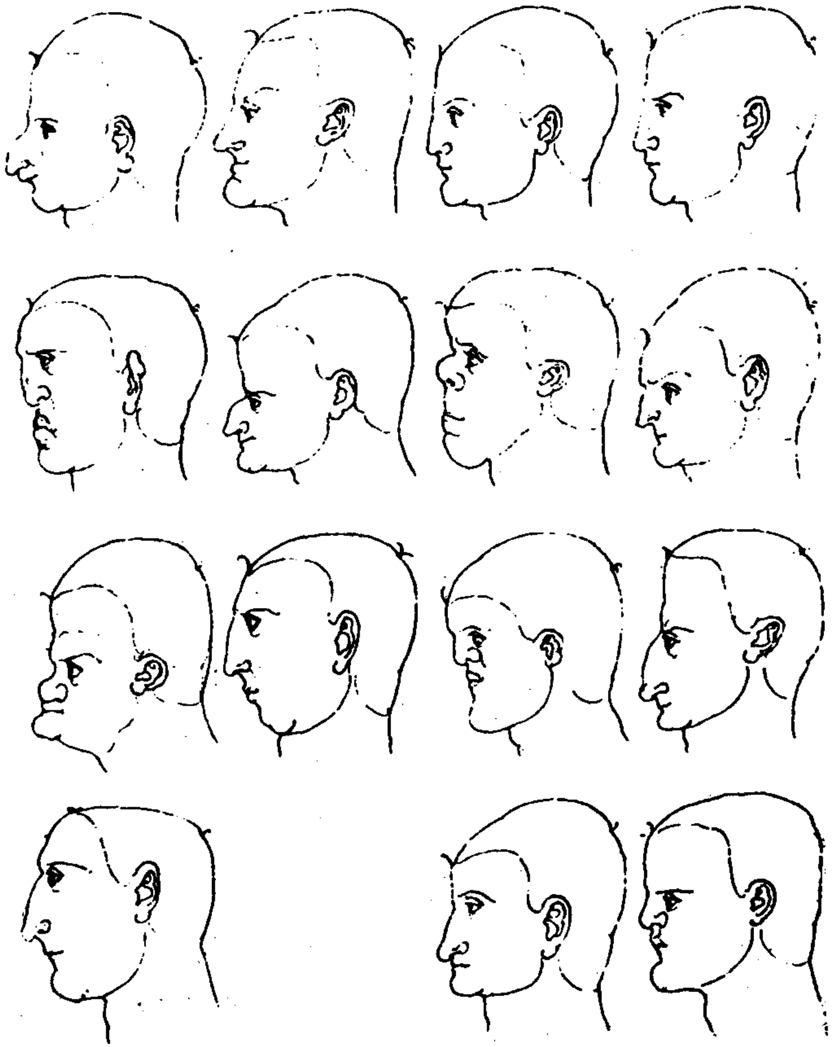
They are topologically almost the same.

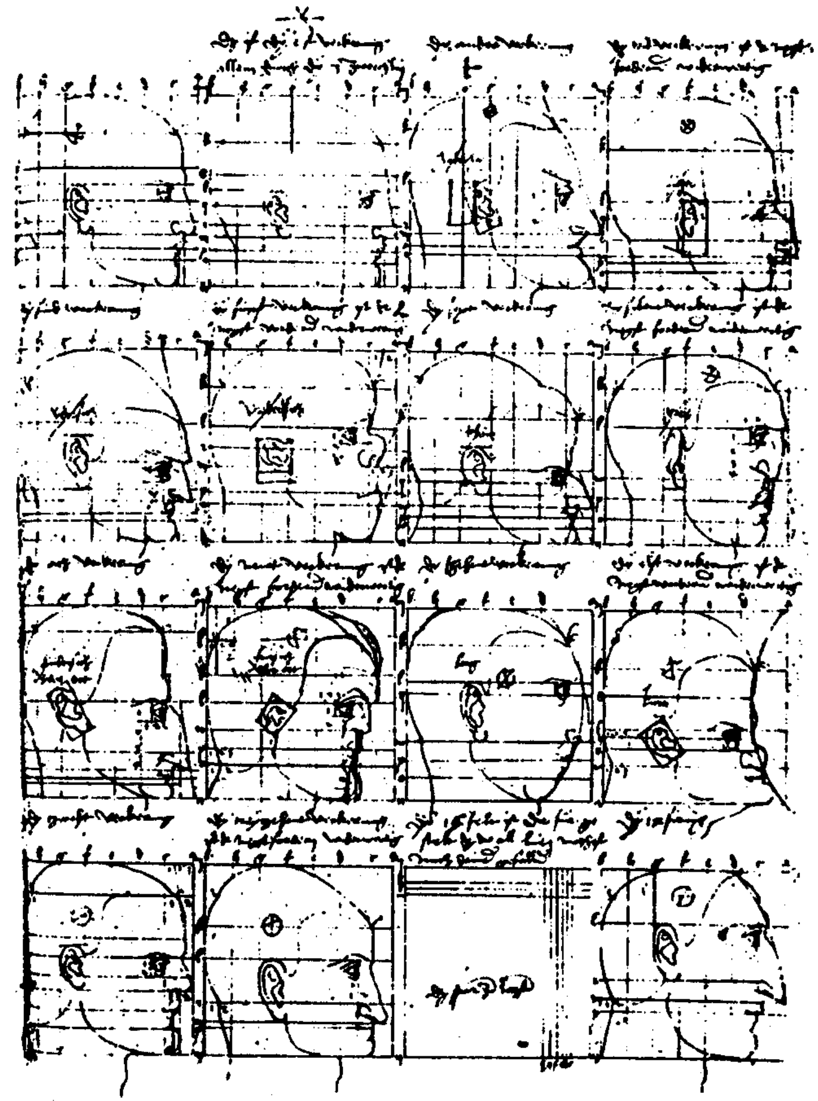


Scorpaena sp.

Antigonia capros.

Abrecht Dürer’ s book
“Vier Bücher von Menschlicher Proportionen”, Nürenberg, 1528, described geometric transformations of human bodies and faces.





Now

Three types of geographic transformations will be discussed

The primary emphasis - the first type of transformation - is on transformations to solve specific geographical problems.

Here one can think of map projections as spherical versions of graph paper: that is, as nomograms.

Logarithmic graph paper is often used to illustrate,

or study, trends over time, or rates of change.

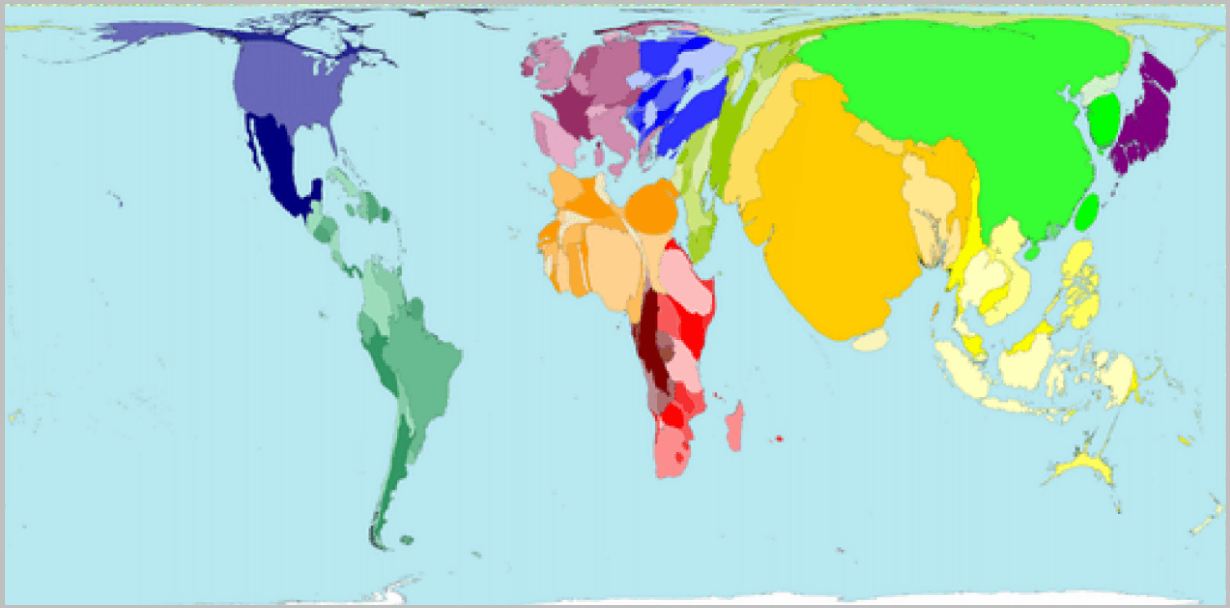
A logarithmic map projection can depict geographical rates of change.

This use of map projection is in the classical

Transform - Solve - Invert paradigm.

A second type of transformation illustrates a property of the world, or a point of view.

The "Atlas of the Real World", here showing the world by population, is a wonderful version of this attribute.
Colors are here used to represent different countries.



This is a type of anaphomose, as are all map projections.
But, when perverted, such transformations could also be a form of propaganda.

A third type of transformation illustrates a more psychological view.

This is a cognitive or behavioral map wherein believed or imagined attributes of the world are depicted and these may be used to influence action.

I will show examples of all three types of transformations.

All map projections result in distorted maps!

Since the time of Ptolemy an objective of map makers has been a high metric fidelity representation of the earth's surface.

Ptolemy asserted the the objective was to make a map of the world so that the distances thereon were nearly as correct as possible and noted that "the difficulty is evident".

That was 2000 years ago.

Distance, area, or angular preservation is still the dominant theme today!

In 1569 Mercator broke this mold and introduced a new paradigm.

His idea was to use a systematic warping to assist in the solution of a particular global problem.

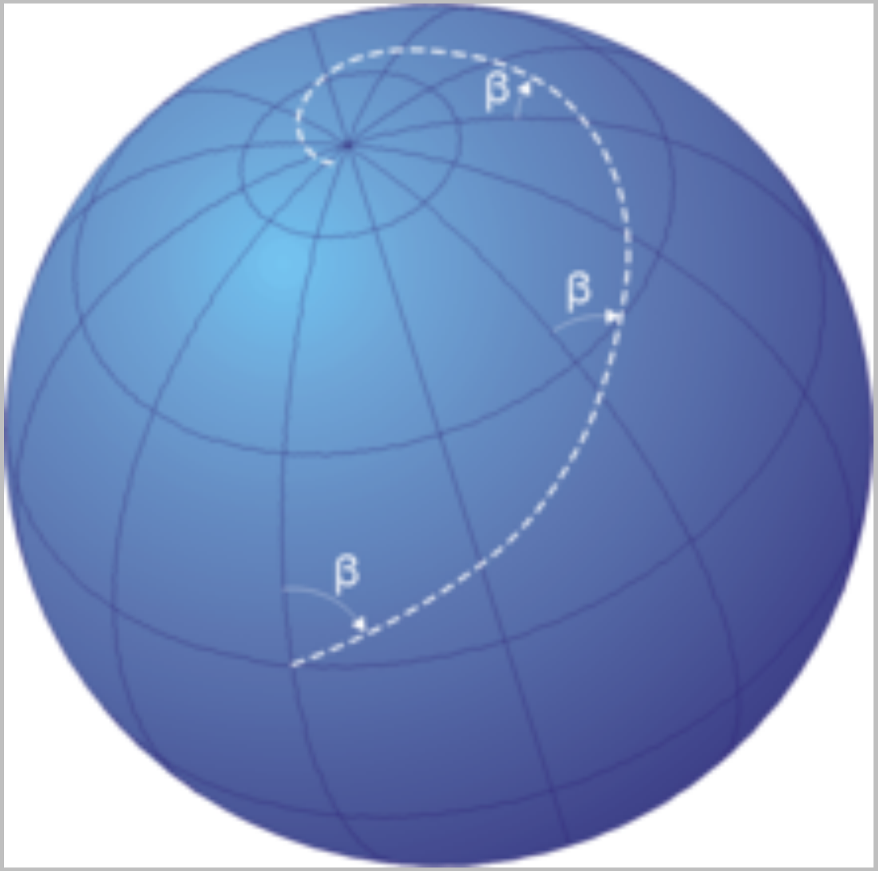
The notion caught on, even though it took almost a century.

The seaman plots a straight line on the distorted Mercator map and follows the indicated directions. The same course is a curve on the earth but it is much easier to draw a straight line on a map.

Today the seaman will hardly accept any other map projection.

Anamorphic projections of the world, or parts thereof, can similarly be used to enlightening effect for a variety of problems, but are also used for interesting displays.

The loxodrome is a logarithmic spiral on a sphere.
It intersects all meridians at the same angle.
Mercator's projection warps it into a straight line.



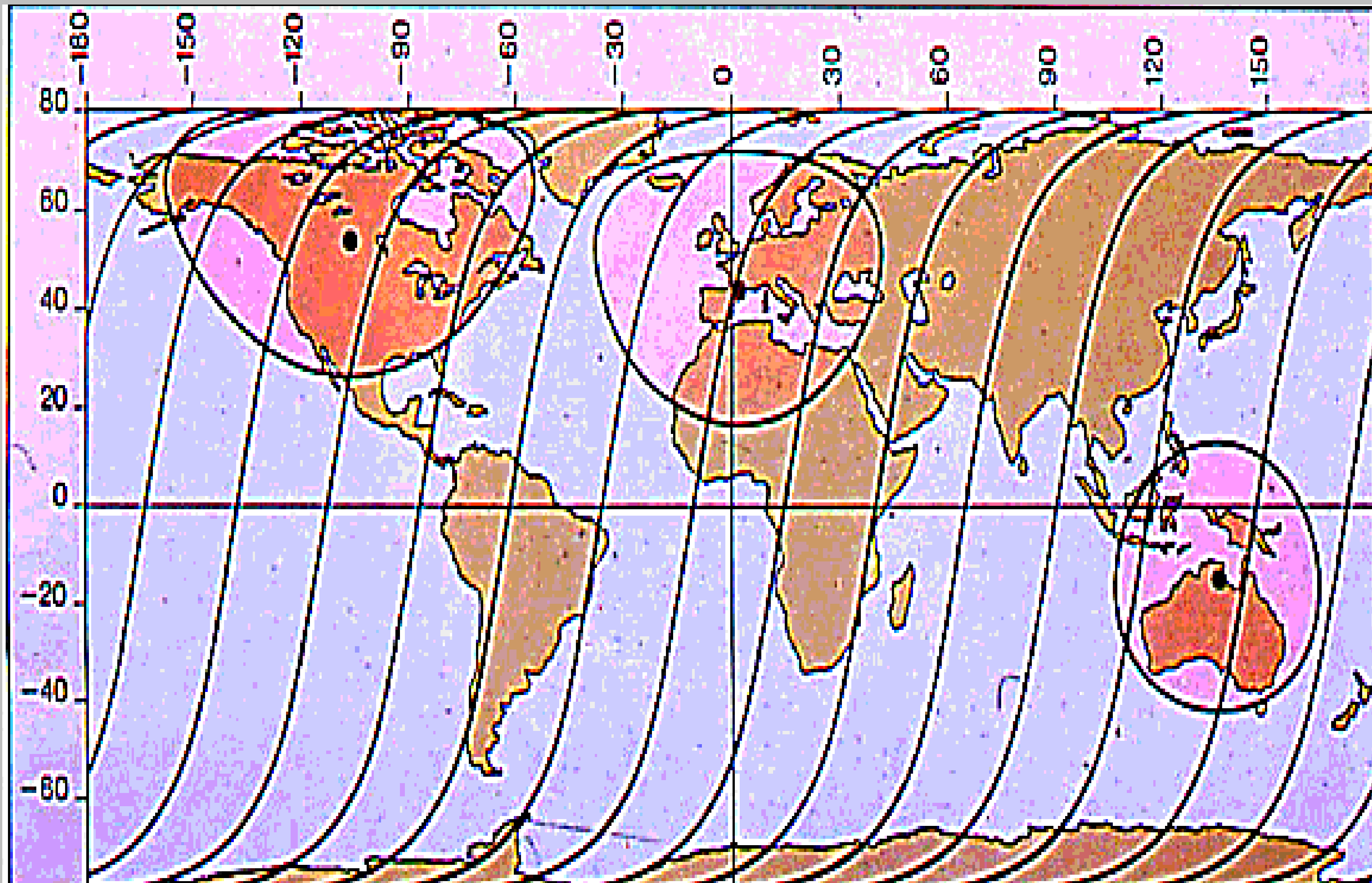
The next several maps are examples of other purposeful distortions introduced in order to provide solutions to particular problems.

These examples are in the spirit of the change introduced by Mercator.

Conventional Way of Tracking Satellites

The range rings on the conventional map are circles on the earth.

Satellite tracks are curves.



An engineer at the Radio Corporation of America was faced with the task of tracking earth satellites.

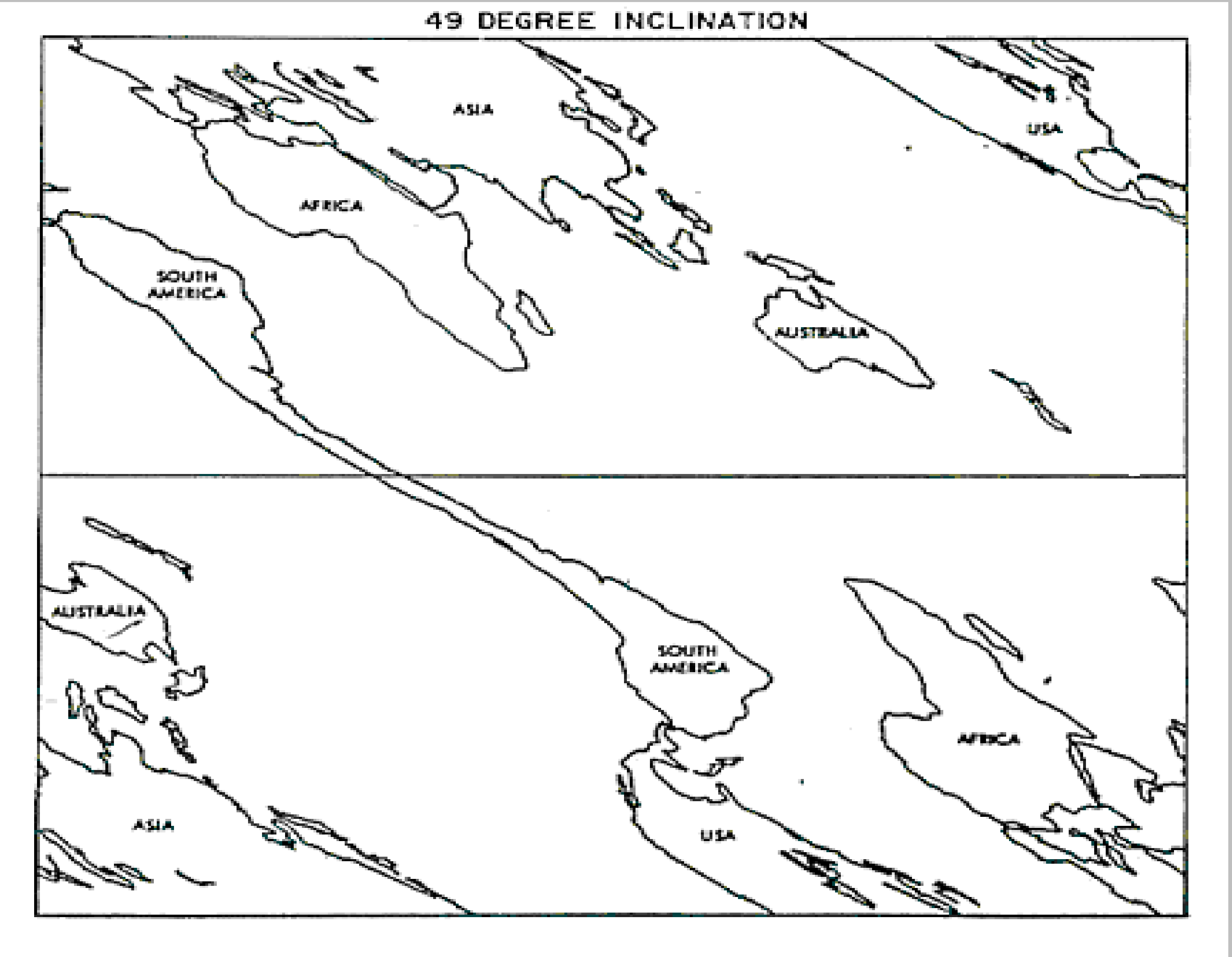
So, instead of straight meridians and parallels with curved satellite tracks, as on the previous map, he bent the meridians so that the satellite tracks become straight lines.

This is convenient for automatic plotting of the satellite tracks.

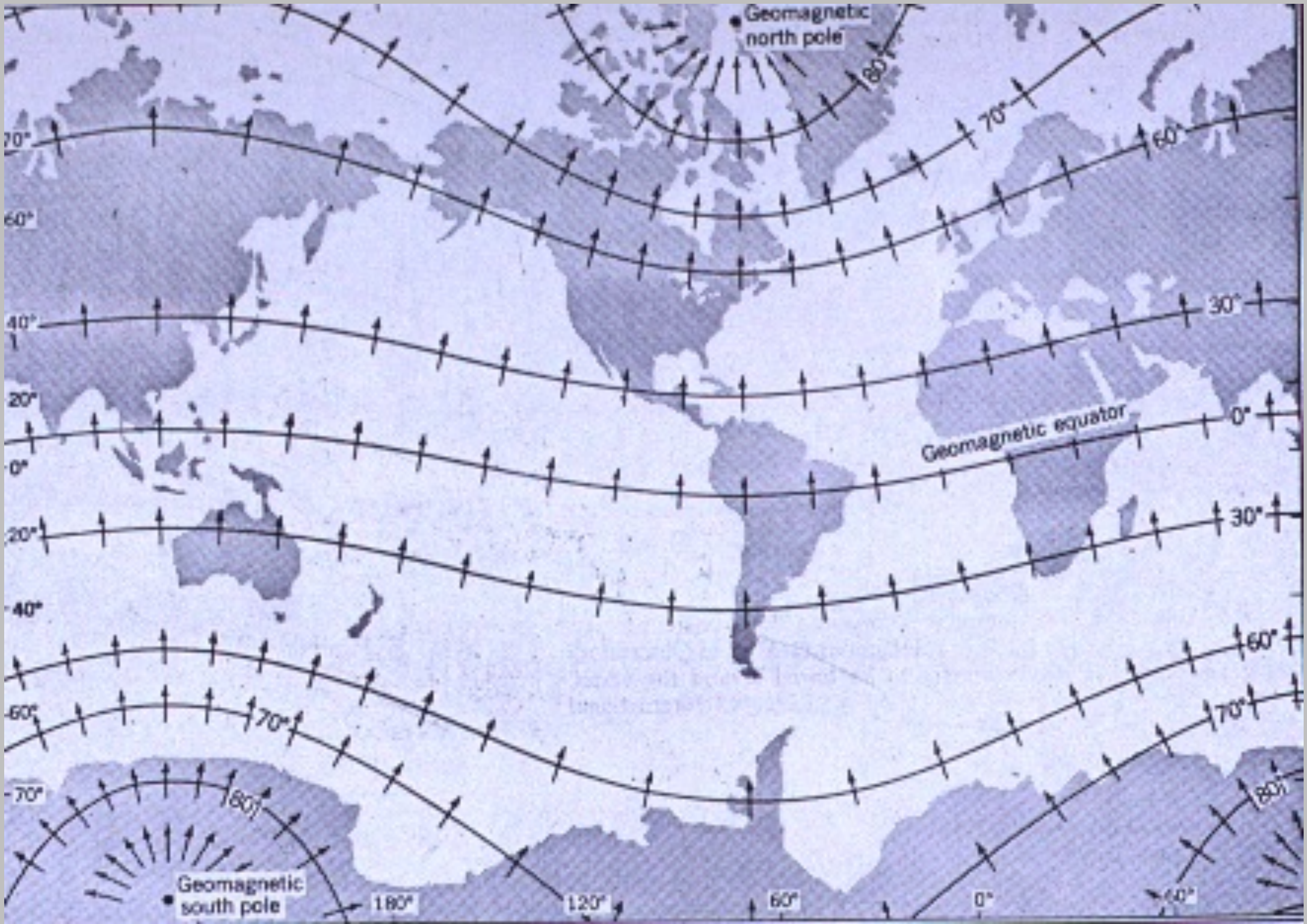
Jack Breckman, 1962, “The Theory and use of B-Charts”, Radio Corporation of America, 18 pp.

Bend The Meridians Instead

The map is also folded about the furthest southern extent of a particular satellite



This Map Shows Magnetic 'Parallels' And 'Meridians'



Choosing the correct coordinate system is often used to simplify a problem

On the next map we straighten the magnetic coordinates in order to simplify the solution of problems involving terrestrial magnetism.

This warps the normal geographic coordinates, but so what?

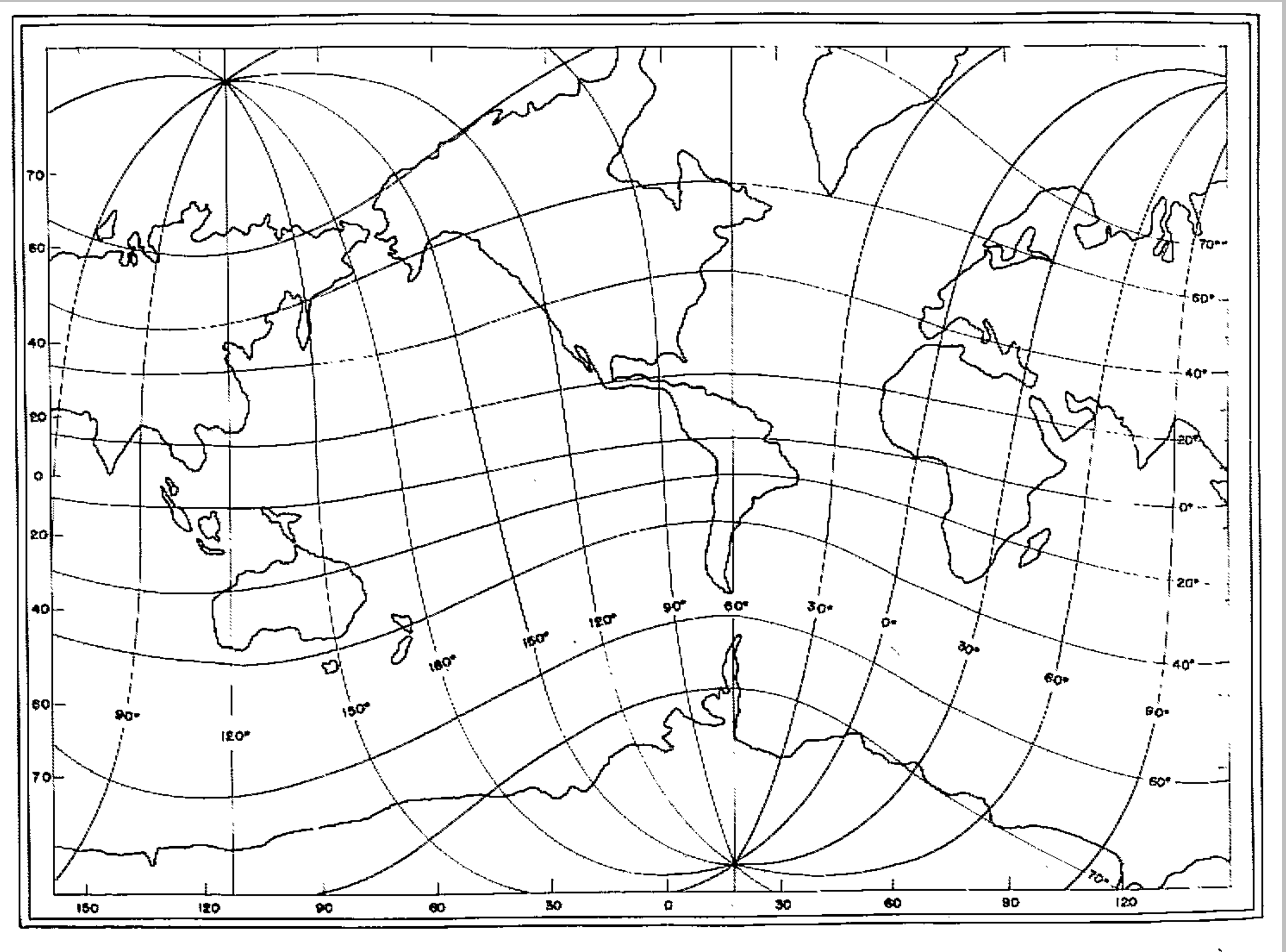
It is not difficult to produce such maps graphically. They can also be computed in a mathematical conformal way.

Mathematical models of historic world wide magnetic intensity go back a few centuries.

What an opportunity for animation!

Map Which Straightens The Magnetic Coordinates.

Student Drawing with geographical latitude & longitude.



Coordinates from Distances

For the next map a student used a table of travel distances between places from a road atlas.

From this table of distances he computed coordinate locations of the places.

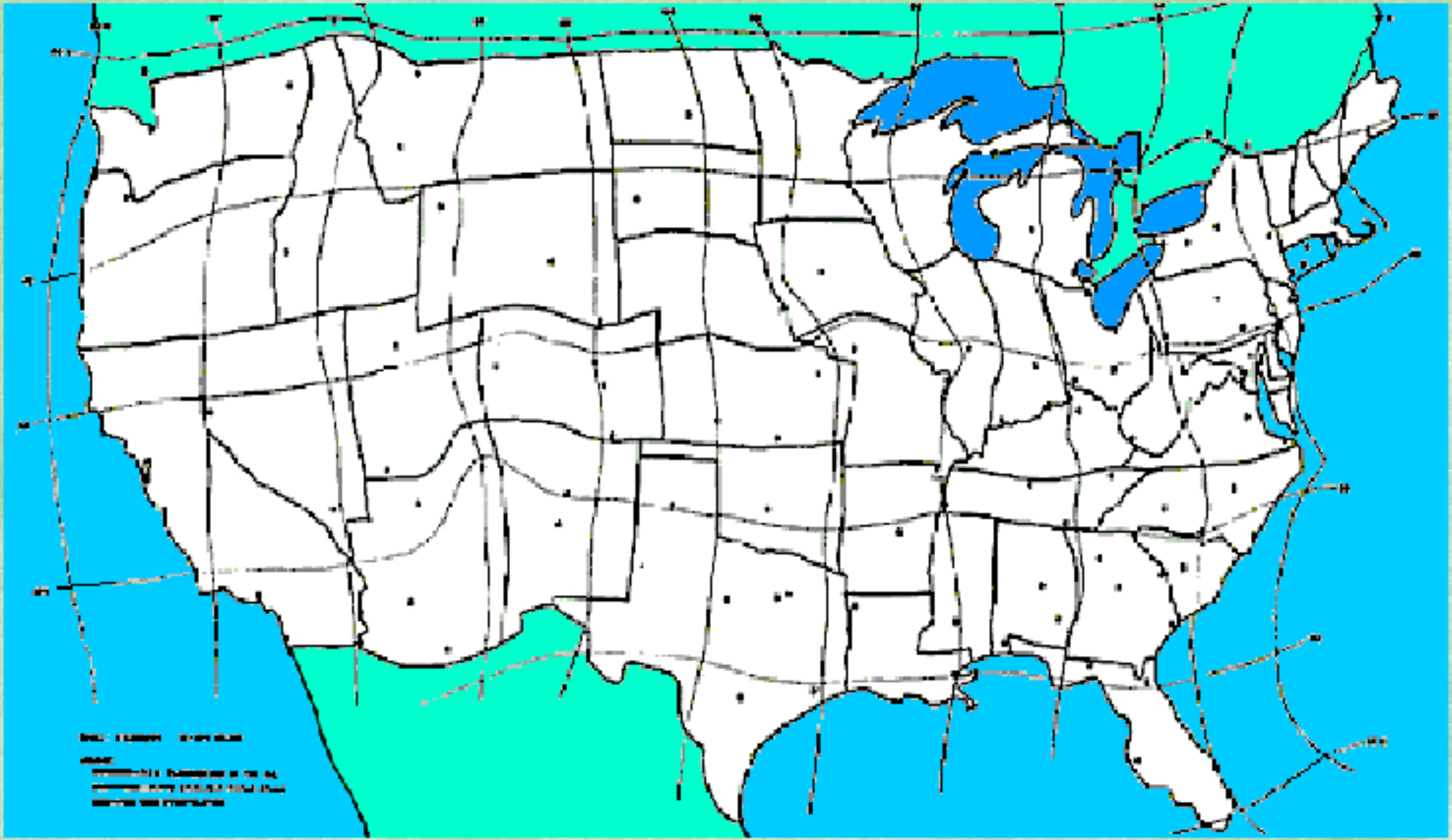
Converting distances to coordinates is a well known surveying problem.

Interpolation was then used to fill in the boundaries.

The locations used are shown by dots on the map.

Road Distance Map of the United States

Student drawing



The “warped” road map, and similar distorted maps, can be evaluated by the methods of Tissot.

M.A. Tissot, 1881, Memoire sur la reresentation des surfaces, Paris, Gautier Villars.

The road system introduces distance distortion, as is obvious, but there are also area and direction distortions and these can also be measured.

In other words, cartographic theory can be used to evaluate quantitatively several geometric impacts of a road system

or of any transportation innovation and many other changes to the world.

Including measuring distortion on mental maps, as on the next slides.

Cognitive-behavioral geography looks at how the world is believed to be by people, not how it is.

It is associated with the work of Kevin Lynch and Peter Gould.

The first map shows distances from Seattle, as estimated in an informal survey, on a north oriented azimuthal equidistant projection centered at Seattle.

