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Using Asymmetry to Estimate Potential

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Abstract

Network analysis is often based on matrices of connections. An asymmetric square array of this type can be decomposed into two components. The anti-symmetric part is especially interesting because this can be used to view influence patterns. A journal to journal table is examined as an example. Another application is to the geographic migration of people.

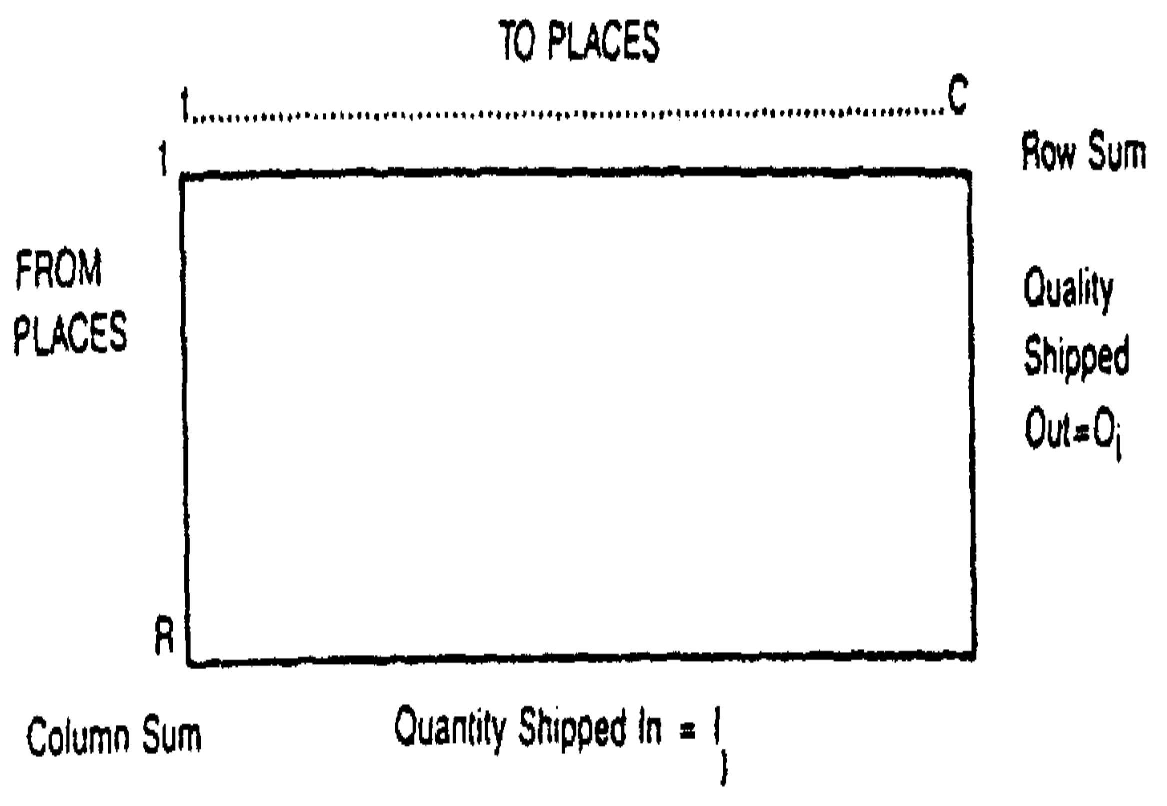
The concern is with complete, square, asymmetric, valued tables, though the procedure may also work with two mode tables.

For this demonstration I have used only small examples.

One example is based on geographic data, the other on journal-to-journal citations.

The form of a movement table M_{ij}

In a non-geographic, network, environment the ‘places’ are sometimes called ‘actors’.



Let M_{ij} represent the movement table, with i rows and j columns. It can be separated into two parts, as follows.

$$M_{ij} = M^{+} + M^{-}$$

where

$$M^{+} = (M_{ij} + M_{ji})/2 \qquad \text{symmetric}$$

$$M^{-} = (M_{ij} - M_{ji})/2 \quad \text{skew symmetric}$$

The variance can also be computed for each component,

and the degree of asymmetry can be computed.

How the two parts are used

I consider the symmetric component as a type of background. The real interest is in the asymmetric part.

In the geographic case the position of the places is known.

But if locations are not given then the symmetric part may be used to make an estimate of these positions.

This estimate is made using an ordination, trilateration, or multidimensional scaling algorithm.

The first example uses a 33 by 33 matrix of commuting in the vicinity of Munich, Germany.

The matrix is shown next.

A map of the regions is given in:

D. Fliedner, 1962, “Zyklonale Tendenzen bei Bevölkerungs und Verkehrsbewegungen in Städtischen Bereichen untersucht am Beispiel der Städte Göttingen, München, und Osnabrück”, *Neues Archiv für Niedersachsen*, 10:15 (April 4): 277-294, (following p. 285).

A geographic example

Munich Commuting 1939

Between 33 districts of known location

NACH ARBEITSSTÄTTE IM STADTBEZIRK .....

VON WOHNSITZ IM STADTBEZIRK ......

	VON WOHNSITZ IM STADTBEZIRK	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	Insgesamt	
1	NO-Altstadt		174	151	455	212	157	48	119	247	74	86	100	192	52	52	47	79	26	70	30	76	91	22	112	23	27	86	24	42	12	29	8	7	2910	1
2	SO-Altstadt	428		335	482	215	153	51	176	414	164	176	170	170	59	48	68	123	57	102	76	87	124	34	232	49	46	128	40	58	16	31	18	15	4545	2
3	SW-Altstadt	209	149		455	114	133	43	143	350	107	85	60	73	35	30	42	58	24	85	39	59	75	29	128	26	38	100	31	48	22	15	10	8	2821	3
4	NW-Altstadt	104	32	55		77	76	13	46	92	25	21	23	41	5	6	7	21	6	20	14	11	22	5	32	12	8	48	6	15	7	1	6	3	860	4
5	Universitätsviertel	549	165	242	998		669	139	284	587	121	110	74	367	61	38	60	99	26	90	72	172	393	59	187	61	207	333	39	95	20	39	22	9	6387	5
6	Techn.-Hochschul-Viertel	508	122	252	765	516		180	697	784	140	124	77	220	66	44	53	86	36	116	89	326	230	111	235	72	160	373	70	51	13	26	41	18	6401	6
7	SW-Schwabing	441	180	348	933	1063	981		625	878	197	121	94	365	106	66	70	133	53	141	112	497	438	95	274	94	386	709	95	110	37	39	53	28	9762	7
8	Viertel nw. Hauptbahnhof	245	123	207	643	405	504	159		880	158	116	66	135	81	57	66	103	40	124	171	469	191	204	262	100	107	336	76	58	26	29	36	72	6233	8
9	Viertel südl. Hauptbahnhof	338	190	429	843	368	426	88	439		350	162	91	194	62	77	77	109	36	188	197	179	145	69	302	141	78	270	33	67	26	21	61	39	6095	9
10	Viertel nö. Südbahnhof	426	366	483	874	394	362	87	447	1312		405	149	241	115	95	186	258	117	432	193	190	196	87	746	123	80	341	46	73	41	45	58	37	9005	10
11	Viertel zw. Isar u. Südfriedhof	553	510	546	1049	465	472	97	456	984	726		347	392	168	131	272	360	151	362	144	228	231	65	740	125	86	422	56	94	91	62	59	34	10478	11
12	Viertel zw. Museumsinsel u. Altstadt	548	415	354	847	370	308	88	256	653	203	332		548	134	123	225	145	82	193	110	132	181	64	289	65	59	258	32	63	69	61	40	22	7049	12
13	Viertel südl. Engl. Garten	836	221	270	1137	526	379	65	266	493	146	153	193		150	107	129	108	44	63	77	115	244	40	149	29	52	220	21	110	37	35	51	19	6485	13
14	N-Haidhausen	508	318	286	793	448	373	108	360	637	204	192	140	741		463	304	295	92	172	85	152	247	62	349	75	81	332	47	348	184	313	38	20	8787	14
15	Mittel-Haidhausen	509	344	331	747	448	401	69	443	726	248	283	199	476	658		461	436	121	203	77	184	226	55	442	110	79	368	61	212	213	397	43	32	9602	15
16	SW-Haidhausen	625	531	478	946	442	431	84	456	843	377	469	322	441	304	399		562	244	257	109	211	272	77	511	128	95	391	58	135	190	188	49	21	10654	16
17	Giesing	380	374	284	582	319	337	111	306	678	353	381	198	257	276	209	375		285	233	111	120	200	44	506	73	55	294	46	77	212	90	33	17	7816	17
18	Harlaching u. Menterschwaige	647	567	522	1013	568	639	99	430	1028	570	642	266	472	303	214	525	1230		407	123	216	286	47	766	120	99	415	86	148	176	38	42	28	12792	18
19	Unter- u. Mitter-Sendling	592	447	571	1178	591	692	65	690	1556	908	422	162	419	234	99	184	540	114		280	254	259	98	2163	310	111	477	82	82	135	52	50	77	13694	19
20	Viertel westl. Theresienwiese	278	216	296	606	281	403	86	722	1756	284	235	81	208	107	73	94	185	59	484		228	172	136	609	415	54	402	64	54	42	40	68	51	8789	20
21	O-Neuhausen	305	166	234	664	390	510	73	987	805	171	125	53	233	114	39	81	109	40	117	150		263	281	266	164	123	427	192	42	32	27	105	37	7325	21
22	O-Schwabing u. Freimann	575	239	327	1042	1053	725	121	428	713	204	166	66	537	189	60	104	102	57	143	71	265		83	284	92	444	1221	127	168	32	55	173	29	9901	22
23	Gern u. W-Neuhausen	559	282	340	1147	616	893	105	1186	1824	226	148	103	397	126	54	81	119	30	169	217	635	561		379	614	156	453	243	86	44	39	181	54	12007	23
24	Ober-Sendling, Solln u. Thalkirchen	290	212	201	447	373	418	69	291	372	396	237	88	178	150	62	100	183	112	717	151	113	182	46		107	39	161	47	71	51	31	16	58	6089	24
25	Laim	518	342	457	1115	567	799	84	946	2312	371	303	147	347	206	84	120	201	74	598	799	424	440	269	897		118	508	153	81	90	52	139	199	13760	25
26	W-Schwabing	494	228	361	1230	1188	1039	278	530	871	211	155	95	519	117	71	67	118	54	124	112	414	1140	100	295	107		1220	79	125	53	41	81	114	11556	26
27	N-Schwabing u. Milbertshofen	212	142	160	403	419	346	93	229	382	133	80	38	235	83	29	63	70	34	99	65	247	802	47	162	65	280		91	90	36	22	51	30	5228	27
28	Moosach	378	272	295	743	499	728	117	839	950	204	129	88	309	129	56	71	134	44	188	135	1074	439	470	401	240	157	1054		81	40	29	175	52	10516	28
29	Bogenhausen, Daglfing etc.	366	169	165	680	452	379	41	190	437	124	101	87	753	321	199	143	101	51	105	52	116	297	24	147	43	79	224	48		76	234	28	14	6246	29
30	Ramersdorf u. Perlach	329	240	248	518	319	317	49	210	480	155	187	130	316	252	419	222	347	78	144	49	123	215	26	273	58	46	214	43	165		262	27	21	6482	30
31	Berg am Laim u. Trudering	300	268	236	457	347	330	55	245	537	221	206	144	337	396	659	238	301	110	197	53	149	286	54	328	92	77	261	63	259	339		28	20	7793	31
32	Allach u. Feldmoching	103	65	95	201	149	178	74	204	389	56	44	34	78	10	17	36	26	33	58	45	200	230	100	123	93	59	1169	320	36	44	12		220	4501	32
33	Pasing u. Menzing	307	195	243	380	356	602	67	521	1255	172	107	69	251	50	54	58	75	67	342	238	215	265	151	564	529	66	184	90	38	33	41	284		8081	33
	Insgesamt	13260	8264	9882	24363	14450	15160	2901	14167	26445	7999	6501	3954	10258	5119	4334	4629	6616	2597	6743	4246	7881	9343	3094	13139	4355	3550	13419	2509	3182	2461	2436	2014	1274	260440	

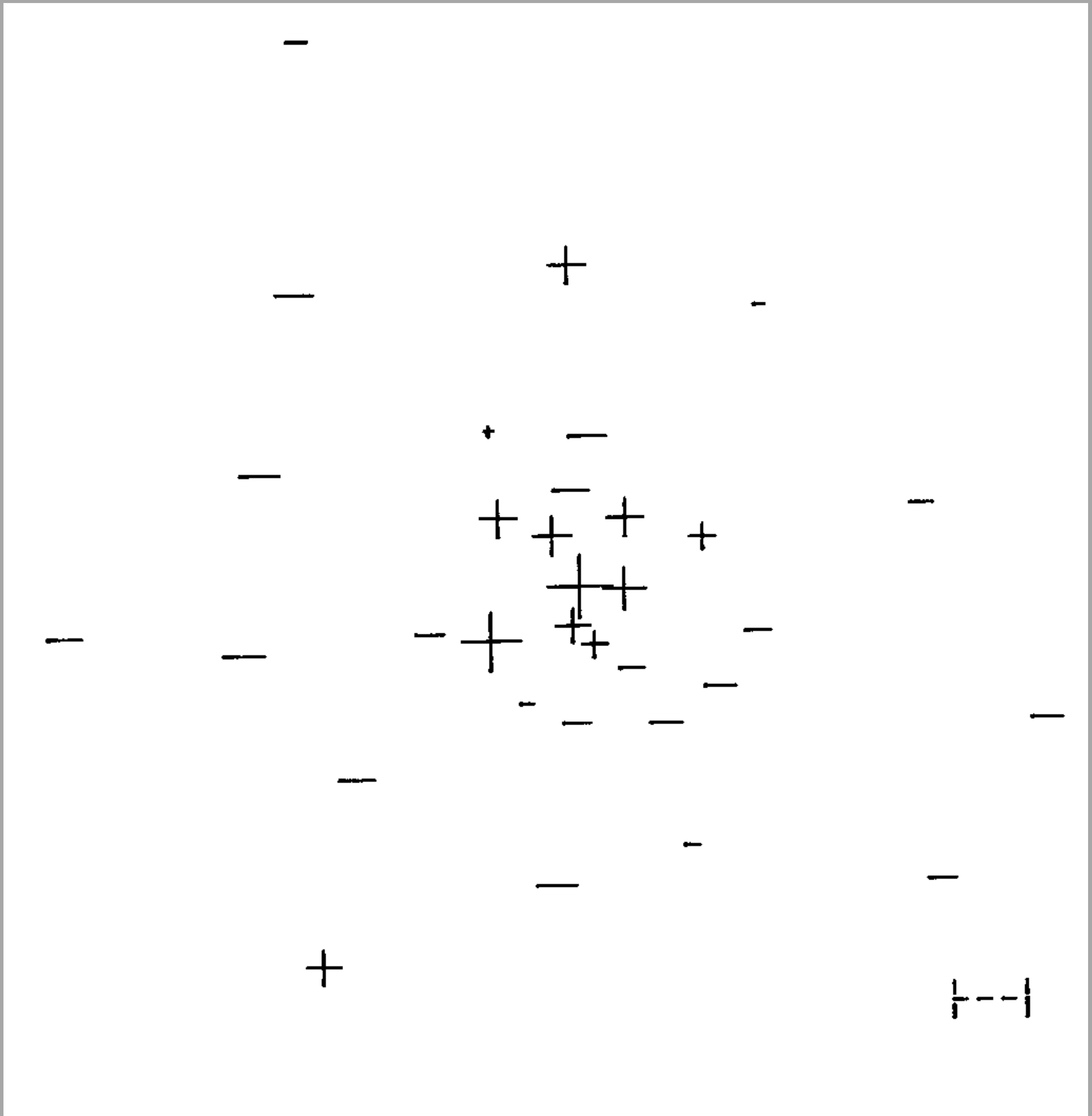
Adding across the table, the column marginals give the outsums (a.k.a. outdegree). Summing down the rows gives the insums (a.k.a indegree).

