The World is Shriveling as it Shrinks

Invited Presentation

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(For my presentation I will hide most of the textual slides)

A slight modification of a presentation at the ESRI International Users Conference, San Diego in July 1999

Please continue

It is a pleasure to be here, and I wish to thank you for inviting me.

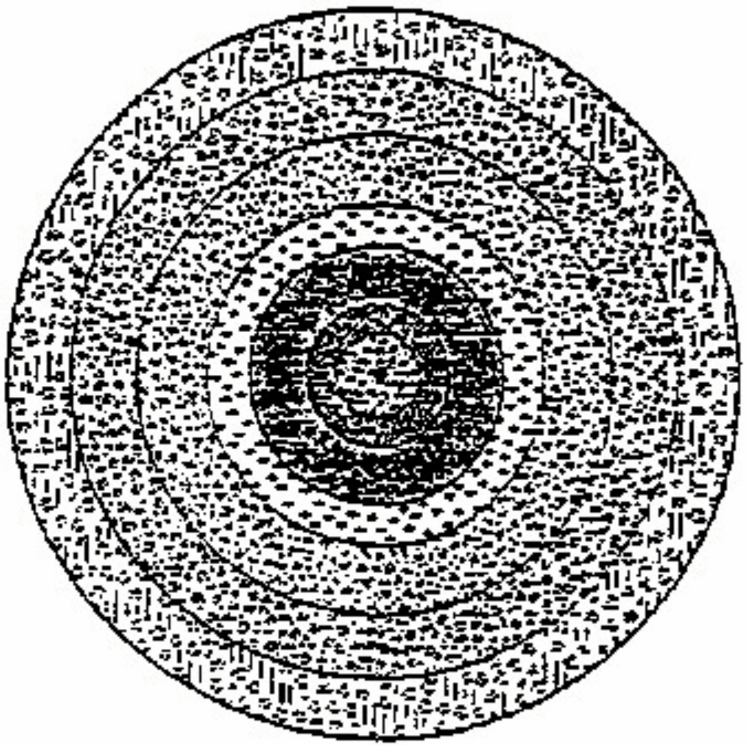
I regret that I cannot stay for the football game.

My talk today relates to transportation, or more generally, to movement. This is a large topic and no single talk can hardly do it justice.

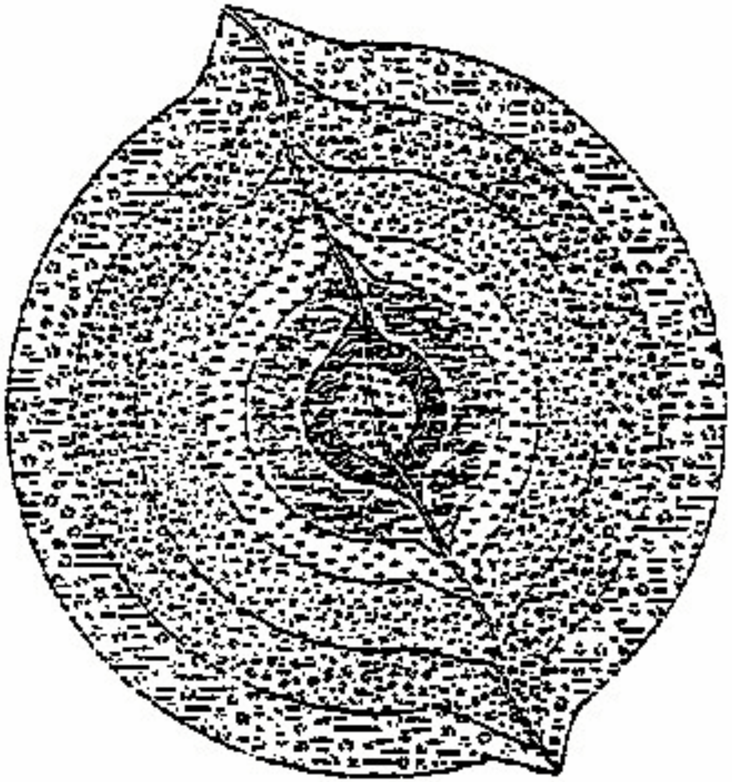
Geographers know a lot about transportation

The Von Thünen model is familiar to most

Agricultural patterns in the Von Thünen model
with a modification due to river transportation