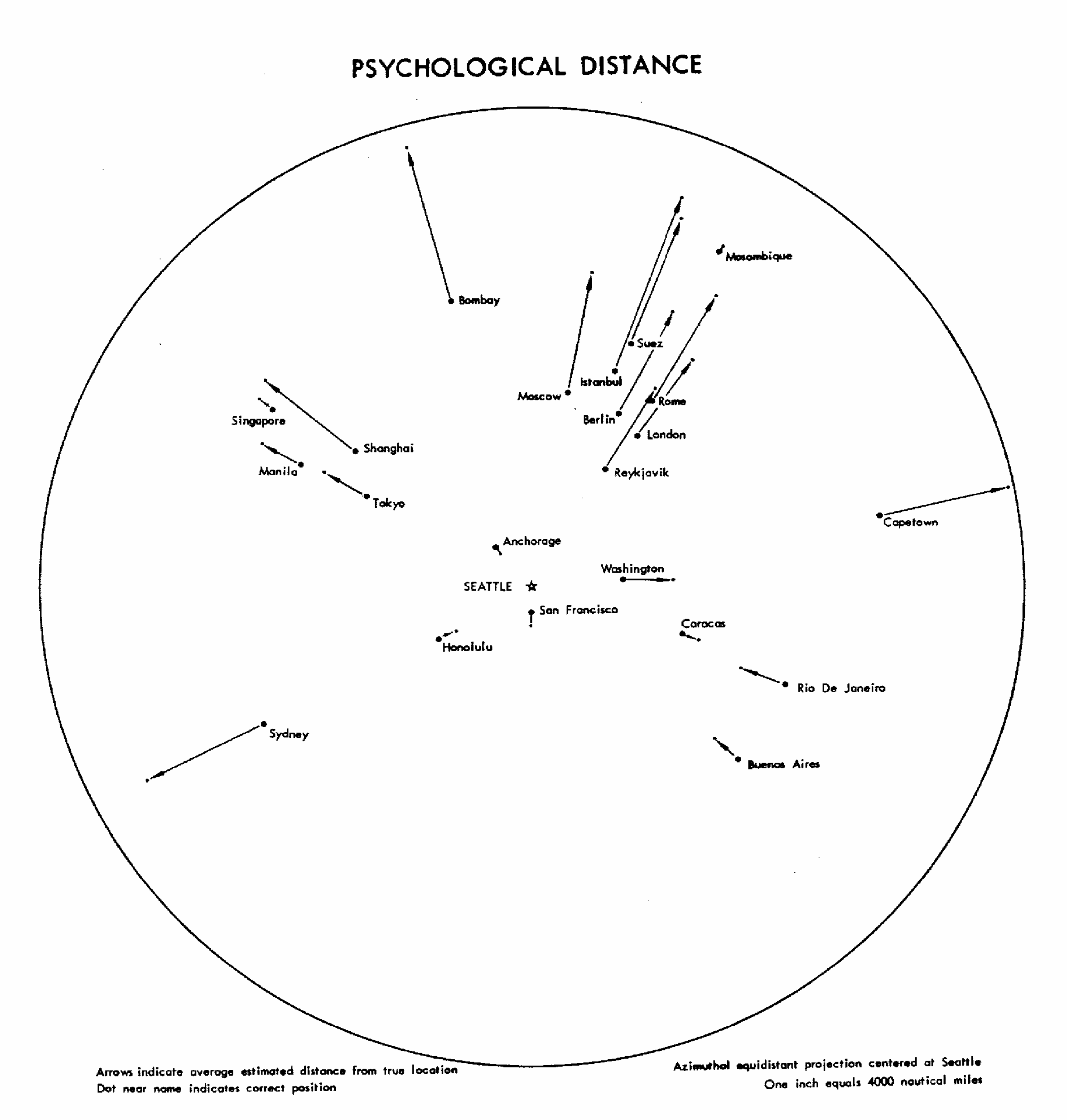
South American places and Hawaii are estimated as being closer, Europe and Asia as further away.

The second map has examples drawn from young students throughout the world.

Both maps can be analyzed using Tissot's method.

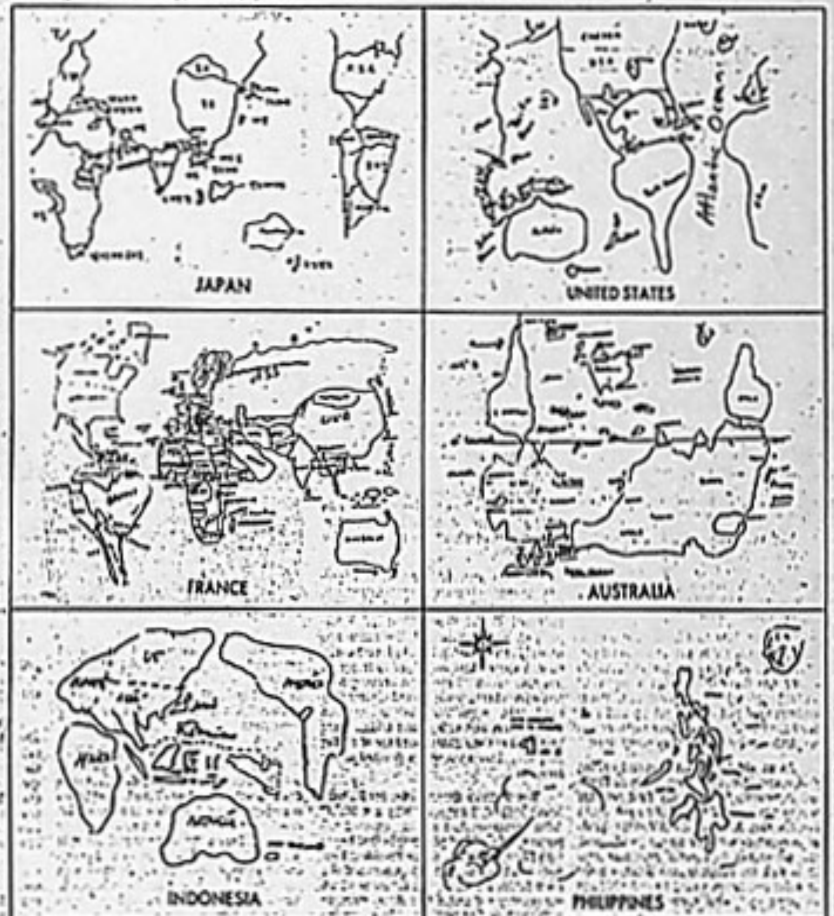
Estimated distances from Seattle

Azimuthal equidistant projection centered at Seattle.



Quick! Draw the World

Joy Aschenbach
GEOGRAPHY



These maps, sketched from memory, are a sample from 4,277 drawn by first-year college students in 54 countries.

You are given a blank piece of paper and asked to draw a map of the world from memory. You are told to label all the countries and include any other features of interest or importance. Time limit: 20 minutes. How would you do?

The 4,277 first-year college students in 54 countries who were assigned this task by geographer Thomas F. Saarinen of the University of Arizona remembered an average of 30 countries per map — fewer than one-fifth of the independent countries in the world.

Saarinen, who personally administered the classroom test in 20 of the countries surveyed last year, says he would give the maps a worldwide grade of "fair." Most students managed to draw and name all seven continents and the major countries, but lacked specific knowledge about the rest of the world.

The 840 U.S. students who were tested ranked with the world average in numbers of countries labeled, he says, but made more mistakes in placing the countries on the map.

The more than 4,000 maps from every continent except Antarctica represent the largest collection of world sketch maps ever assembled, Saarinen says, and will form an archive of computerized information about mental images of the world.

"Before we can have a shared world image," he says, "we have to understand what images are at present, and why." The college mapping project was sponsored by the International Geographical Union and financed by the National Geographic Society.

Saarinen selected a cultural cross section of students enrolled in introductory geography courses, because they already had completed their countries' basic educational process and were interested enough in geography to take a college course in it.

What was totally unforeseen, Saarinen says, was "the perceived importance of Europe everywhere. It has to be part of the colonial legacy, at least in the educational system. I thought there'd be a home-centered bias, with it pictured bigger, but more important was the exaggerated size of Europe.

"Our mental images," he observes, "don't seem to have caught up with the reality of a world of free and independent nations."

Eighty percent of the students placed Europe in the center of their worlds. This positioning, with the Americas on the left and Asia on the right, has been the most commonly used world map projection since the discovery of the New World.

Even in countries such as India, Pakistan, Bangladesh and Saudi Arabia, virtually all students drew "Eurocentric" maps, where "Sinocentric" maps would put Asia in the middle.

Saarinen says that, with the exception of certain European countries, the nations most often included were those of continental dimensions such as the Soviet Union, Canada, China, the United States, Brazil, India and the continent-country, Australia.

In contrast to the exaggerated size of Europe, Africa was generally sketched smaller than it should have been, with lots of blank space where countries should be — reflecting a general lack of awareness of the Third World.

Among the diversity of maps and map information, Saarinen notes [illegible]

■ Several maps from Australia turned the world upside down. Most Australian students sketched Sinocentric maps, which centered Australia as well as Asia. Some also made the switch from "down under" to "upover" maps. On one, a student wrote: "Remember, Australia is on the top of the world, not the U.S.A., and especially not Russia."

■ Hungarian maps, the product of an educational system that stresses learning all the countries of the world, were among the best.

■ An extremely Philippinocentric map, drawn in the Philippines, prominently outlined the island nation and left the rest of the world blank, labeled "terra incognita."

■ A Spanish map made the United States as big as the Soviet Union and bigger than Canada, and moved New York to the Arizona-Mexican border.

What do you do with 4,277 sketch maps when all the information they contain is finally fed into a computer and analyzed? Saarinen says he hopes to publish the [illegible] most representative ones as "an atlas of world images." ■

Joy Aschenbach wrote this article for National Geographic.

The next map is obtained by converting 862 ocean shipping distances between world ports into coordinate locations.

The coastlines are then interpolated.

Observe that the poles go to the outside of the map since there is no shipping across these locations.

Also the route from New York to Seattle passes very close to Panama, all three nearly lying on a straight line.

Also look at the route from Rio de Janeiro to Bombay as it passes Cape Town.

The scale in the lower right corner measures shipping distances.

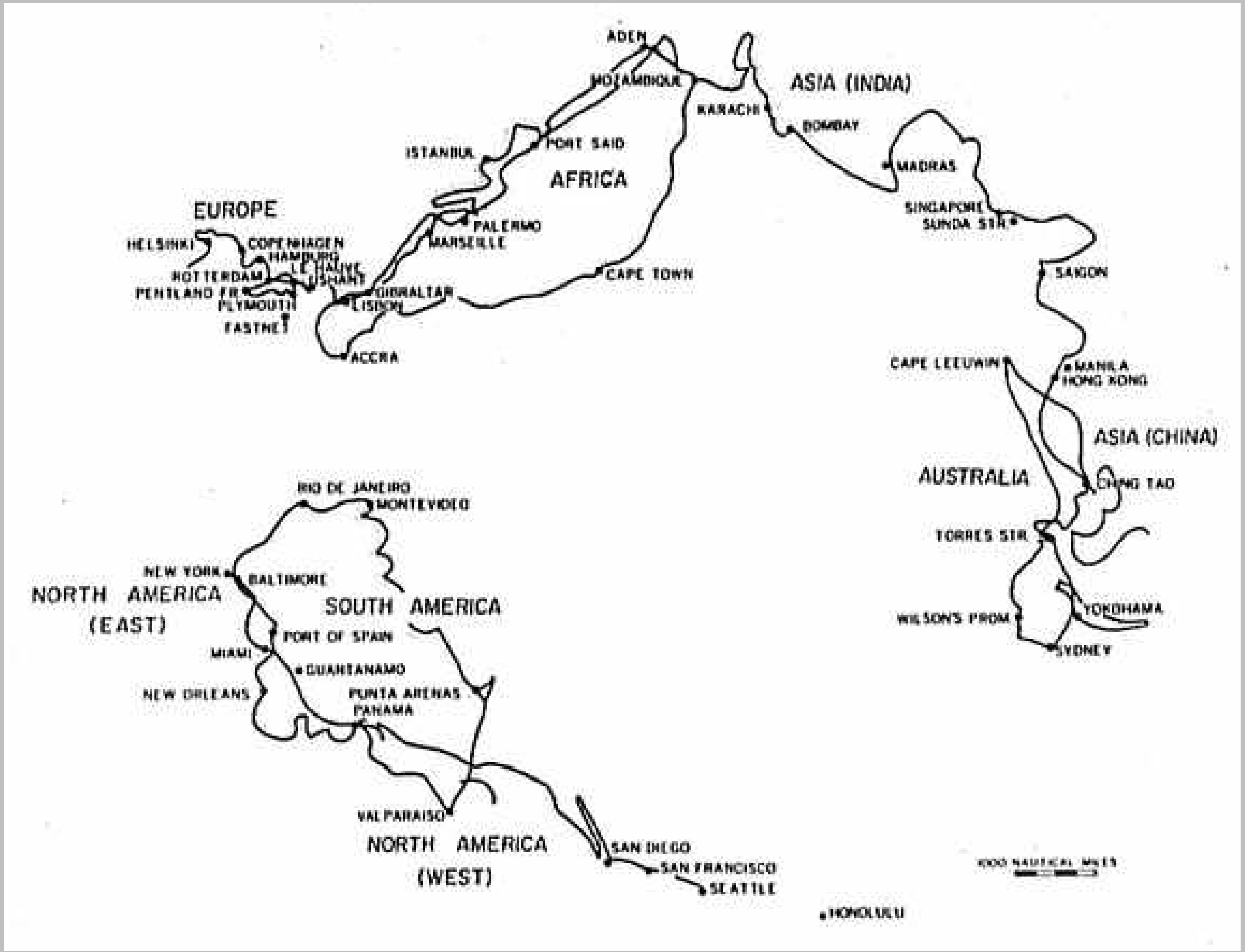
Think about estimating the latitude & longitude graticule.

Map by W. Tobler 1964

World Ocean Distances Map

Based on Shipping Distances Between 42 Ports

Observe the scale bar in the lower right corner



The London Underground Map

breaks the mold again, as did Mercator, this time by not preserving the metric properties.

Would Ptolemy be shocked?

The map preserves the topology, which is perhaps even more important.

And this famous distorted map serves a very useful purpose.

As evidenced by the many worldwide imitations,

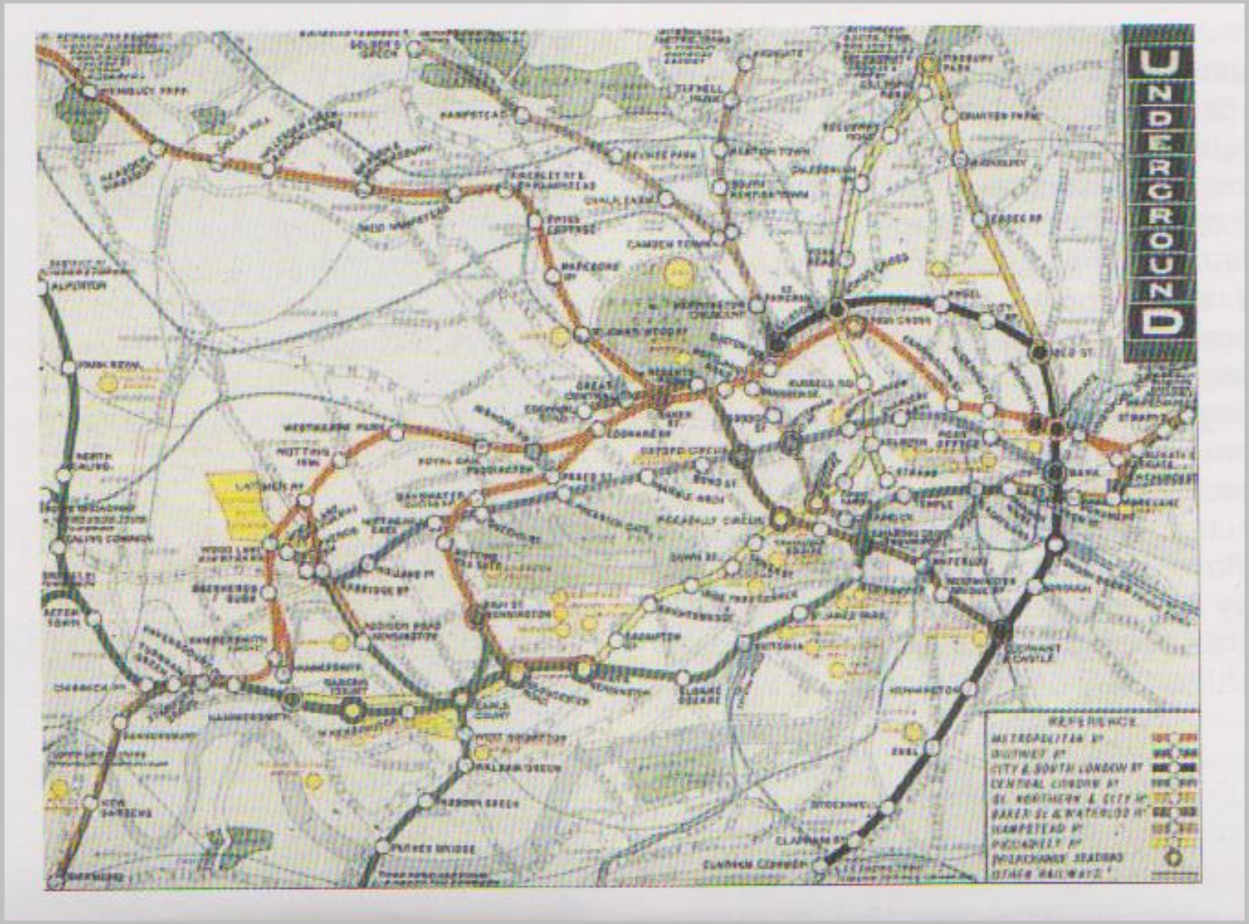
even though in 1933 H. Beck had a difficult time getting it accepted!

H. Beck’s 1933 Map

Can, and has been, analyzed using Tissot’s method

Compare with the old underground map.

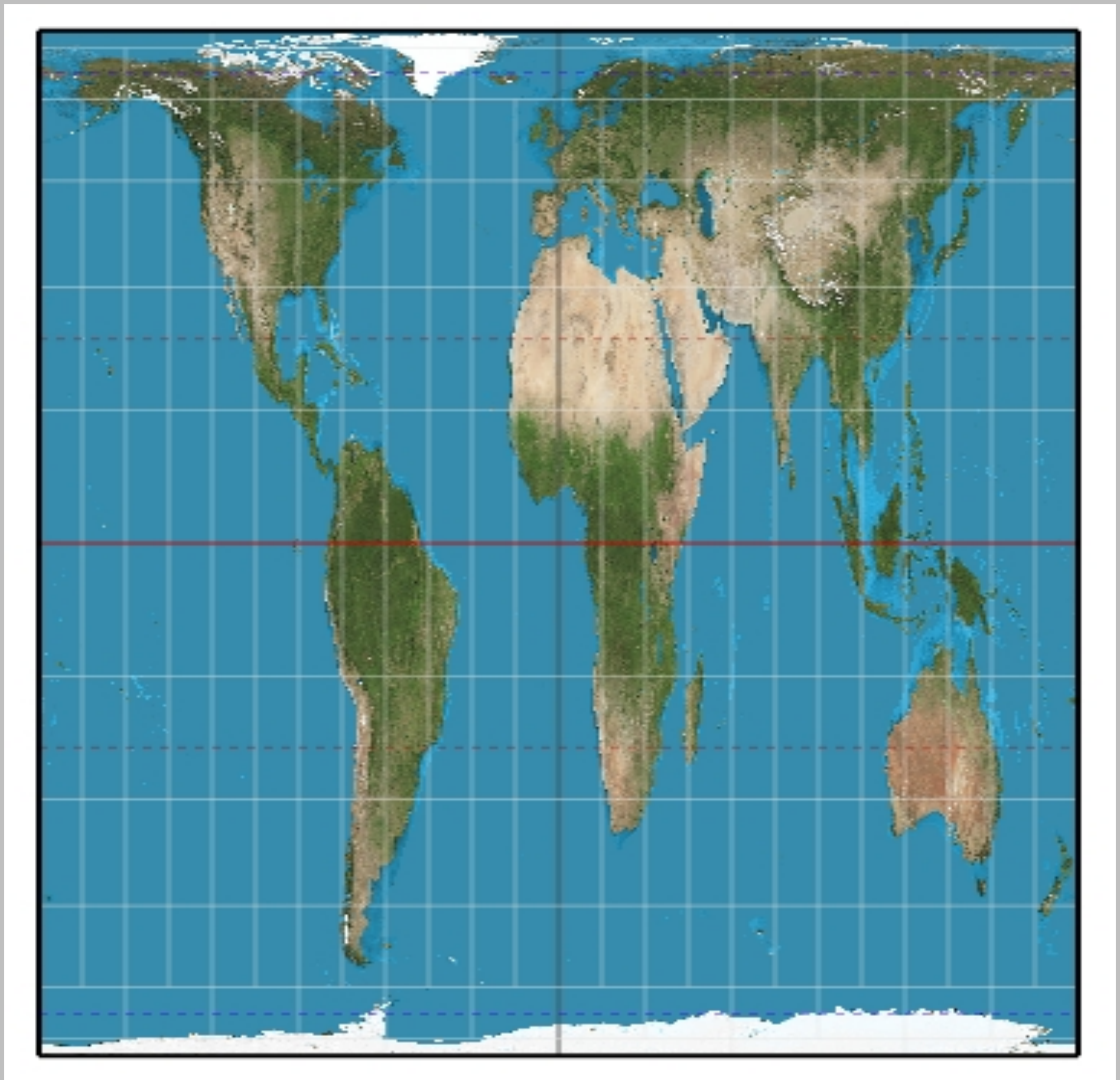
From 1910



Equal area world in a square

Design for use as a global data indexing scheme.

W. Tobler, Chen, Z.,1986, "A Quadtree for Global Information Storage", *Geographical Analysis*, 18,4 (Oct):360-371.



A Map Projection To Solve A Special Problem

The next illustration shows the U.S. population assembled into one degree latitude-longitude quadrilaterals instead of ephemeral political units.

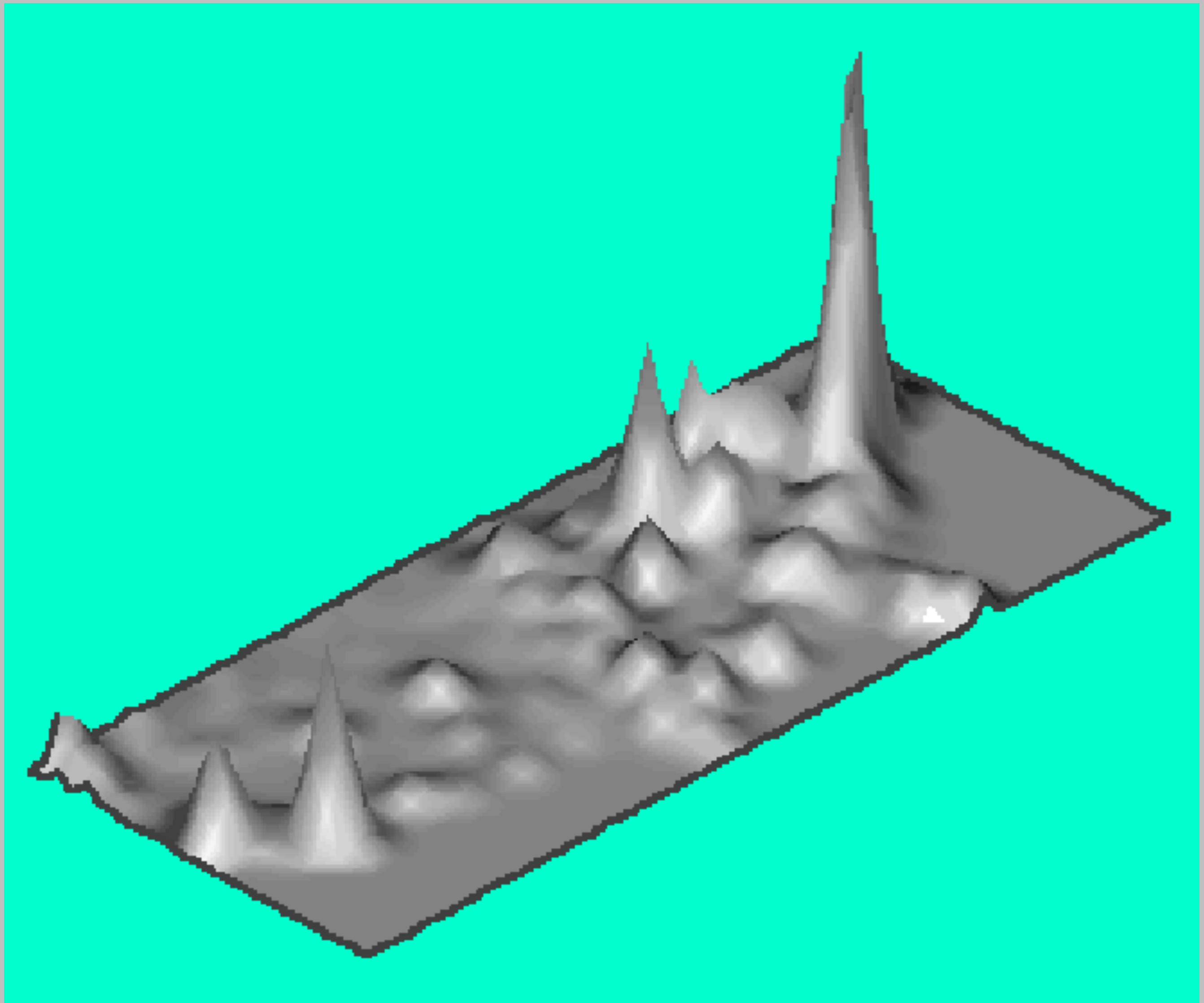
We would like to partition the U.S. into regions containing the same number of people.

There follows a map projection (anamorphose) that may be useful for this problem.

Think political districting! Or central place theory!

US Population By One Degree Quadrilaterals

The population is indicated by the height of cities



Now use the Transform-Solve-Invert paradigm

Transform the graticule, and map, to obtain areas of equal population.

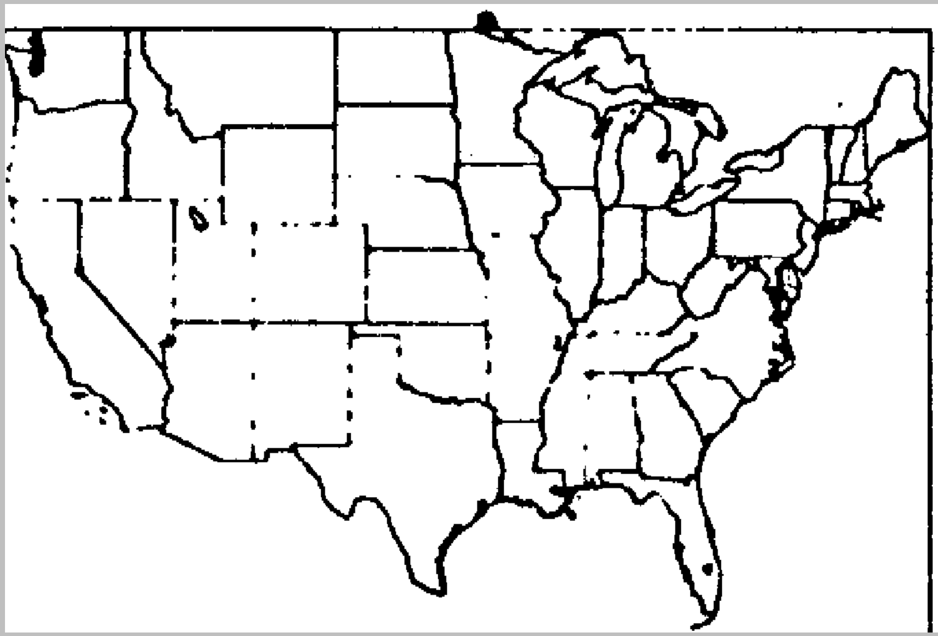
Then position a hexagonal tesselation on the distorted map.

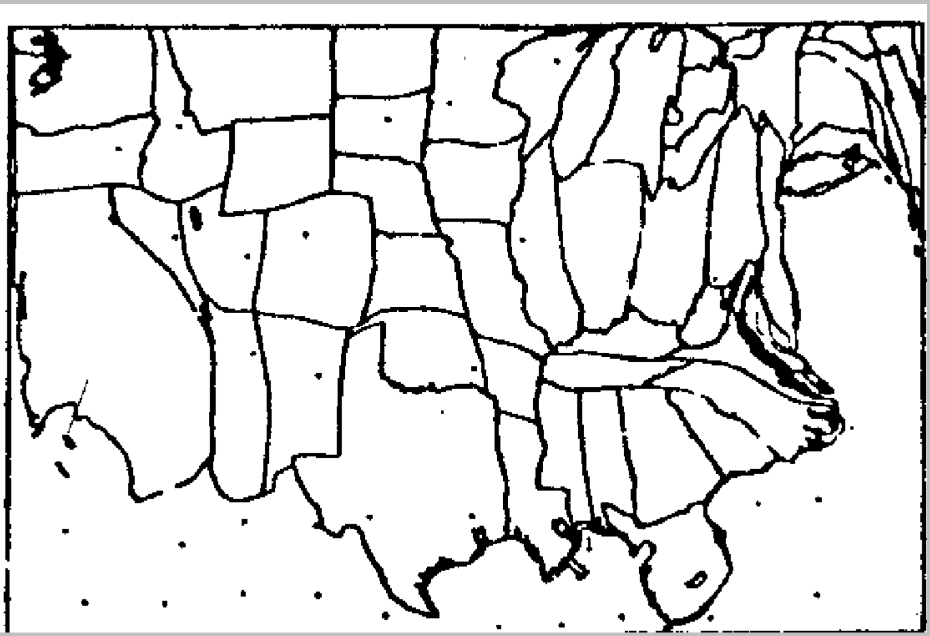
Then take the inverse transformation.

This partitions the US into approximately equal population areas.

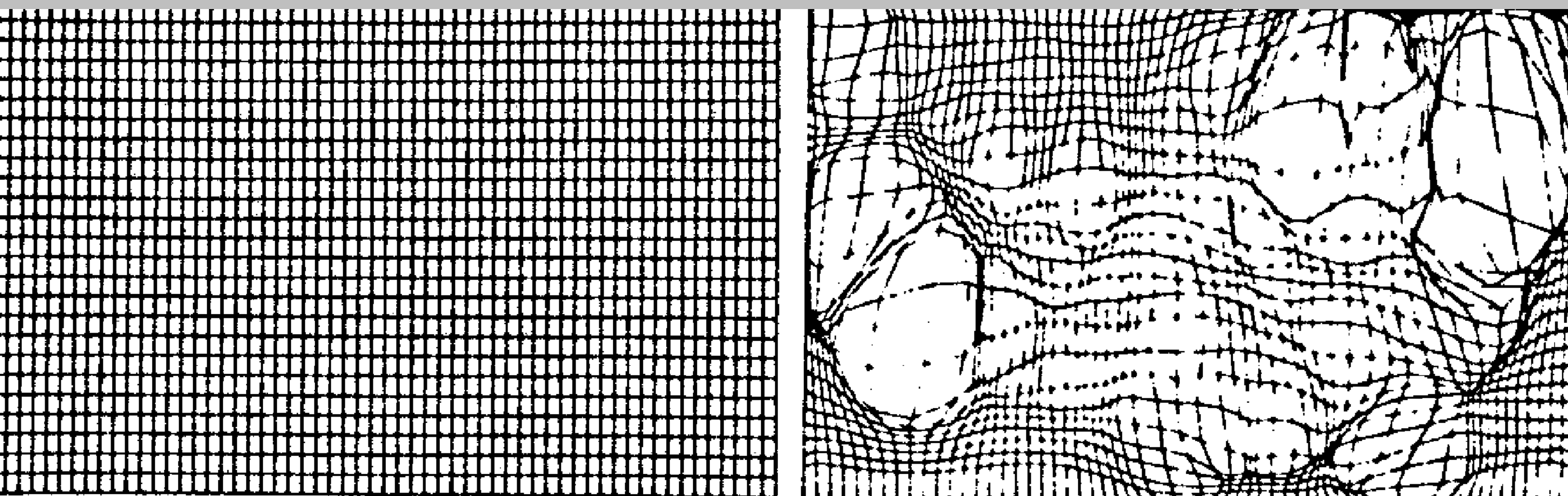
W. Tobler, 1973, "A Continuous Transformation Useful for Districting", *Annals*, N.Y Academy of Sciences, 219:215-220.

Left: Regular U.S. map
Right: Warped to population

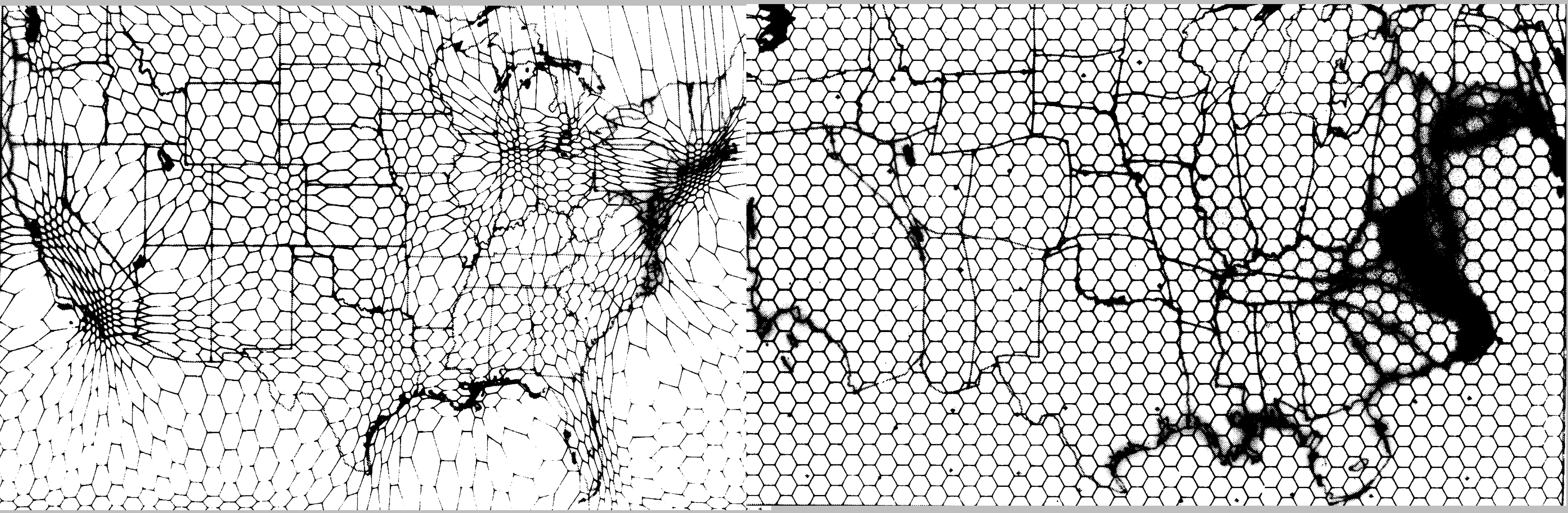


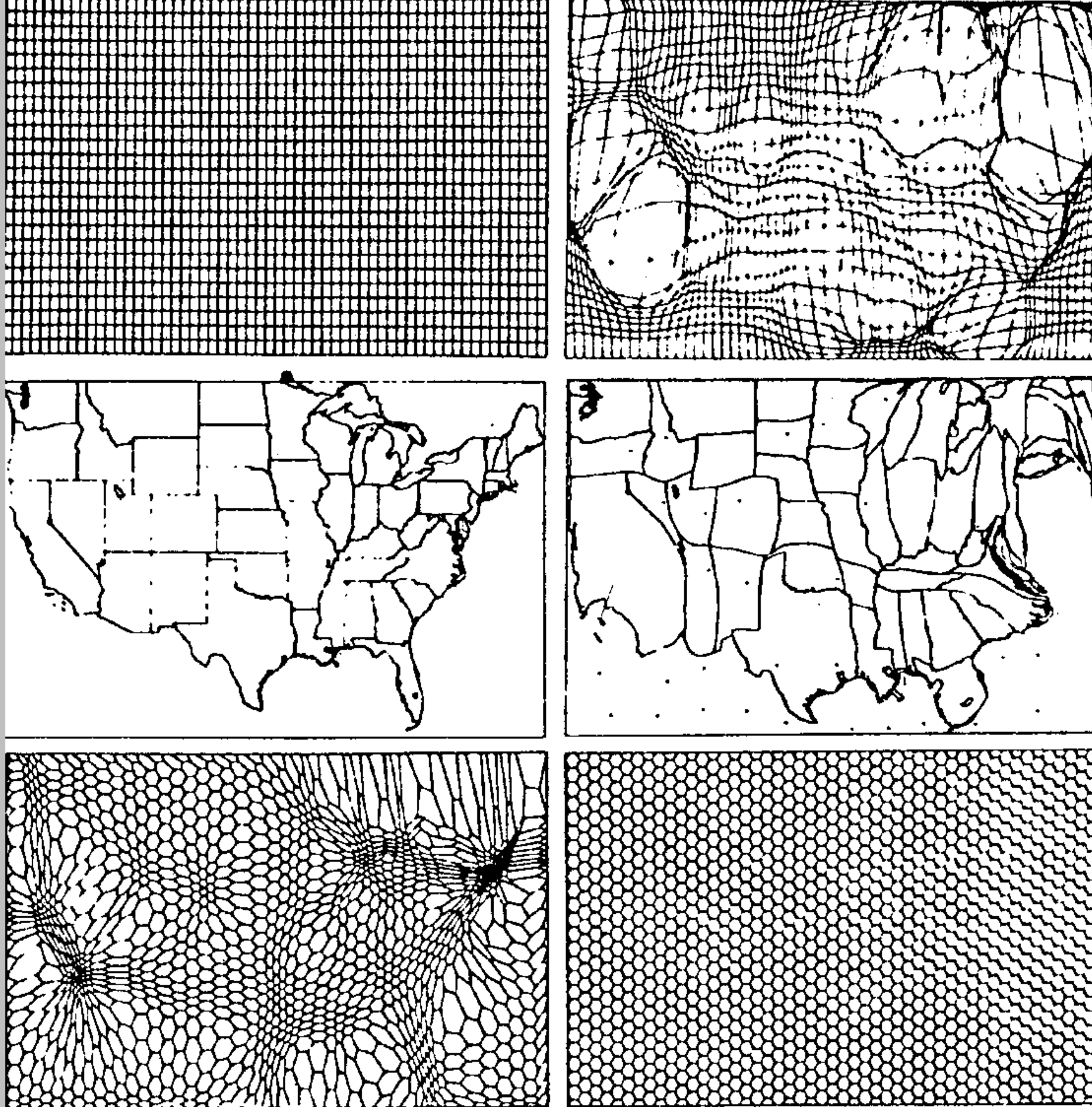


Left: One degree Latitude–Longitude cells
Right: Same, warped by population



Right: Hexagons on population map.
Left: Hexagons inverted to normal map.
To estimate equal population cells.
Compare Nevada with California.