The ‘sending’ places (rows) are known as ‘sources’, and are shown on the map as negative signs.

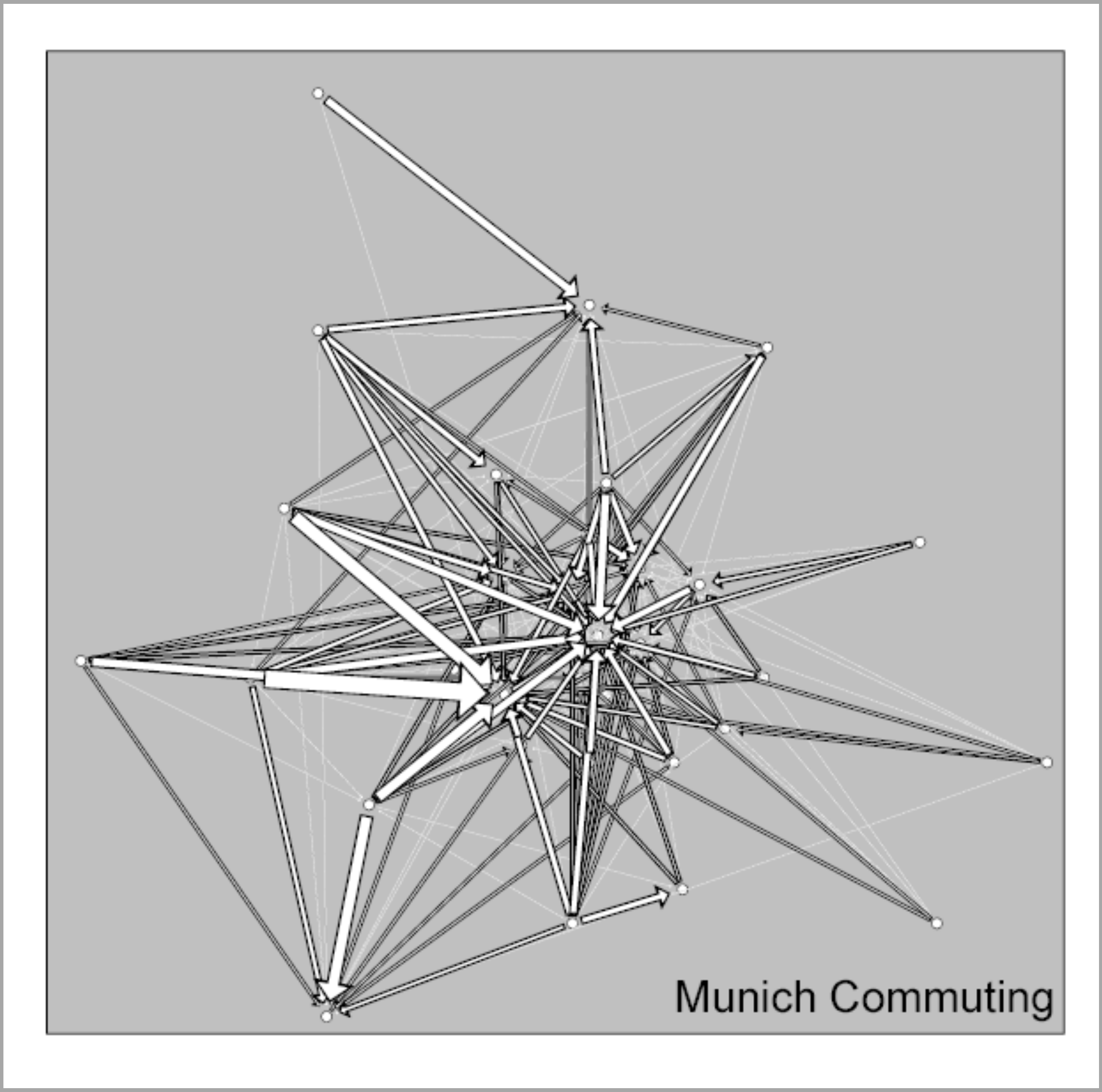
The ‘receiving’ places (columns) are the ‘sinks’ and are shown as plus signs.

The size of the symbol represents the magnitude of the movement volume.

Munich Commuting (1939)



1939 Net Communting



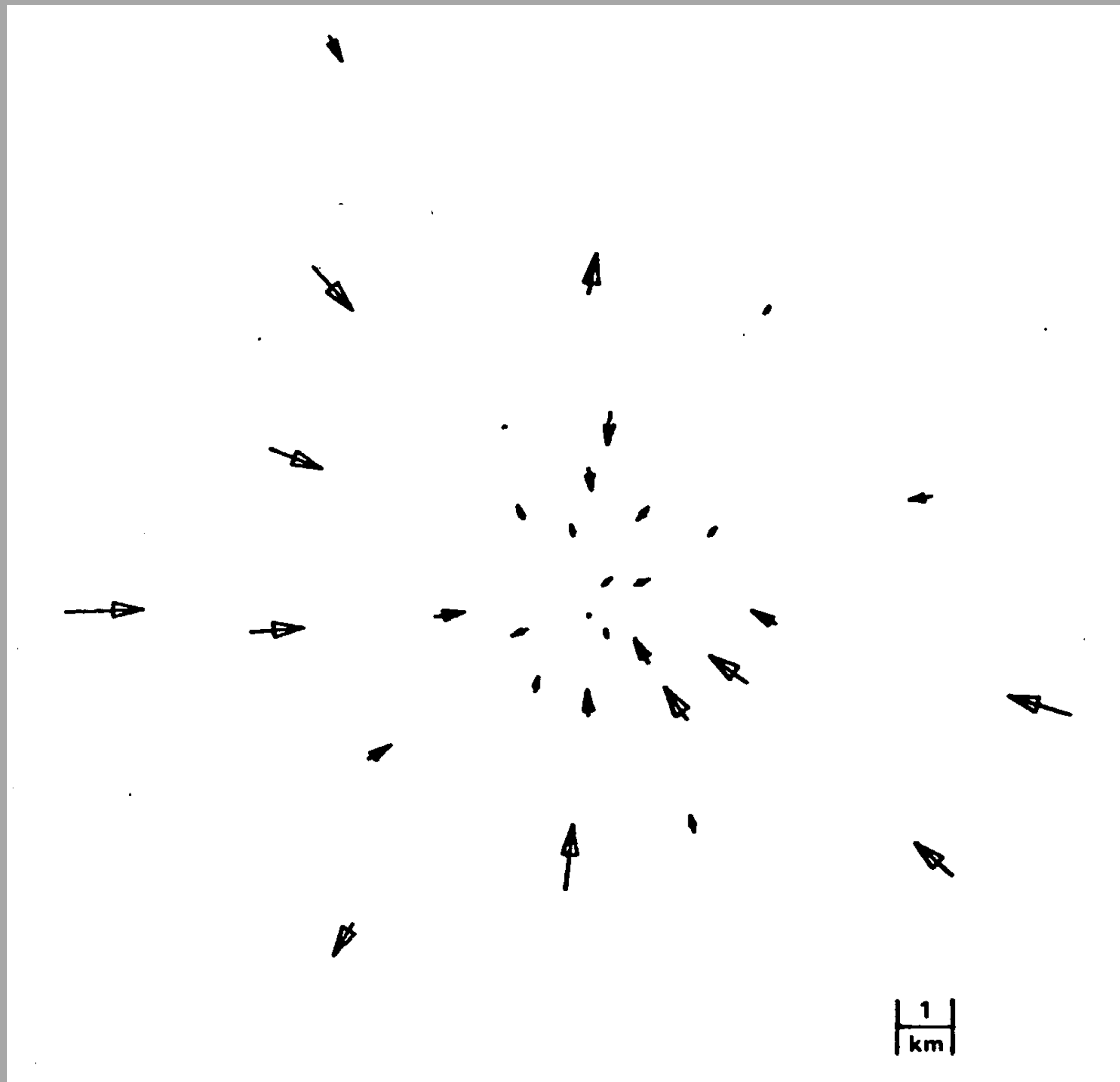
An alternative is a local vector to indicate the movement from each source location, showing the direction and magnitude of the net movement.

The computation is based on the asymmetry of the movement table.

Small directed vectors represent this movement on the next map.

Munich Commuting

Displacement vectors

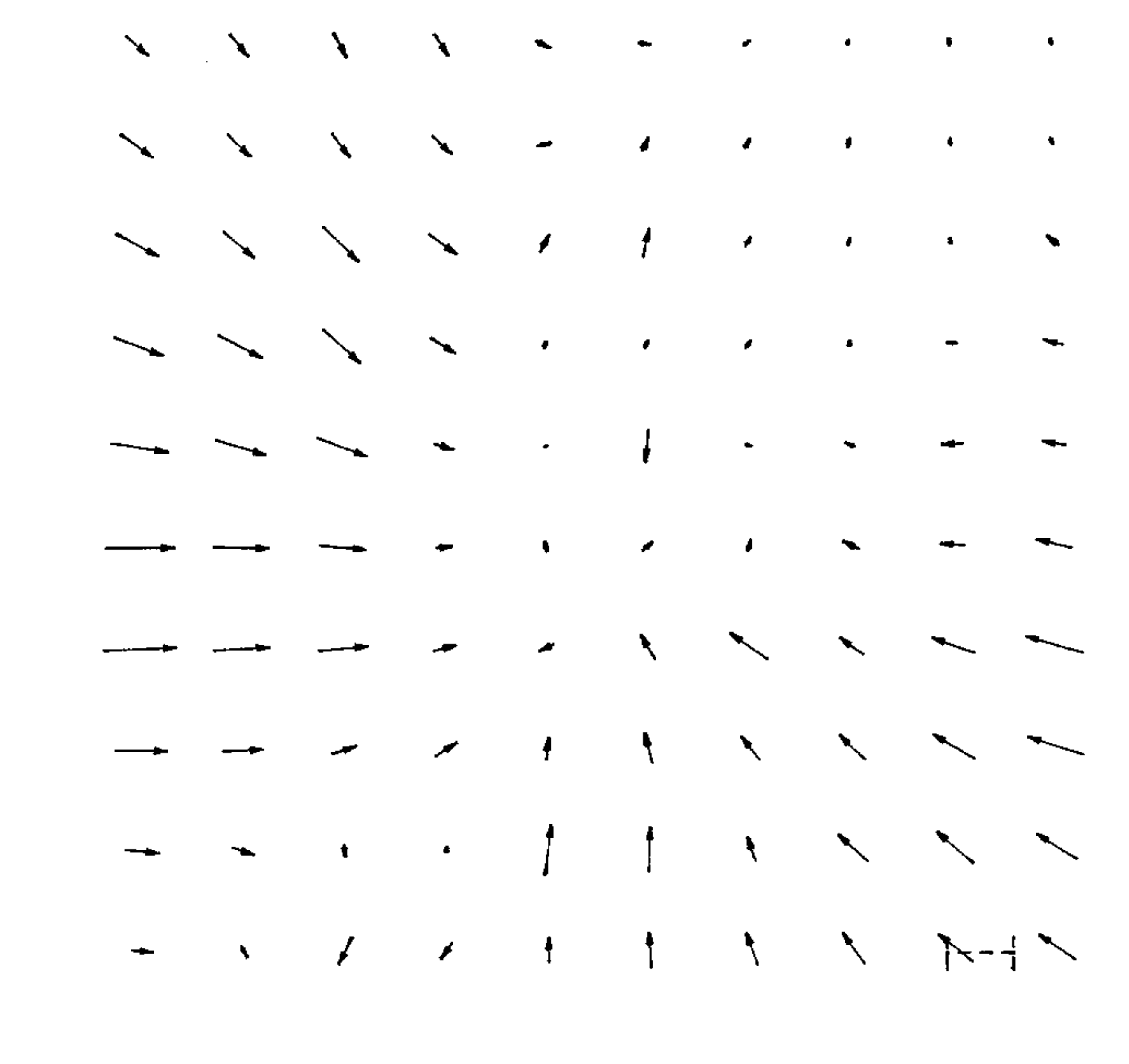


An interpolation is then performed to obtain a vector field from the isolated individual vectors.

This is done to simplify the mathematical integration needed to obtain the forcing function.