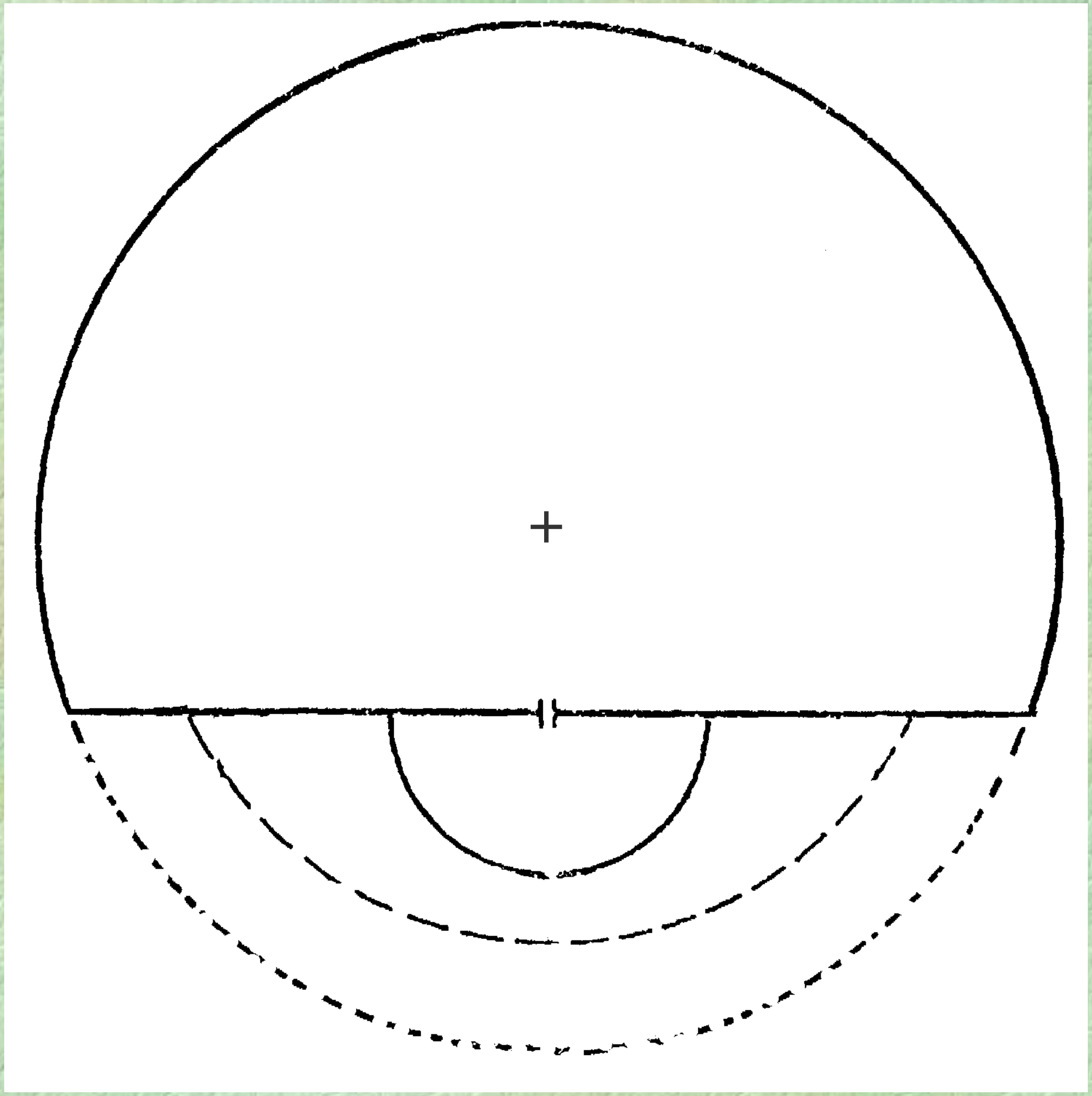


SCHEMATIC OF VON THUNEN LAND USE RINGS



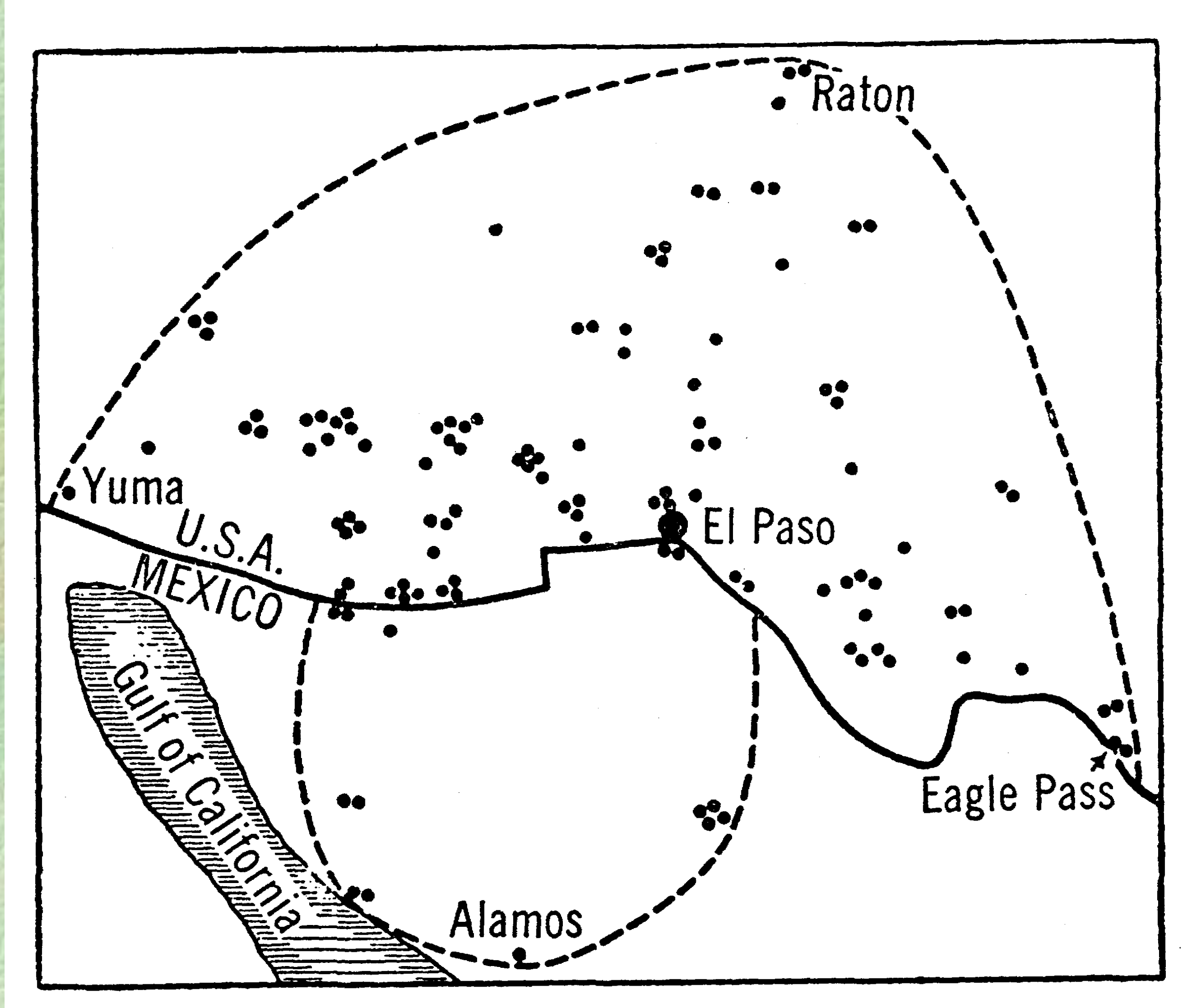
SCHEMATIC OF VON THUNEN RINGS MODIFIED BY RIVER

Some effects of a boundary on a trade area



Modified from A. Lösch, *The Economics of Location*

El Paso bank customers



A. Lösch, *The Economics of Location*

What I hope to do in this talk is to give you some other ways of looking at the world.

My interest in movement modeling is because most geographical change comes about because of movement.

What is important is the movement of ideas, of people, money, disease, energy, or material.