How to use area cartograms.

As in the example just illustrated.

Cartograms are to be used just as is Mercator's projection, that is, as a problem solving device.

As such, their primary purpose is not visualization, just as the Mercator projection is not for visualization.

For epidemiologists and poll takers, after viewing their population of interest in the demographic-cartogram space, devise a sampling scheme, then implement it using the inverse transformation.

Another class of distorted maps attempts to present a point of view

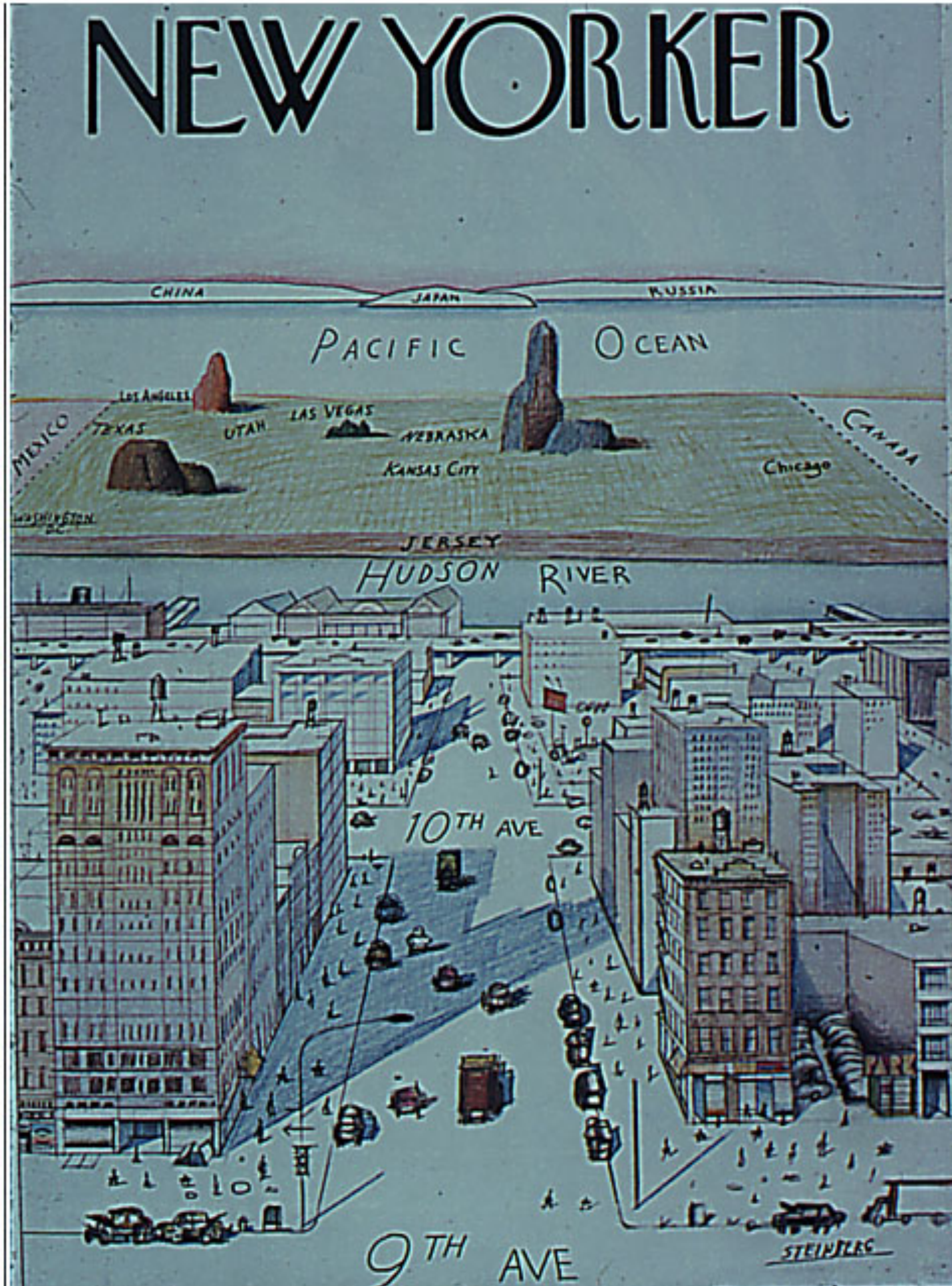
This may be to influence
or it may be simply to reflect an opinion
or to illustrate a property of the world.

The “Atlas of the Real World” contains many examples of this.

There are also many psychological caricatures of state’s views of the US.

(I have a large collection of the state post cards - but would appreciate your sending me more of them.)

Also included in this category is Steinbeck’s
“The View from New York”
Every major city in the world has a replica.


NEW YORKER
CHINA
JAPAN
RUSSIA
PACIFIC OCEAN
LOS ANGELES
MEXICO
TEXAS
UTAH
LAS VEGAS
NEBRASKA
KANSAS CITY
CANADA
Chicago
JERSEY
HUDSON RIVER
10TH AVE
PARK
9TH AVE
STEINBERG

Wallingford's New Yorker's View of America

Dating from 1939 this is an example of a type of quasi-psychological impression of a world view.

Many of the tourists (and retirees) from New York travel to Florida, thus it looms large.

Spatial interaction and distance decay are easily recognized.

"Cousins live in the West - in Delaware".

Thus areal, angular, and distance distortion are all included.

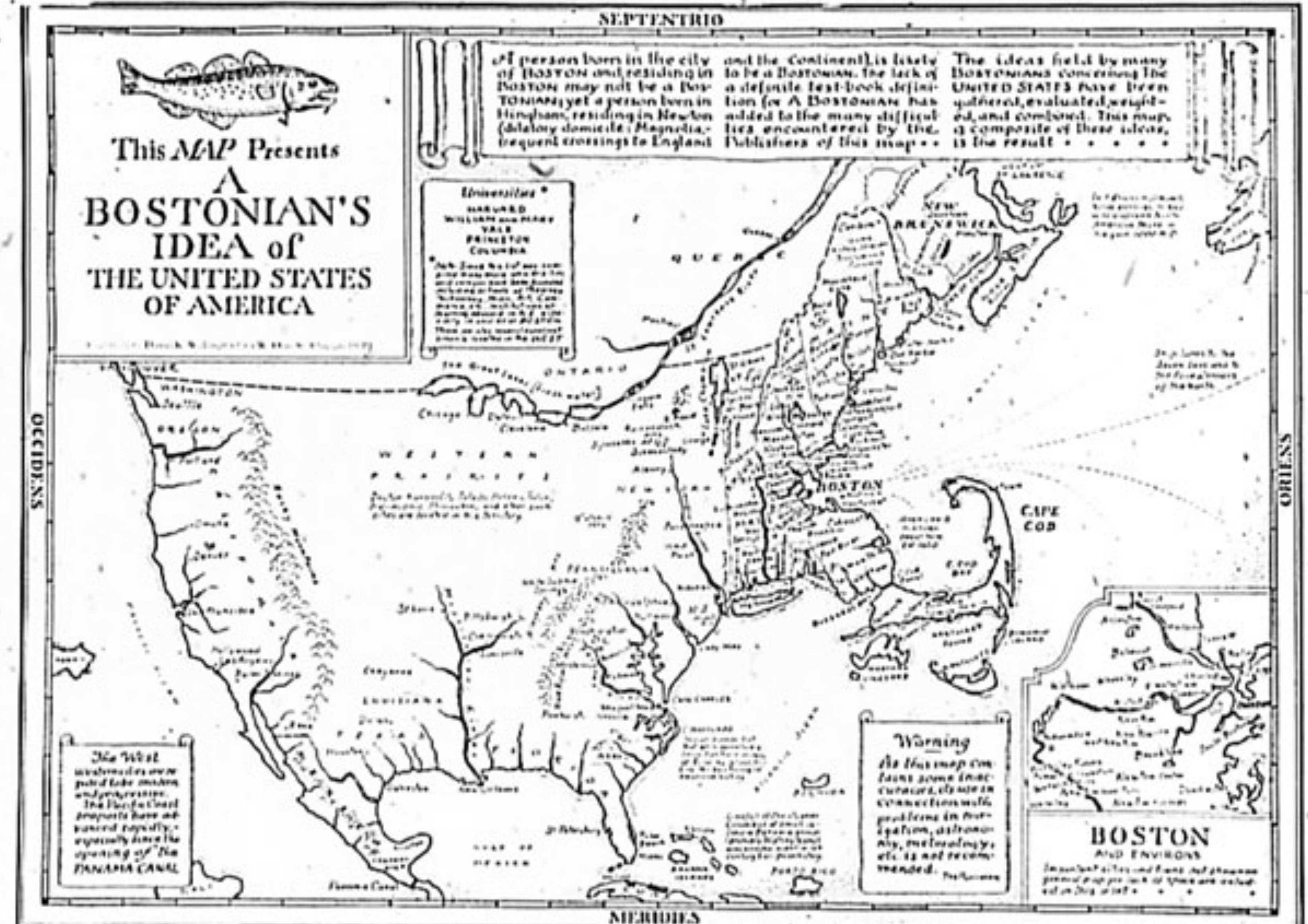
Inversions, tearings of space, & topological violations, occur:

Idaho is a city in the state of Oregon, which is to the North of the state of Washington, etc.

Wallingford, an architect, also did a "Bostonion's View".

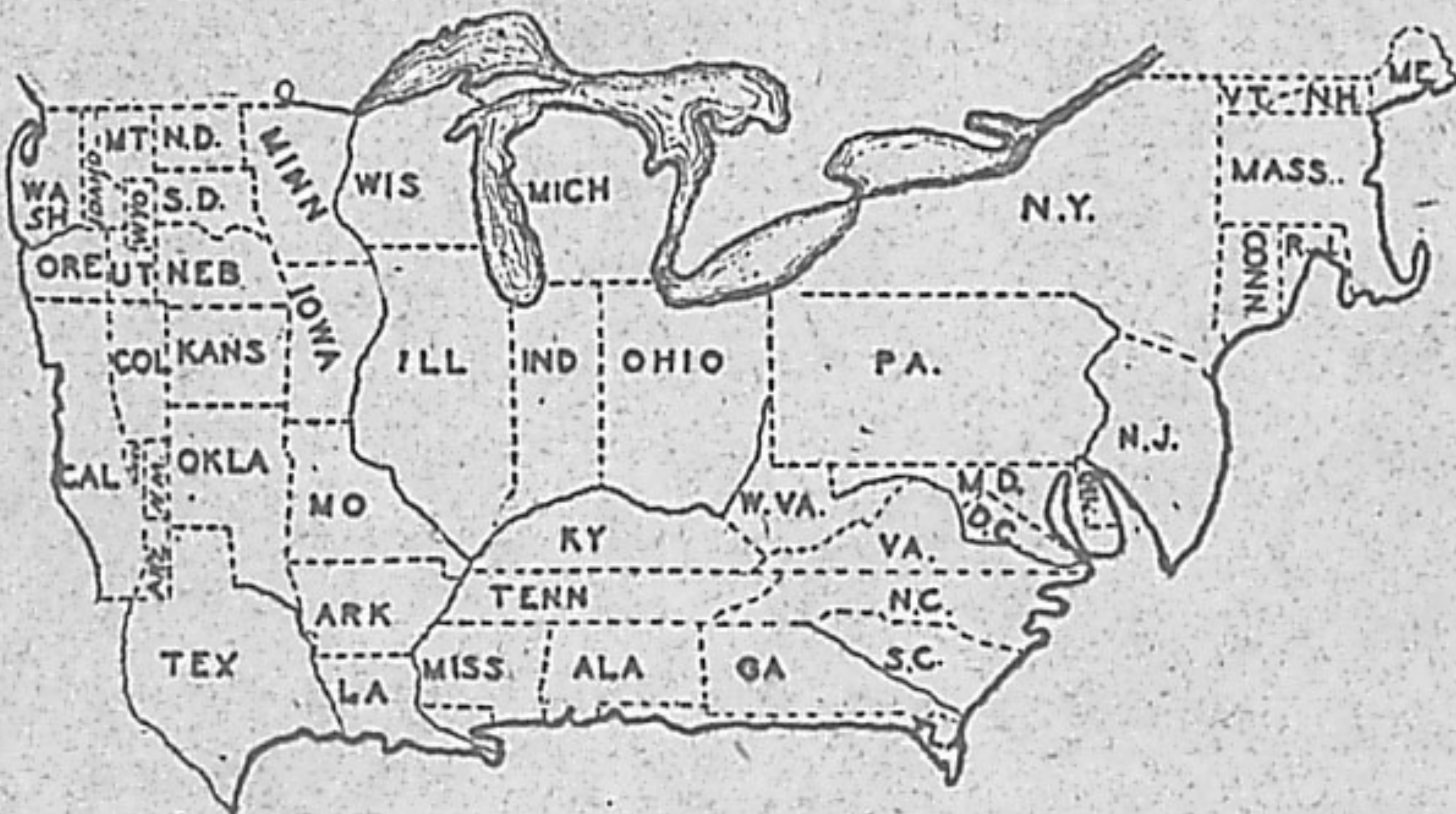
This kind of map needs serious study as a thesis topic!.

Courtesy of Daniel K. Wallingford


SEPTENTRIO
This MAP Presents
A BOSTONIAN'S IDEA of THE UNITED STATES OF AMERICA
A person born in the city of BOSTON and residing in BOSTON may not be a BOSTONIAN; yet a person born in Hingham, residing in Newton (dilatory domicile: Magnolia,- frequent crossings to England and the Continent) is likely to be a BOSTONIAN. The lack of a definite text-book definition for A BOSTONIAN has added to the many difficulties encountered by the Publishers of this map • •
The ideas held by many BOSTONIANS concerning the UNITED STATES have been gathered, evaluated, weighed, and combined. This map, a composite of these ideas, is the result • • • • •
Universities *
HARVARD
WILLIAM AND MARY
YALE
PRINCETON
COLUMBIA
QUEBEC
NEW BRUNSWICK
ONTARIO
WASHINGTON
OREGON
Chicago
BOSTON
CAPE COD
OCCIDENS
ORIENS
PANAMA CANAL
Warning
As this map contains some inaccuracies, its use in connection with problems in navigation, astronomy, meteorology, etc. is not recommended.
BOSTON
AND ENVIRONS
MERIDIES

A businessman wants taxes reduced in “important” states.

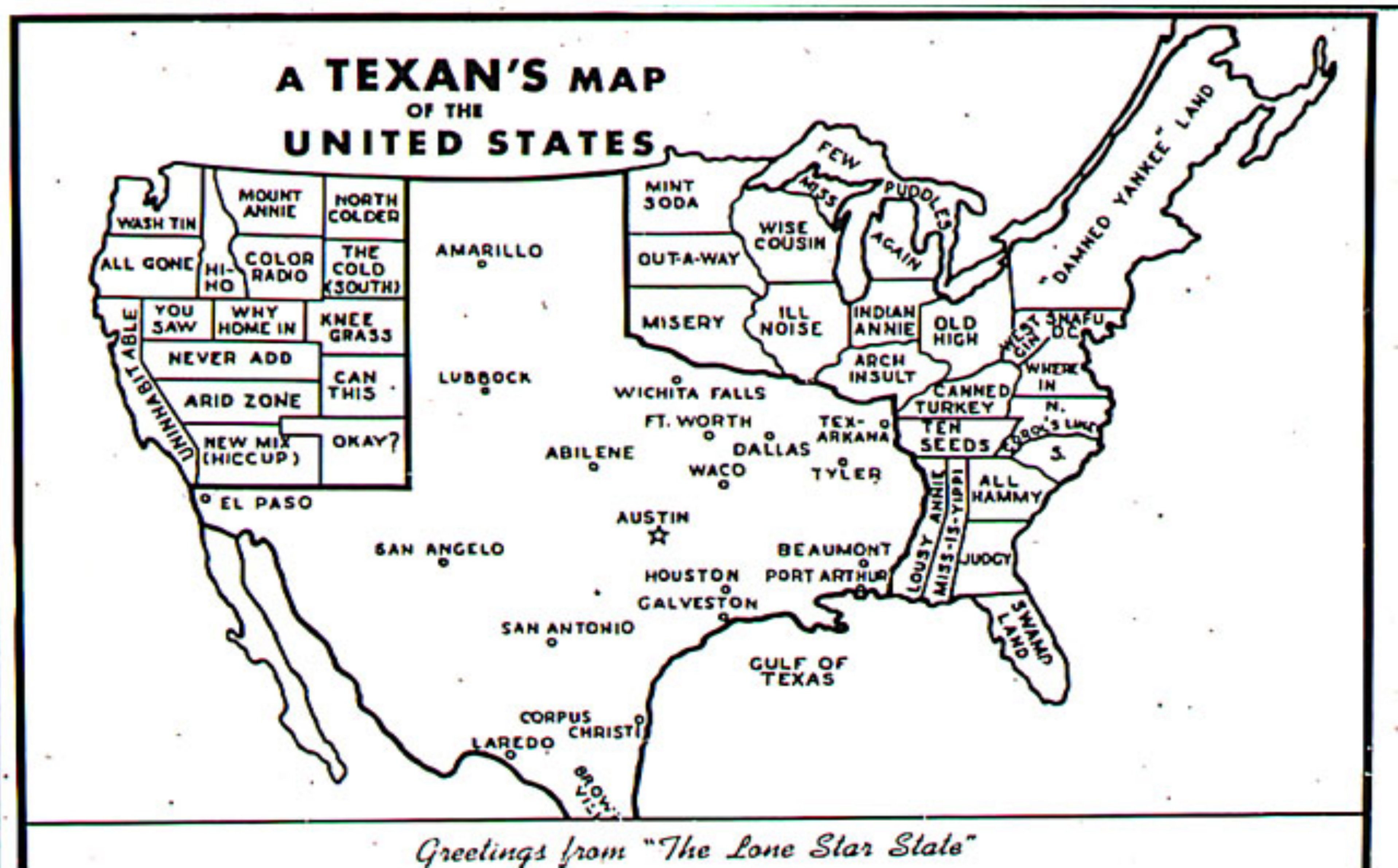
GRUNDY'S MAP OF THE UNITED STATES



Joseph R. Grundy, Pennsylvania manufacturer, suggested to the Senate lobby committee that the present equal power of States in voting on tariff bills is unfair because of differences in voting strength. Here's a map of the United States showing the size of each State on the basis of population and Federal taxes.

Every state seems to have at least one of these.

Often as a postcard, in color, and "humorous"



The next map is “azimuthal”, in the sense that it focuses on a particular location and has a ‘fish eye lens’ effect.

Think of rotating a globe
until your home town is in the center in front of you.

Now take a picture of the globe in this position, thus simulating a map of the world centered on this location.

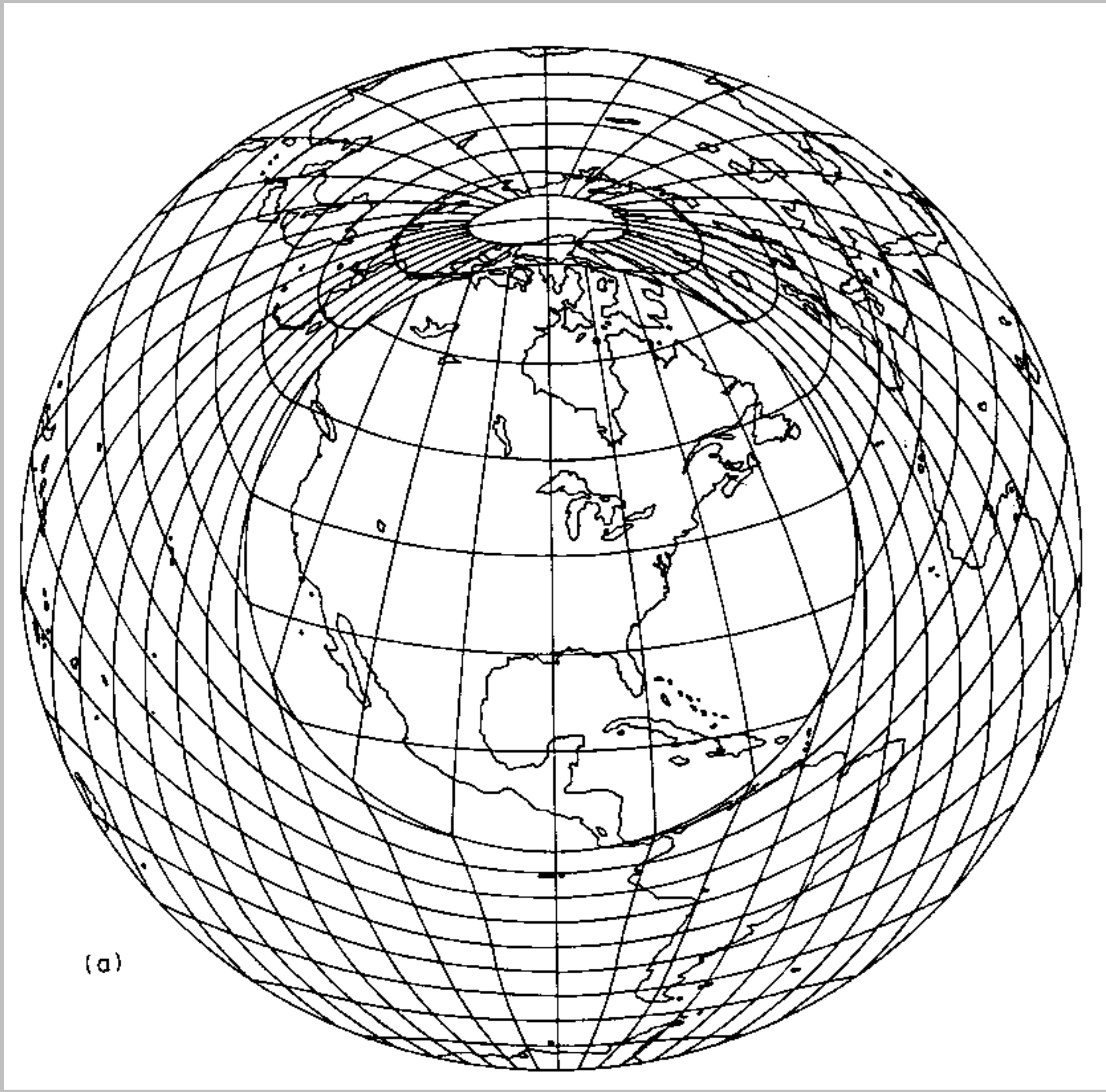
On such a map directions from the center are correct.

The distances from the center can be stretched.

Such maps are easy to make and are quite common.

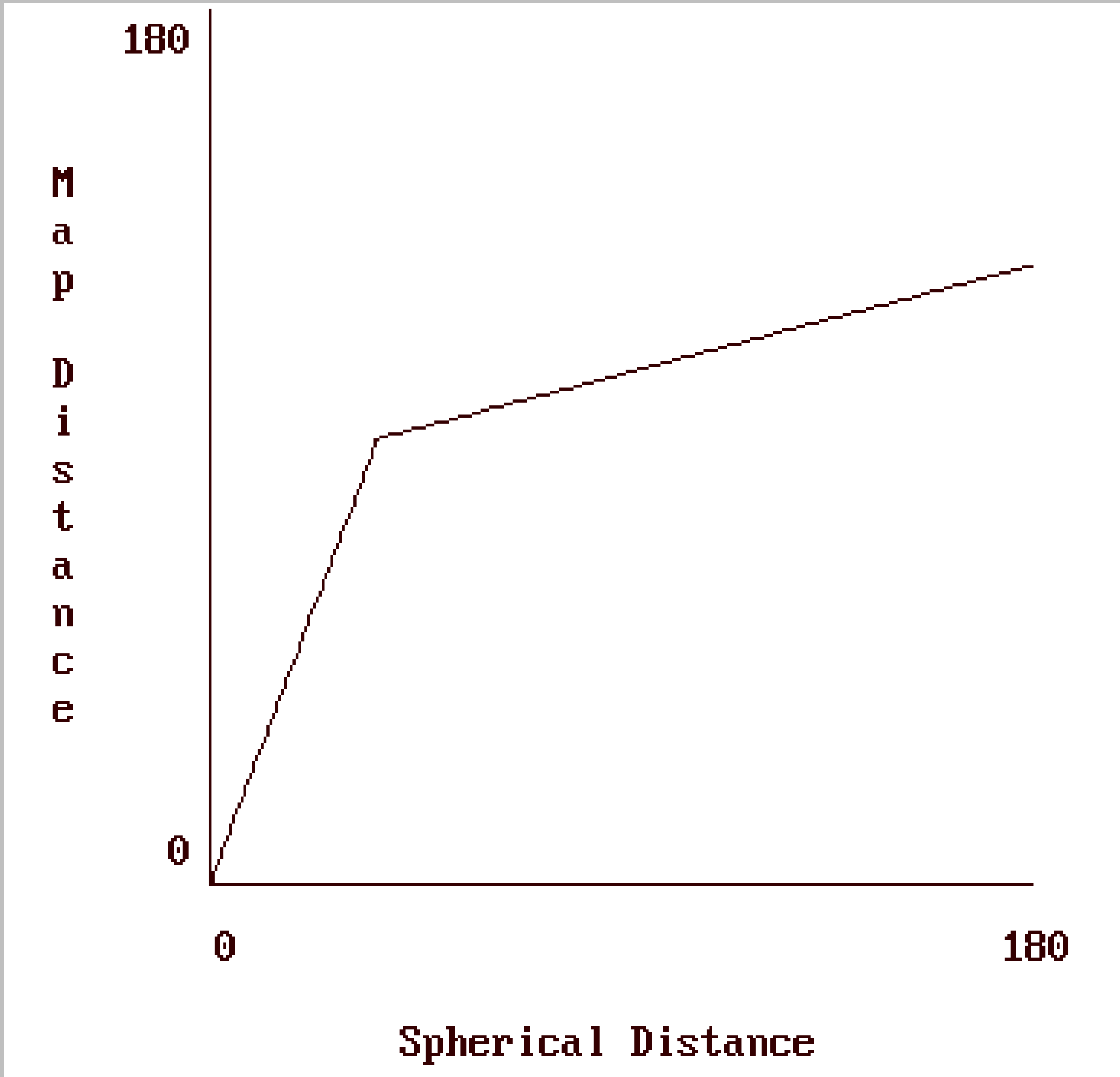
Snyder's Magnifying Glass Projection

J.P.Snyder, 1987, "Magnifying Glass Azimuthal Map Projection", *Am. Cartographer,* 14(1):61-68



Snyder’s Magnifying Glass Projection

In the radial function display, with two scales and a discontinuity.



In studying migration about the Swedish city of Asby, Hägerstrand used the logarithm of the actual distance as the radial scale.

This enlarges the scale near the center of Asby, where most of the migration takes place.

The map nicely shows the spatial rate of change.

In other words, the distance decay function.

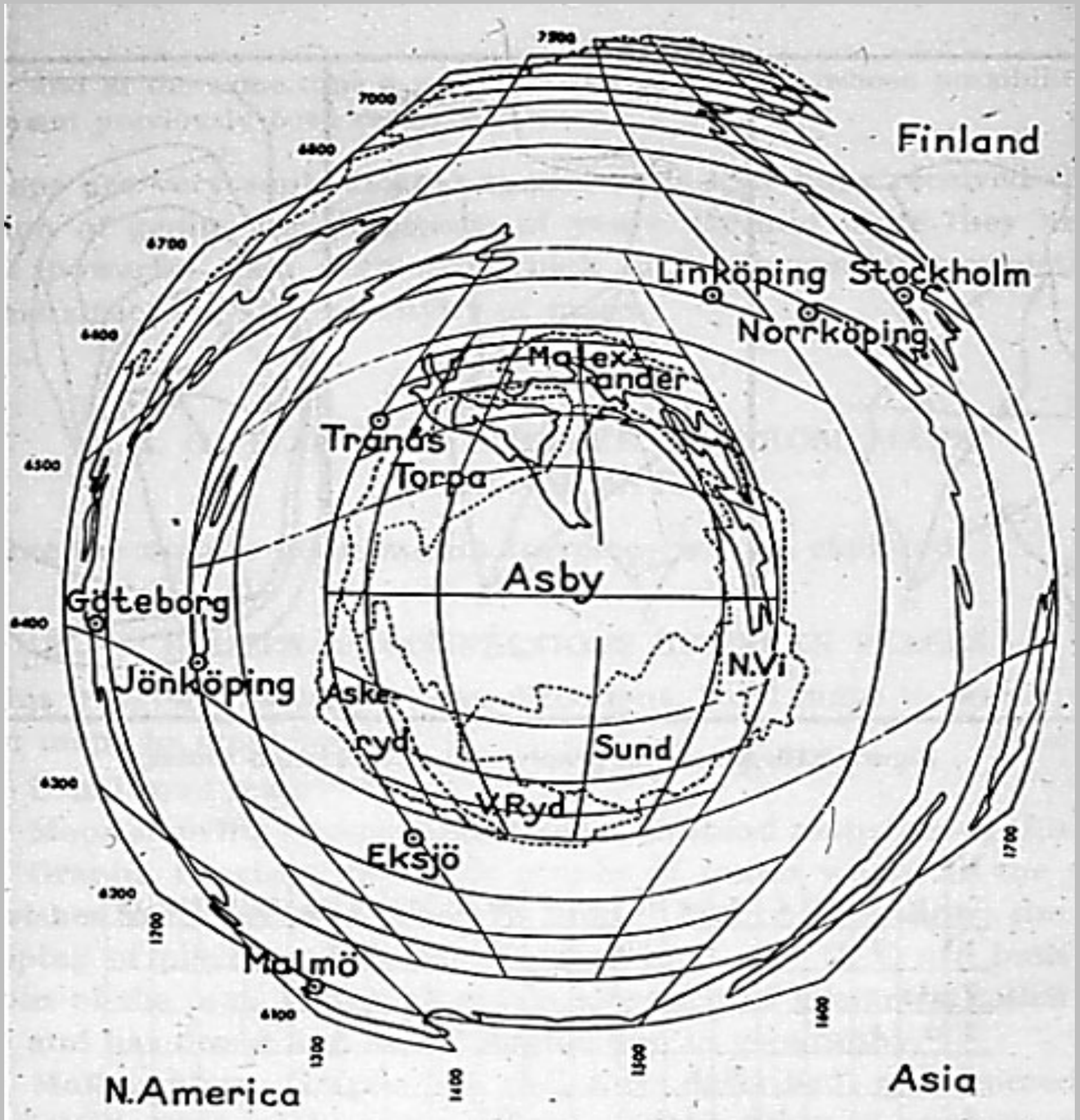
Plotting the migrations on the map allowed users to see, but more importantly to **count,** the instances of interest.

This is another instance of using a map projection as graph paper to solve a problem.

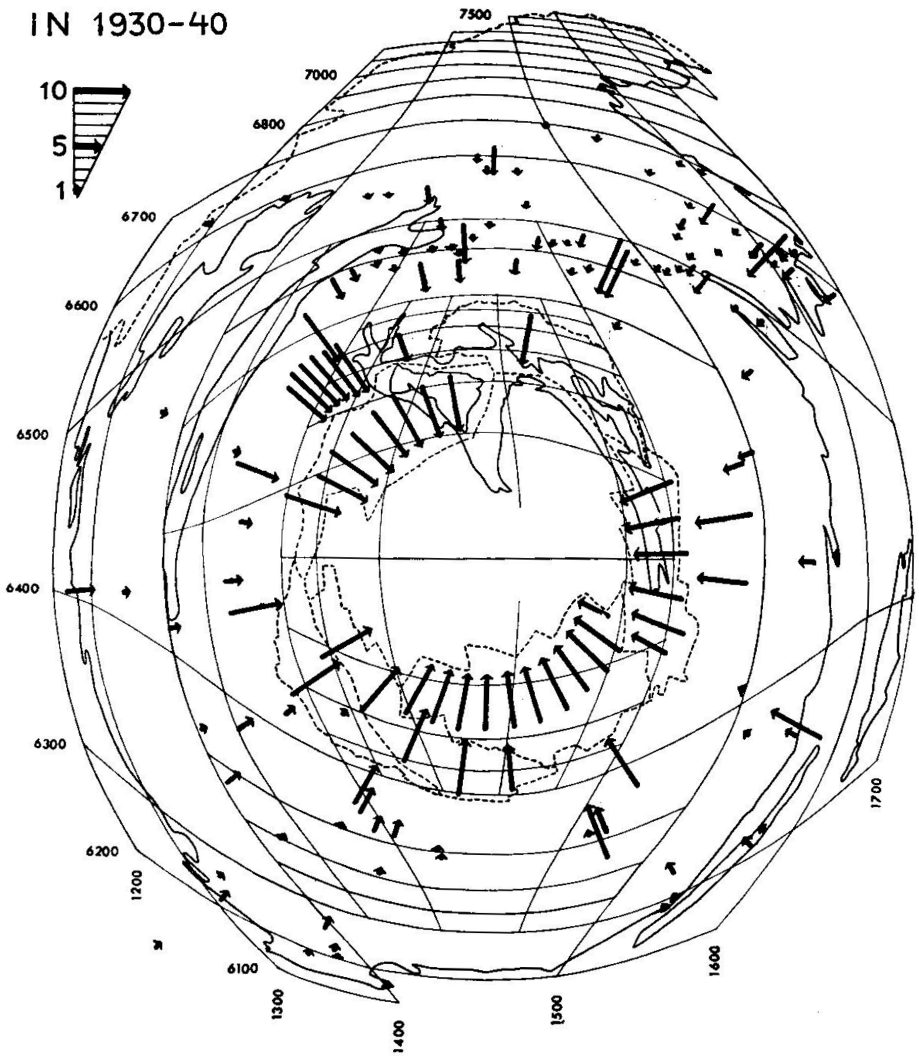
Actually, but not shown, there is a small hole in the middle of the map since the logarithm of zero is minus infinity.