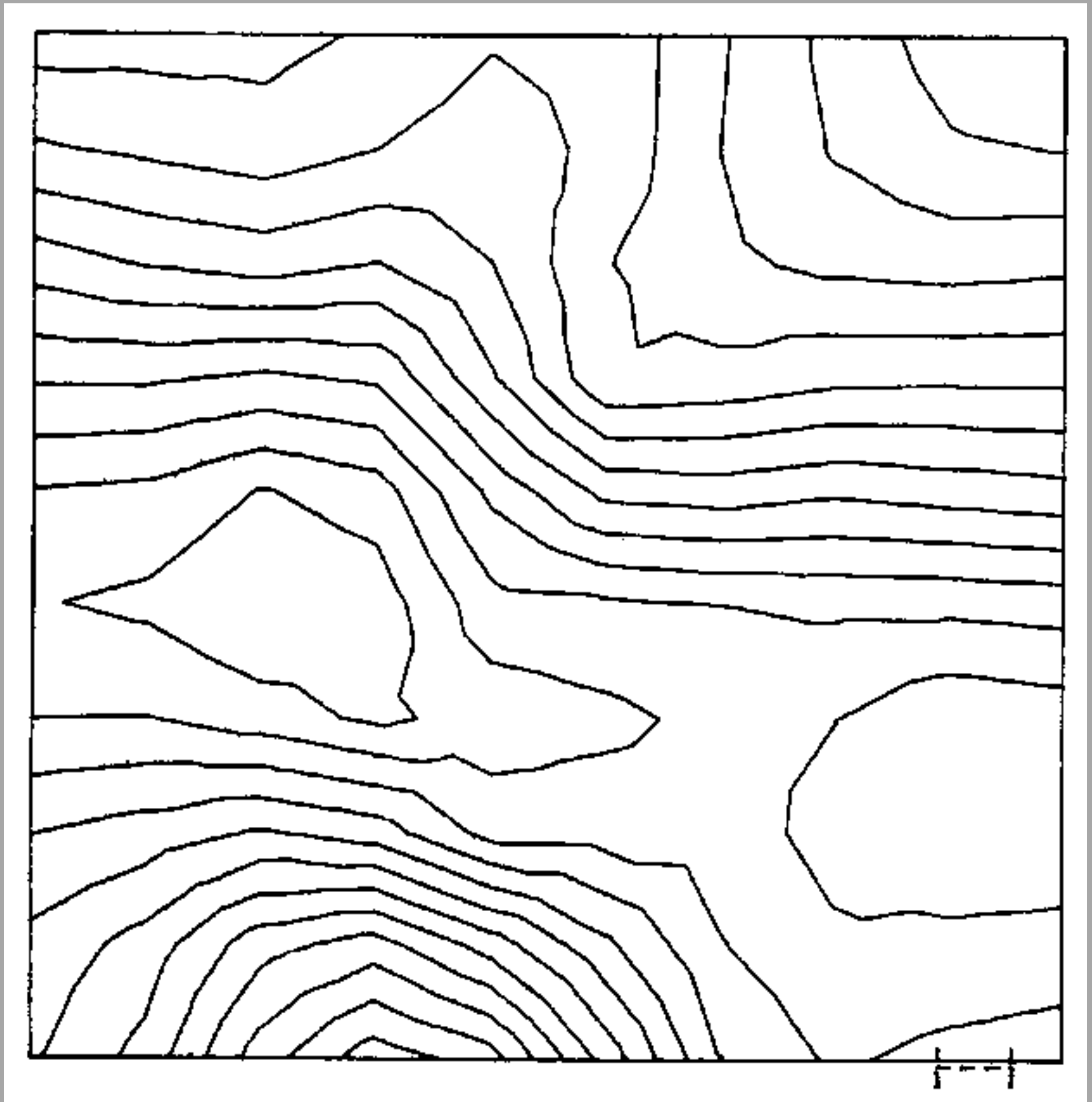
Interpolated Field

of displacement vectors



Computed Potential

based on the displacement vectors



The computed potential should have the vector field as its gradient.

This is a hypothesis that can be tested.

The base level of the potential is determined only up to a constant of integration.

The vector field, to be a gradient field, must be curl free. This can also be tested.

The attempt is now made to apply these ideas in a social space.

This can be considered a development of Lewin's *Topological Psychology* or his *Field Theory in the Social Sciences*.

The data represent citations between a small set of psychological journals. Larger citation tables are now also available.

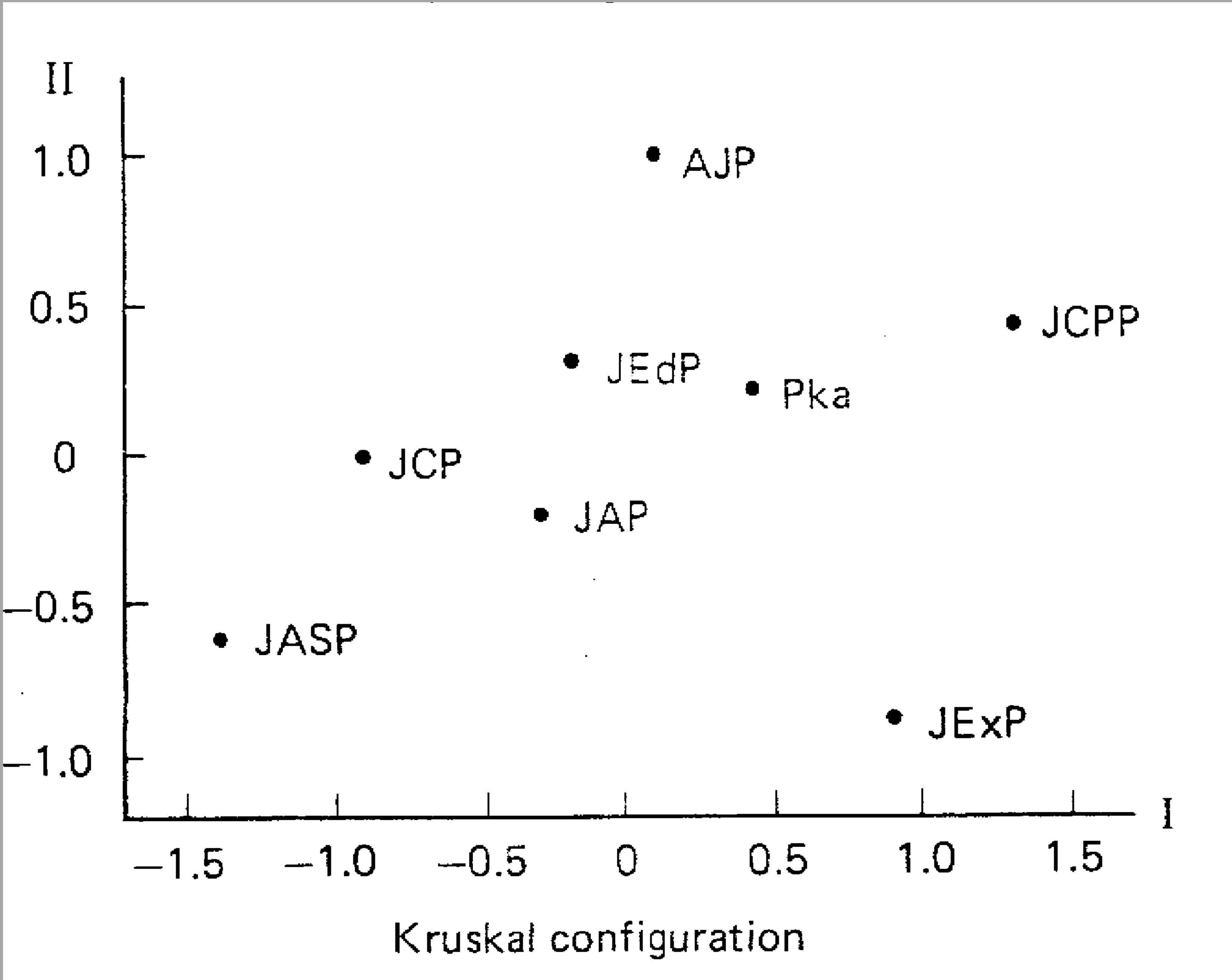
Citations among psychology journals

Coombs et al 1970

Data from 1964

	AJP	*JASP*	*JAP*	*JCPP*	*JCP*	*JEdP*	*JExP*	*Pka*	*Total*
American Journal of Psychology	119	8	4	21	0	1	85	2	240
Journal of Abnormal and Social Psychology	32	510	16	11	73	9	119	4	774
Journal of Applied Psychology	2	8	84	1	7	8	16	10	136
Journal of Comparative and Physiological Psychology	35	8	0	533	0	1	126	1	704
Journal of Consulting Psychology	6	116	11	1	225	7	12	7	385
Journal of Educational Psychology	4	9	7	0	3	52	27	5	107
Journal of Experimental Psychology	125	19	6	70	0	0	586	15	821
Psychometrika	2	5	5	0	13	2	13	58	98
Total	325	683	133	637	321	80	984	102	3,265

In Journal Space



To **Journal to Journal Citations**

From									X	Y	Net Flow
AJP	119	8	4	21	0	1	85	2	125	910	-85
JASP	32	510	16	11	73	9	19	4	-1382	-644	91
JAP	2	8	84	1	7	8	16	10	-261	-237	3
JCPP	35	8	0	533	0	1	126	1	1302	366	67
JCP	6	116	11	1	225	7	12	7	-924	-2	64
JEdP	4	9	7	0	3	52	27	5	-180	324	27
JExP	125	19	6	70	0	0	586	15	904	-924	-163
Pka	2	5	5	0	13	2	13	58	416	207	-4

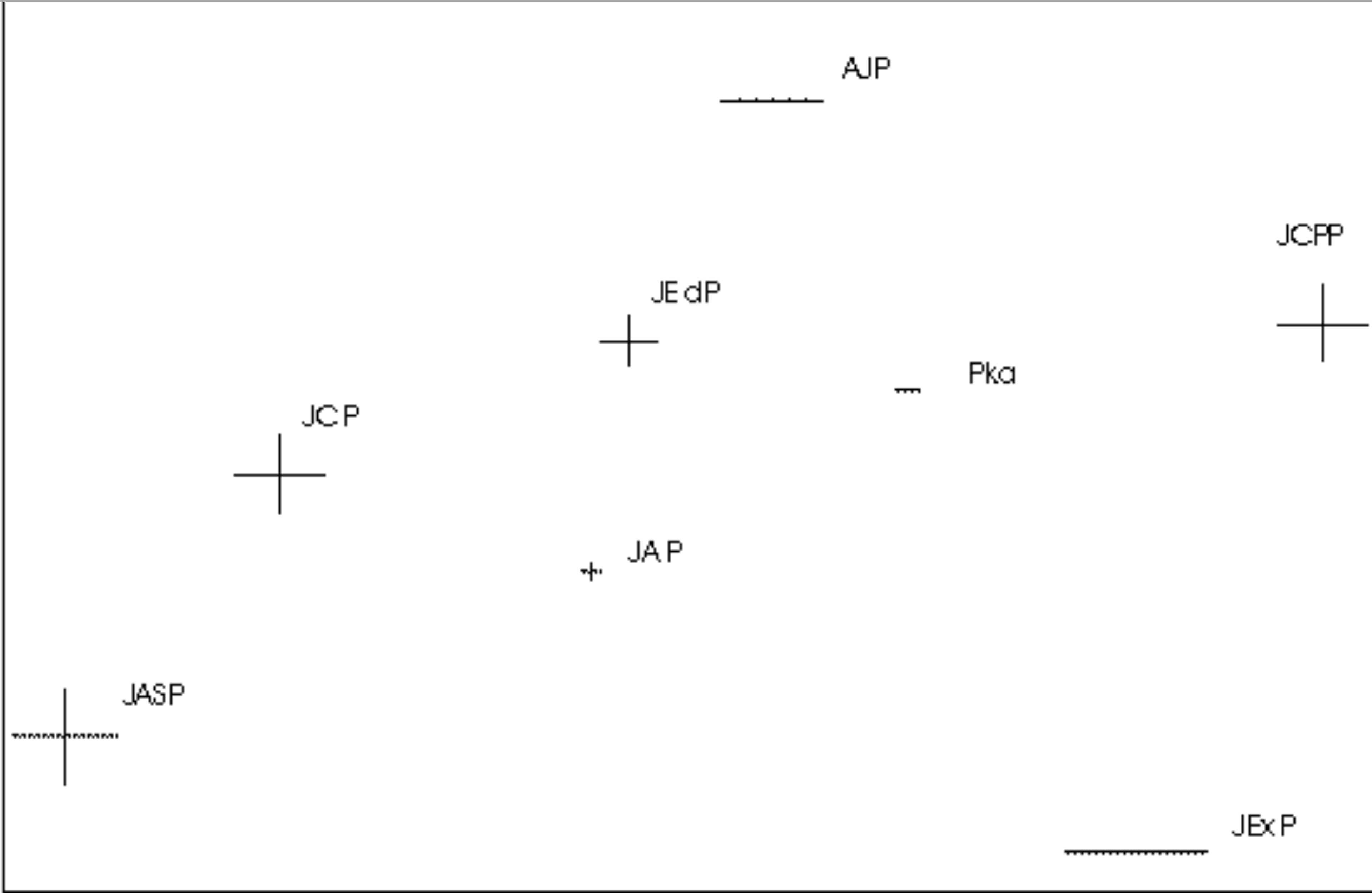
AJP Am J of Psychology
JASP J of Abnormal & Social Psychology
JAP J of Applied Psychology
JCPP J of Comparative & Physiological Psychology
JCP J of Consulting Psychology
JEdP J of Educational Psychology
JexP J of Experimental Psychology
Pka Pyschometrika

C. Coombs, J. Dawes, A Twersky, 1970, *Mathematical Psychology*, Prentice Hall, Engelwood Cliffs, NY, Pages 73-75

The table gives the being-cited journal across the columns. But the information can be considered to move from that journal to the citing journal.

Therefore the transpose is used to produce the source to sink map.