The often used phrase “The World is Shrinking” really refers to measurement in cost or time. 100 years ago the transportation cost for me to get here would have been prohibitive, and the travel time probably measured in weeks.

The 80 days to travel around the world have been reduced to about 24 hours. So we all know that the world is shrinking.

I hope to show you that this shrinking is very uneven and that the world is shriveling as well as shrinking.

To this end, and for my own understanding, I use a simple topological definition of transportation systems.

There are two parts to this classification scheme.

One concerns the domain in which the transportation takes place.

The other part is the boundary, the place where one can change from one mode to another.

As domains I consider two types:

The first type is a domain in which movement is everywhere possible.

The other type is one in which movement can only take place on a network.

In terms of boundaries, there are again two types.

One type of boundary allows exit from the domain anywhere along the boundary.

In the second boundary type exit from the domain can only occur at nodes.

Transportation Systems

DOMAIN	BOUNDARY	EXAMPLE
Go anywhere	Leave anywhere	walking, rowboat, helicopter
Go anywhere	Leave at nodes	Ship, airplane, seal under ice
On network	Leave anywhere	Automobile on street
On network	Leave at nodes	RR, freeway, subway

Consider the first case. Walking can generally take place everywhere and anywhere on land. And one can move to a rowboat at any place along the edge of the land.

Similarly one can row in any direction in a lake and can leave a rowboat at virtually any place along the shore. To this first transportation type, movement anywhere, and exit anywhere, one might also add tanks and ICBMs.

We can add radio, TV, cell phones to the second class.

Bicycles travel on networks but one can get off anywhere.

Look quickly at the other cases and you will get an idea of what I have in mind. If you can think of modes that do not fit the scheme, let me know. Obviously my classification is somewhat of a simplification, but I believe it is still a useful set of categories.

Going through the classes one has to recognize that the ways of moving things or people is extremely large.

In this century we have added automobiles, airplanes, and radio.

Within my lifetime rockets, television, and the internet have been added.

But also hovercraft, dune buggies, snowboards, skateboards, snowmobiles, inline skates, jet skis, and many more.

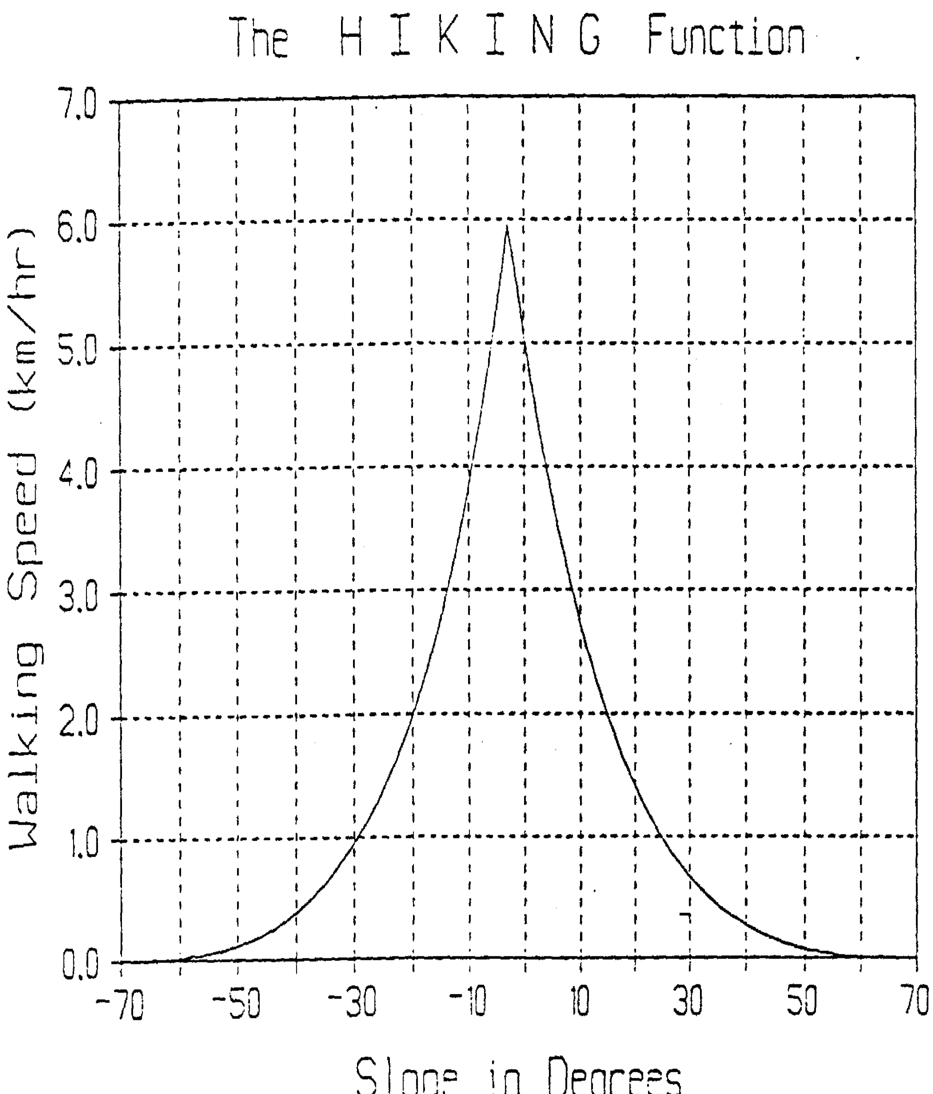
It is really quite amazing.

Looking in more detail at the first category, we recognize that walking speed is quite variable.

It is less easy to walk in hilly country.

To take this into account I constructed a “hiking function”, which gives the speed of walking at every slope steepness.