Hägerstrand's Logarithmic Map

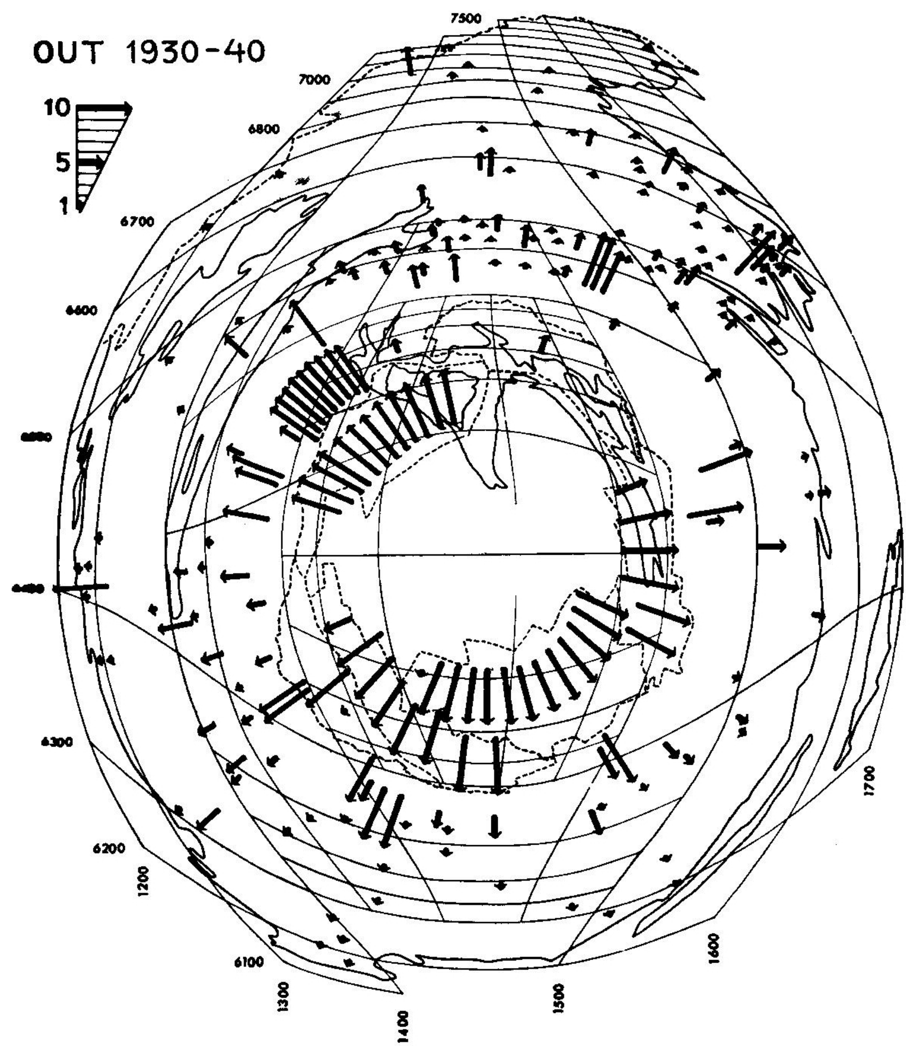
Centered at Asby, Sweden



Hägerstrand's Asby maps, with data



In-migration to Asby from 1930 up to 1940.



Out-migration from Asby from 1930 up to 1940.

The Ultimate Azimuthal "Fish-Eye" Projections

ρ is spherical distance. θ is direction.

Using quarter circles.

The myopic view

$$r = (\pi\rho/2 - (\rho/2)^2)^{1/2}$$

$$X = r \sin\theta,\ Y = r \cos\theta$$

The Anti-Myopic view

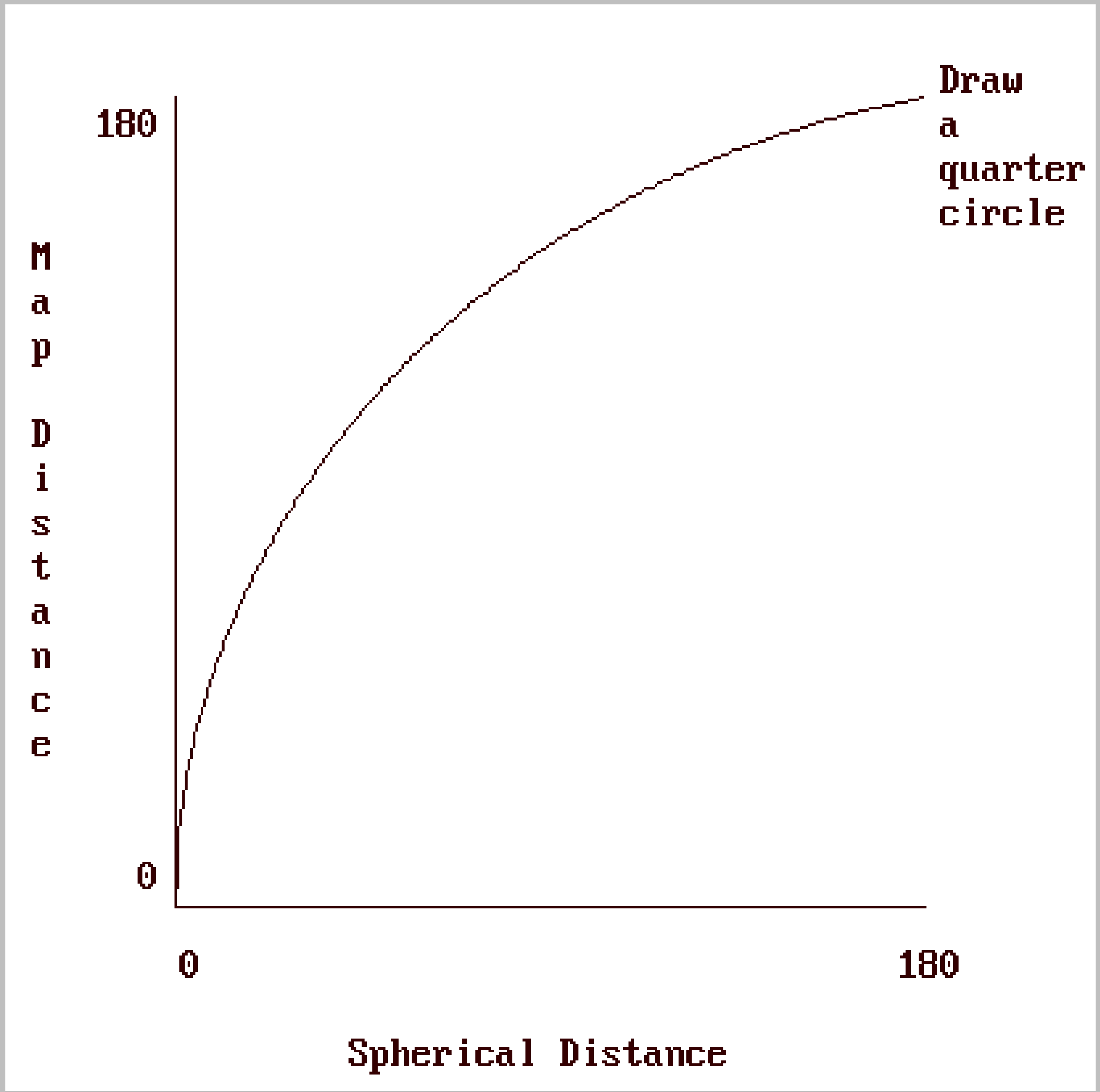
$$r = \tfrac{1}{2}\pi - (\pi^2/4 - (\rho/2)^2)^{1/2}$$

$$X = r \sin\theta,\ Y = r \cos\theta$$

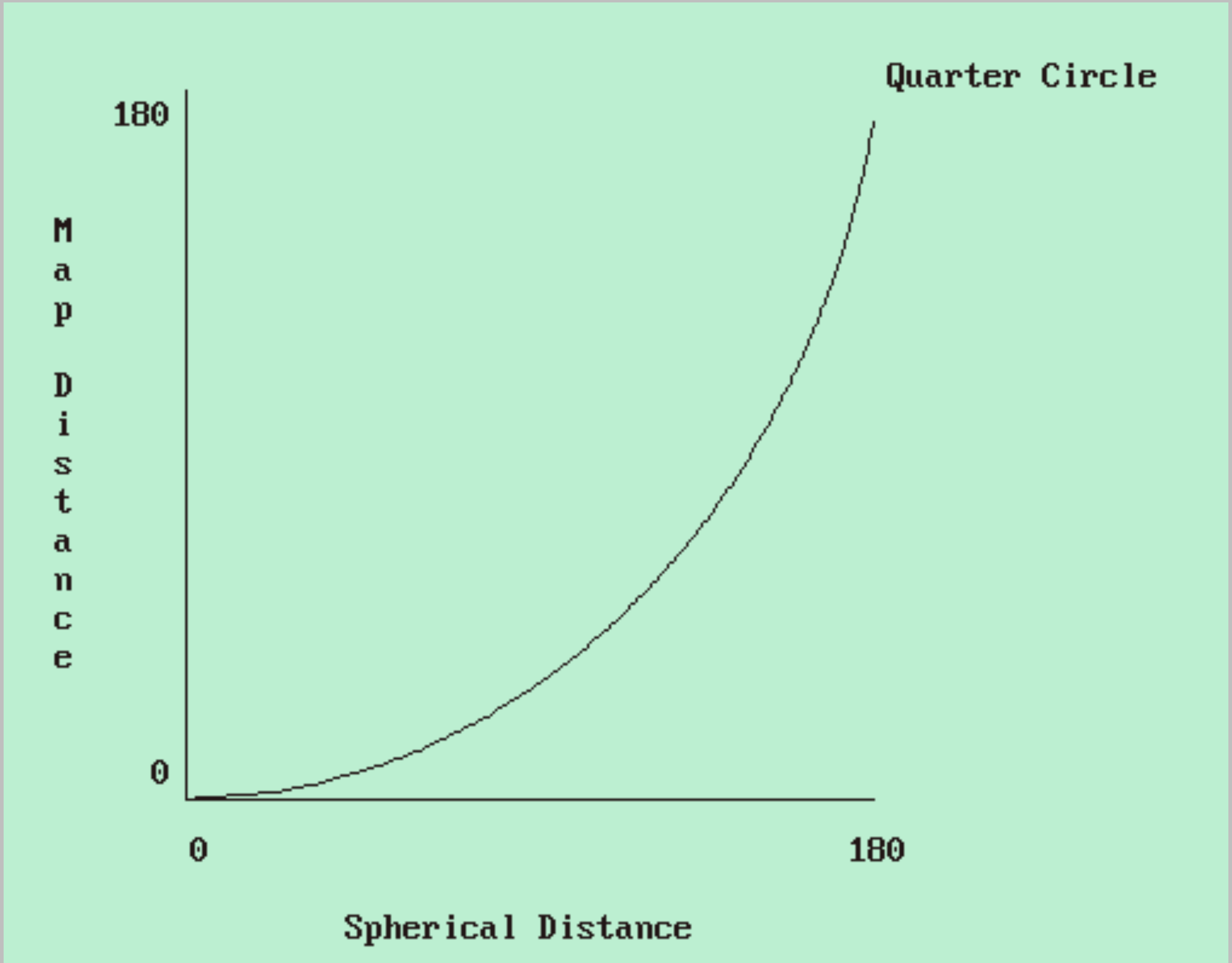
Another possibility: use half an ellipse instead of quarter circles,

Draw Your Own

I've drawn a quarter circle, but you can invent your own azimuthal map projection. This one encourages myopia.



Or reverse the effect to combat myopia.



Experiment with this type of map.

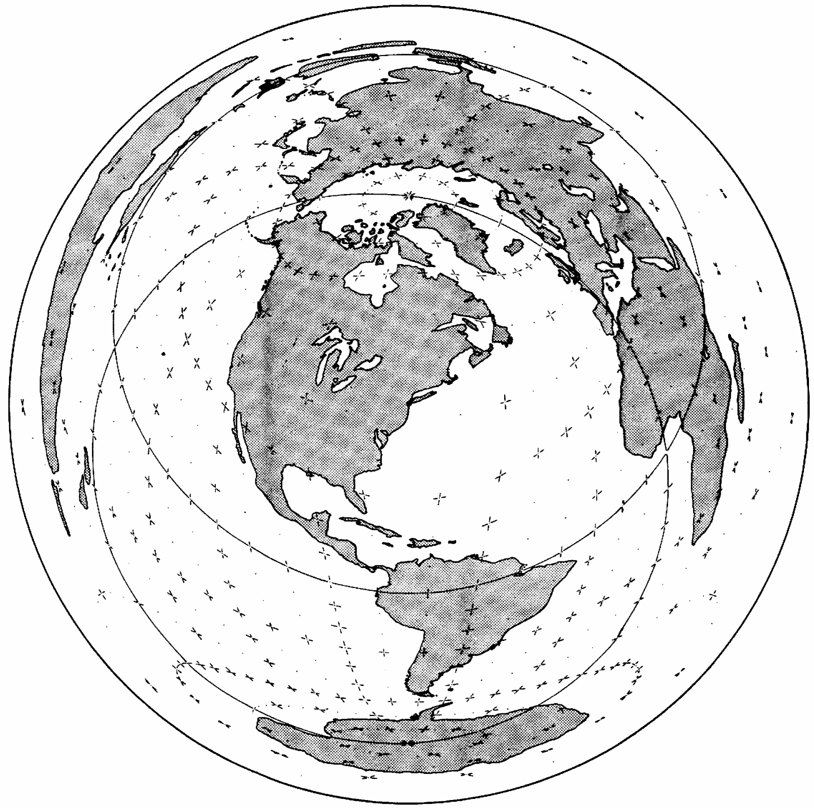
Can you see how to get an azimuthal map with the following property:

Show the center and the periphery in reduced size,

and with the middle portion enlarged?

Or the opposite effect?

SQUARE - ROOT PROJECTION



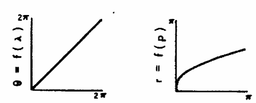
An Azimuthal Projection of the World

$$\left(\theta = \lambda^{*} \quad r = \sqrt{\rho^{*}} \right)$$

CENTERED ON NEW YORK CITY

(40° 45' N, 74° W)

DIAGRAMMATIC REPRESENTATION OF THE EQUATIONS FOR THIS PROJECTION

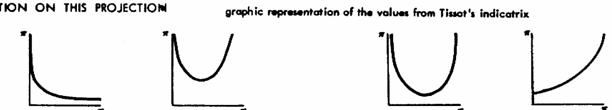


Azimuth Equation   Radius Equation

The scale of these diagrams corresponds to a globe of one radian. The map scale can also be obtained from this value for any point on the map or can be obtained from the graphs of the distortions.

DISTORTION ON THIS PROJECTION

graphic representation of the values from Tissot's indicatrix

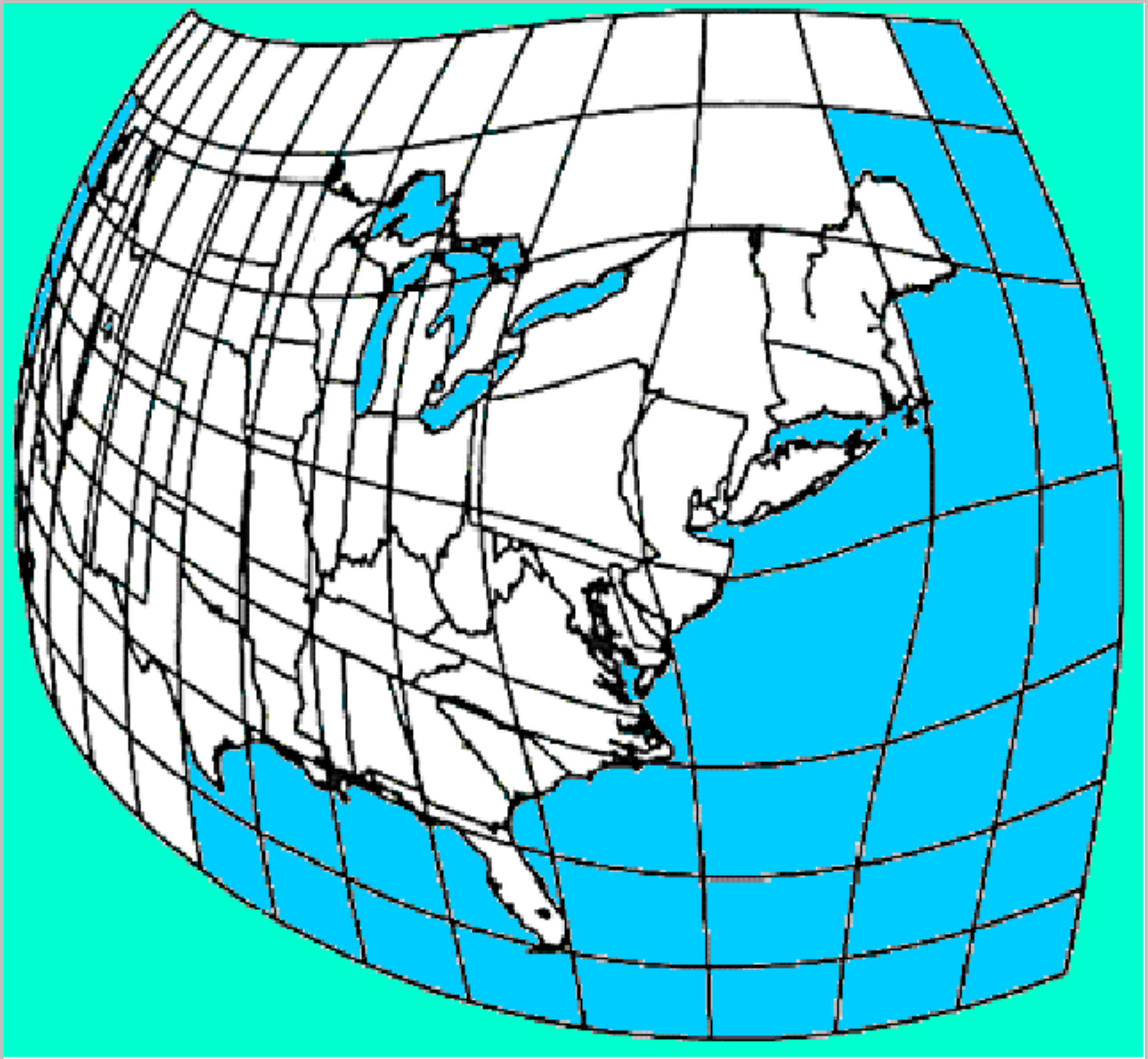


Radial   Circumferential   Areal   Angular

Cartographic Laboratories, University of Washington, Seattle   Tobler

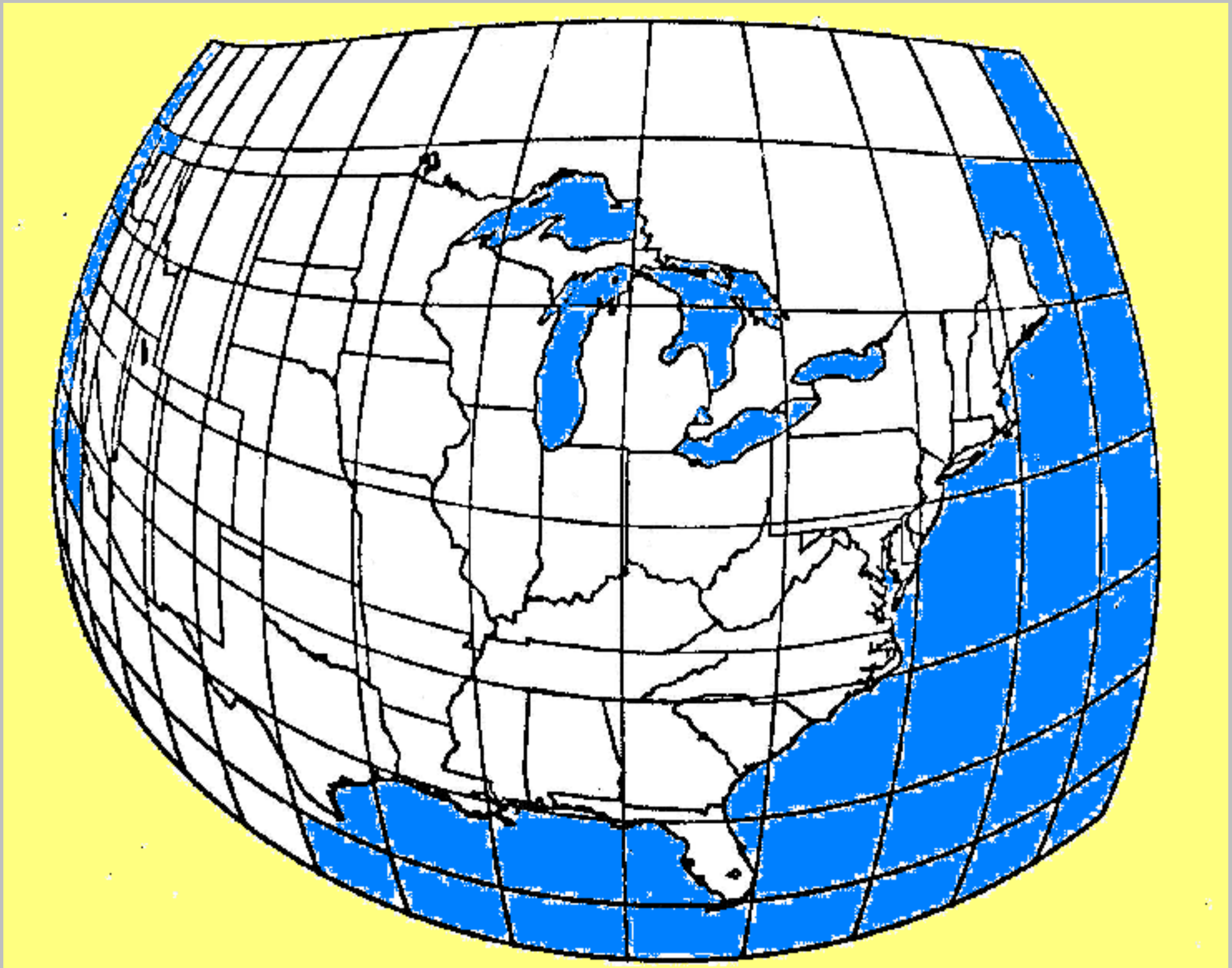
A New Yorker's View

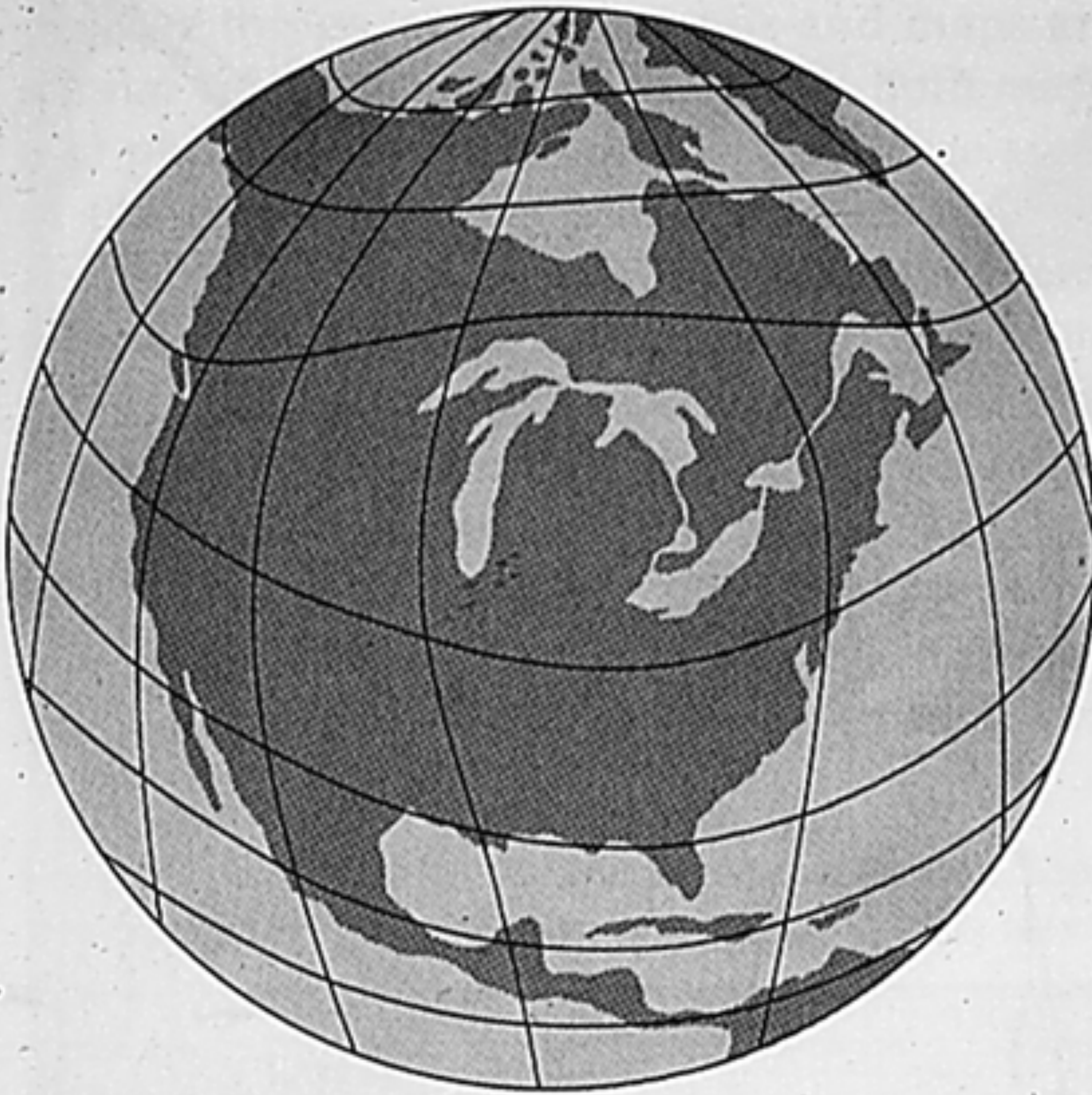
Square root azimuthal projection, with obvious distortion



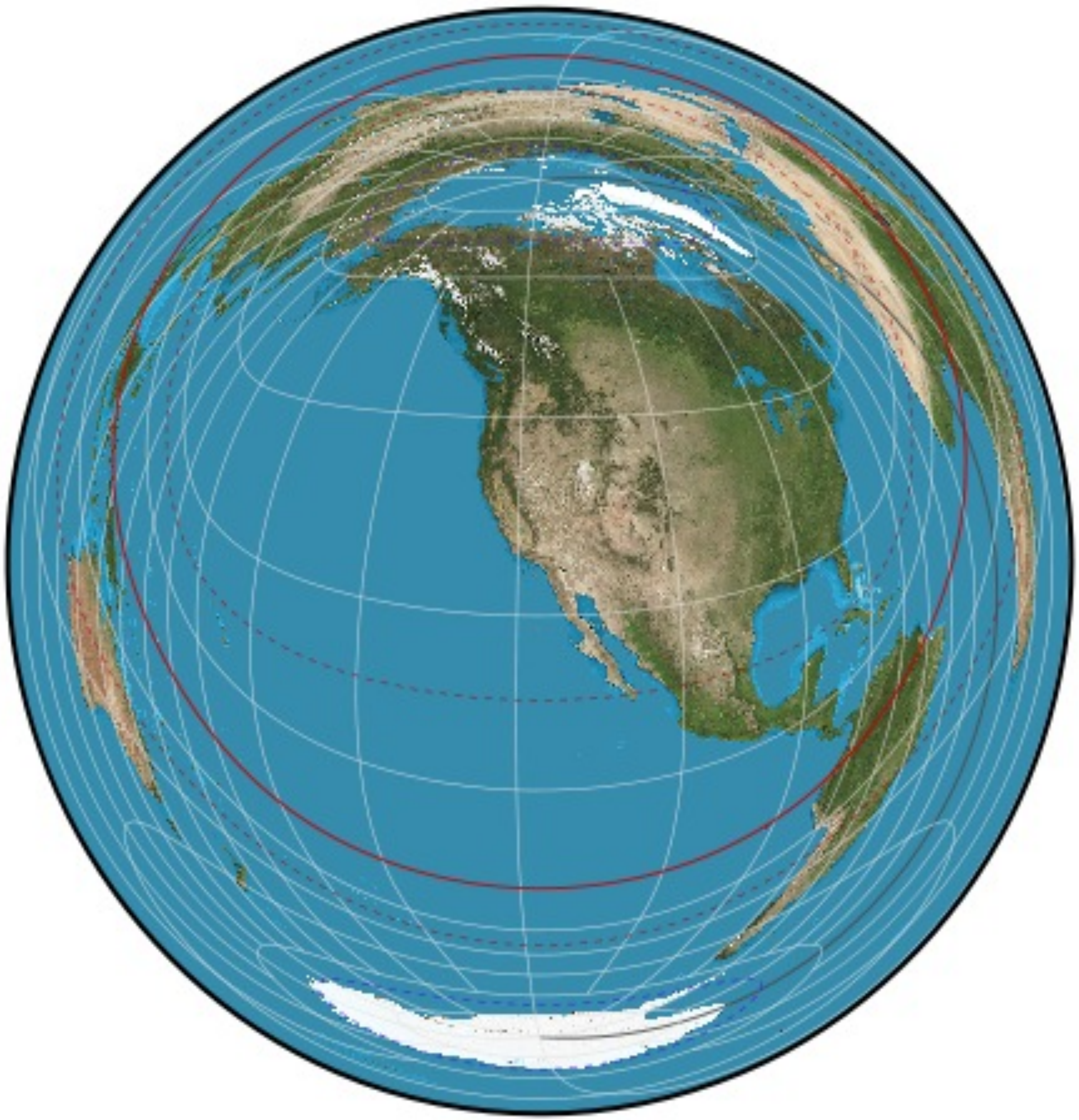
The View From Michigan

with less obvious distortion





Centered at Santa Barbara, California

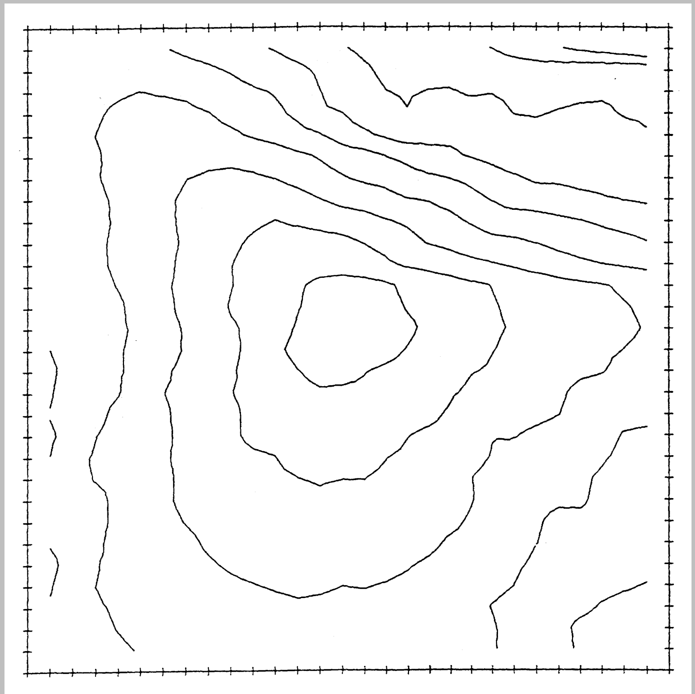


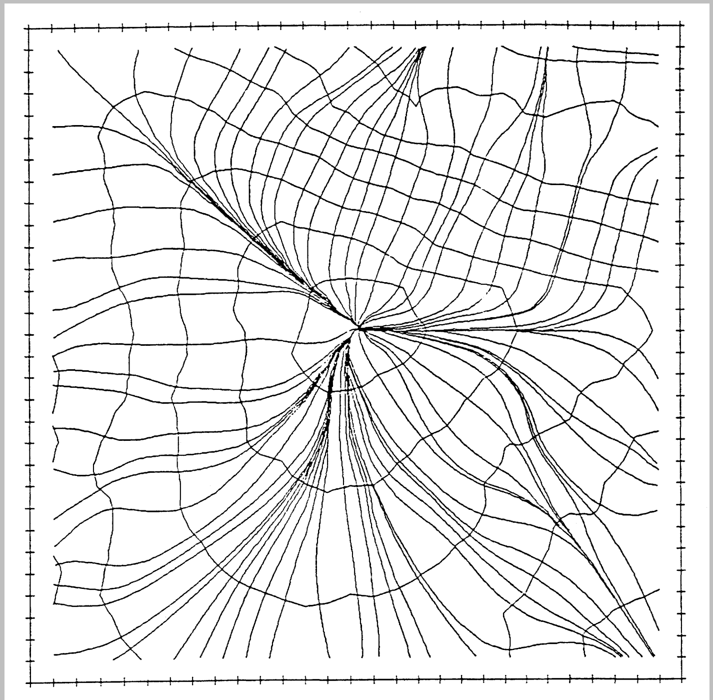
A variant of this type of map uses isochrones: travel times, or costs, from a center.

Travel times can be warped into circles.

Use the orthogonals to the isolines as the directions.

This gives one a map in polar coordinates.