Journal Sources and Sinks



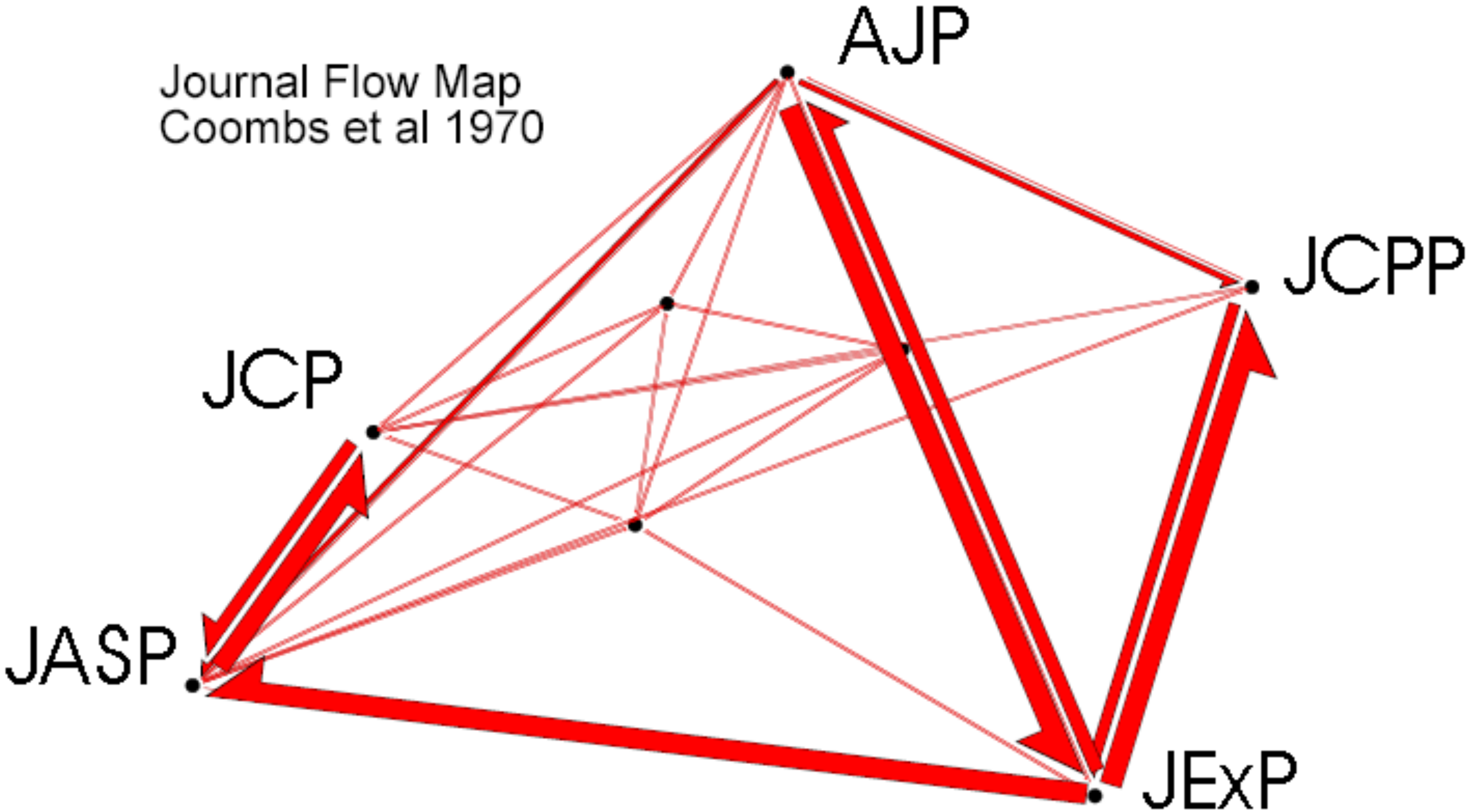
We now have an assignment problem. How to get 163 citations from JExP, 85 from AJP, & 4 from Pka to the 5 receiving journals, using only the marginals. There are obviously many possibilities

One solution is to use the "Transportation Problem" (Koopmans, Kantorovich, ~1949): Minimize M..d.., subject to $M_{.J} = O_I$, $M_{I.} = I_J$, $M_{IJ} >= 0$, given the distances computed from the coordinates and using the simplex method for the solution.

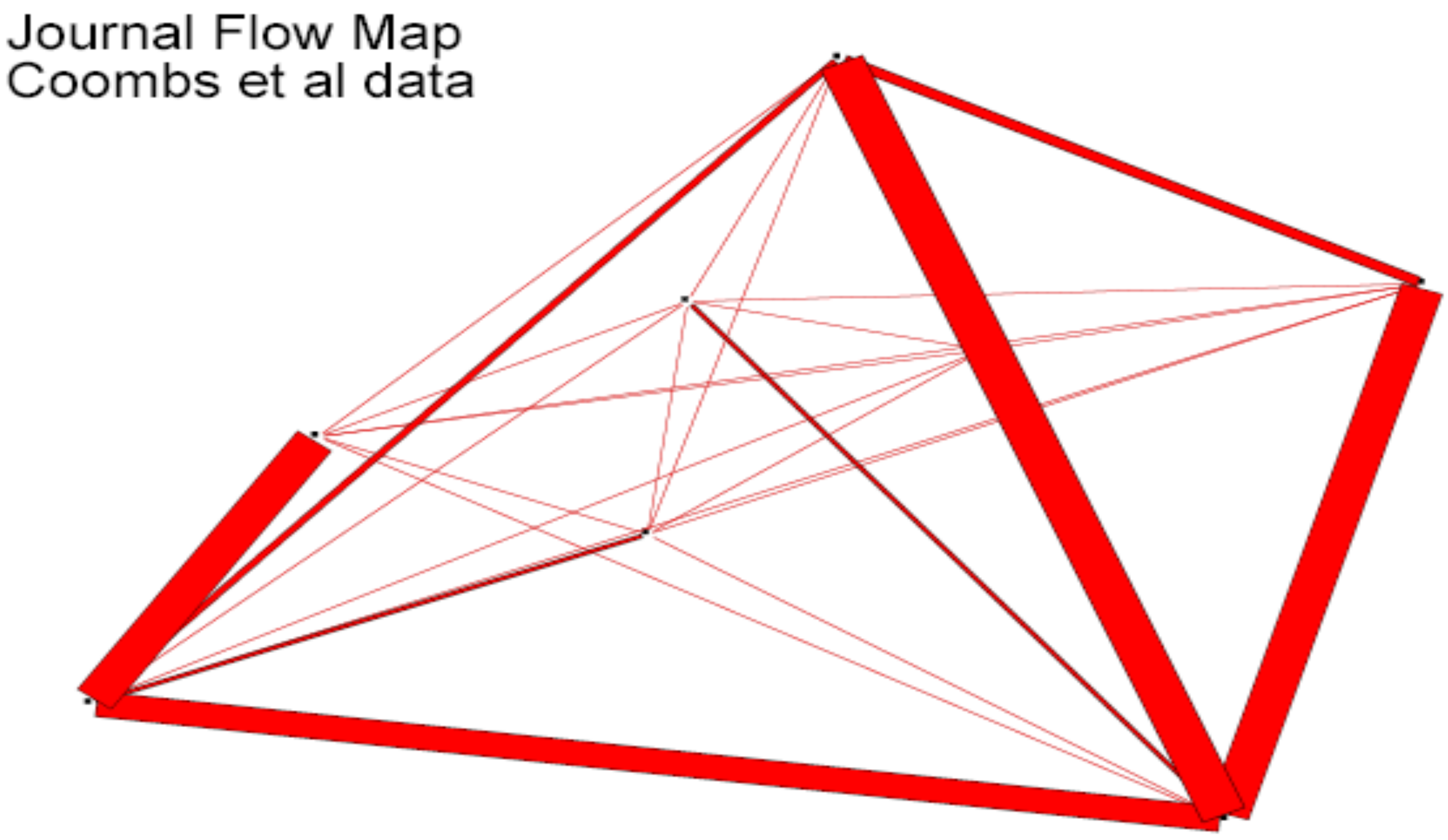
A more realistic solution is given by the quadratic transportation problem: Minimize M^2..d.., subject to the same constraints.

Both of these solutions result in discrete answers, and 'shadow prices'. We are looking for a spatially continuous solution that allows vectors and streamlines, in order to determine spatial flow fields and a continuous potential.

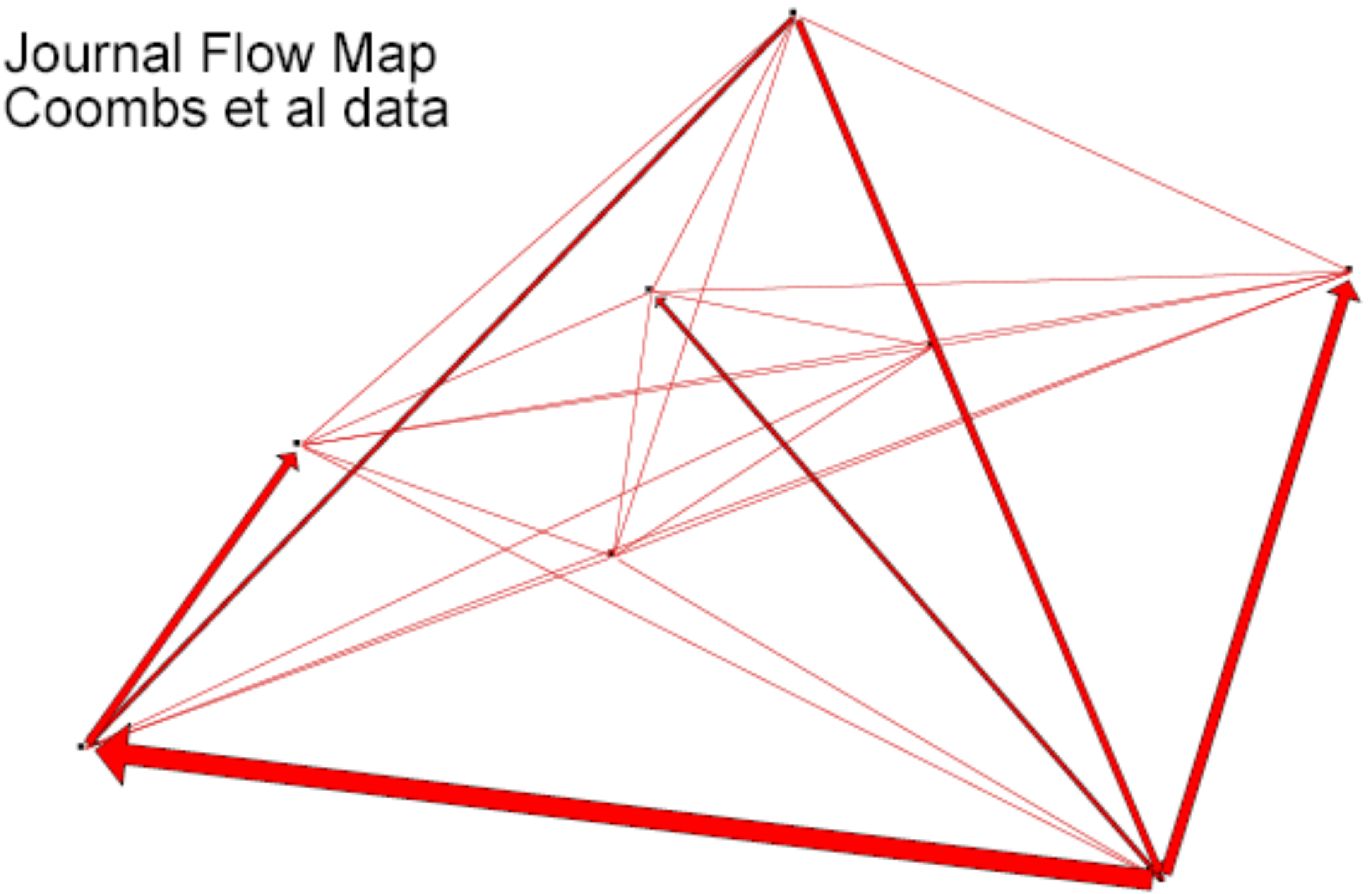
The observed two-way flow between the journals



The total flow between the journals



The net flow between the journals



The next step is to compute the displacements between the cited journals.

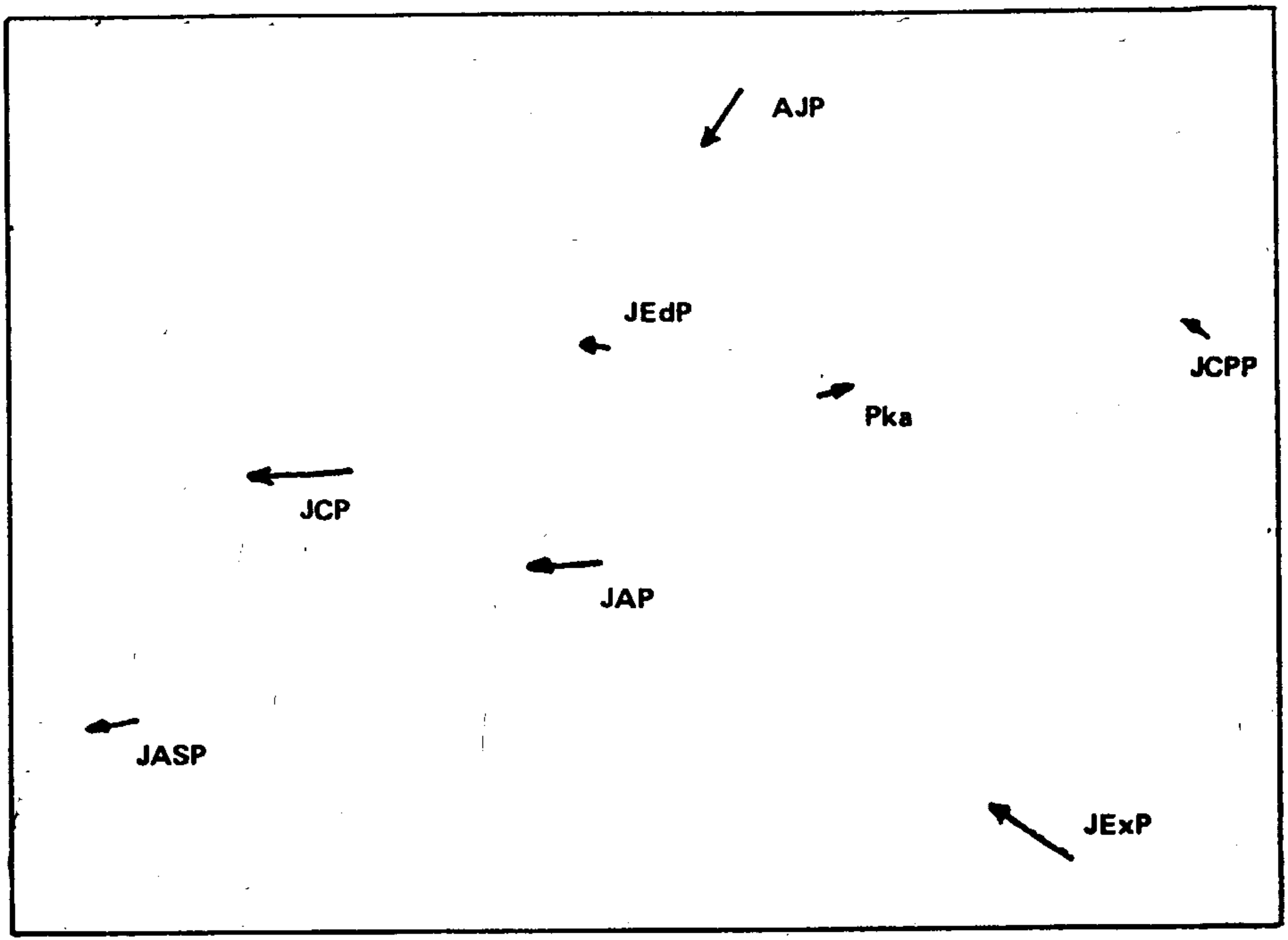
This is based on the asymmetry of the citations table.