An Estimated Hiking Function



For walking on footpaths in hilly terrain:

W = 6 exp (-3.5 * abs(S + 0.05))

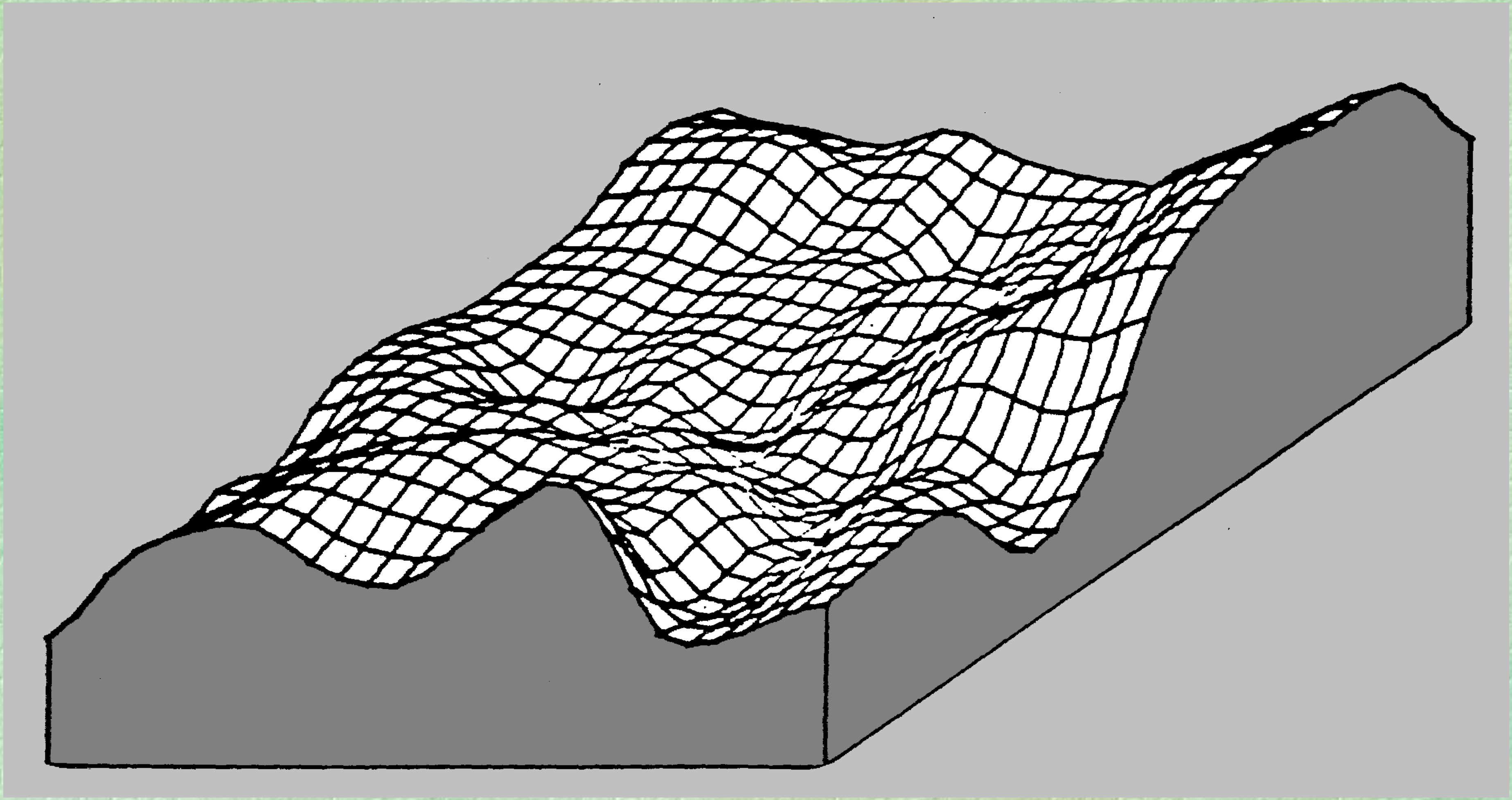
where W is the walking velocity, S is dh/dx = slope = tan (theta); dh and dx must be measured in the same units. The velocity is given in km/hr. On flat terrain this works out to 5 km/hr. For off-path travel multiply by 3/5 (= 0.6). For horseback, multiply by 5/4 (= 1.25). The travel time is computed as distance/velocity.

The graph shows walking speed, with slope on the horizontal axis, up hill on the right and downhill on the left (it is not centered on zero).

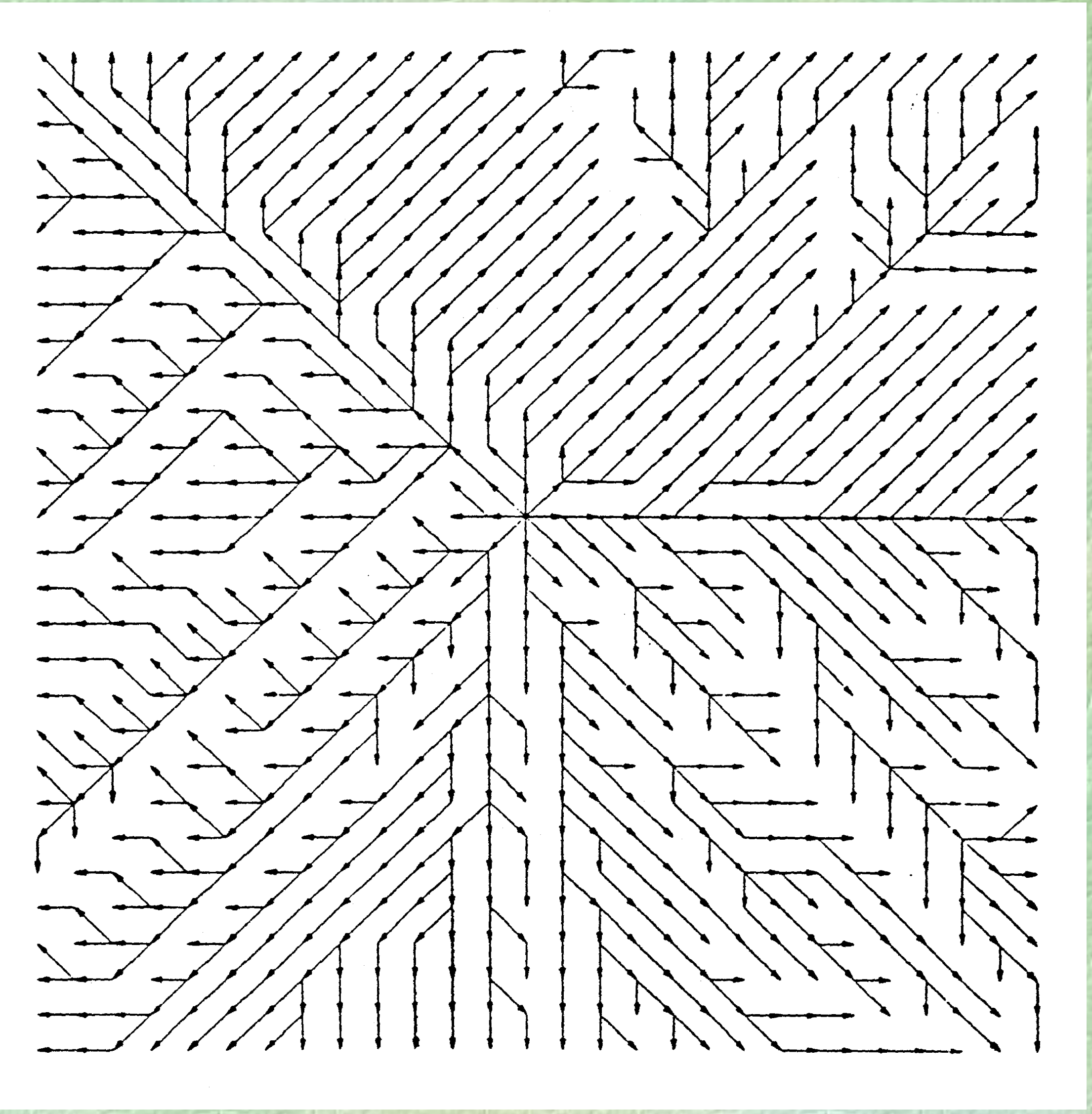
The vertical axis gives the speed in kilometers per hour. It is over simplified – the weather affects walking speed, as does vegetation and altitude.