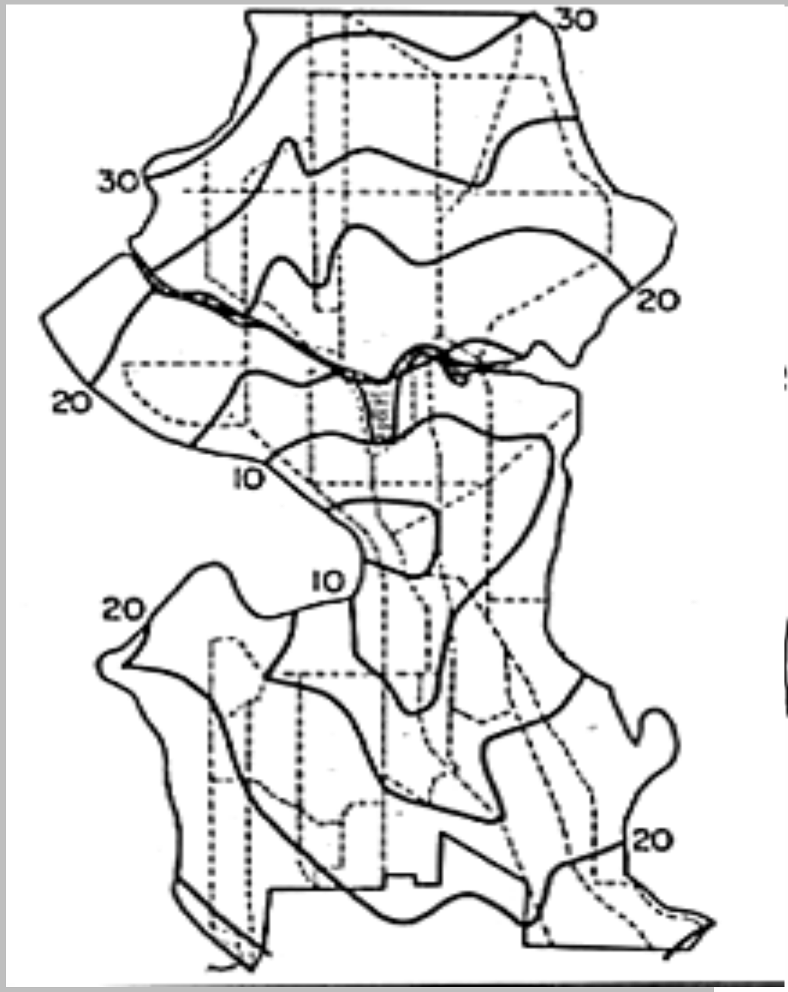


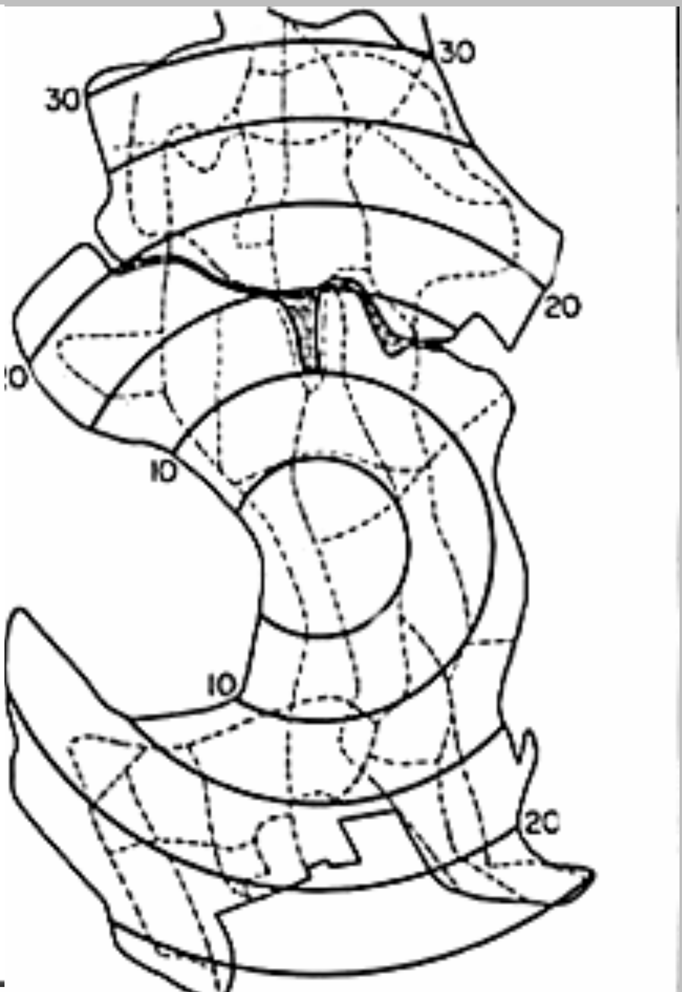


Travel time from center of Seattle

as isochrones and then circles on the warped map.

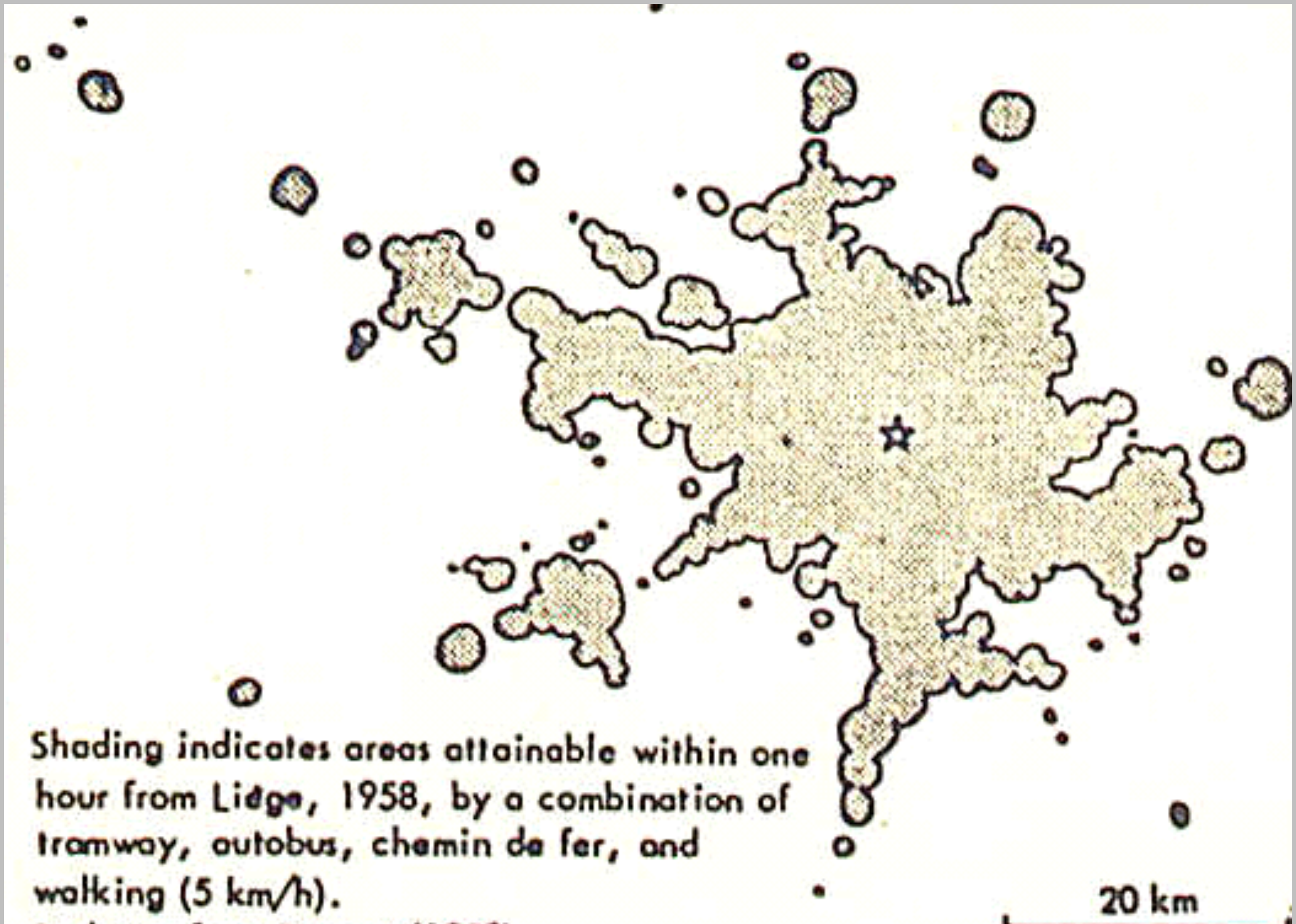
after W. Bunge





But this can get complicated,

as shown by a one hour distance circle around Liege



Shading indicates areas attainable within one hour from Liège, 1958, by a combination of tramway, autobus, chemin de fer, and walking (5 km/h).
Redrawn from Dussart (1959).

Parcel post rates in the United States are broken into distance zones.

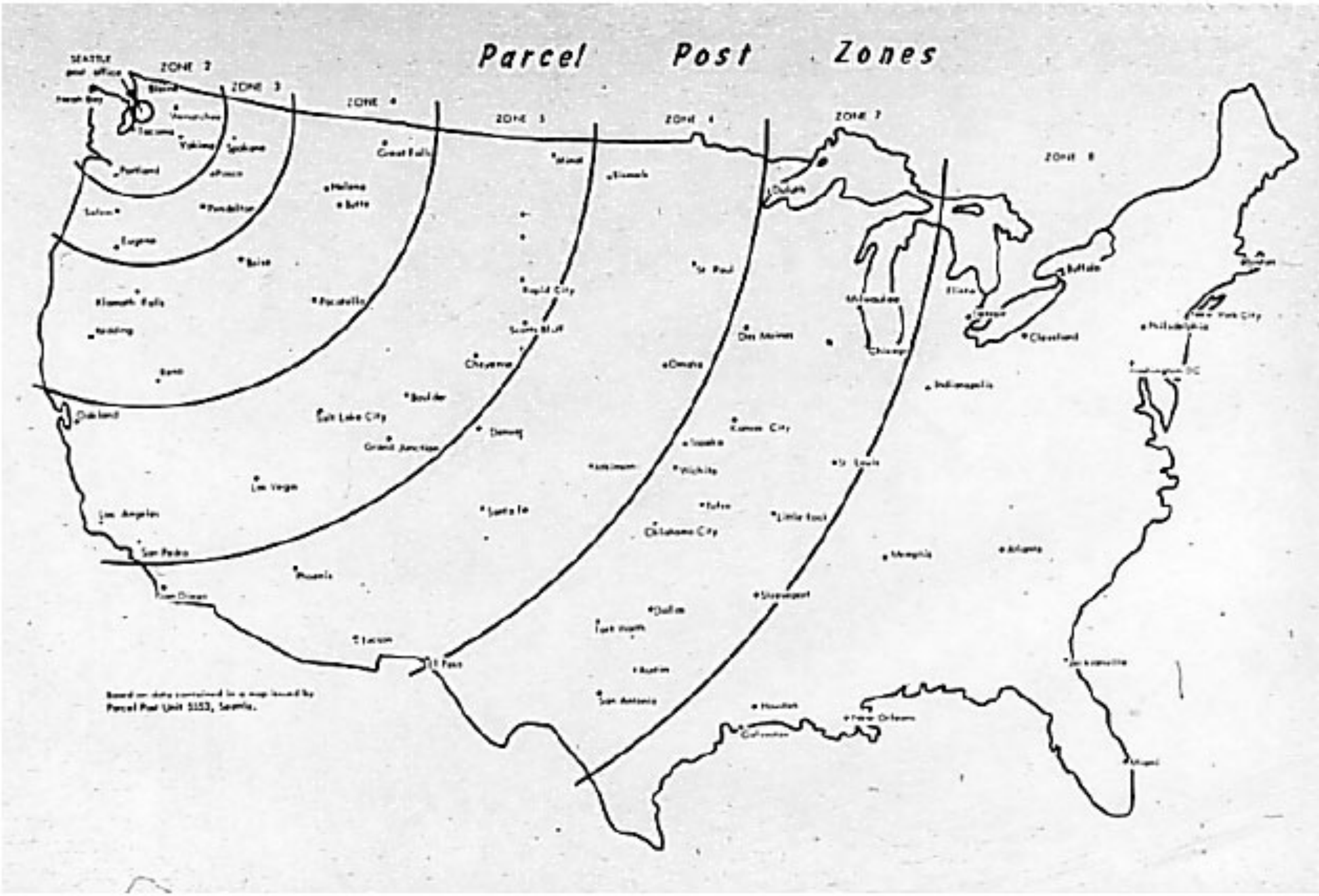
The rate increases with distance at a decreasing rate.

This is a common downward convex tariff structure in transportation

The largest cost jump is in the first zone.

Within each zone the rate is constant.

Parcel postage rate classes from Seattle



The next map is an azimuthal map projection with all places shown at their correct direction from Seattle.

But the distance scale is in parcel postage cents.

Look at the radial distance function in the lower right hand corner of the map, where

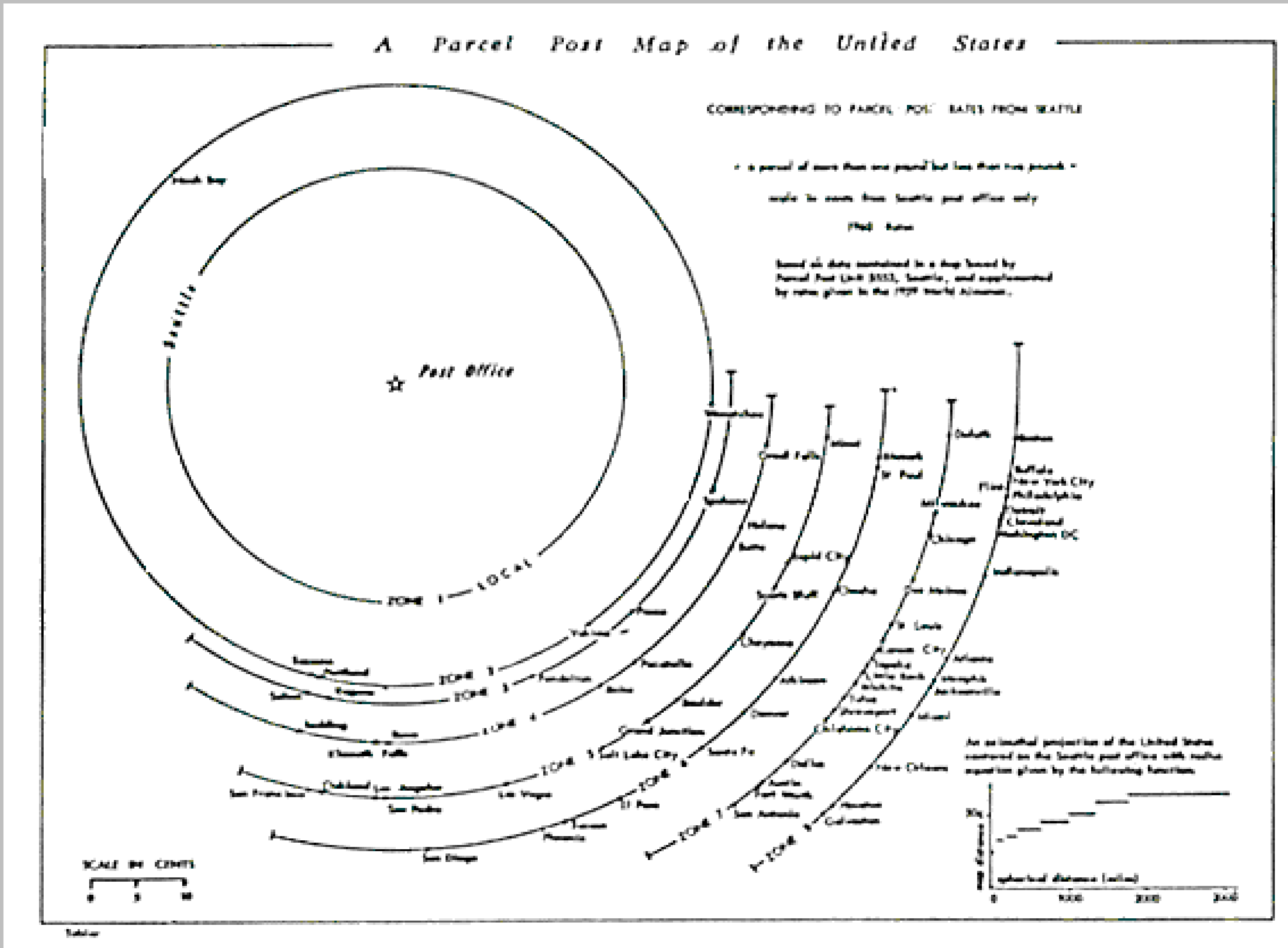
the x-axis is the spherical distance from Seattle, and the

y-axis is the cost in dollars to send a one pound package within the United States as a function of distance.

The graph shows a step function and this collapses much of the geographic space. The places are on the arcs!

Spherical directions from Seattle are correct.

Parcel Post View From Seattle



Can other convex down transport systems be represented cartographically?

Consider the class of azimuthal map projections with radial equations

$$r = (\rho)^{1/n},\ n > 1,\ \text{where}$$

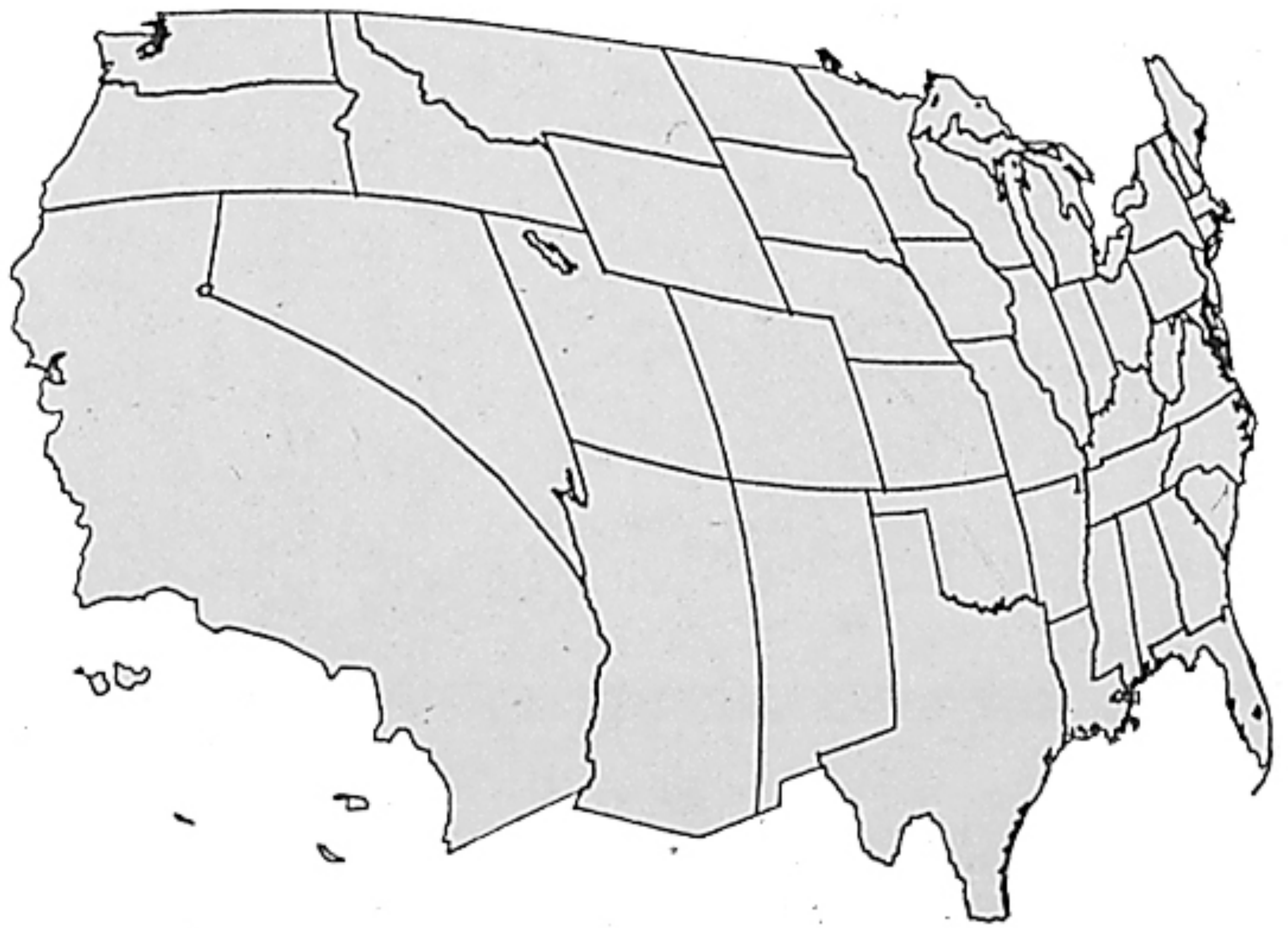
ρ is the spherical distance and r is the map distance.

Can this be related to geographical distance decay?

Consider another map with Santa Barbara as center.

The Santa Barbaran View

A cube root distance azimuthal projection



Consider first maps with correct distances from the center

The azimuthal equidistant projection comes to mind.

Then

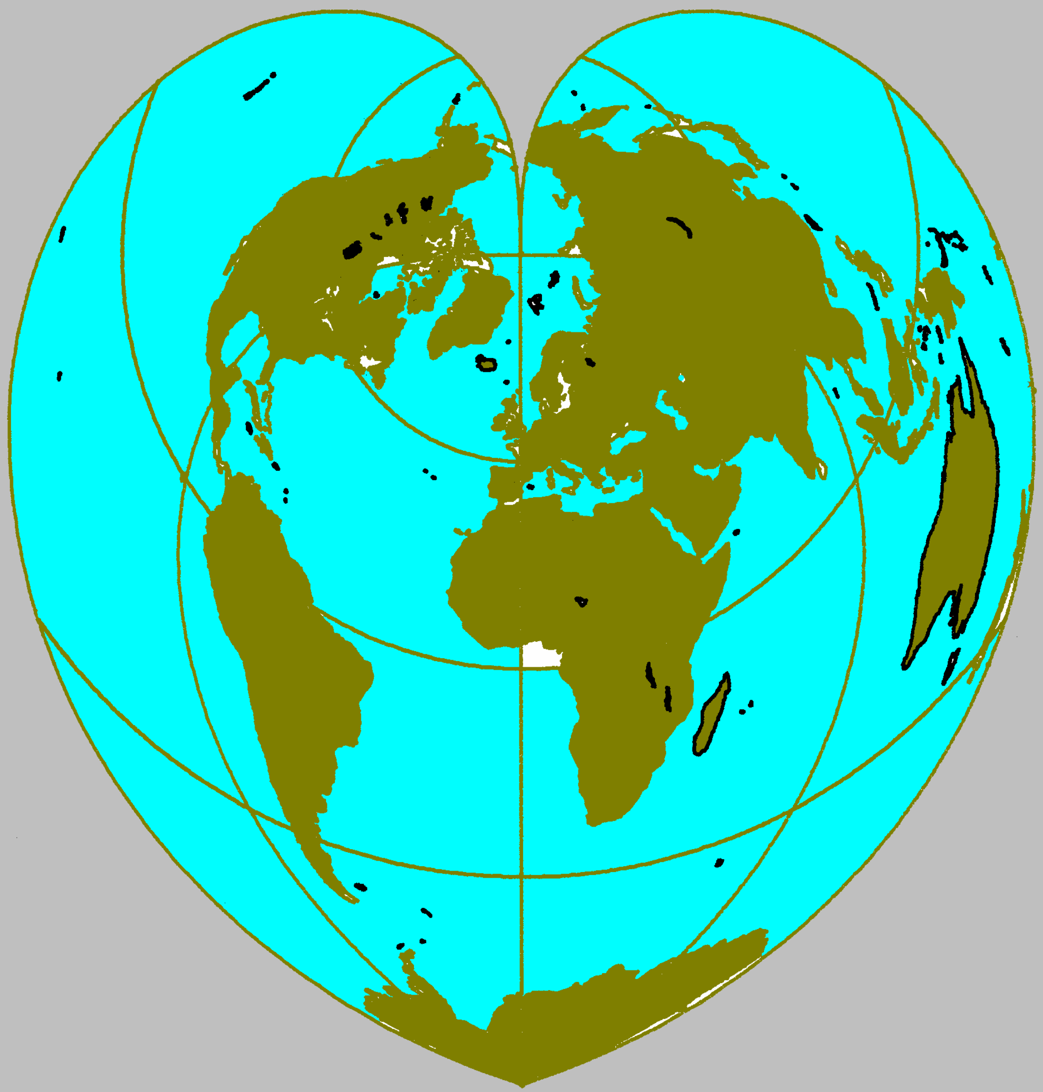
Add the equal area requirement.

To get

Werner's projection.

Werner's Projection, from 1515

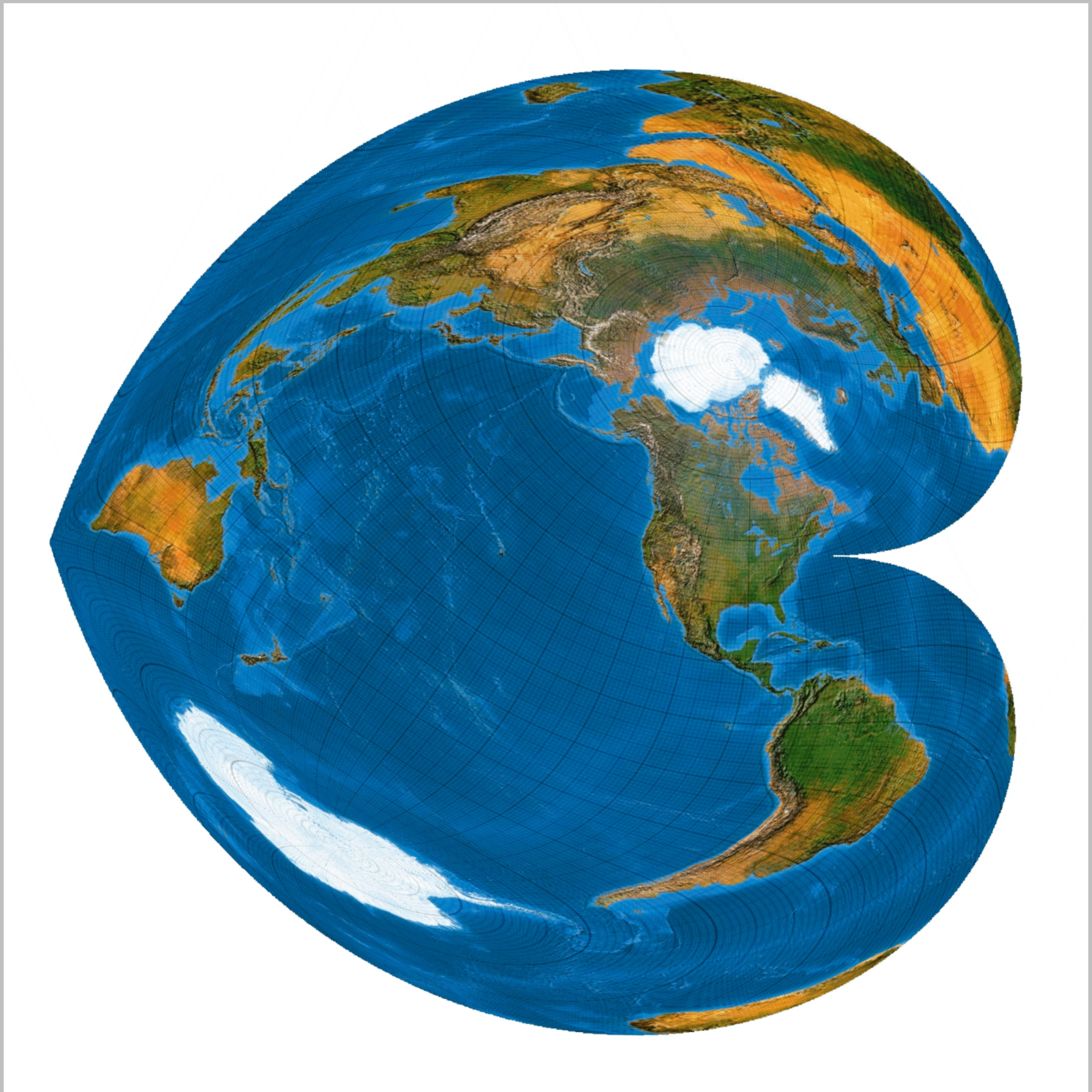
Distances from center (North Pole in this case) are correct and areas are correctly rendered..



Oblique equidistant & equal area Werner projection

Great circle from New York City through Seattle is the horizontal axis.

A rarely seen view, courtesy of R, Böhm.



Can a map with a different distance decay be made equal area?

With a convex down transport function?

Try n = 2, square root distance decay.

Then use

$$X = R(\rho)^{1/2} \sin(\lambda \sin \rho)$$

$$Y = R(\rho)^{1/2} \cos(\lambda \sin \rho)$$

where

R is the assumed earth radius and

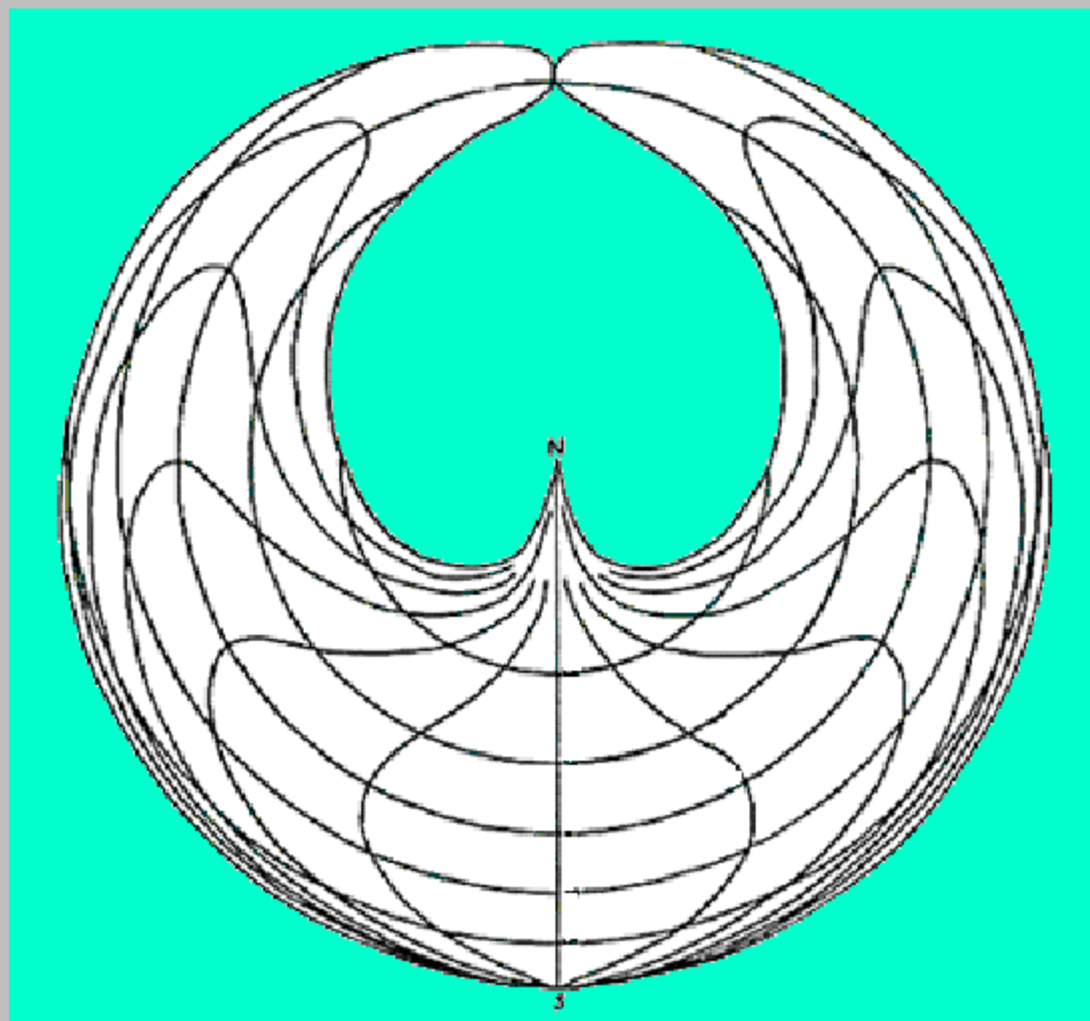
λ is the longitude, ρ is spherical distance.

The result is an equal area map projection with shrinking distances.

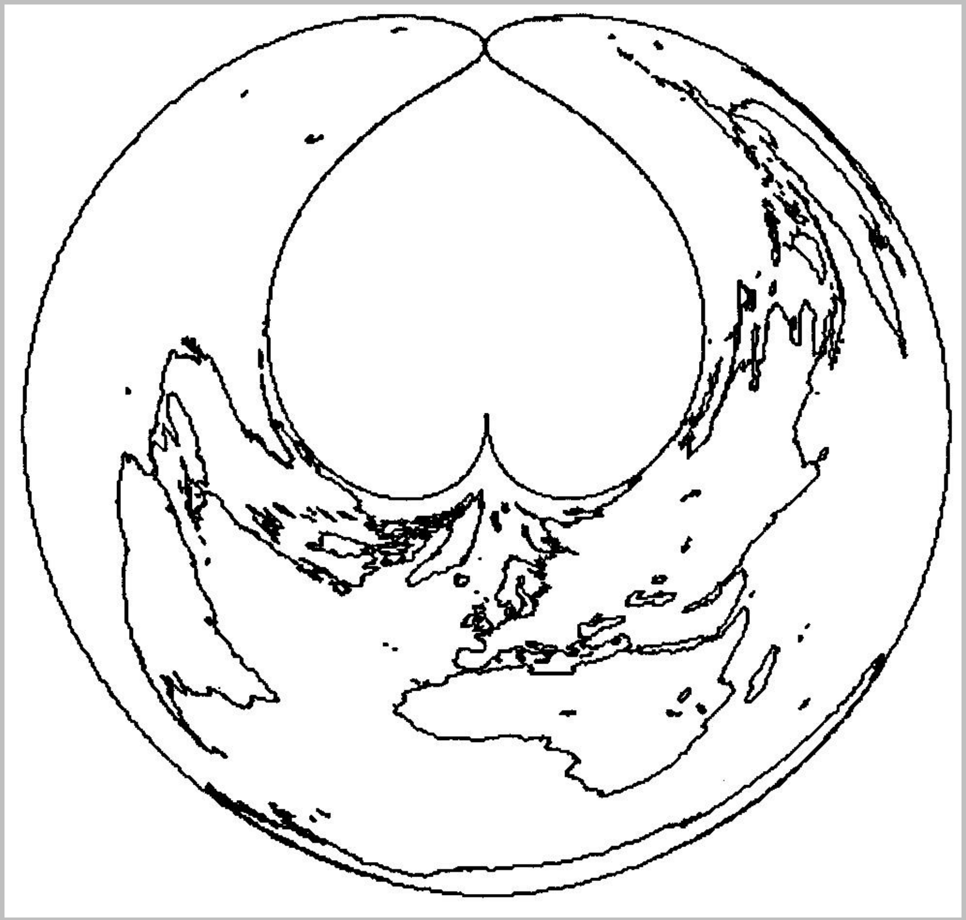
But this map no longer preserves directions (azimuths)!

Equal Area Projection
with Square Root Distances from Center

Polar Case



Here are the coastlines.