The fundamental idea being that there exists a 'wind' making movement easier in some directions.

The mathematical details are given in a published paper.

W. Tobler, 1976, "Spatial Interaction Patterns", *J. of Environmental Systems*, VI(4):271-301

Displacement between Journal Citations

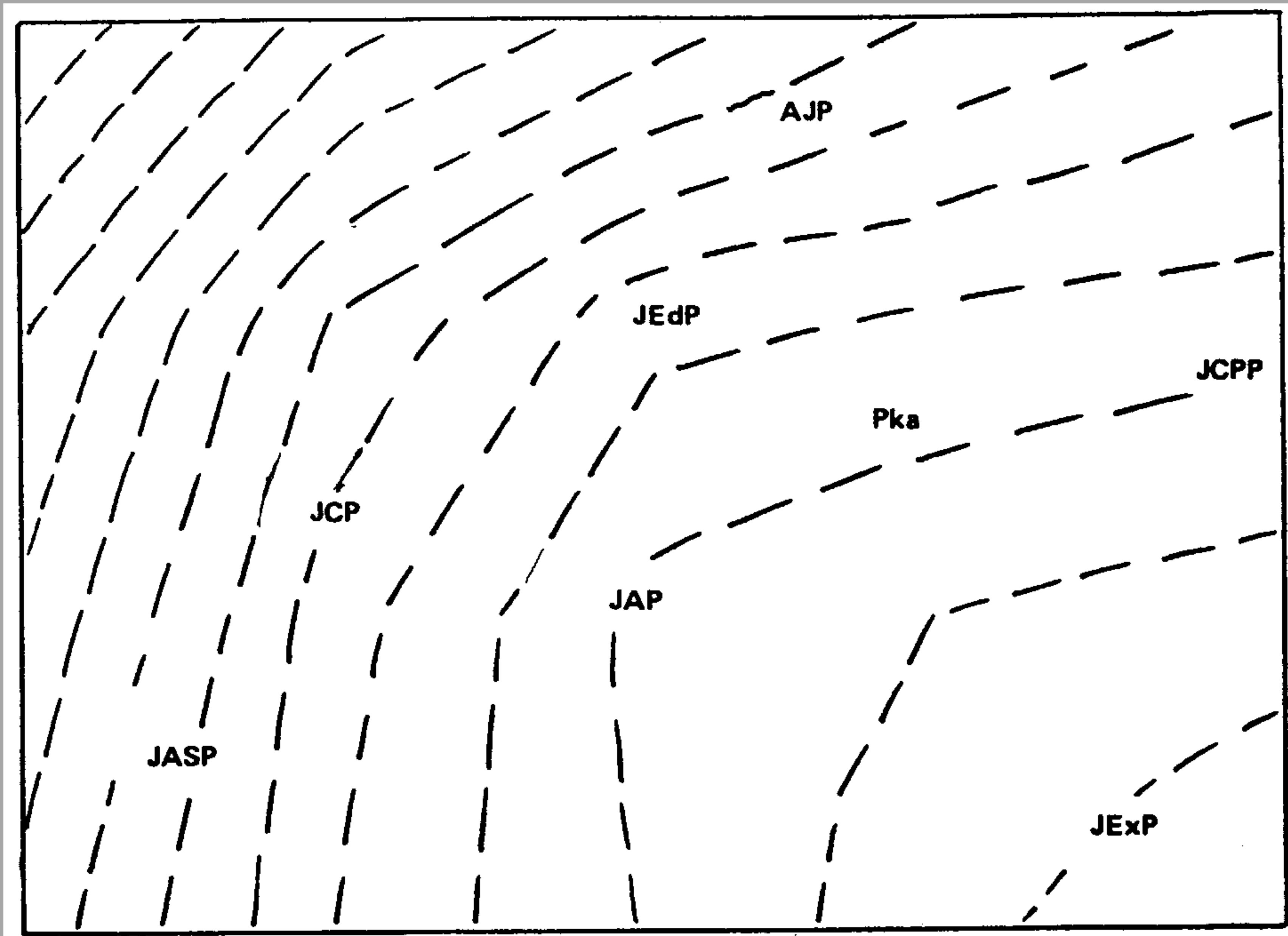


Then the potential is computed by integration.

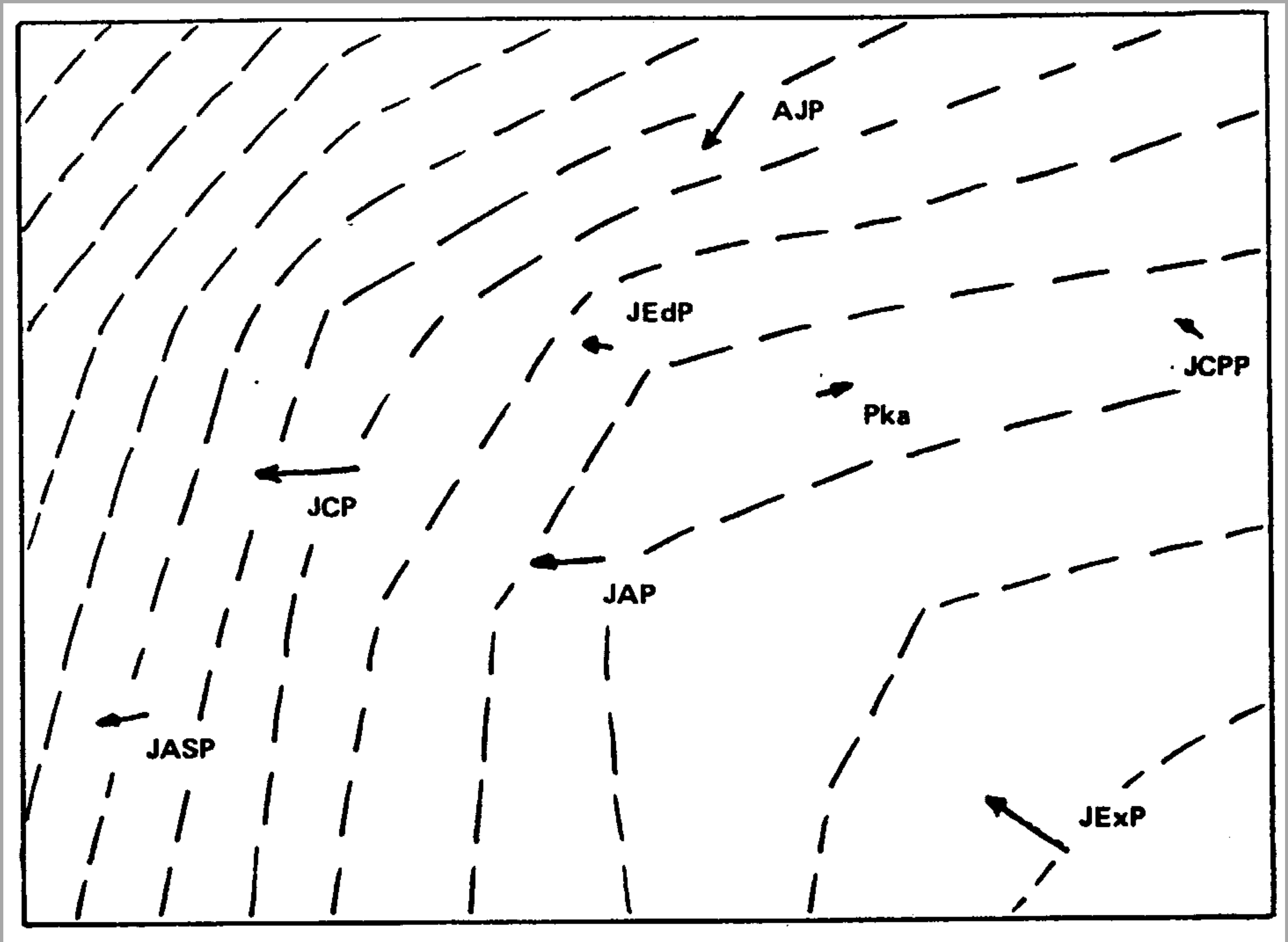
This potential should be such that its gradient coincides with the displacement vectors.

It may be necessary to use an iteration to obtain this result.

Journal Potential Function



Flow and Potential between Psychological Journals



Some questions

Suppose a new psychological journal were started. Where should it be inserted into in this space?

Does it make sense to treat journal citations as being located in a continuous two-dimensional social space?

Can other social data be treated in a similar fashion, for example social mobility tables?

And more general network data?

CONCLUSION

I have given some speculative thoughts on how one might represent network relations with vectors, fields, and scalar potentials in a continuous social space.

Still needed are error estimates.

Your comments are desired.

Thank you for your attention.

http://www.geog.ucsb.edu/~tobler

References

K. Boyack, 2004, XXX, *Proceedings*, National Academy of the United States, 101, Supplement 1, (April 6): 5192-5199.

C. Coombs, J. Dawes, A. Twersky, 1970, *Mathematical Psychology*, Prentice Hall, Englewood Cliffs, NY.

D. Fliedner, 1962, “Zyklonale Tendenzen bei Bevölkerungs und Verkehrsbewegungen in Städtischen Bereichen untersucht am Beispiel der Städte Göttingen, München, und Osnabrück”, *Neues Archiv für Niedersachsen*, 10:15 (April 4): 277-294 (Table 2, p. 281, map following p. 285).

K. Lewin, 1936, *Principles of Topological Psychology*, McGraw Hill, New York

K. Lewin, 1951, *Field Theory in the Social Sciences*, Harper, New York.

W. Tobler, 1976, "Spatial Interaction Patterns", *J. of Environmental Systems*, VI (4) 1976/77, pp. 271-301.

W. Tobler, 1981, "A Model of Geographic Movement", *Geographical Analysis*, 13 (1): 1-20.

W. Tobler, 1996, “A Graphical Introduction to Surveying Adjustment”, *Cartographica*, 33-42.

S. Wasserman, Faust, K., 1994, *Social Network Analysis: Methods and Application*, Cambridge University Press, Cambridge.

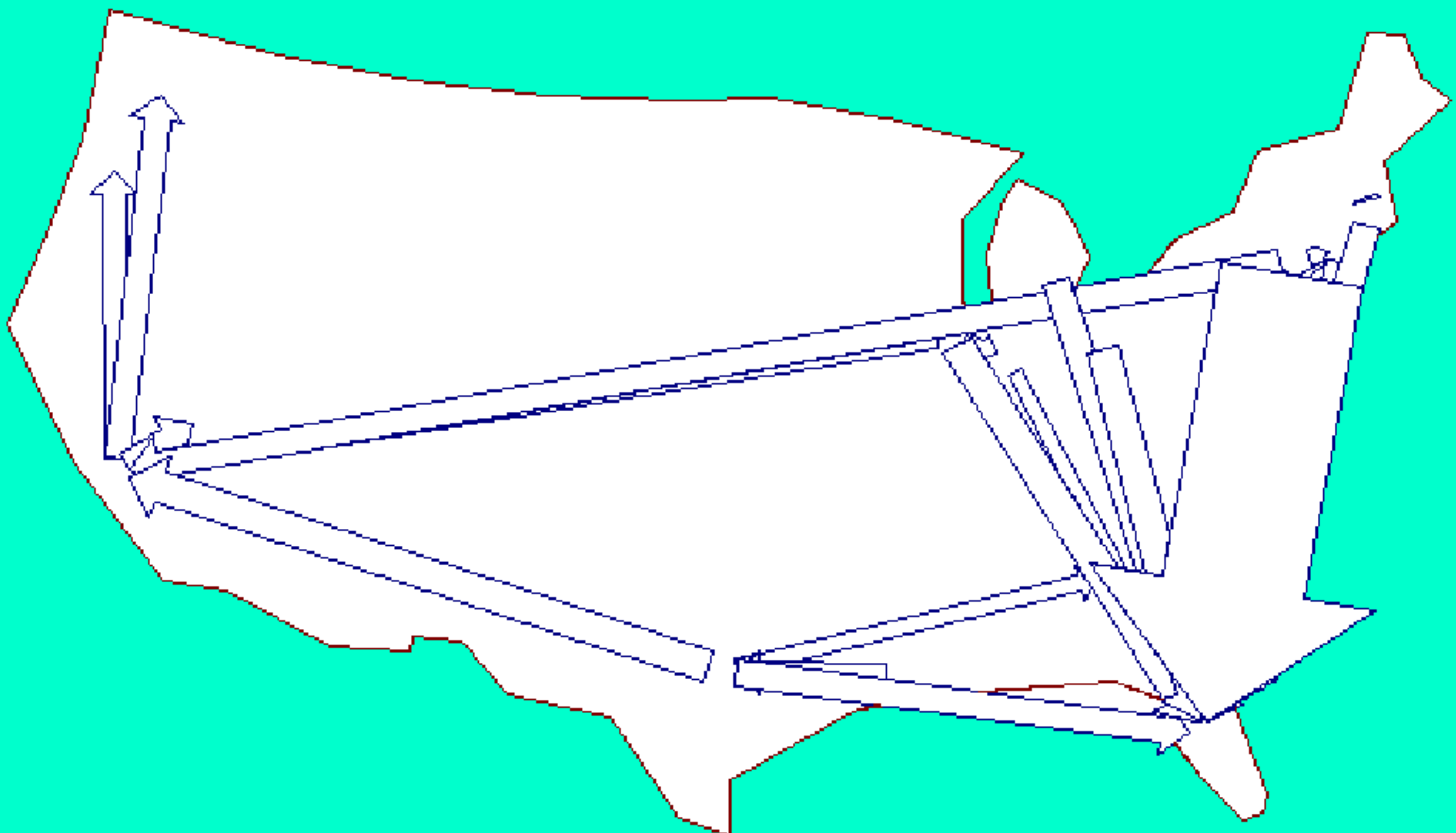
Another geographic example as motivation