Still this function should be useful as a first approximation to archeologists relating ancient sites, and possibly to hikers.

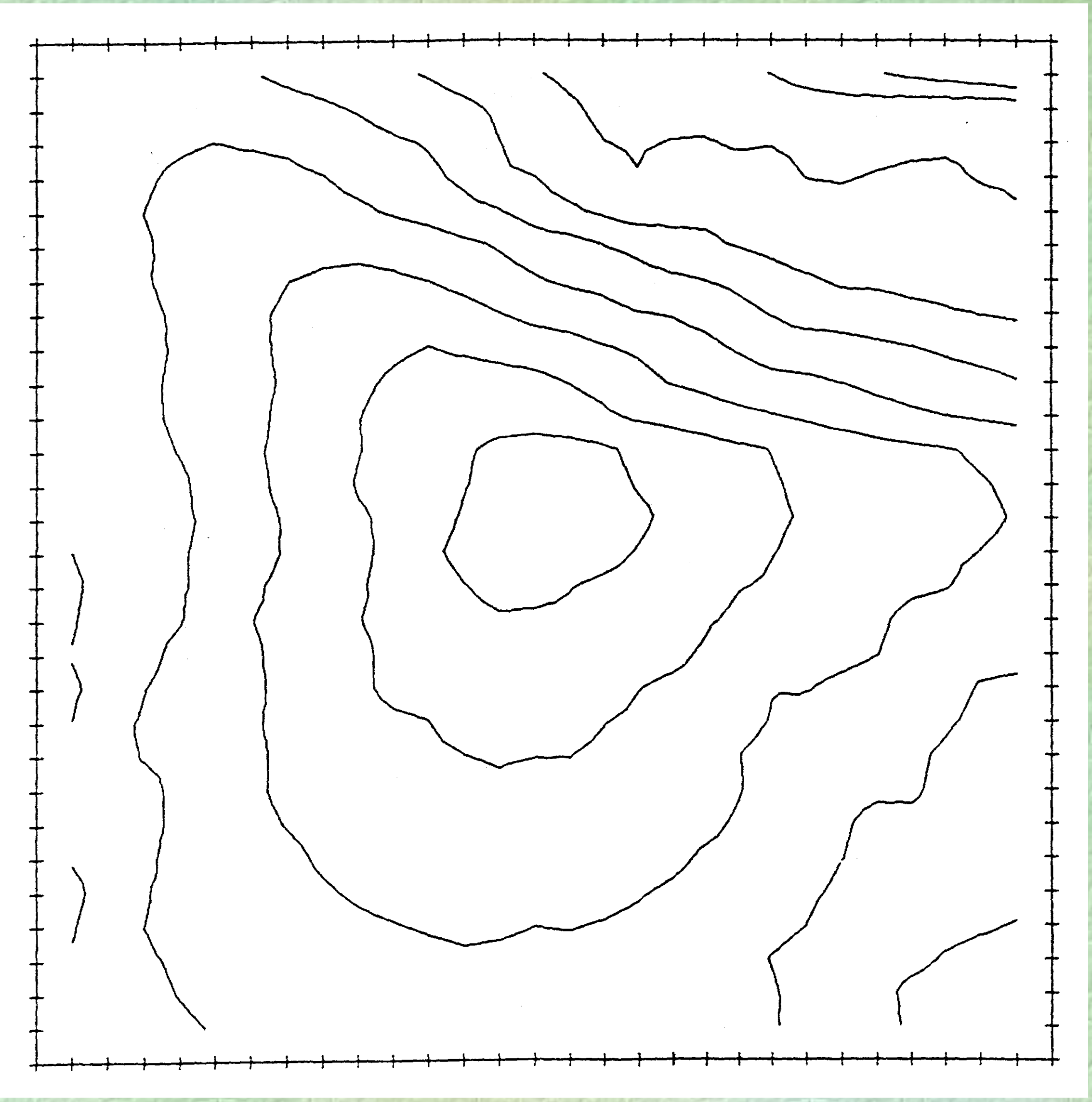
A simple topographic surface



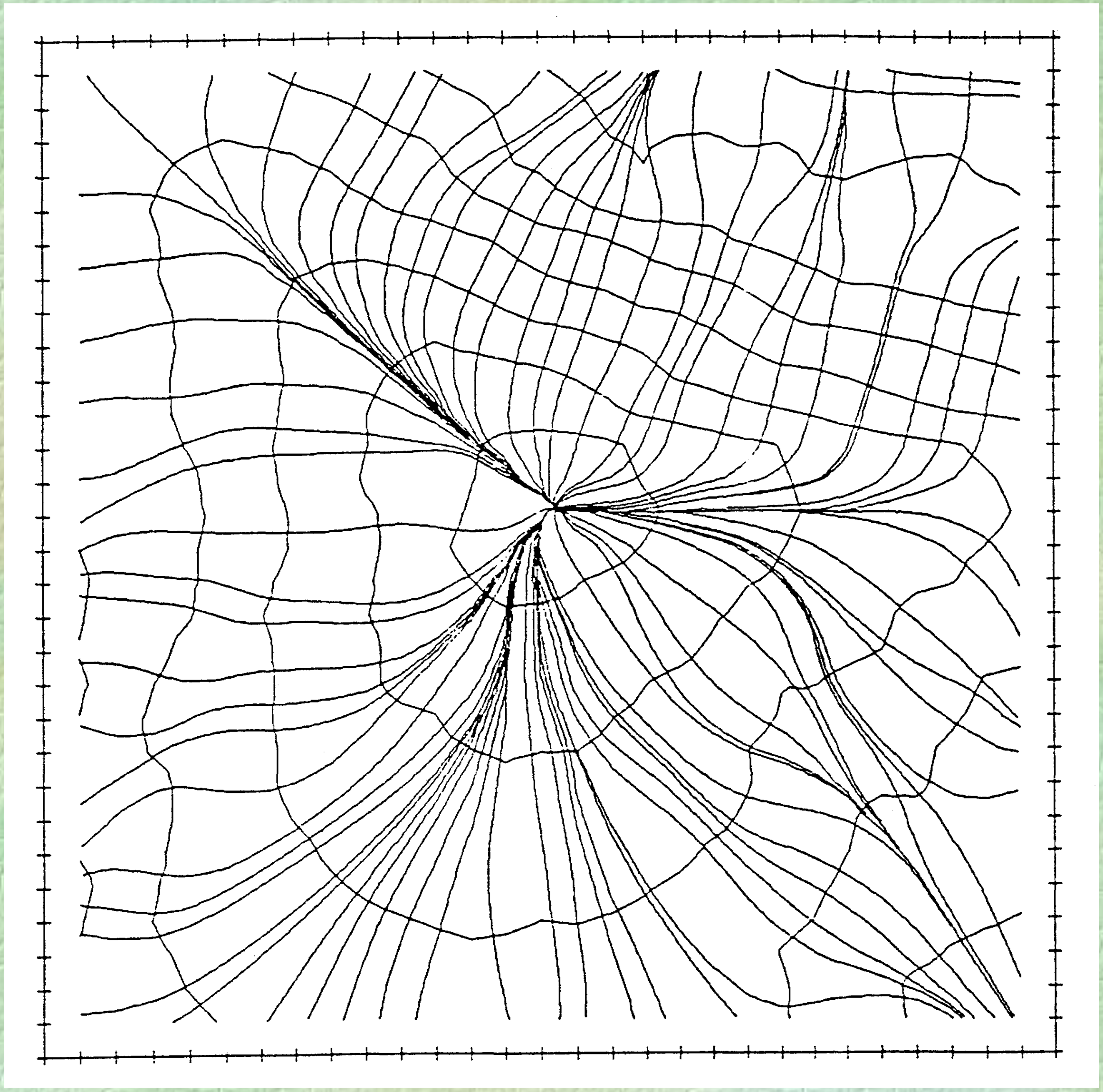
Least time paths on the surface
from the center



Isochrones - constant time contours
about the center