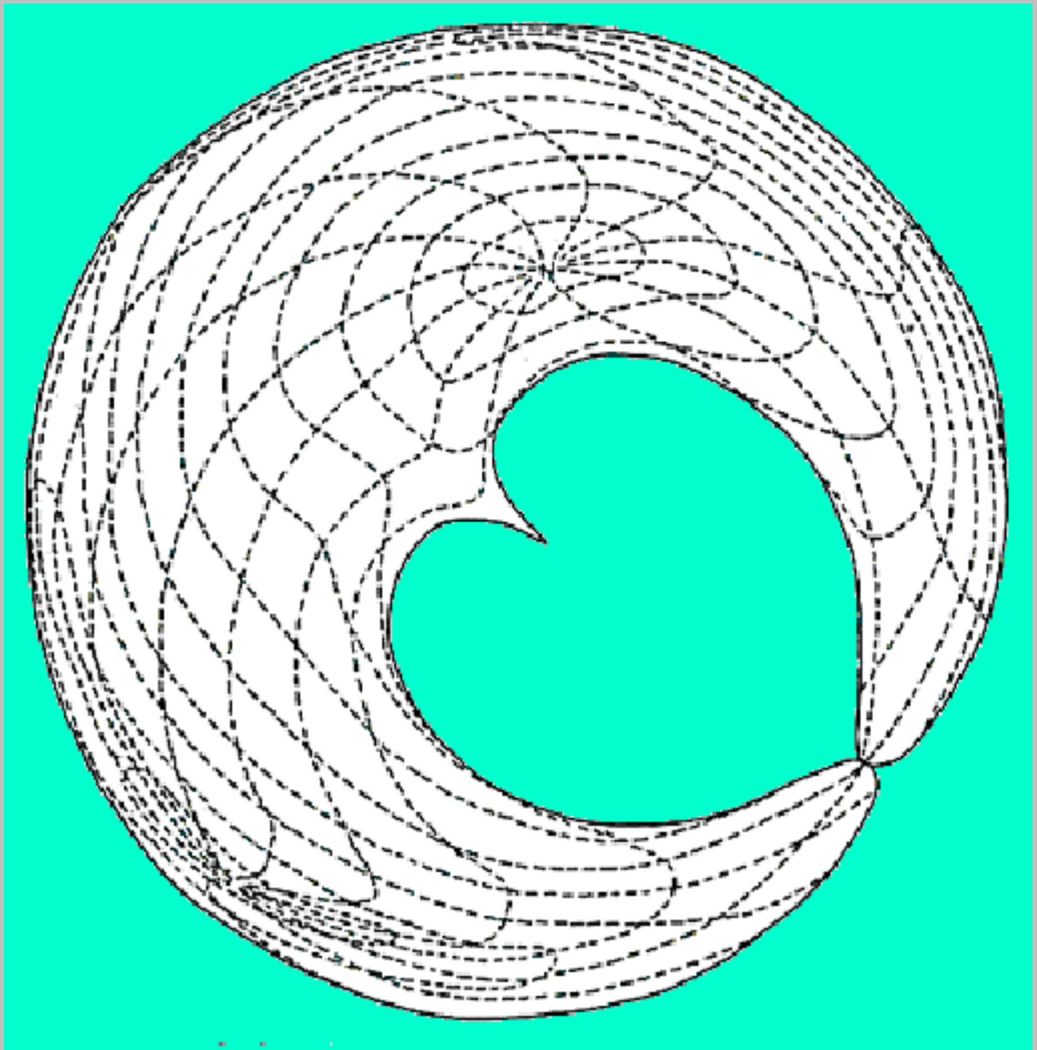


Next is the oblique case of this equal area projection, with the great circle from New York (at the center) passing through Seattle as a straight line.

Oblique Version Of The Previous Projection

Centered on New York and Directed Toward Seattle

North orientation



Mercator's projection is not the only one that can render loxodromes (rhumb lines) as straight lines and correct directions

But it is the only one that does it for all loxodromes.

The projection shown here has all loxodromes **from one location** as straight lines, going in their correct direction and with correct length.

This can be considered an azimuthal loxodromic projection.

The present map is focused on 45 degrees north latitude and the Greenwich meridian, near Le Havre.

When centered on the equator the north and south hemispheres are symmetrical.

When centered at the pole this becomes the azimuthal equidistant projection.

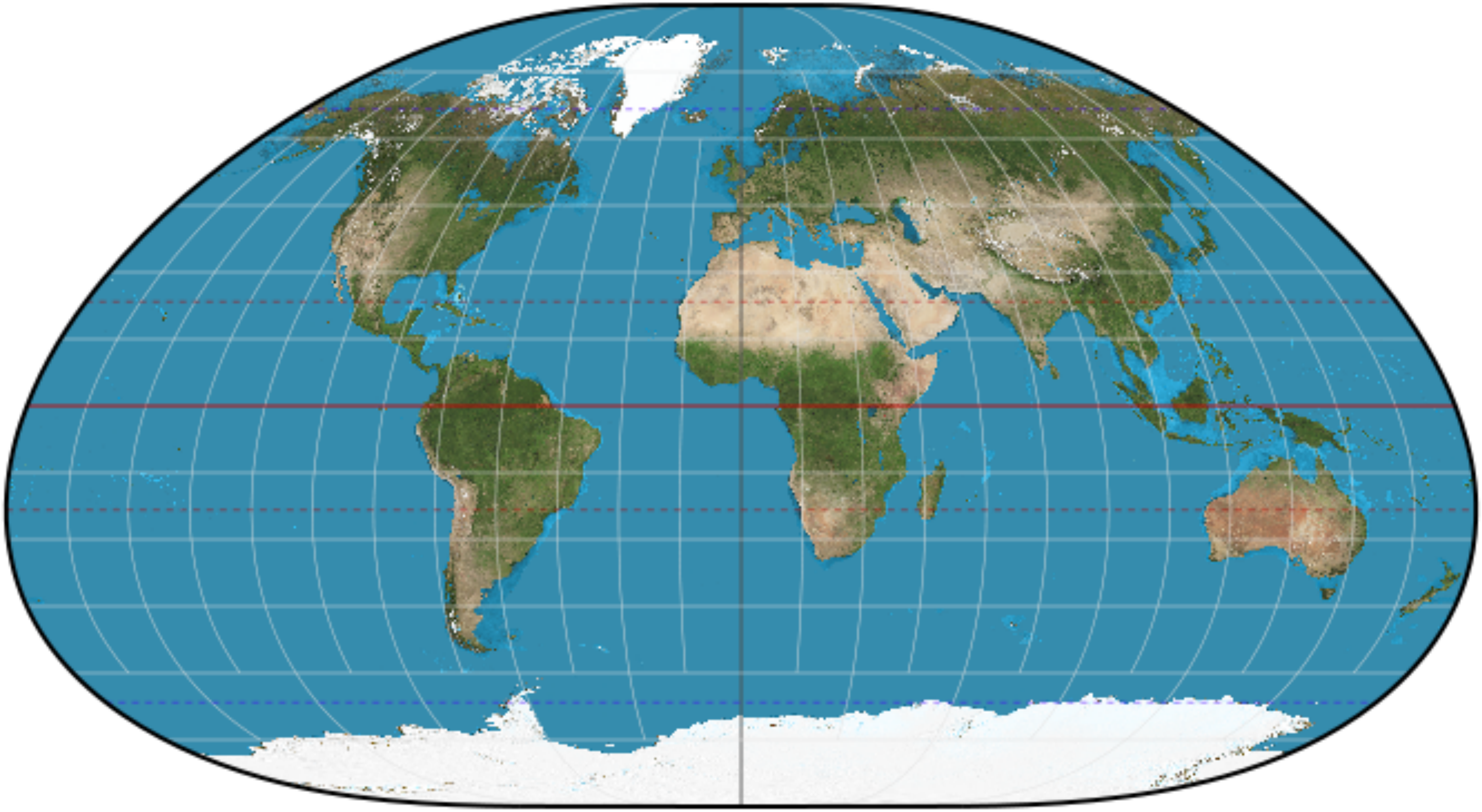
W. Tobler, 1966, "Notes on Two Projections: Loximuthal and Two Point Equidistant",

The Cartographic Journal, 87-89.

As an exercise, consider the possibility of a two-point loximuthal projection.

The Loximuthal Projection

Centered off the west coast of France, at 45° N & 0° E



As you know, the even numbered highways of the US interstate system run East-West and the odd numbered ones run North-South.

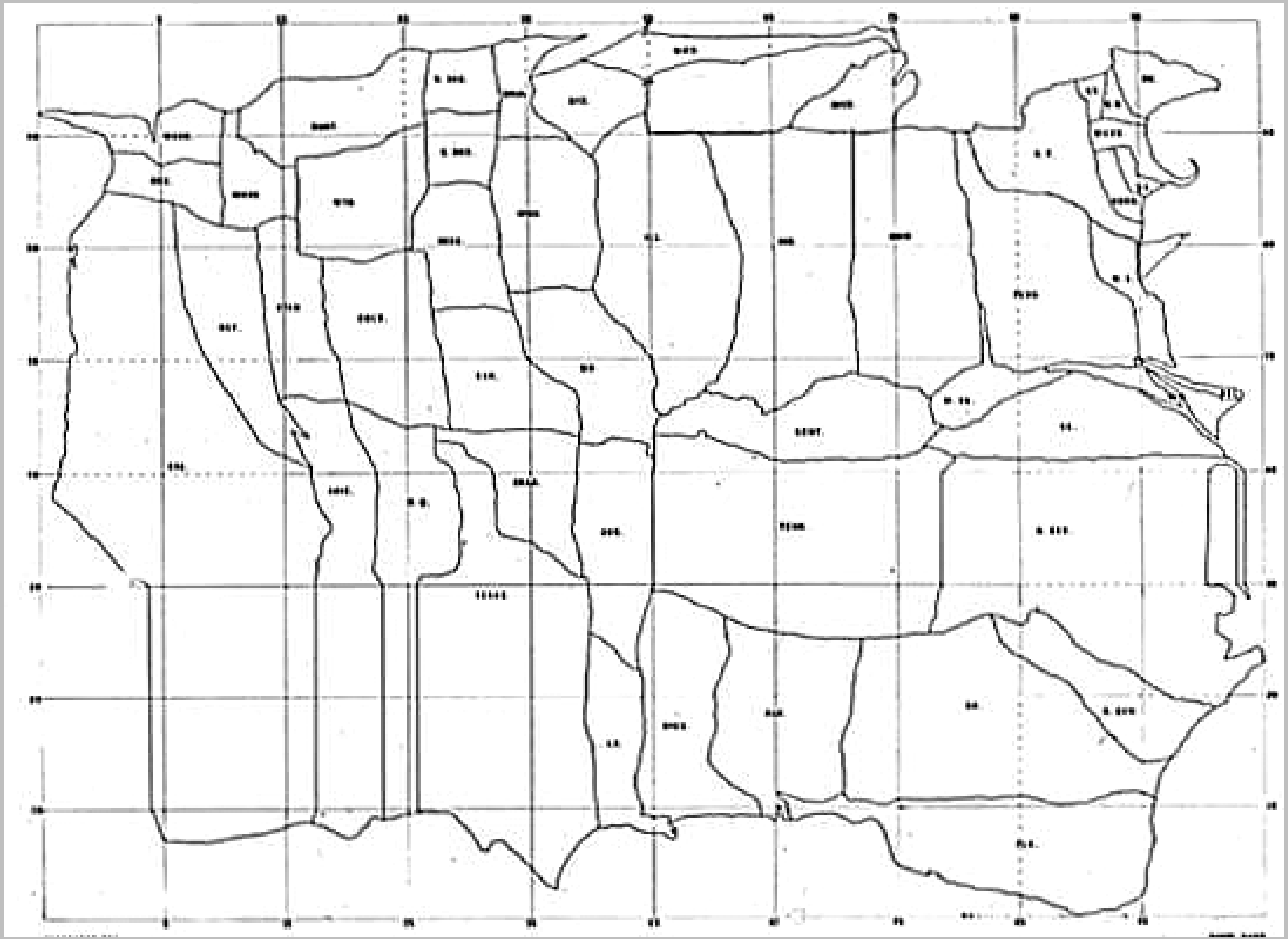
Therefore the interstate highway system can be drawn as orthogonal equidistant lines on a map and the meridians and parallels bent to fit this. Try it!

The result will look like the next map.

This map could be analyzed using Tissot's method.

US Highway Coordinates

Student Drawing. Use it to travel?



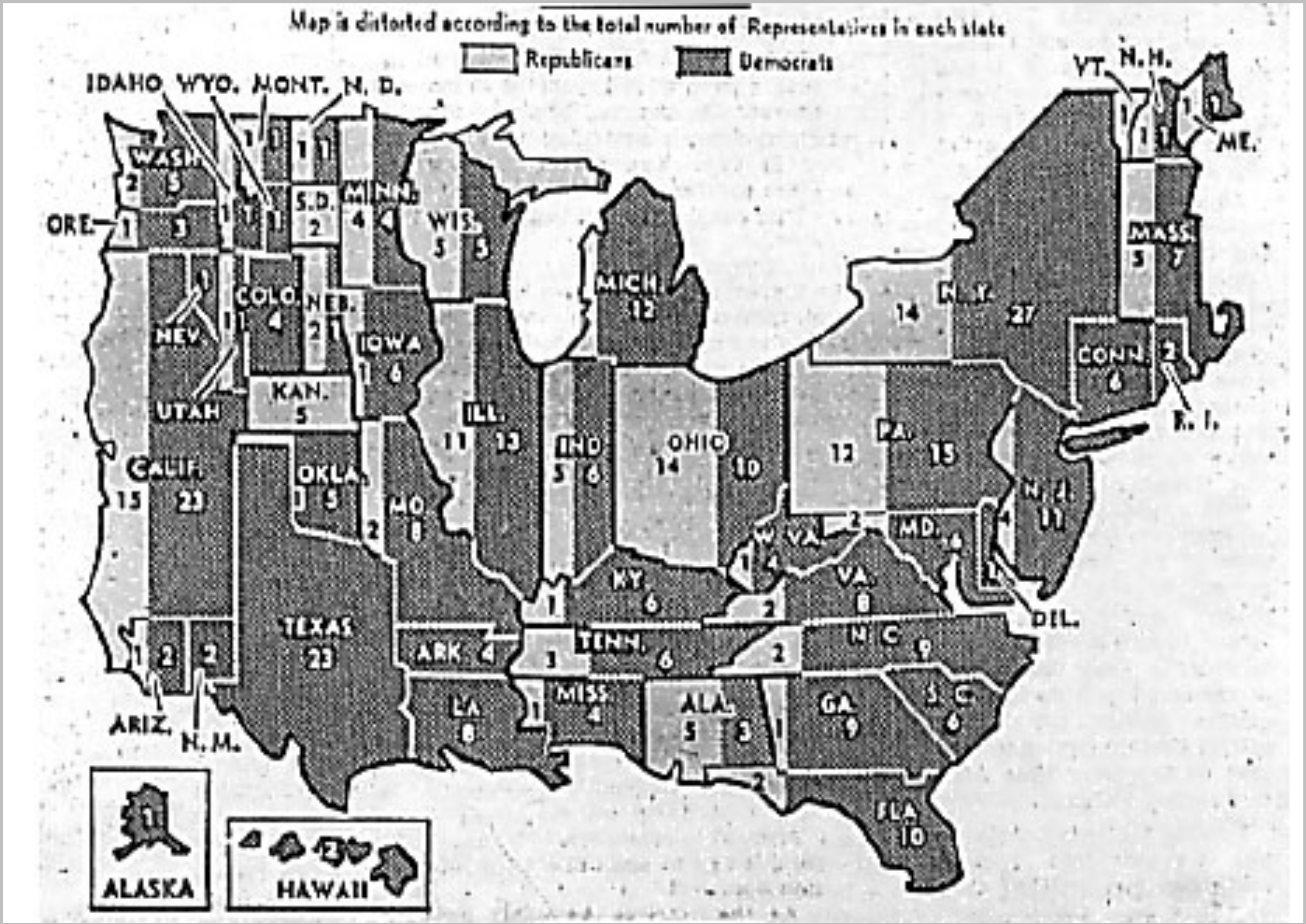
Area Cartograms often depict political concepts.

Numbers of members of congress by state.

Or the vote by party, sometimes colored in red or blue.

Expect one, or more, of these maps in the newspapers every voting season.

Numbers of members of congress by state.



Area cartograms are anamorphoses - a form of map projection designed to solve or demonstrate particular problems.

They represent map area proportional to some distribution on the earth, through a 'uniformization'.

This property, and the inversion, are useful in studying distributions.

Cartograms are such a departure from Ptolemy that some people don't even consider them maps.

The equations show that all equal area projections are a special case of area cartograms.

Area cartograms can also be displayed on a globe.

Map Projections and Cartograms

The equal area condition for a map projection in spherical and plane rectangular coordinates is

$$\frac{\partial x}{\partial \lambda}\frac{\partial y}{\partial \varphi} - \frac{\partial x}{\partial \varphi}\frac{\partial y}{\partial \lambda} = R^2 \cos(\varphi)$$

The condition equation for an areal cartogram is

$$\frac{\partial x}{\partial \lambda}\frac{\partial y}{\partial \varphi} - \frac{\partial x}{\partial \varphi}\frac{\partial y}{\partial \lambda} = R^2 D(\varphi, \lambda) \cos(\varphi)$$

Where $D(\phi, \lambda)$ is the density distribution on the earth, considered spherical.

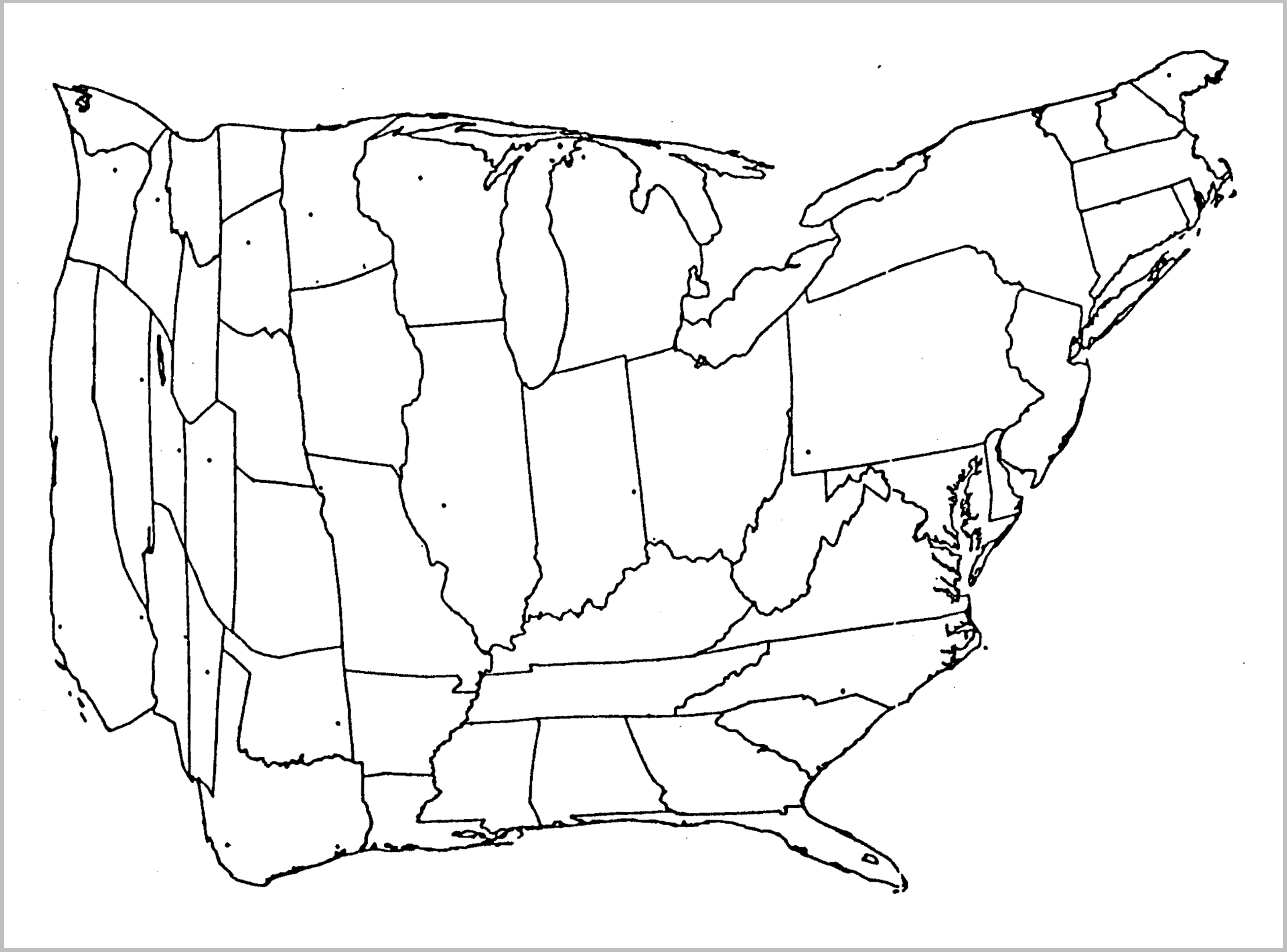
Clearly, when the density distribution is constant, then the cartogram becomes an equal area map projection.

In both cases the one condition does not suffice to yield the two equations $x=f(\phi, \lambda)$, $y=g(\phi, \lambda)$ needed to completely define a map projection. The obvious second condition is to require that the angular distortion be minimized.

US Population Cartogram

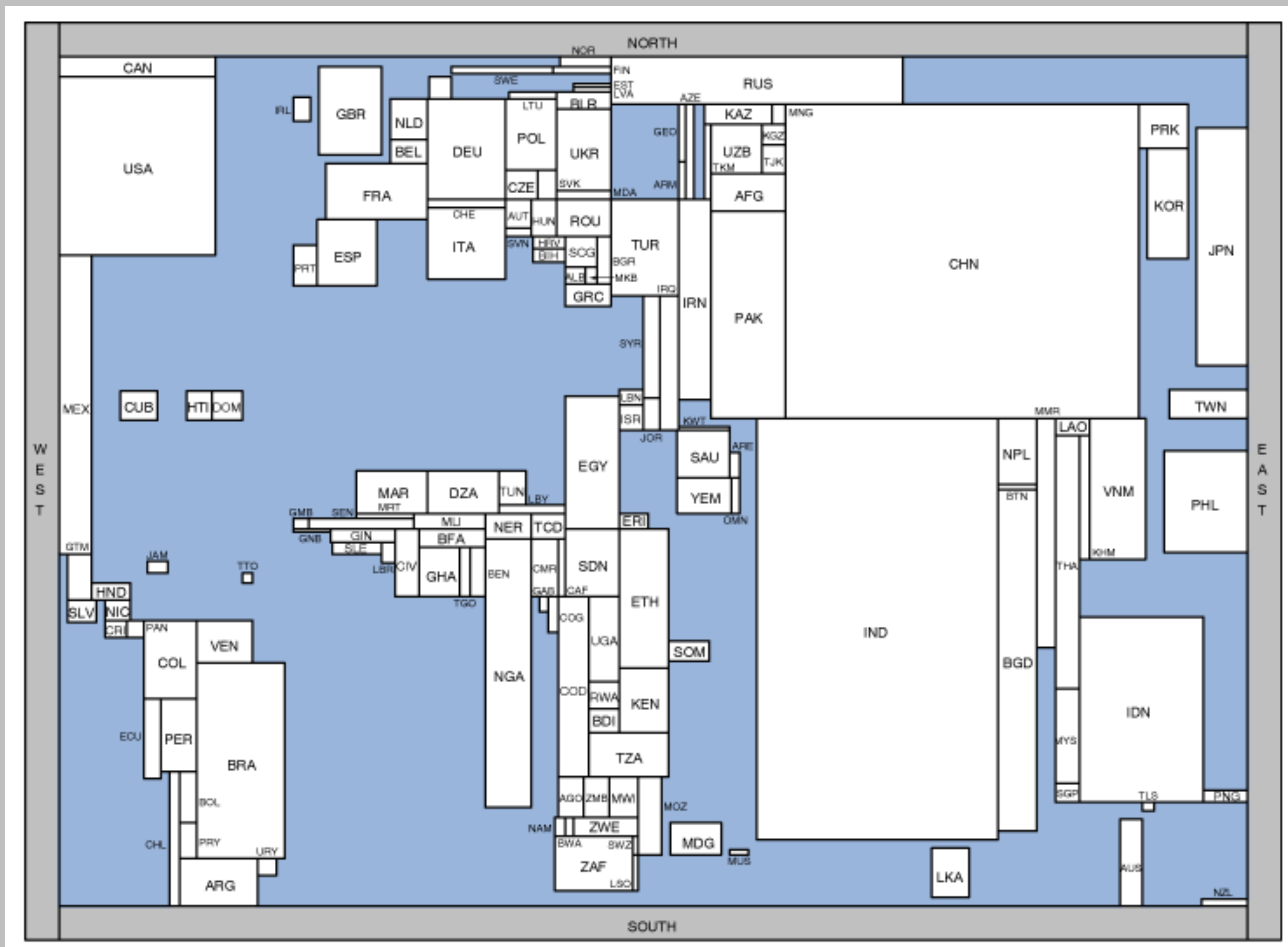
depicts states with sizes proportional to 1960 population.

The first computer generated cartogram, 1967, using a program written by W. Tobler.



World Population “Value-by-Area” Map

A world map projection computed and rendered by Van Kreveld & Speckmann, 2007



There Are Many More Unusual Projections

Of these the strangest may be the retro azimuthals, on which the map may fold over on itself.

The size of the overlap, and the void, depends on the latitude of the center.

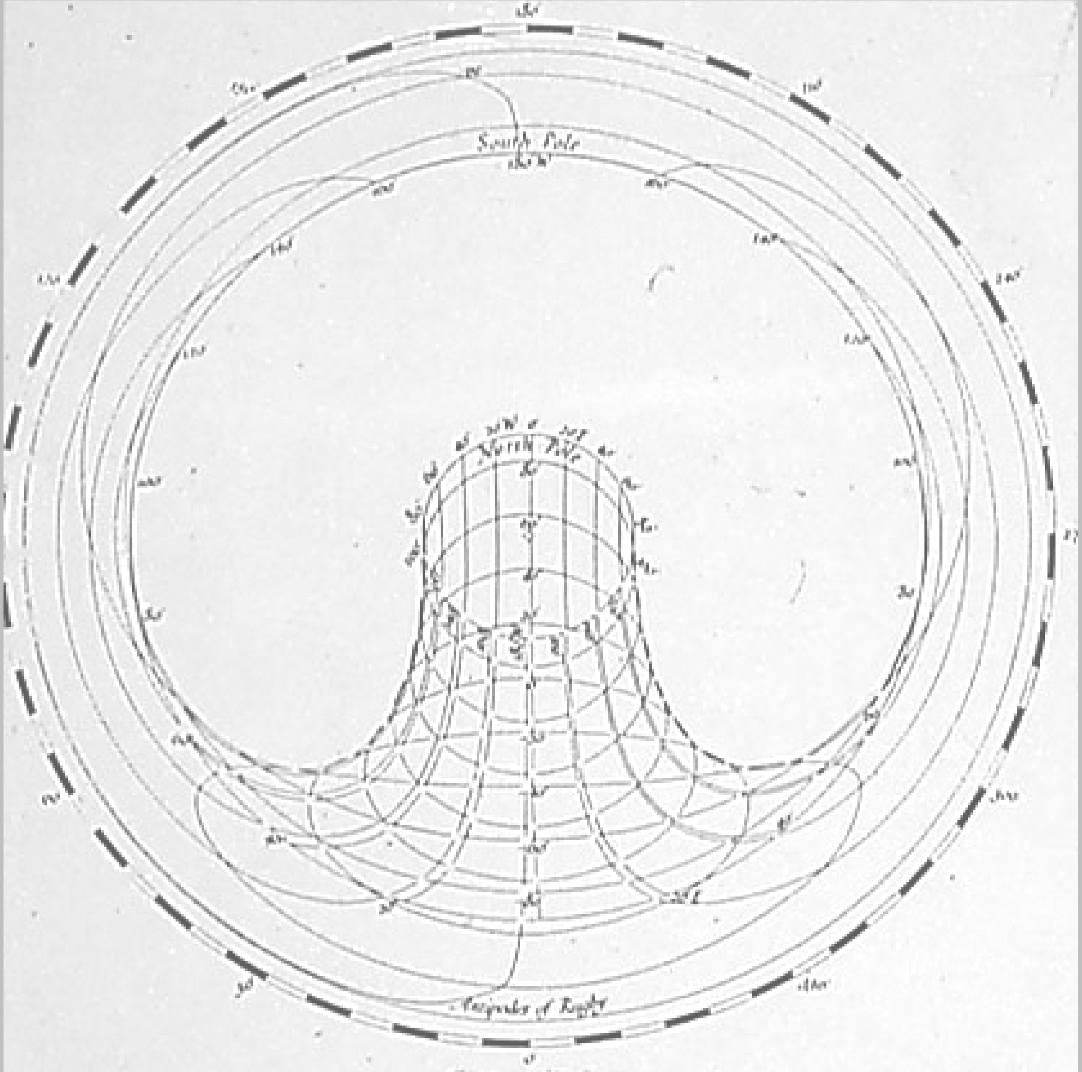
A radio station was established at Rugby (UK) to broadcast a time signal to British colonies overseas. The equidistant retro-azimuthal projection was used to let the colonials know in which direction to point their radio antennas. The result, which served its purpose, resulted in a very strange map. Only the coordinate outline is shown here.

A. R. Hinks, 1929, "A retro-azimuthal equidistant projection of the whole sphere", *Geogr. J.*, 73(2):245-247.

E. A. Reeves, 1929, "A Chart Showing the true bearing of Rugby from all Parts of the World, *Geogr. J.,* 73:247-250

Hinks' Retro Azimuthal Projection

Centered at Rugby, UK



Retro-azimuthal projections show reverse directions to a center. This property can also be combined with correct distances to the center.

J.I.Craig, 1910, *Map Projections*, Cairo, Ministry of Finance.

E. Hammer, 1910, "Gegenazimutale Projektionen", *Petermanns Mitteilungen*, 56(3):153-155+Plate.

C. F. Arden-Close, 1938, A polar retro azimuthal projection", *Geogr. J.*, 92(6):536-537.

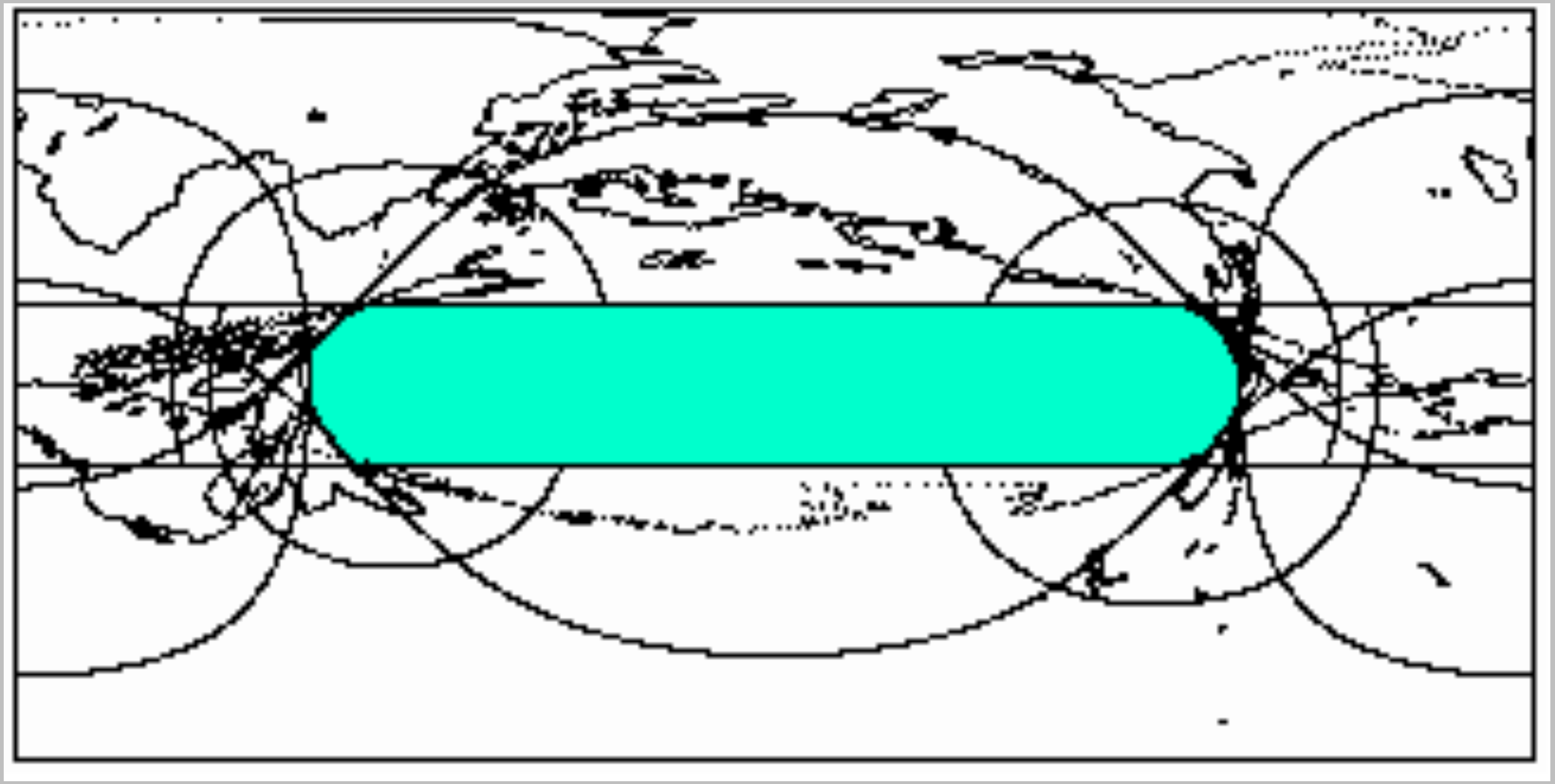
D. A. King, 1999, *World Maps for Finding the Direction and Distance to Mecca,* Leiden, Brill.

Here is a New Retro-Azimuthal Projection

Centered at 20N, 40E, Close to Mecca

Mecca along top. Up is distance to Mecca, left to right is direction to Mecca.

The map contains a hole and overlaps itself.



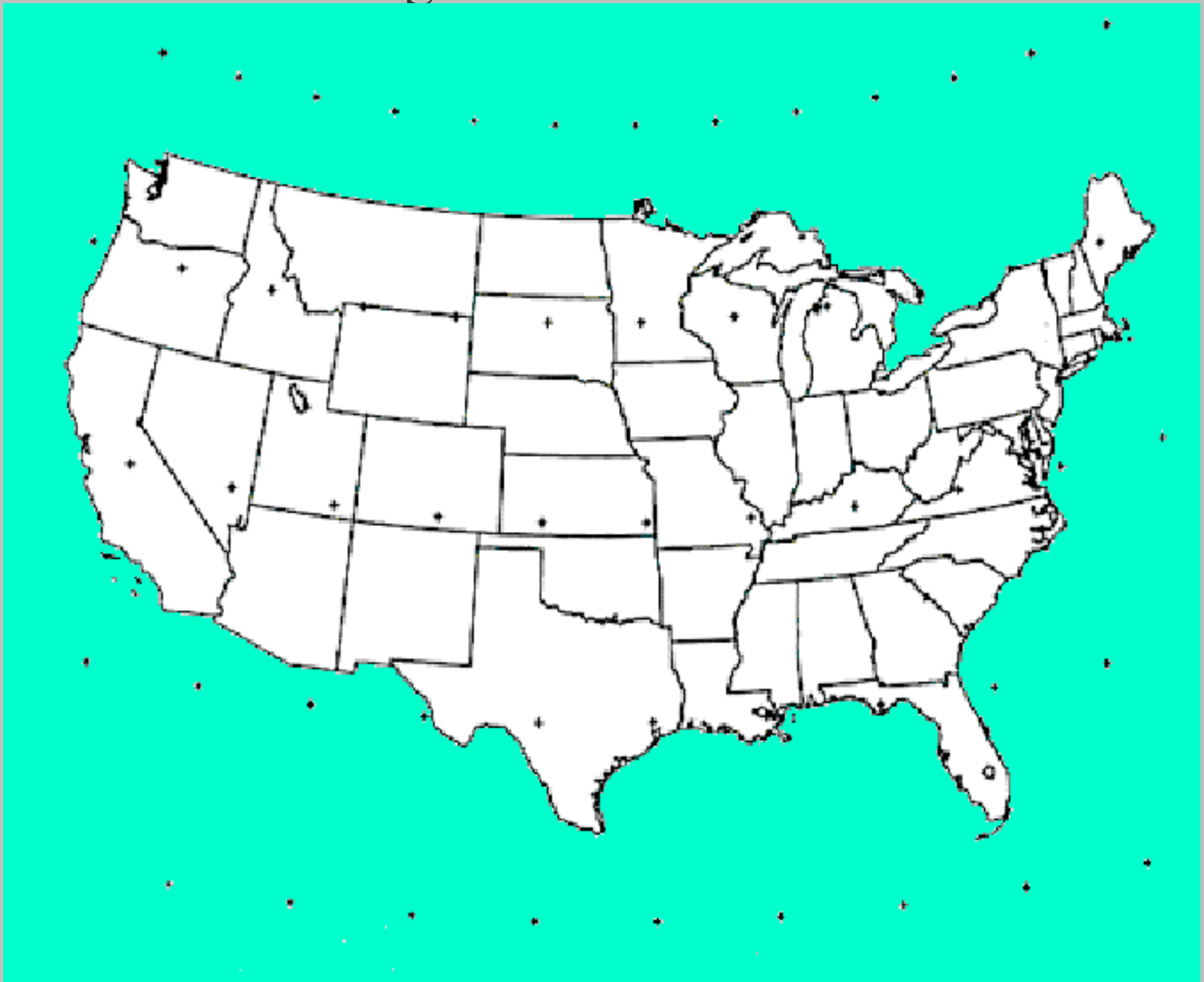
Ptolemy vindicated!
The best spherical distance preserving map.