This was not presented at Sunbelt XXV

The conventional net movement map

Based on movement between state centroids

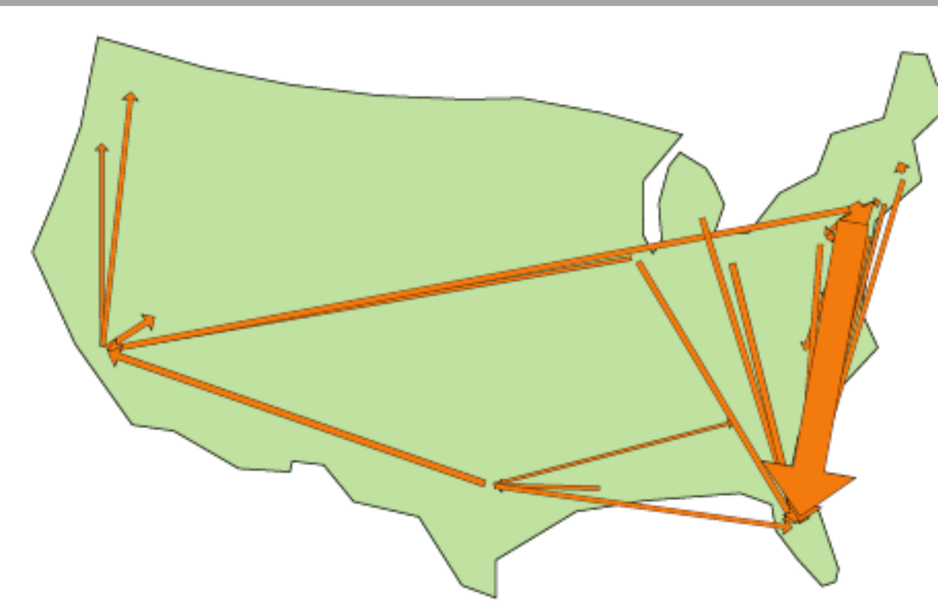
(Computer sketch. Optimum deletion: values below mean ignored)



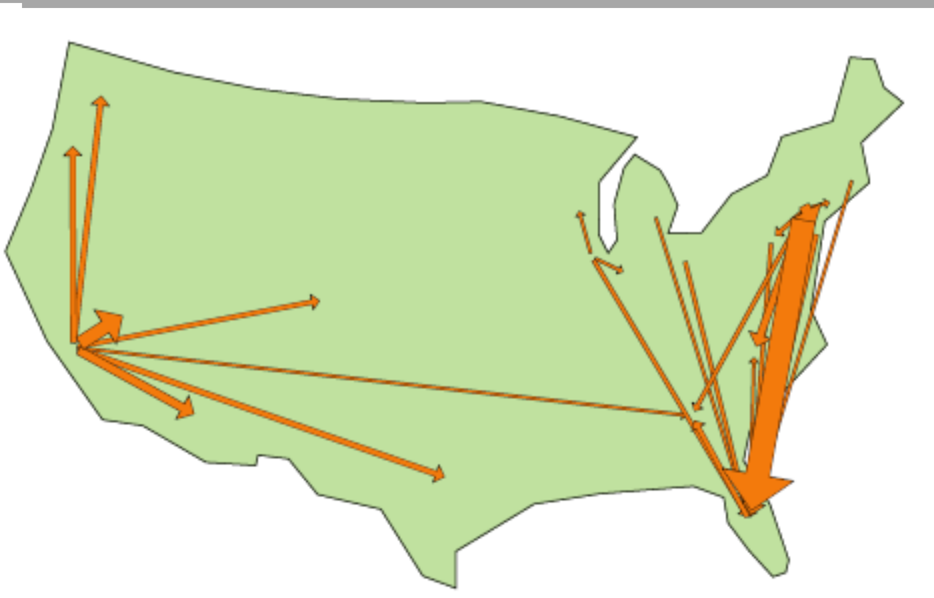
Net Migration in the United States

These patterns persist for a long time

1985-1990



1995-2000



Several ways of representing geographic space in a continuous fashion for migration (or other) studies are:

Writing scalar values as continuous functions of latitude and longitude (or rectangular or polar plane coordinates), perhaps estimated by least squares, as two dimensional algebraic or trigonometric polynomials, splines, eigenfunctions, or spherical harmonics or wavelets. This can be considered as an elaboration of spatial trend analysis. See:

W. Tobler, 1969, Geographic filters and their inverses, *Geog. Anal*., 1:234-253

W. Tobler, 1992, Preliminary representation of world population by spherical harmonics, *Proc. Natl. Acad, Sci USA* , 89: 6262-6264.

Writing vector fields, or interaction data, in a similar fashion as a four dimensional spline or polynomial function of the origin & destination location coordinates. See:

P. Slater, 1993, “International Migration & Air Travel: Smoothing & Estimation” *Appl. Math. & Comp*., 53: 225-234

Expanding regression coefficients in a geographically weighted manner. See

J. Jones, E. Casetti, 1992, *Applications of the expansion method*, Routledge, London

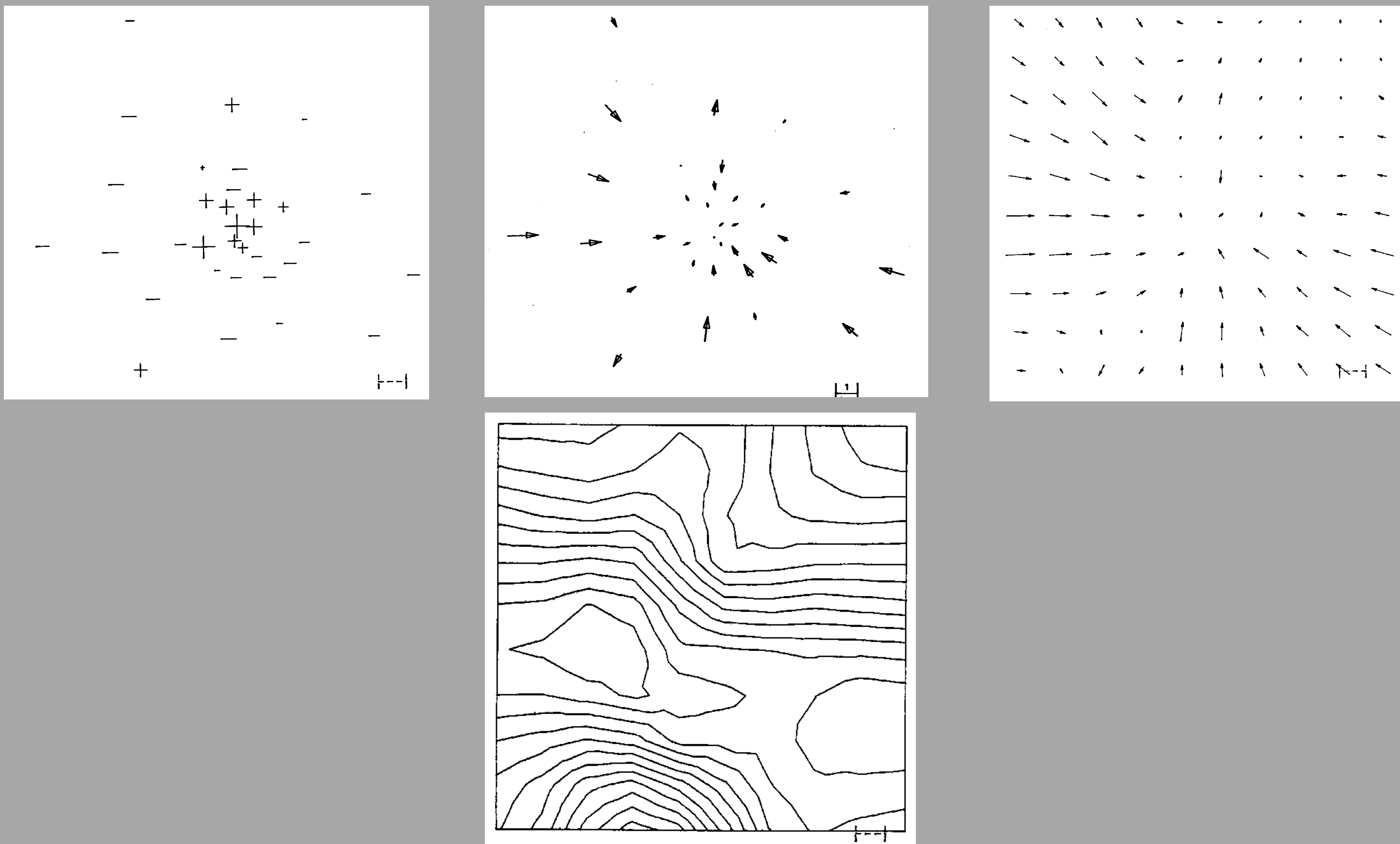
S. Fotheringham, et al, 2002 , *Geographically weighted regression*, Wiley, Chichester.

Approximation by a two dimensional lattice, as in the present study.

You have just seen this simple example

Commuting in Munich 1939

Left to right: from places (sources -) and to places (sinks +), vectors, and interpolated vectors, and the implied potential field



In the previous slide the source places (origins) were used to make a set of vectors pointing towards the sinks (destinations). These were then interpolated to obtain a field of vectors. Integration (in the mathematical sense) was then used to construct a potential field, shown by contours. The magnitude and direction of the vectors correspond to the gradient of the potential surface.

In the next example something similar is done. Except that the model is set up on the basis of interpolating the sources (out-flows) and sinks (in-flows) for the contiguous US. Then the potential is computed directly. The gradient vector field is obtained from this potential field.

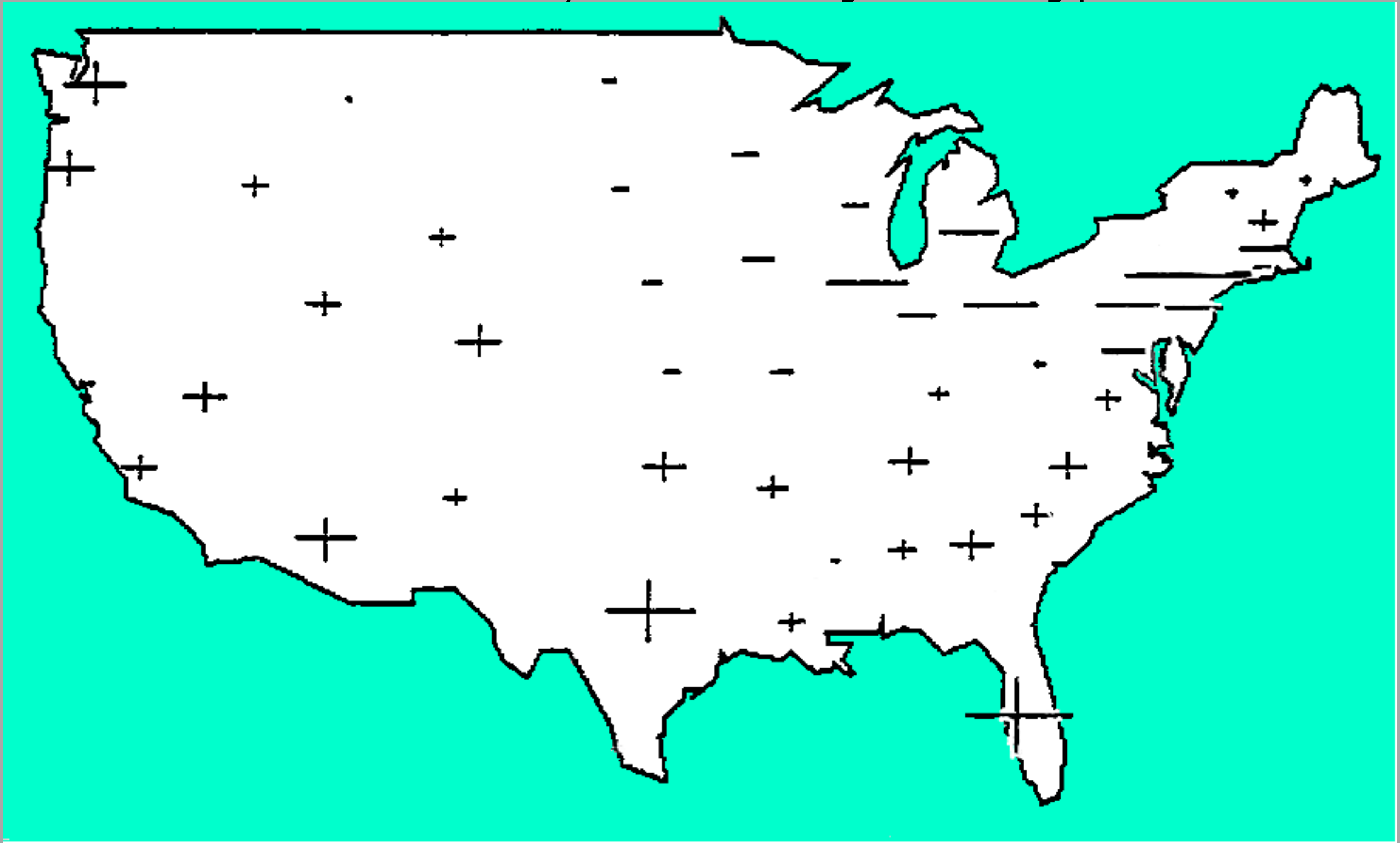
To carry out this operation first assign the in-migration and out-migration totals to each state. Then 'rasterize' the region of interest into a large set of equally spaced nodes and spread the population change over the nodes in each state. This allows the treatment to approximate a **continuous** migration surface and is illustrated on the next slides. This is one of several ways to treat geographic space in a continuous fashion.