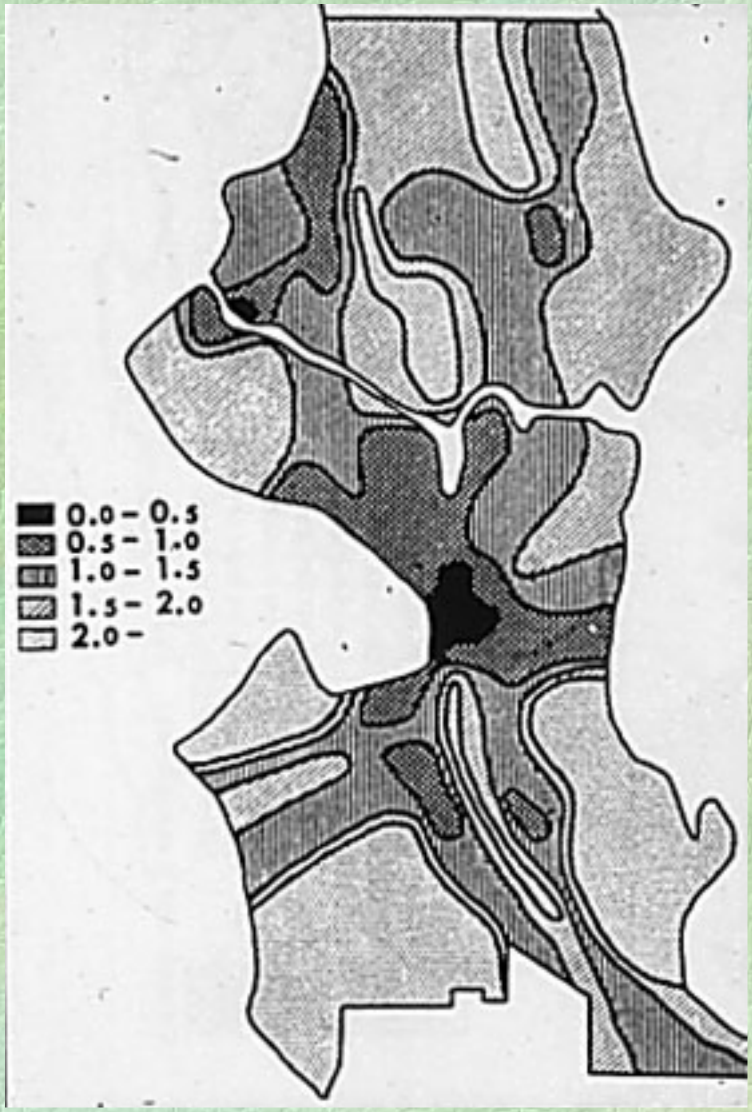
Gradients added to produce polar coordinates
about the center



Or consider mechanized transportation. During the second world war the Germans had Gelaendebefahrbarkeitskarten or terrain trafficability maps. Here, in the next diagram, is an approximate equivalent for automobile travel in a city, from Bunge's book "Theoretical Geography".