An "empirical" map projection.

The projection is produced by covering the United States with a lattice of latitude and longitude points, then computing the spherical distances between these points, and then computing plane coordinates for these points. The plane coordinates are computed to minimize, in the least squares sense, the difference between the plane distances and the spherical distances. The map information is then interpolated by cubic splining and drawn by computer using a ~10,000 coordinate digital file. The meridians are slightly curved, resulting in a 'polyconic' type map.

W. Tobler, 1971, "Numerical Approaches to Map Projections", pp. 51-66 of I. Kretschmer, *Studies in Theoretical Cartography*, Vienna, Deuticke

Optimal Distance Preserving Projection Of The Contiguous United States



Another ‘empirical’ map projection.

Instead of great circle distances one can construct a map to preserve, in the least squares sense, loxodromic (rhumb line) distances, an hypothesis being that Portolan charts made prior to 1500 AD might have used such distances in their construction.

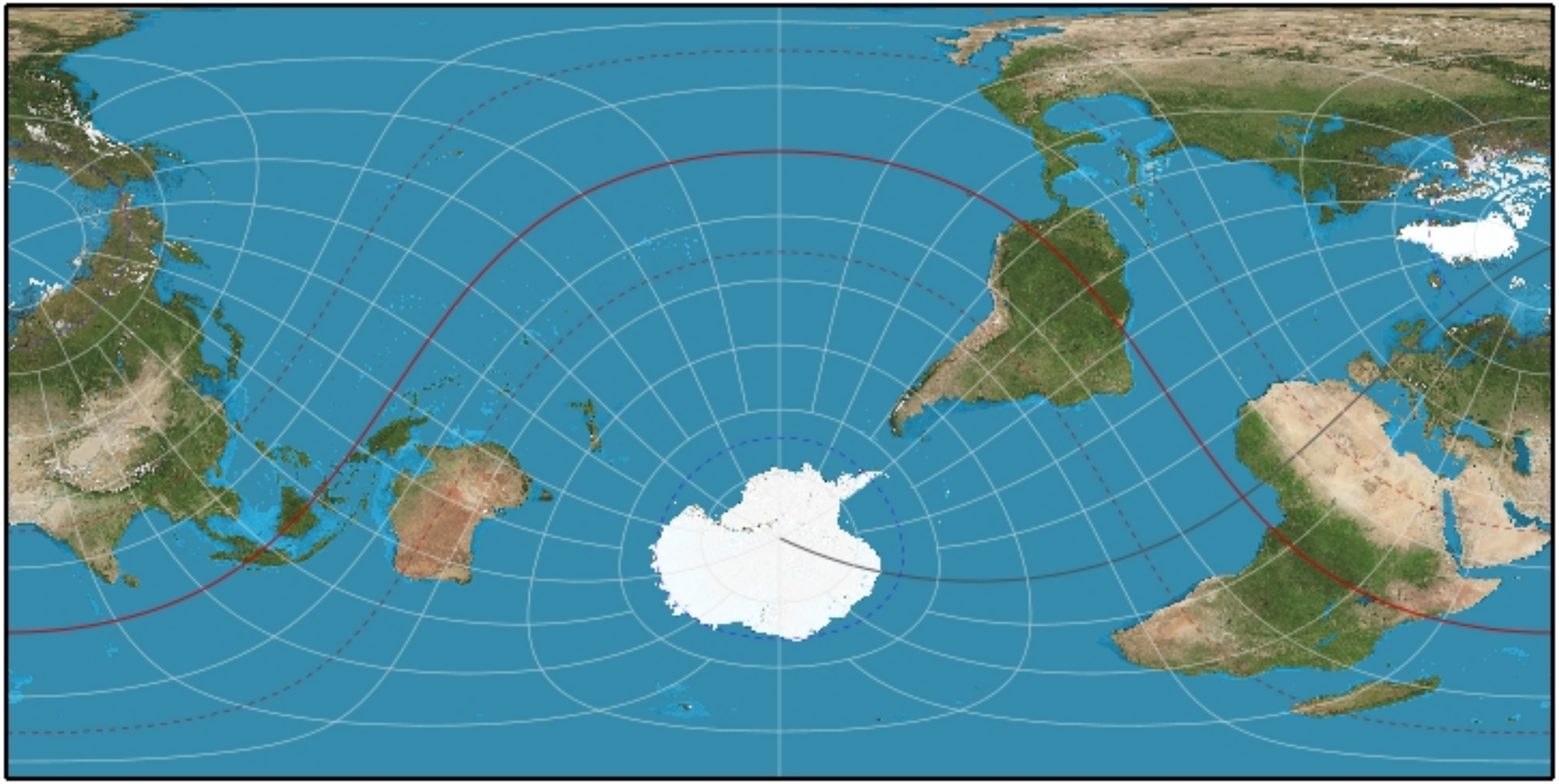
W. Tobler, 1971.

Mediterranean Sea Preserving Loxodromic Distances

Distance-Direction Diagram

Distance from Santa Barbara read down. Direction from Santa Barbara read across.

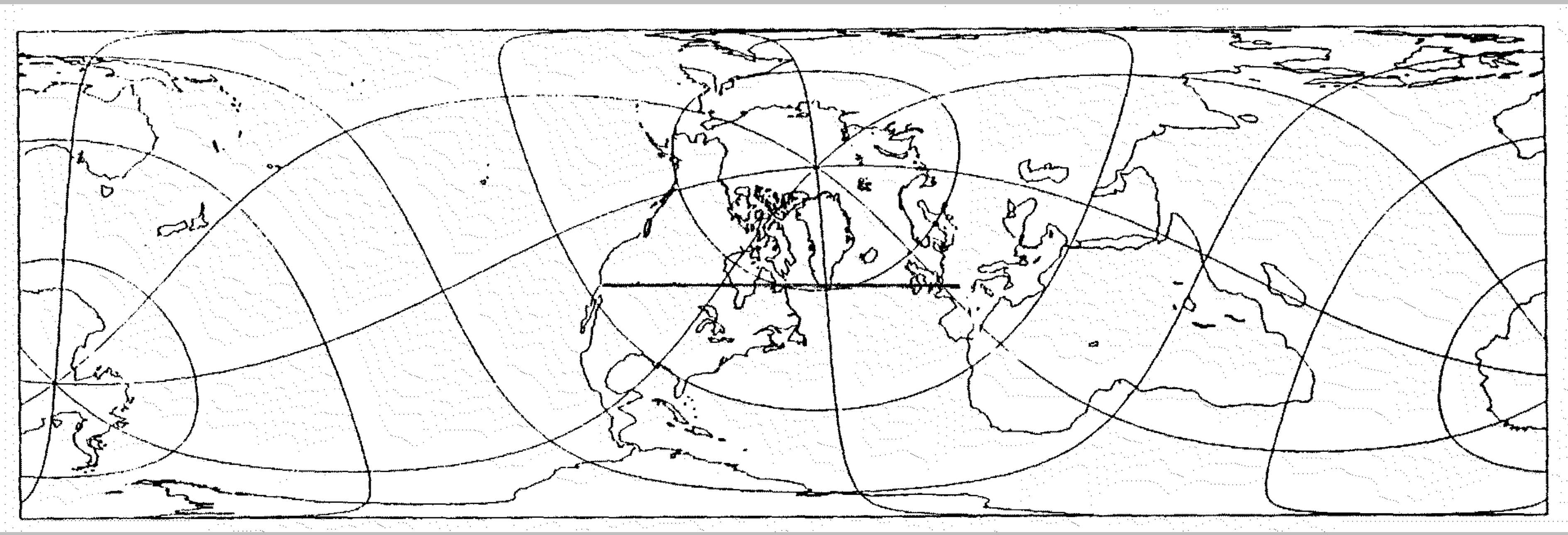
The line across the top represents Santa Barbara.



Oblique rotation to put a great circle through the middle.

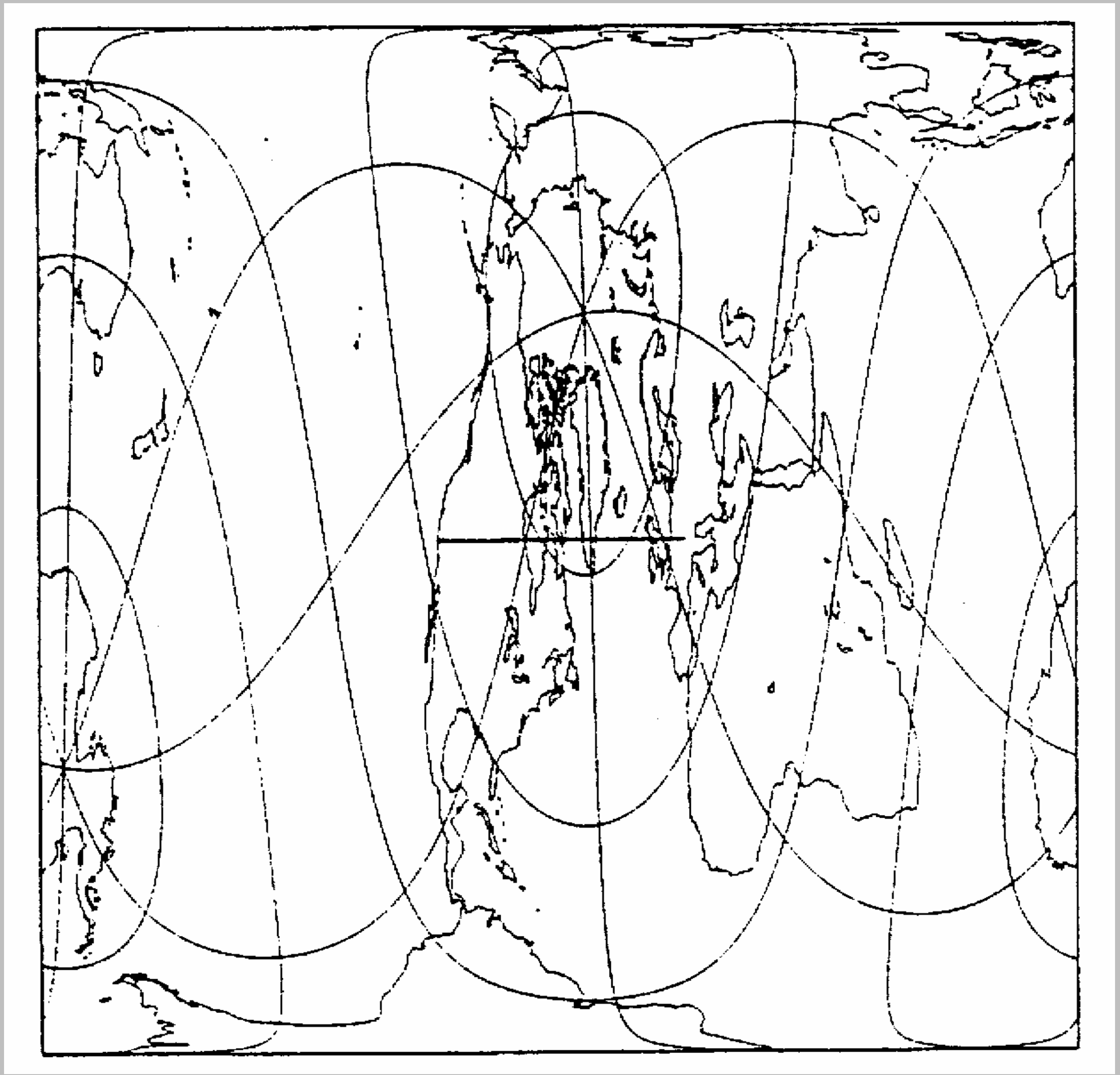
Lambert cylindrical equal area projection.

The central line connects Santa Barbara with Zürich & represents 9600 km.



Same view, just another map projection.

Equal area within a square. The great circle arc connects 34°N, 120°W with 47.5°N, 8.5°E.



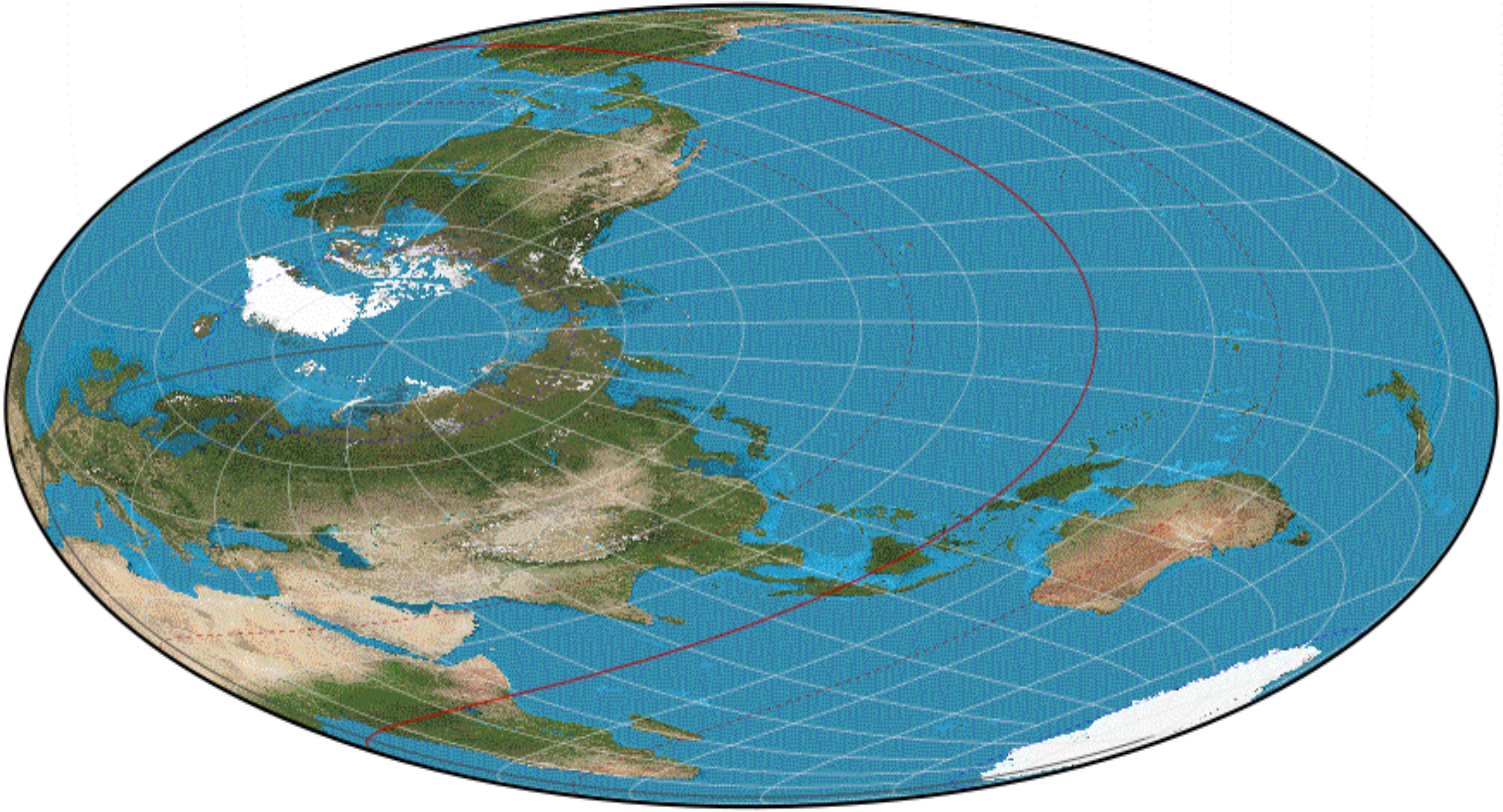
The azimuthal equidistant projection has the property that all distances from the center are rendered correctly, to scale. It is also possible to do this from two points, yielding the so-called two-point equidistant projection. The world map is then contained within an ellipse, the eccentricity of which depends on how far apart the two points are. The points are at the foci of the ellipse. Can you see why it must be an ellipse?* When the two points coincide the map is circular; when they are antipodal the map is a straight line. Distances from each node are hyperbolae. Azimuths are no longer correct, although there is also a map projection with directions correct from two points.

C.F. Close, 1934, "A Doubly Equidistant Projection of the Sphere", *Geogr. J.*, 83(2):144-145.

*Recall the method of constructing an ellipse by fixing the ends of a length of string to two points on paper and then using a pencil constrained by stretching the string to its limit to draw a shape.

Two Point Equidistant Map

London, UK and Wellington, NZ



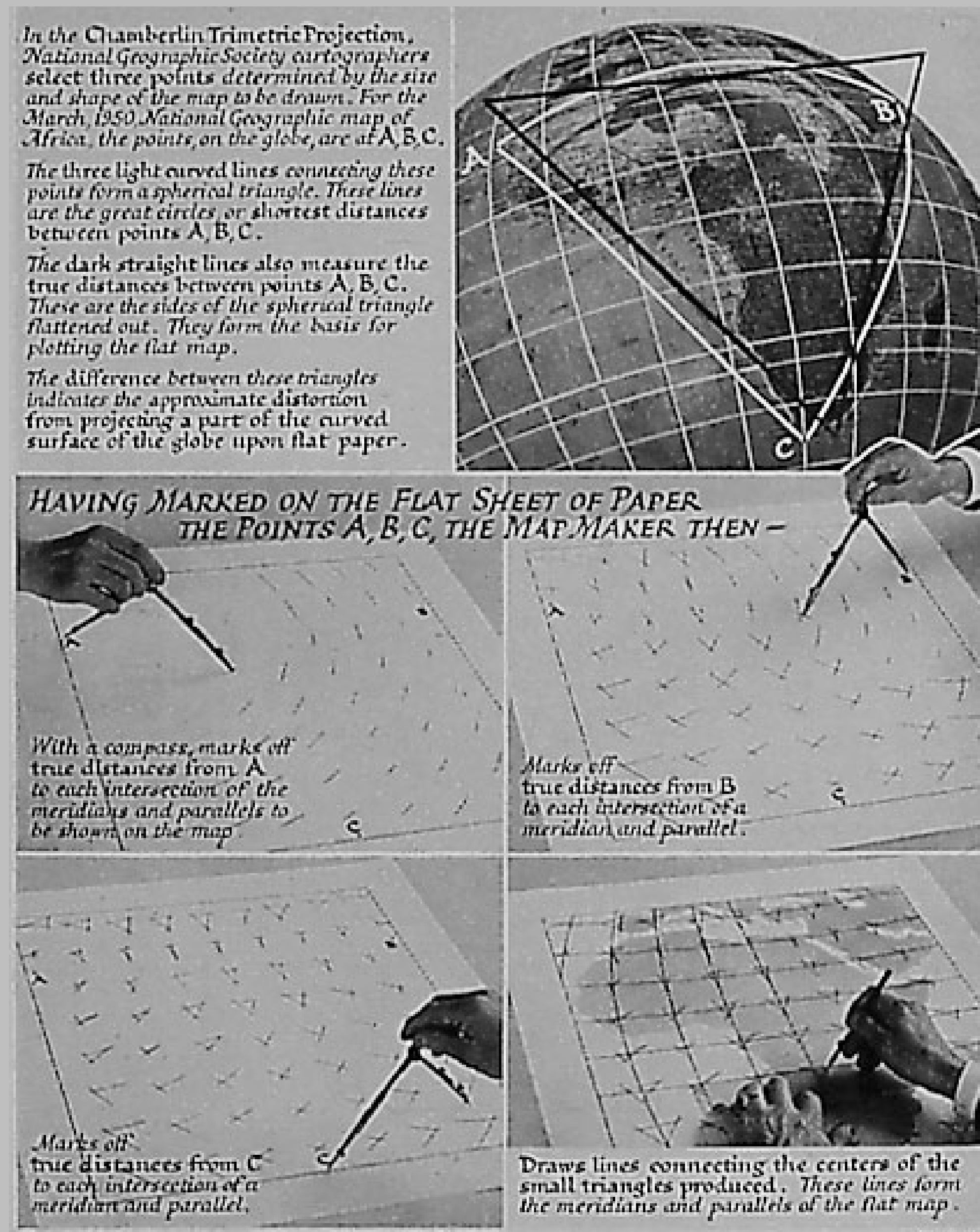
For three points Wellman Chamberlin of the National Geographic Society invented the "trimetric" projection.

His diagram is pretty self explanatory. The arcs of distance from the three points chosen to bound the area do not meet exactly, but rather form a small curvilinear triangle. The centroid of this triangle is used as the position of the graticule intersection. This would appear to be a least squares solution, but is not so described in the literature.

A map with a tolerable amount of distortion.

W. Chamberlin, 1947, *The Round Earth on Flat Paper*, Washington D.C., Nat. Geogr. Soc.

Constructing A Trimetric Projection



Trimetric Projection

Nodes at: 40N, 110W; 40N, 70W; 25N, 95W

A really strange thing to want!

It is possible to draw a map of the world on a Moebius strip. The map must be drawn on both sides of the paper, inverting the image on one side. Then twisting and gluing the ends together.

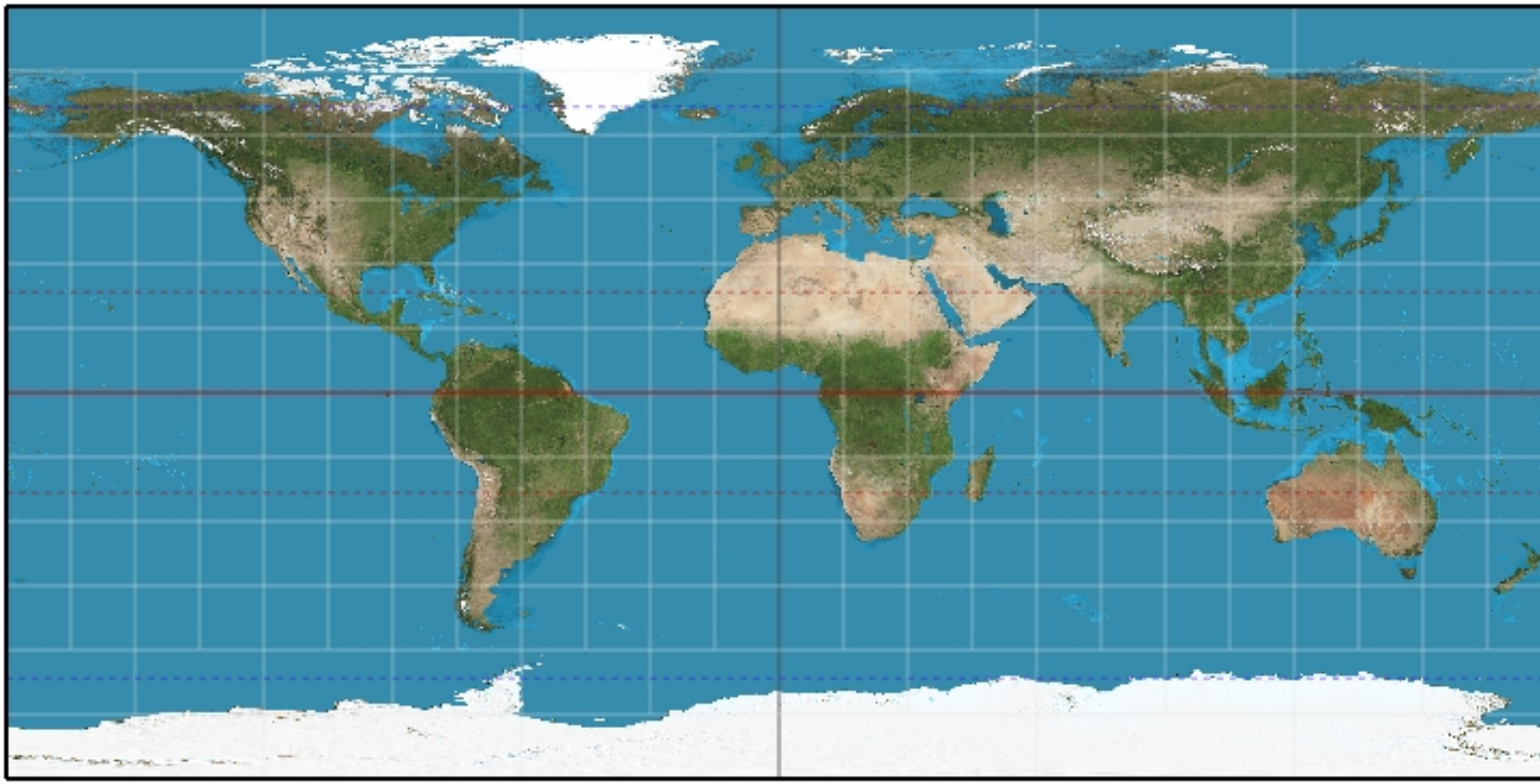
The resulting map, when one uses a cylindrical projection, with the poles at the edges of the strip, has the curious property that a pin pushed through the map exits at the antipodal point.

W. Tobler, & Kumler, M., 1986, "Three World Maps on a Moebius Strip", *Cartography and Geographic Information Systems,* 18(4):275-276.

The World On A Moebius Strip

Print upside down on back and give it a twist then glue the ends together.

Any cylindrical projection will do but this one makes locating places easy.



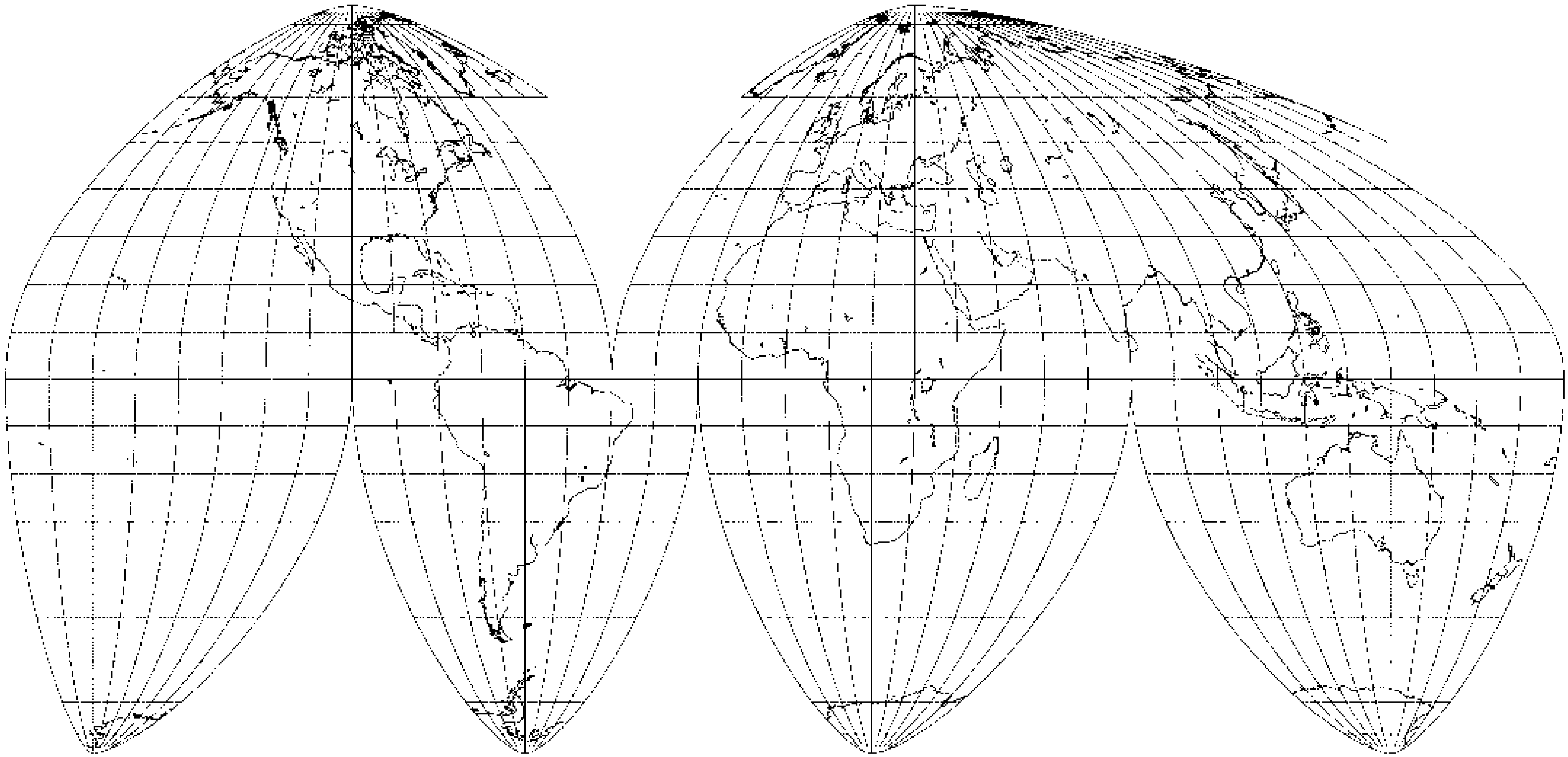
World map on a Moebius strip

Courtesy of Professor Mark Kumler

A blended projection is a splendid projection.

Goode's homolosine (Sinusoidal & Mollweide) but without the kink.

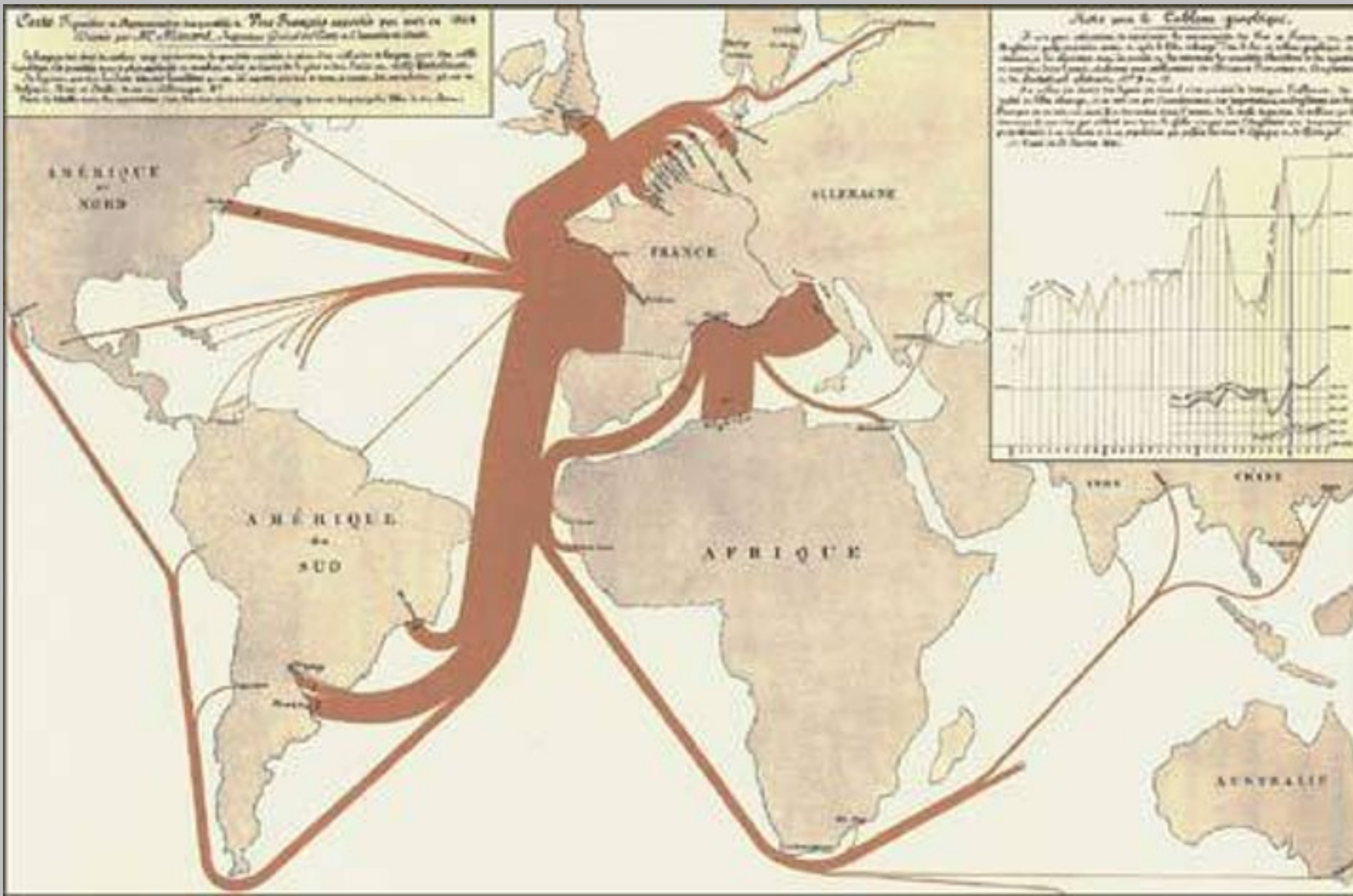
Linear blending of the two from the equator to the poles.



Minard's 1864 Map of Wine Exports from France

Move continents to make room for volume through Gibraltar..