Gaining and Losing States

Based on the marginals of a 48 by 48 migration table,

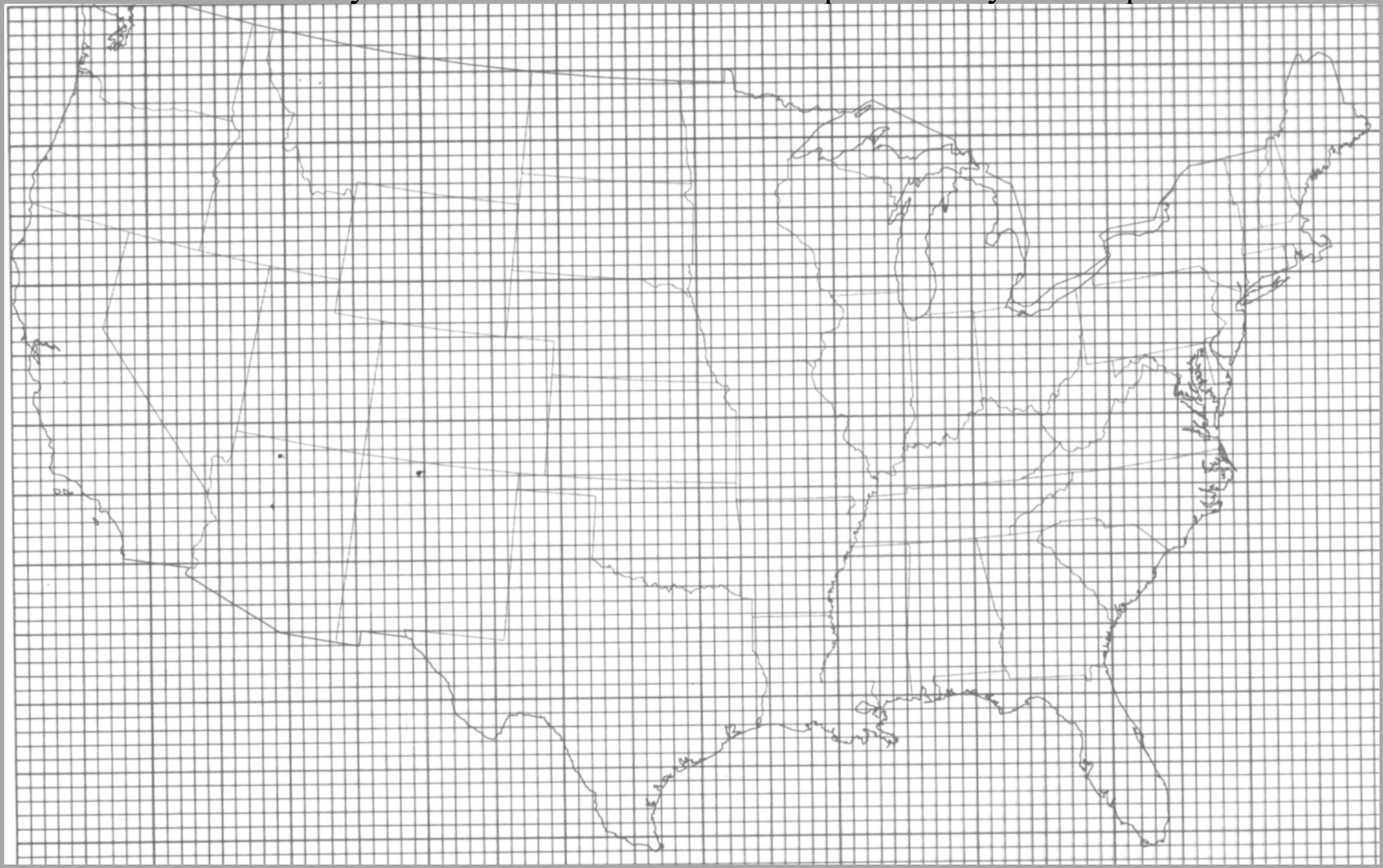
1965-1970 data.

Sketch in the boundary between leaving and arriving places.



“Rasterize” the USA to form a lattice.

Use a point-in-polygon program to assign nodes to individual states. Then assign in and out values to these nodes. There will be one equation for each node on this raster.
Then solve the system of ~6000 simultaneous equations to yield the potential.



In the U.S. example both the in-migration and the out-migration amounts were spread over all of the nodes making up each of the individual states.

Pycnophylactic reallocation was used to do this.

As a related item, world population estimates are now available by fine geographic (lat/lon) quadrangles.

Why does the census not release migration data in this format, by latitude and longitude quadrangles?

If that were done then the spherical version of the model to be described could be used directly.

Studies of urban commuting can also benefit from data recorded in a raster format instead of irregularly shaped traffic zones.

W. Tobler, 1997, "Movement Modeling on the Sphere", *Geographic and Environmental Modeling*, 1(1): 97-103.

Now we need to derive the continuous version of the Push-Pull model.

In the discrete case there is one equation for every pair of places:

$$M_{ij} = (R_i + E_j) / D_{ij}$$

obtained by solving the simultaneous pair for the Lagrangians:

$$R_i \sum_{j-1}^{c} 1/ D_{ij} + \sum_{j=1}^{c} E_j / D_{ij} = 2\ O_i$$

$$\sum_{i=1}^{r} R_i / D_{ij} + E_i \sum_{i=1}^{r} 1/ D_{ij} = 2\ I_j$$

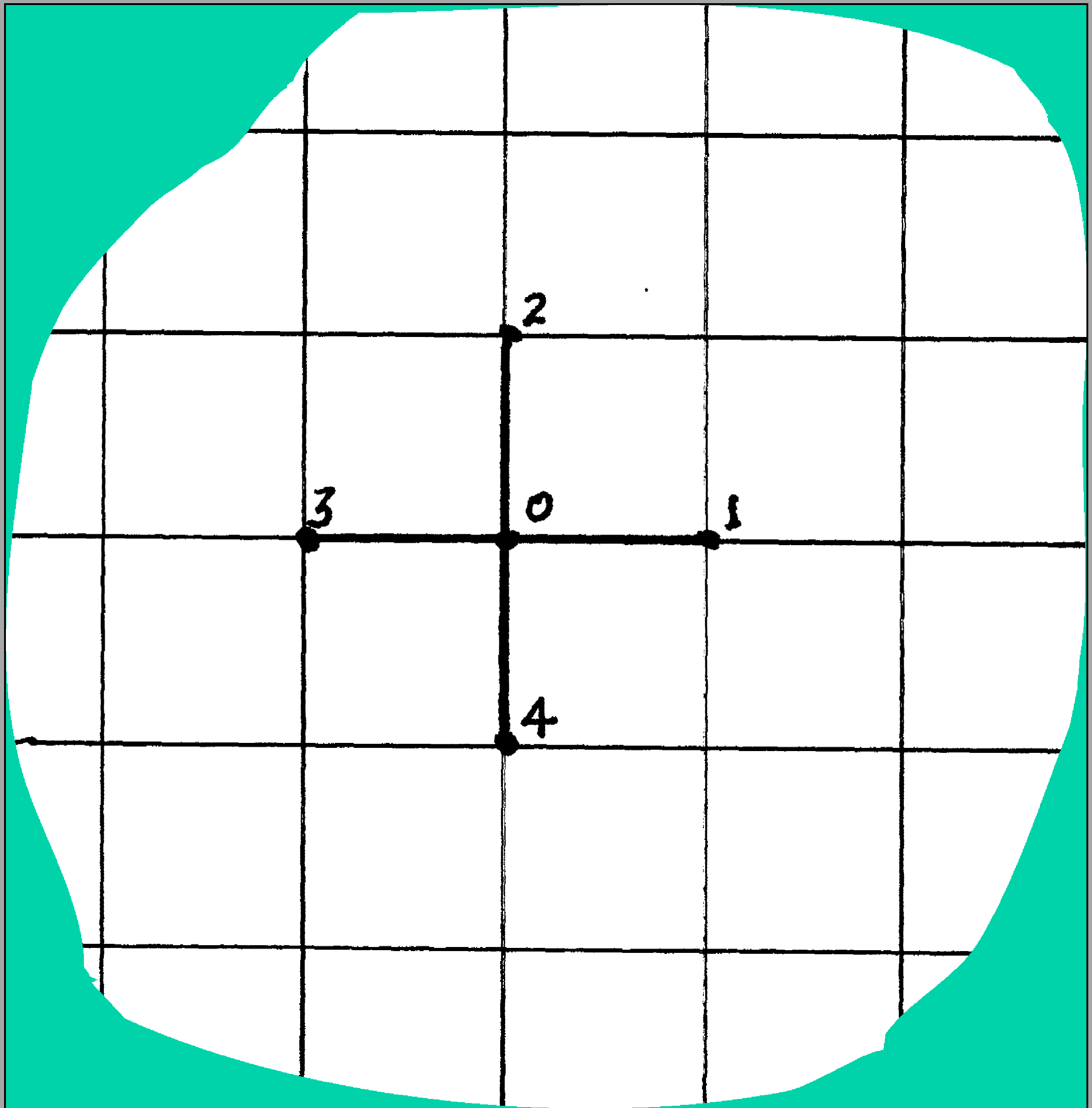
The E ('pulls') and R ('pushes') are the Lagrangians.

These simultaneous equations are solved for the pushes and pulls.

Also obtained were the 'Attractivity' A = E - R and the 'Turnover' T= E + R.

In the raster look at one node and its neighbors

A raster is a special kind of network where movement takes place between neighboring nodes



Derivation of a continuous model for the grid

In the push-pull model $M_{ij} = (R_i + E_j) / D_{ij}$

For the square mesh take all D_{ij} to be the same. Set them equal to 1.

Use the subscript 0 for the center node, and index the neighbor nodes from 1 to 4

Then the moves **from** the center to the neighbors is

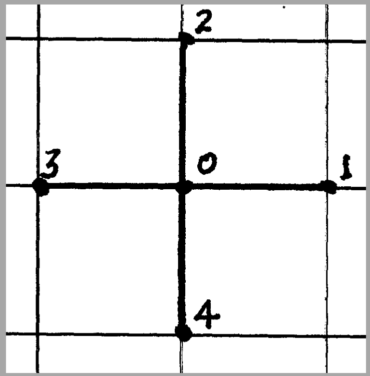
$M_{01} = R_0 + E_1$

$M_{02} = R_0 + E_2$

$M_{03} = R_0 + E_3$

$M_{04} = R_0 + E_4$

$M_{0j} = 4\,R_0 + E_1 + E_2 + E_3 + E_4$



But M_{0j} are the moves out of node 0, and this is O_j the outsum.

In the same way $M_{10} = R_1 + E_0$, etc for M_{20}, M_{30}, M_{40}.

These are the moves **into** node 0 from the neighbors, and this is I_i.

Thus the pair of equations become

$O_j = 4\,R + E_1 + E_2 + E_3 + E_4$

$I_i = 4\,E + R_1 + R_2 + R_3 + R_4$

after dropping the subscript for the central node.

There is one pair of equations for each node.

(A non-regular set of neighbors could also be used)..

An aside:

Incorporating differential transport disutilities into the model.

From the previous slide we can insert a differential transport weight factor into the movement, as follows:

$M_{01} = R_0 + E_1$ becomes $= (R_0 + E_1)/W_{01}$ where W_{01} is the equivalent of d_{01} but more realistic (for example road distance, or travel time or cost). Then similarly for all M_{0j}.

Now do the same for M_{10} inserting a W_{10}, etc. Recognize that W_{01} is not the same as W_{10} and that the weights will be different across every edge, and that they may change rapidly with time. Adjacent cells will naturally have two common, but differentially directed, link values.

It might be helpful to draw and label weights for a system of nine cells.

Doing this naturally leads to a rather more complicated system of equations.

(aside continued)

As a result:

$$R_0 = [O_J - (\Sigma\, E_k/w_{0k})] \,/\, \Sigma\, 1/w_{0k}$$

$$E_0 = [I_I - (\Sigma\, R_k/w_{k0})\,] \,/\, \Sigma\, 1/w_{k0}$$

The summations are over $k = 1$ to 4

All w_{pq} and I_i and O_j are assumed known.

The same set of equations hold for all cells except those on the borders of the region.

Known are 2 w' s per edge + 2*p*q - 1 in and outsums (I' s and O' s) minus 4*(p + q) (Dirichelet or Neumann) values at the edges.

Unknown are 2*p*q pushes (R 's) and pulls (E' s).

Can this system be solved for all R' s and E' s?

The distance values D_{ij}, as constants, have been dropped in the square mesh, for pedagogic purposes, but not a mathematical necessity.

Each place, except along the margins of the region, will have four neighbors.

Just derived were the two equations at each node:

$$4E = I - (R_1 + R_2 + R_3 + R_4), \qquad 4R = O - (E_1 + E_2 + E_3 + E_4).$$

The central E and R require no subscript; their neighboring locations are indexed from one to four - or if you wish - North, South, East, and West directions.

Now add - 4R to both sides of the first equation and - 4E to both sides of the second, rearrange slightly, and using T = E + R, to obtain

$$R_1 + R_2 + R_3 + R_4 - 4R = I - 4T, \qquad E_1 + E_2 + E_3 + E_4 - 4E = O - 4T,$$

The left-hand sides are recognized as finite difference versions of the Laplacian. Thus we can write, approximately and for a limiting uniform fine mesh, the pair

$$\partial^2R/\partial u^2 + \partial^2R/\partial v^2 = I(u,v) - 4T(u,v),$$

$$\partial^2E/\partial u^2 + \partial^2E/\partial v^2 = O(u,v) - 4T(u,v),$$

assuming that R and E are differentiable spatial functions and that I and O are continuous densities given as functions of the Cartesian coordinates u and v.

Now, making use of the continuous movement model.

http:/www.geog.ucsb.edu/~tobler/presentations/A Flow Talk.pps

In this continuous model, we have a coupled system of two simultaneous partial differential equations covering the entire region. These equations can be combined to yield either gross movements or net movements.

For the <u>simultaneous movement in both directions</u> at each pair of places <u>**add**</u> the two equations to get the ‘turnover’ potential.

For the <u>net movement</u> we need only the difference between the ‘in’ and ‘out’ at each node for the ‘attractivity’ potential, as follows:

By <u>**subtraction**</u> from the two previous equations we have the single partial differential equation

$$\partial^2 A/\partial u^2 + \partial^2 A/\partial v^2 = I(u,v) - O(u,v),$$

where A can be thought of as the <u>attractivity</u> of each location. This is the well-known Poisson equation for which numerical solutions are easily obtained. Once A(u,v) - the potential - has been found from this equation, the net movement pattern is given by the vector field

$$\mathbf{V} = \text{grad } A,$$

or by the difference in potential between each pair of mesh nodes.

The result is a system of linear partial differential equations

The number of simultaneous equations depends of the mesh size

These are solved by a finite difference iteration to obtain the potential field (after specifying a boundary condition).

This potential can be contoured and its gradient computed and drawn on a map.

In other words a map is computed using a continuous movement model.

Estimates of the potentials for two different populations (male & female for example) can be added to get the correct potential for the sum.