It is a speed, or velocity, map of Seattle, showing how fast one can travel, by automobile, in any part of the city. As expected the CBD is the slowest part of the city. It is a fiction of course in that it pretends that one can travel through buildings, that travel effort across any piece of territory is the same in all directions, and so on. It should really show only the road network. But from this map one can draw lines of equal time distance - technically known as isochrones, from any point in the city.

Travel Speed in Seattle
In Miles per 5 minutes



Measured by automobile driving time.

This shows a scalar function, one value at each location. In reality it should be a tensor function, a different value in each direction at every location.

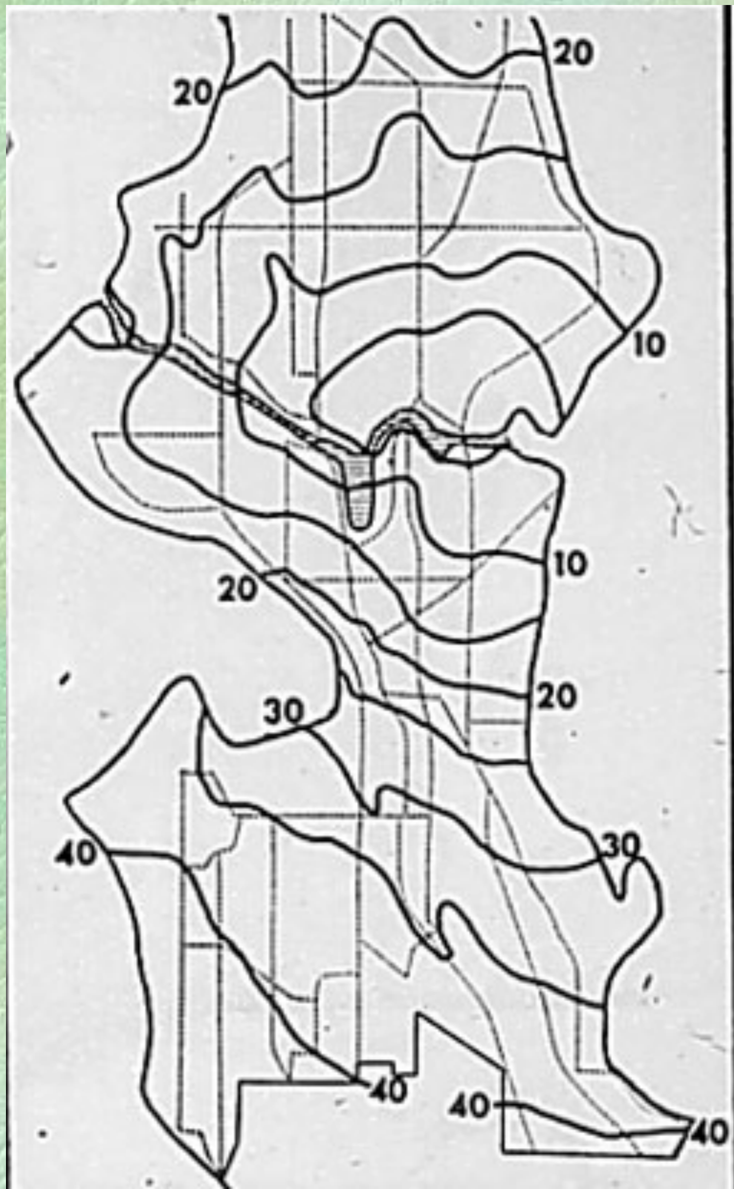
The next map shows concentric isochrones, lines of equal travel time, from the University of Washington. This is followed by a map that shows travel times from the CBD.

Both maps were made from the same travel velocity (speed) map. Such maps are easily created from the rate maps by summation over paths. This type of map can be used, in a computer, to count the number of people within travel time or travel cost rings, and this is perhaps useful for business geographics. Again the travel times should not really be shown as isochrones but rather as road segments, best rendered in different colors.

And, as we all know, travel times vary rapidly in a 24-hour time period. So we need to update the velocity map in real time, and this is now becoming possible. For example, when riding in a vehicle equipped with a GPS and a radio, a voice synthesizer could present optimal driving directions taking into account current traffic conditions.