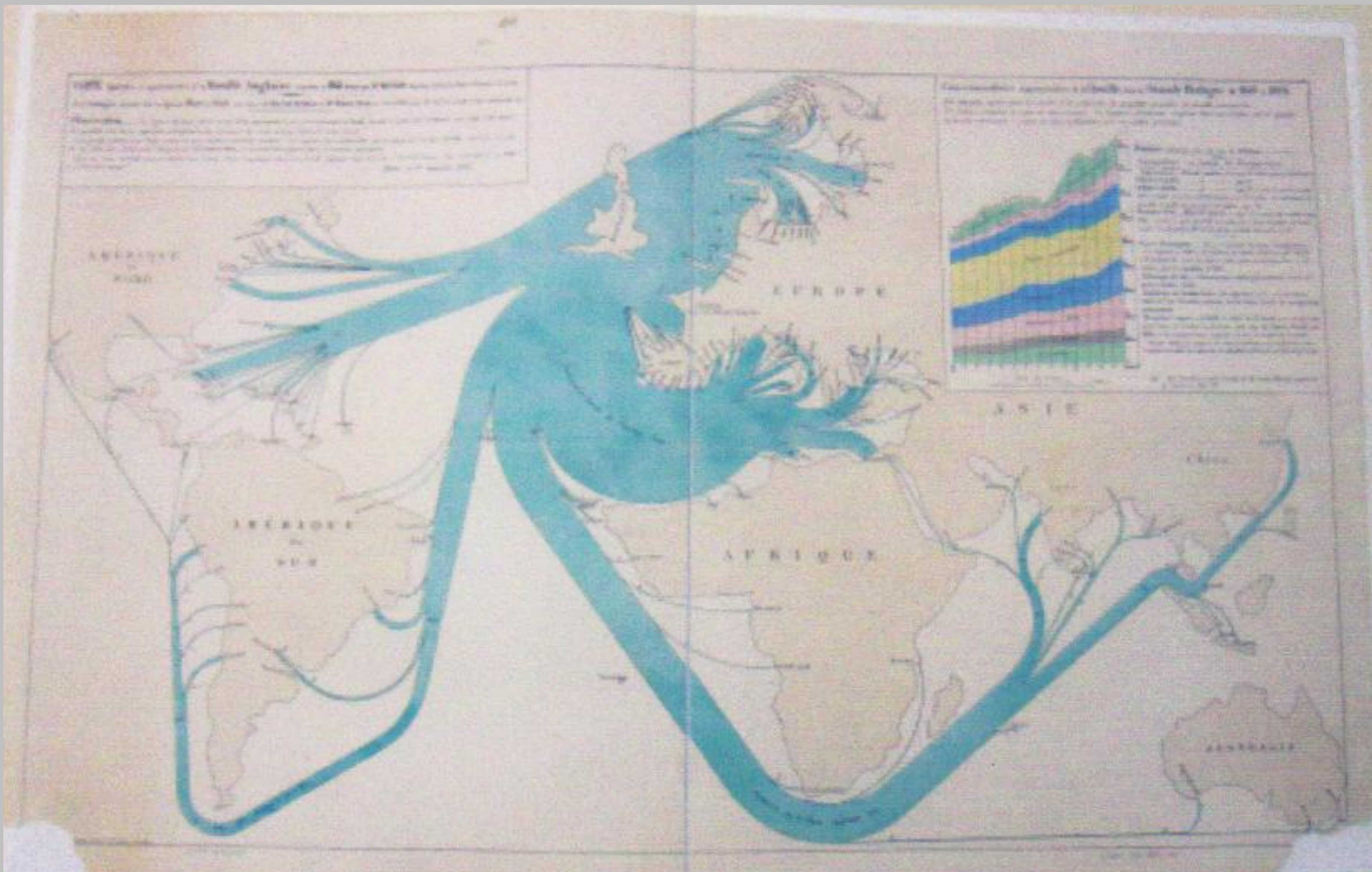
What would Ptolemy think!



C. J. Minard compresses Africa to make way for coal

British coal exports for 1864.

What would Ptolemy think of this one?



Thus far neglected were

The gnomonic projection.

A classic “graph paper” projection.

Which transforms great circles into straight lines.

And another classic

The stereographic projection.

Which represents all spherical circles as circles.

And

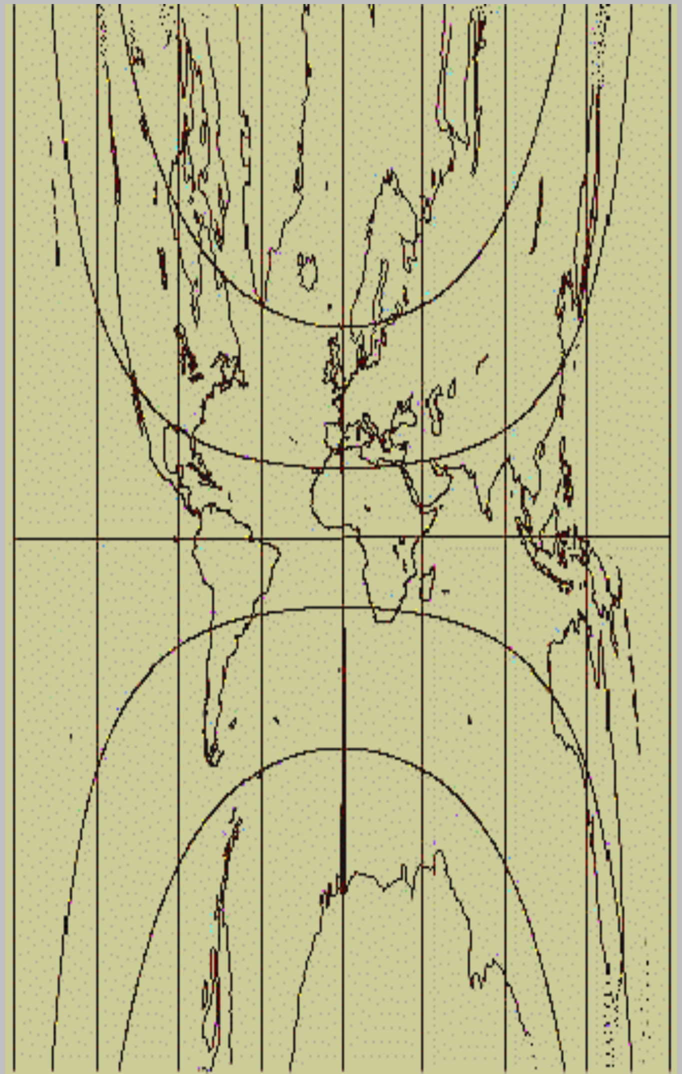
the Schmidt Net used by geologists

(Lambert’s equal area azimuthal projection).

But not all Azimuthal Projections are Circular.

Directions are correct from the intersection of Greenwich and the Equator. Based on an idea by J. Craig, Cairo, 1910.

The map could be centered anywhere, including your home town.



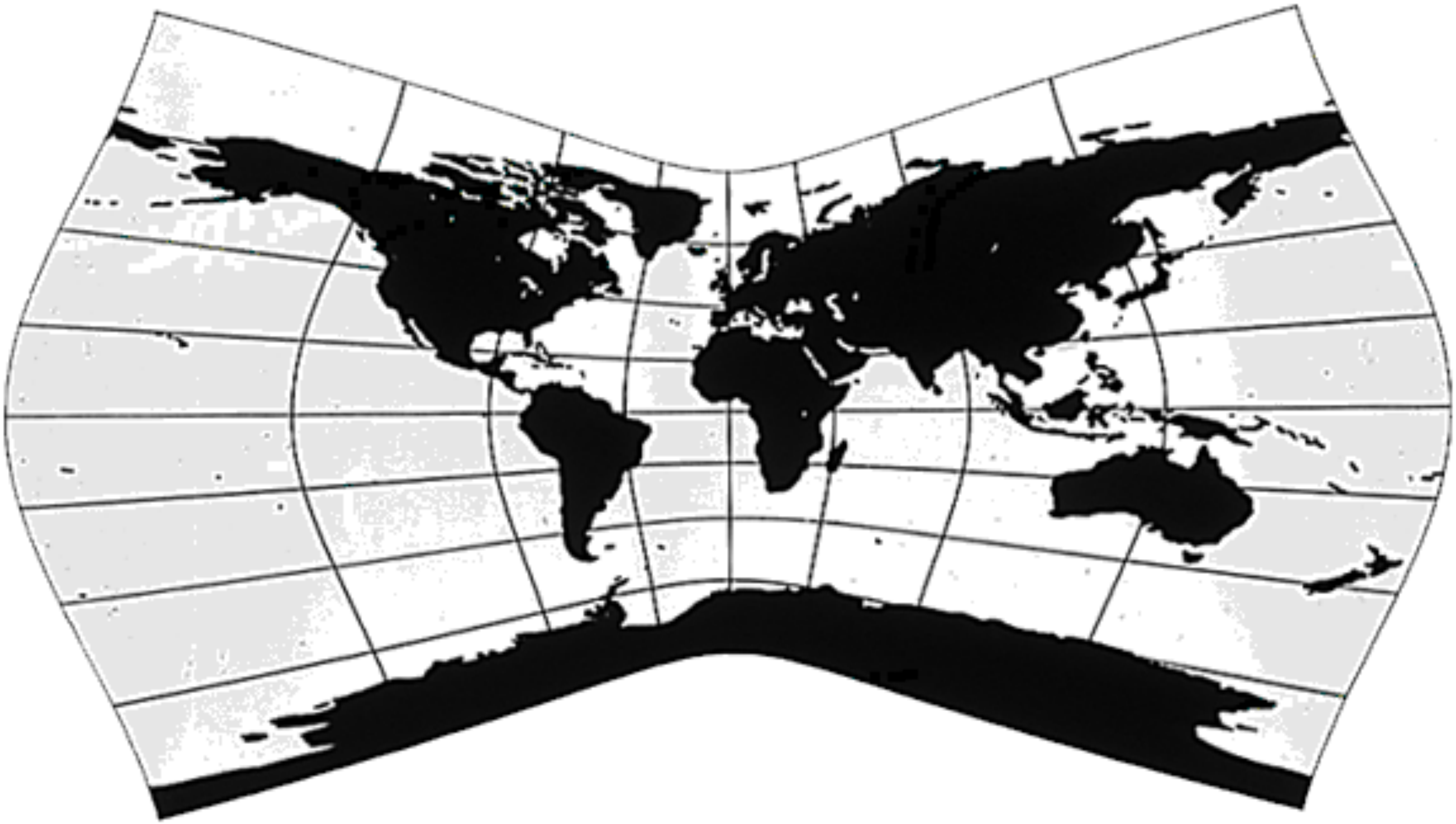
Another use of map projections.

Maps are frequently used in advertisements
in newspapers and magazines.
As part of a design.

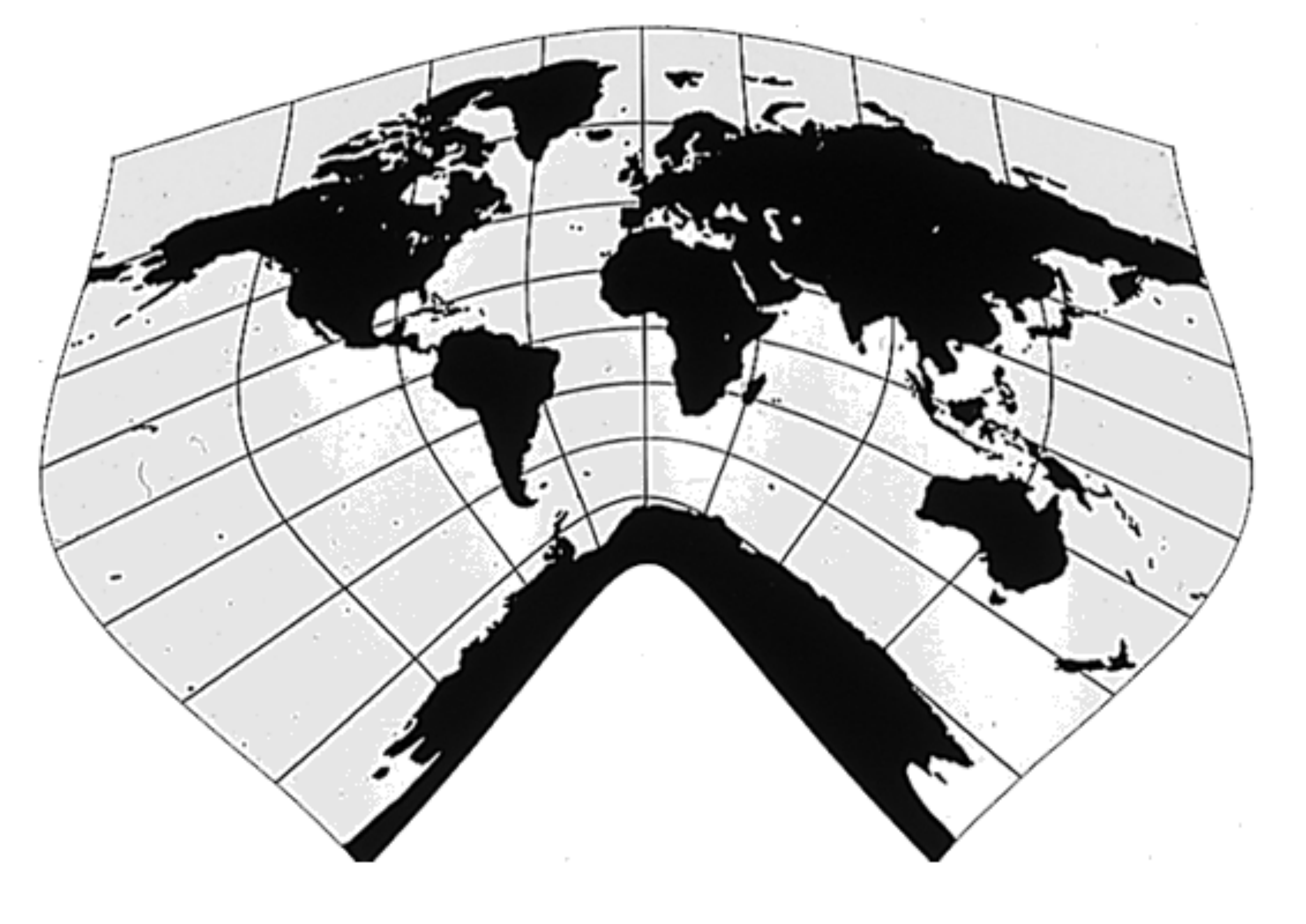
Thus one also finds unusual map projections
In these publications.

Here' s one of several possibilities:

A haberdashery could use this to advertise ties.

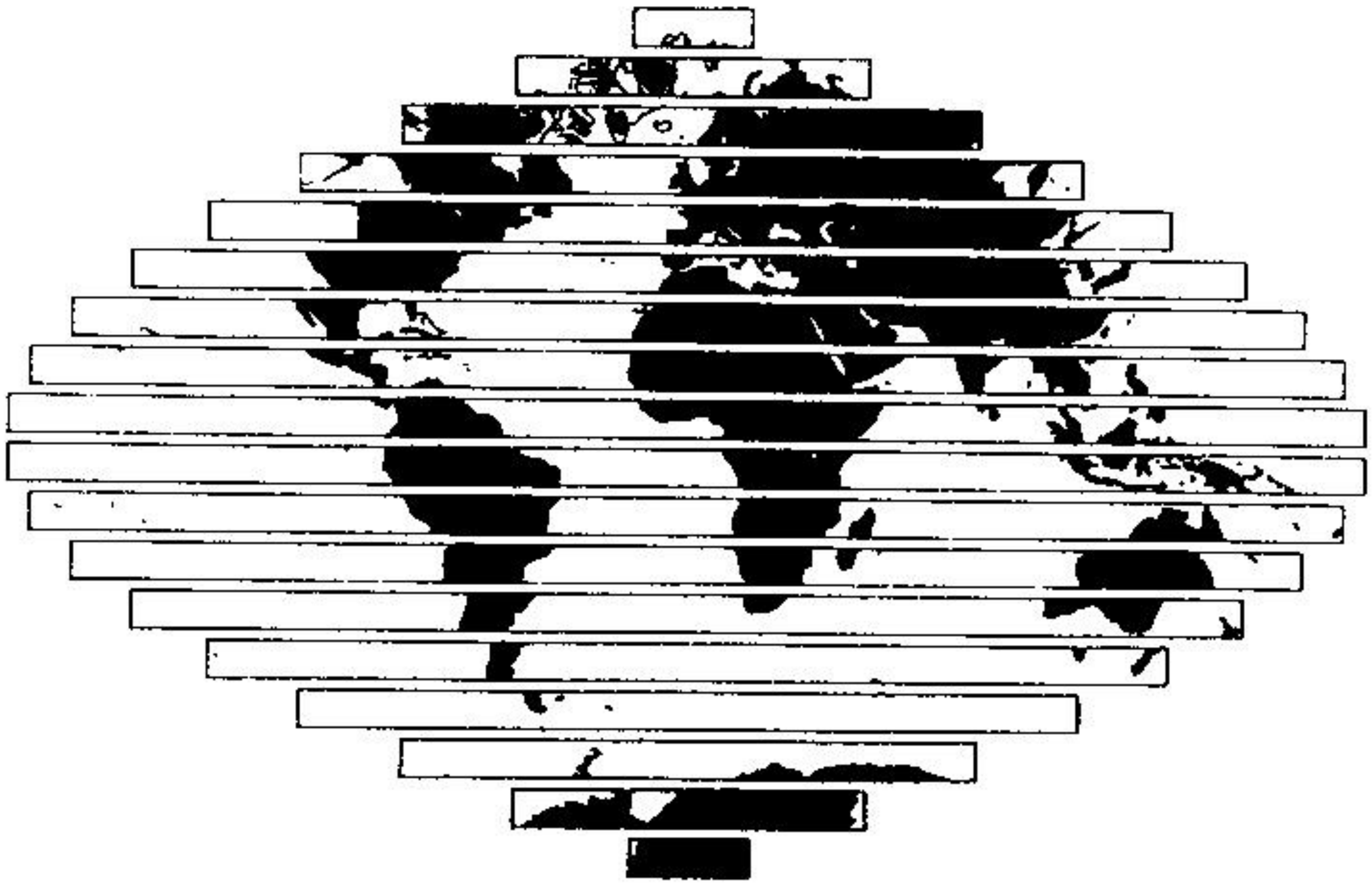


Or perhaps this floppy one, which is a bit more elegant



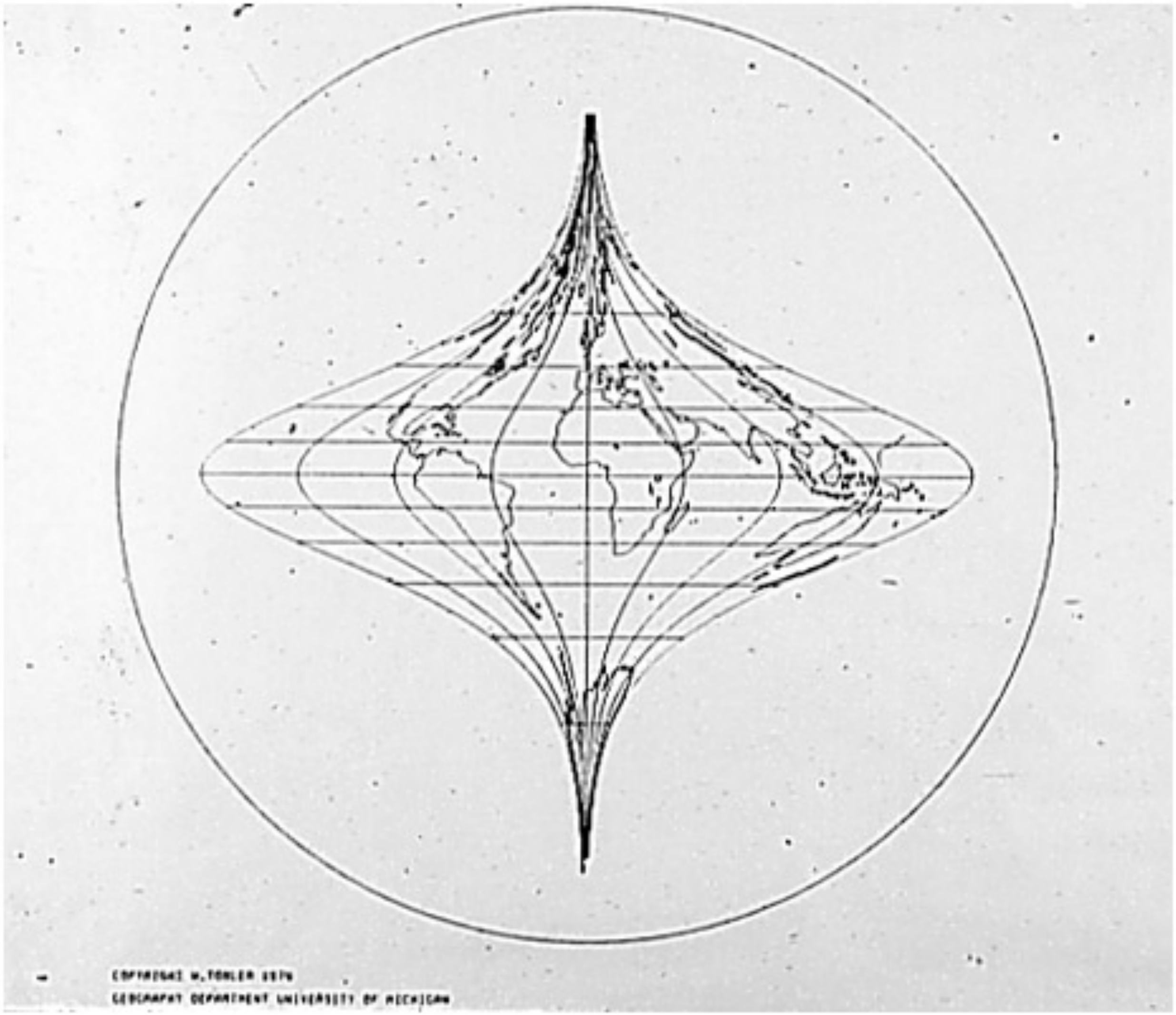
Design for a Japanese Lantern

Waldo Tobler & Miklos Pinter

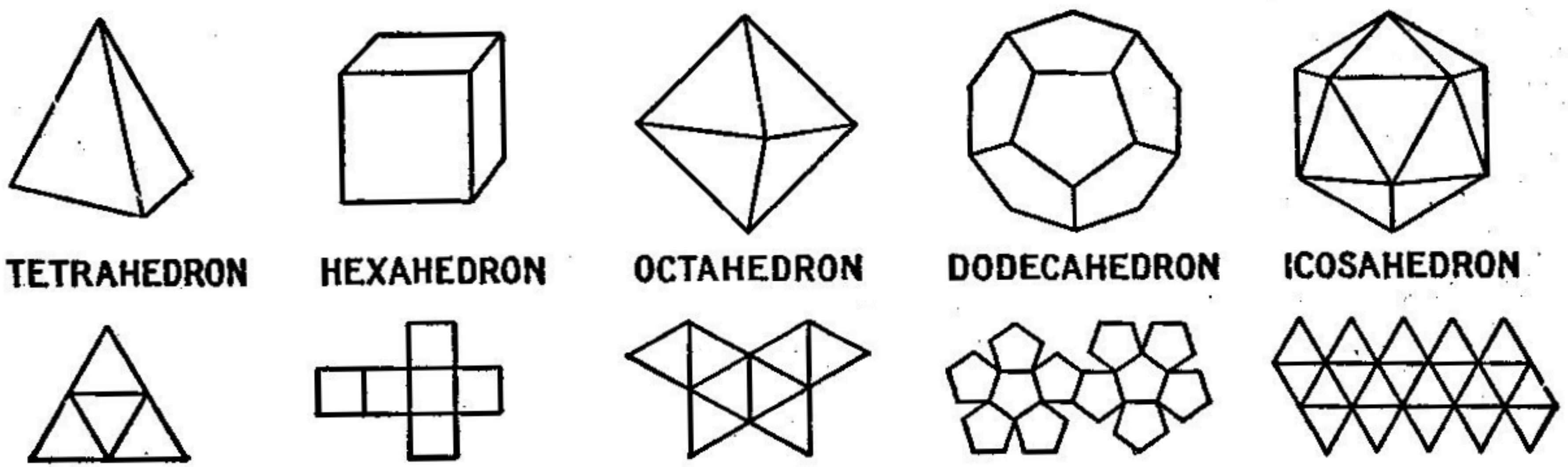


Alternately, one of several possible Christmas ornaments.

An equal area projection with latitude spacing as on the Mercator projection.



Maps on the five platonic solids have been known for a long time. They can be equal area or conformal. The gnomonic projection is particularly easy to do on the surface of these solids.

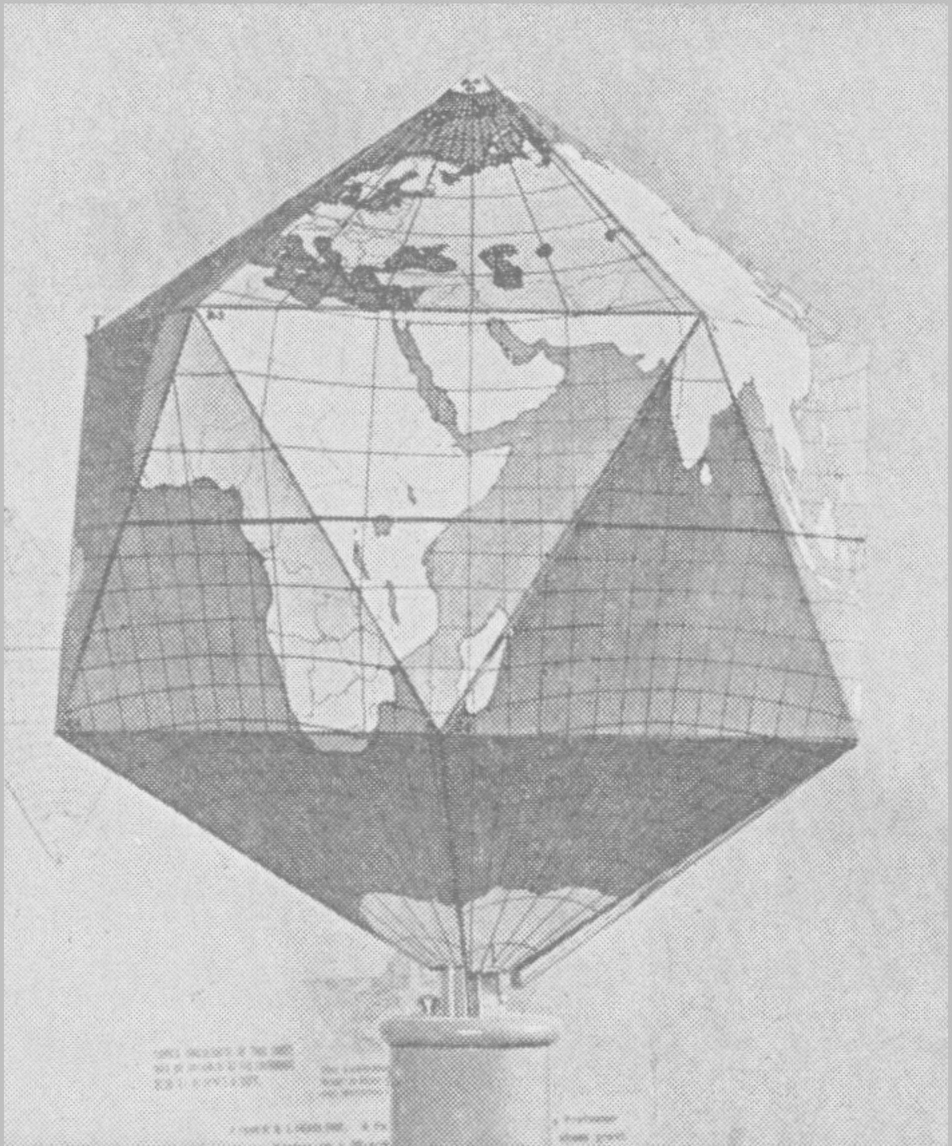


THE FIVE PLATONIC BODIES

Above the names of the bodies are the solids, and below are the faces of the solids laid out flat.

Conformal facets for a map on an octahedron

Icosahedron Globe



An equal area map on a pyramid

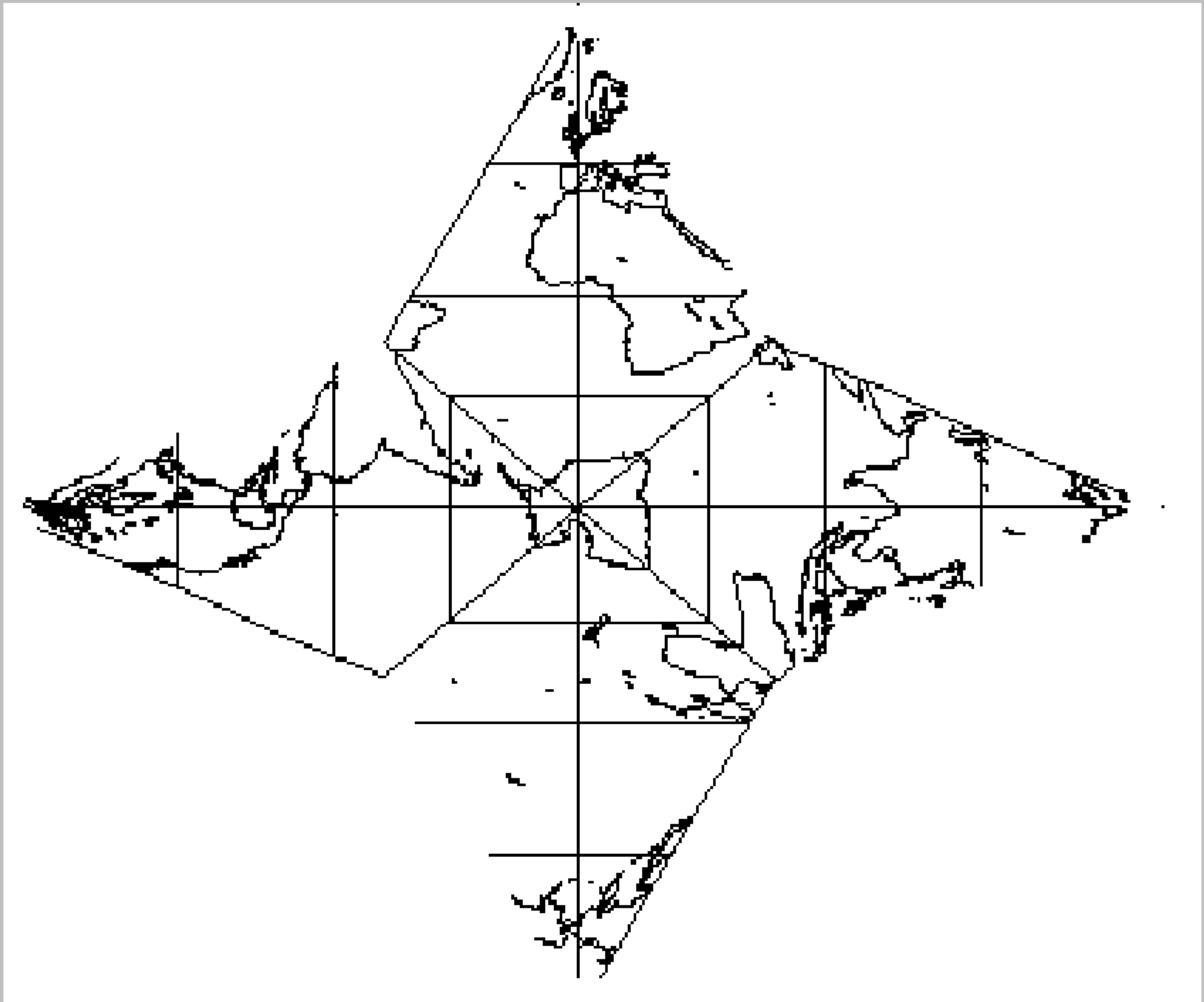
Apparently a map on the surface of pyramid has never been done. The next illustration is a special case of an equal area projection having N pointed triangular protrusions on an N sided base. For three lobes the base is a triangle (this folds into a tetrahedron). With four lobes we get the pyramid. For six lobes the base is a hexagon, etc. All can be drawn with one computer program, with N as a parameter.

This item is a bonus, and does not solve any serious problem.

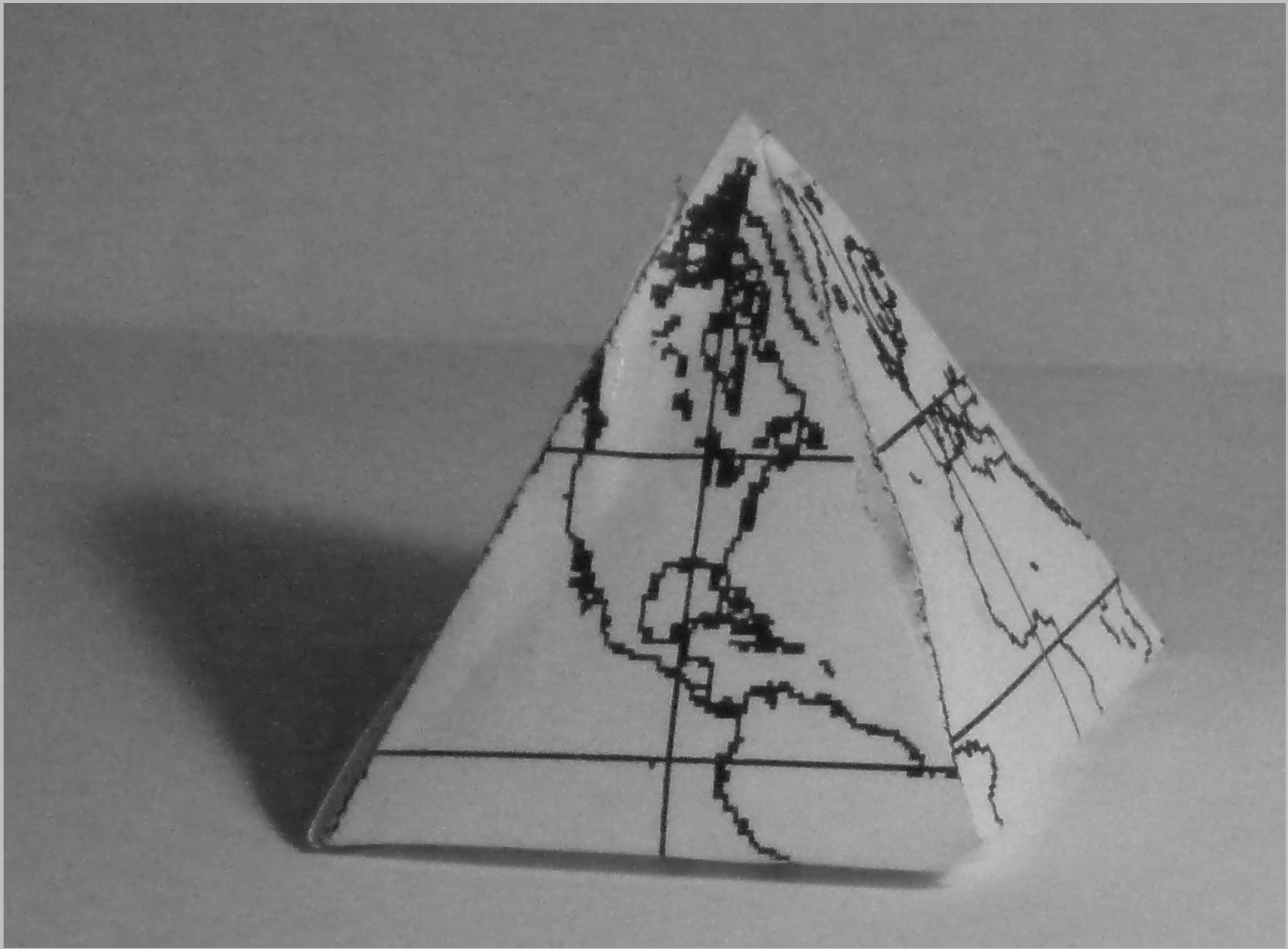
The gnomonic on a pyramid is easy. The stereographic works well for the conformal version.

An Equal Area Projection on a Pyramid

Cut out and glue together



Equal area world on a pyramid.



Certainly there are more analytical uses of map transformations.

Send me suggestions for other projections to be displayed in this theme.

With sources and illustrations.

Thank you.

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