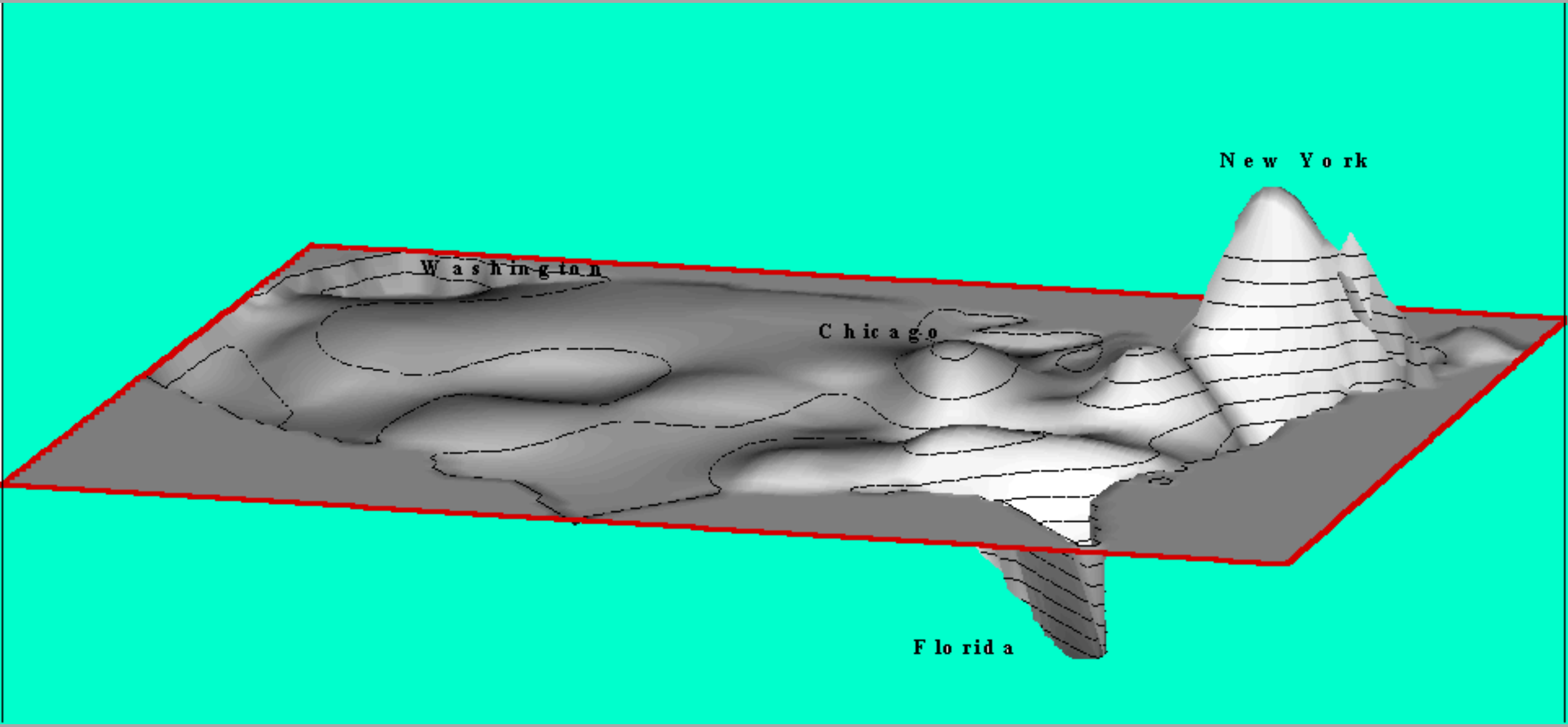
W. Tobler, 1981,"A Model of Geographic Movement", *Geogr. Analysis*, 13 (1): 1-20
G. Dorigo, & Tobler, W., 1983, “Push Pull Migration Laws”, *Annals*, AAG, 73 (1): 1-17.

The potential gives

The Pressure to Move in the US

Based on the continuous spatial model

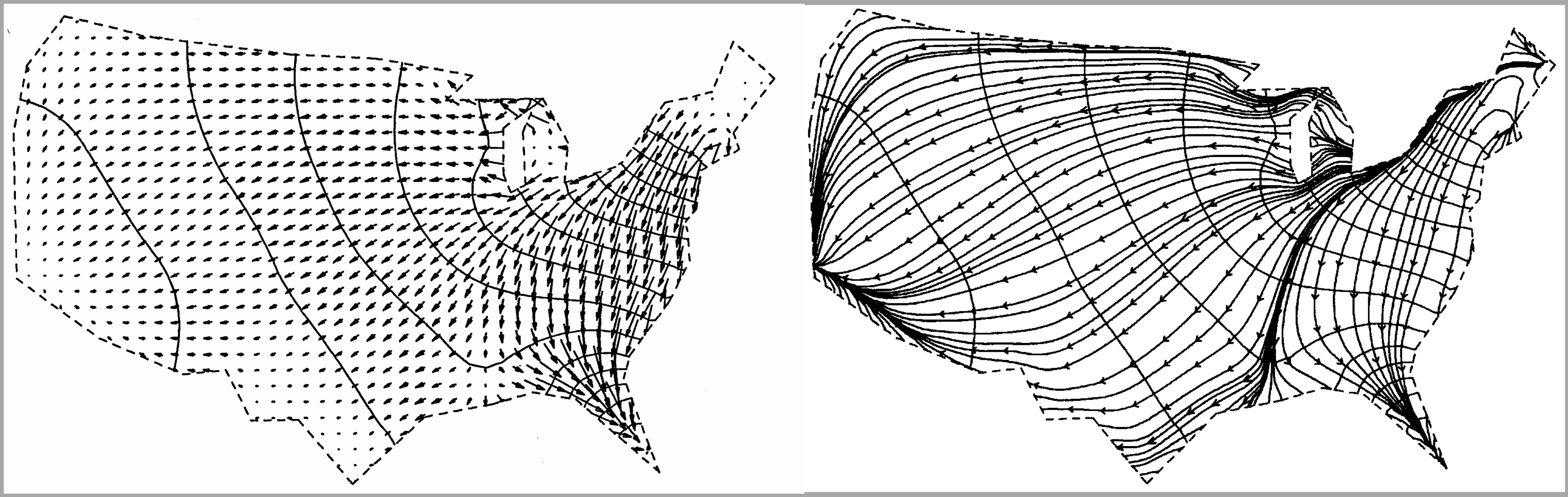
Using state data



Another view

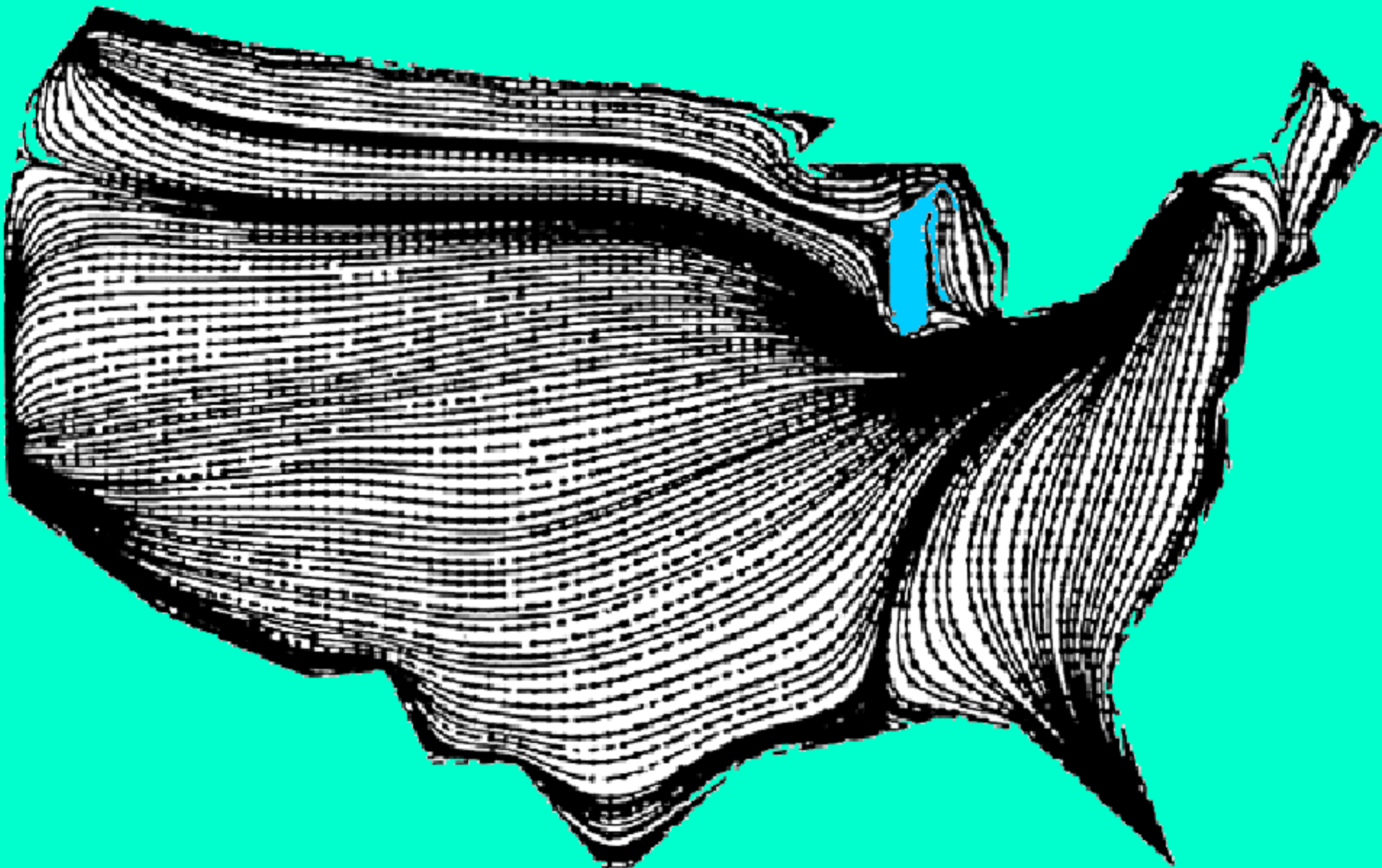
The migration potentials shown as contours

and with gradient vectors connected to give streaklines



16 Million People Migrating

An ensemble average. Note the distinct migration domains.

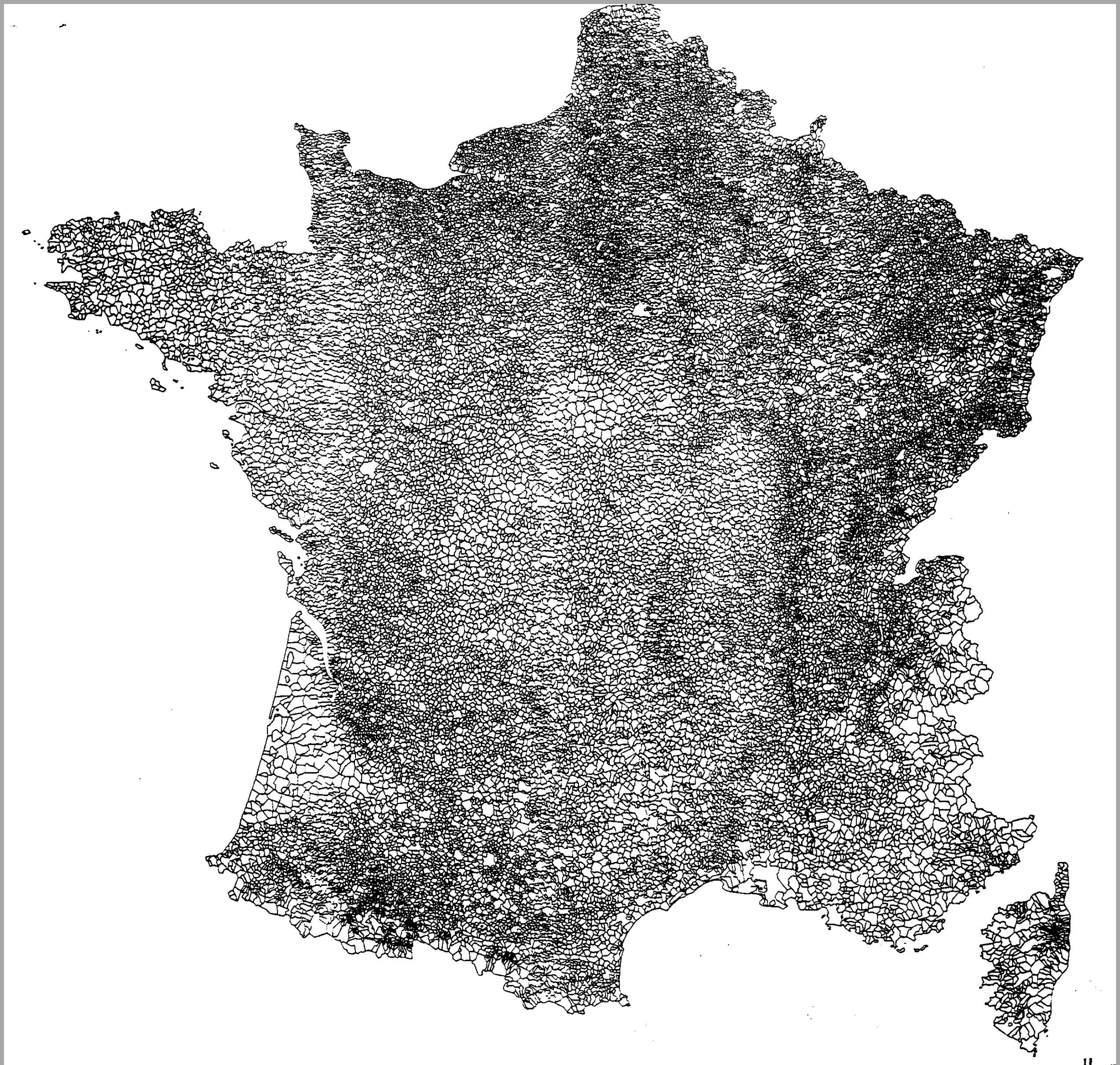


That these migration maps resemble maps of wind or ocean currents is not surprising given that we in fact speak of migration flows and backwaters, and use many such hydrodynamic terms when discussing migration and movement phenomena.

The foregoing equations have captured some of this effect in a realistic manner.

One advantage of the continuous potential model is in the clarity that it provides of the overall pattern and domains.

France's 36,545 Communes



Think Big! Think High Resolution!

The 36,545 communes of France could yield a migration or interaction table with as many as 1,335,537,025 entries. (3 km average resolution)

My assertion is:

Looking at a conventional flow map in this amount of detail would not be useful, but a vector field could show divergences, convergences, and reveal interesting domain patterns. And the potential surface would yield further insight.

Thank You For Your Attention

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