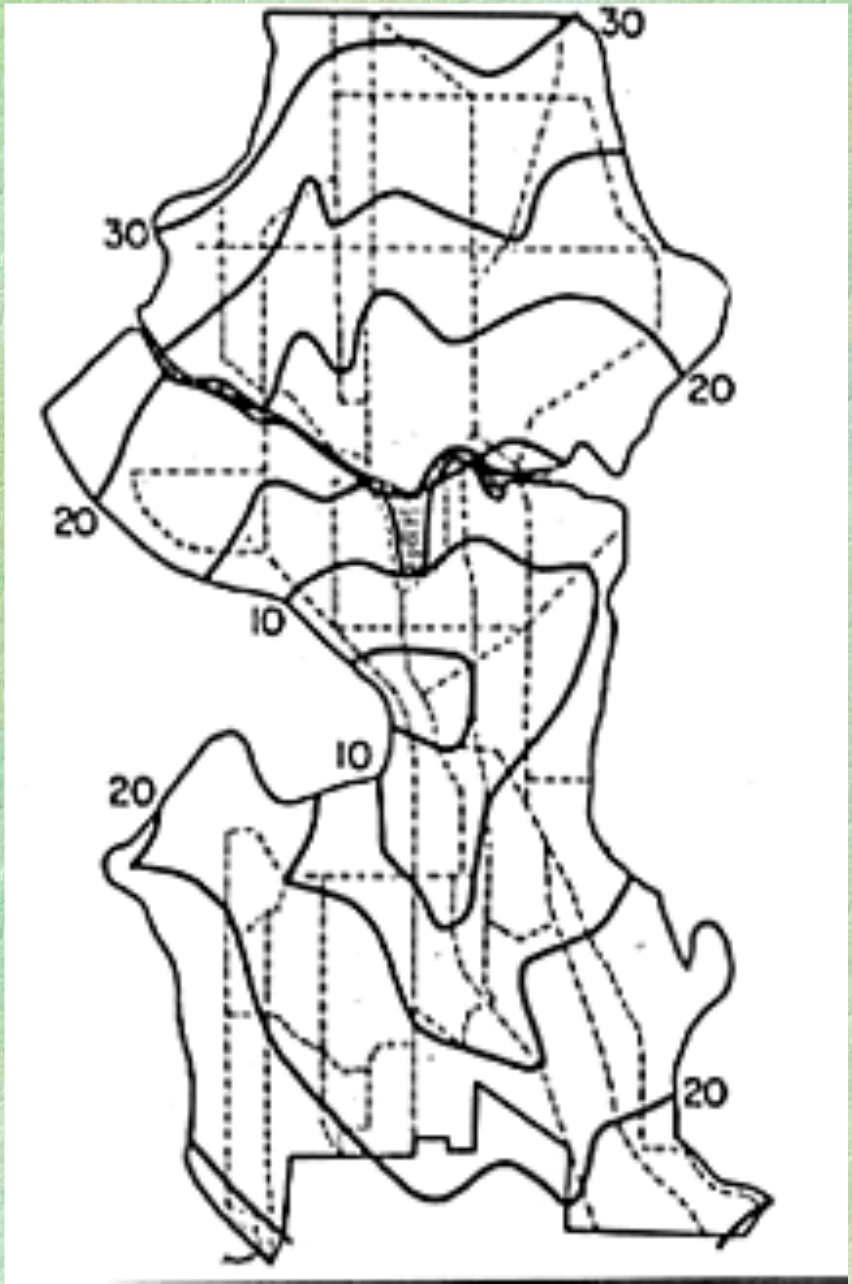
Peak Hour Travel Time from UW

In Five Minute Intervals



Draw orthogonal trajectories to the isolines. Think of these as the gradients to the contours. The two sets of curves form a system of curvilinear coordinates, known as polar geodesic coordinates. This coordinate system not only identifies locations but can be used analytically for metrical operations.

Five Minute Isochrones from CBD



- Travel time from Seattle's central business district, in five minute intervals.
- Draw in the orthogonal trajectories, to turn the isolines into a system of polar coordinates.

W. Bunge, 1966, *Theoretical Geography*, 2nd ed., Gleerup, Lund University

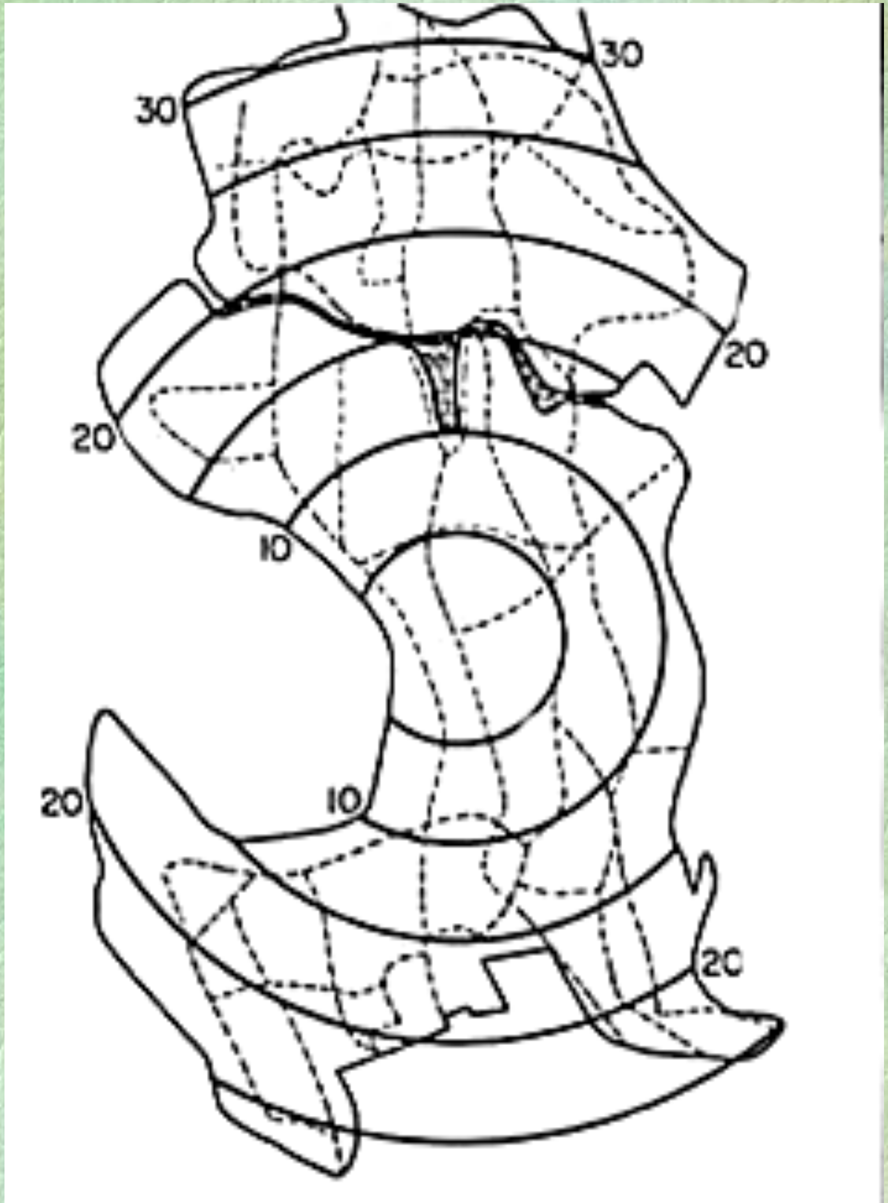
One can go one step further, cartographically.

The concentric travel time isochrones can be converted to equally spaced “normal” circles by warping the geographic background, to give “realistic” time or cost distance maps, as in the next diagram.

Again this might be done in real time.

Time Distance from CBD

Background Warped



The travel times are now concentric circles. The geographic background has been warped to fit this.

The orthogonals are now equally spaced straight lines (not drawn) radiating from the center to give the usual polar coordinates.

W. Bunge, op. cit.

Next is another example: Travel time, in days from Berlin, 1909.

Look at Africa.

The gradients represent the minimum time travel routes – construct them mentally.

How can the isochrones be converted to “normal” circles?

To render this cartographically Africa would have to be turned inside out!

Travel Time from Berlin in Days

Eckert, 1909

Another familiar example is the airline time distance from London map.

Parts of the United Kingdom are farther away than locations in Europe, even North America.

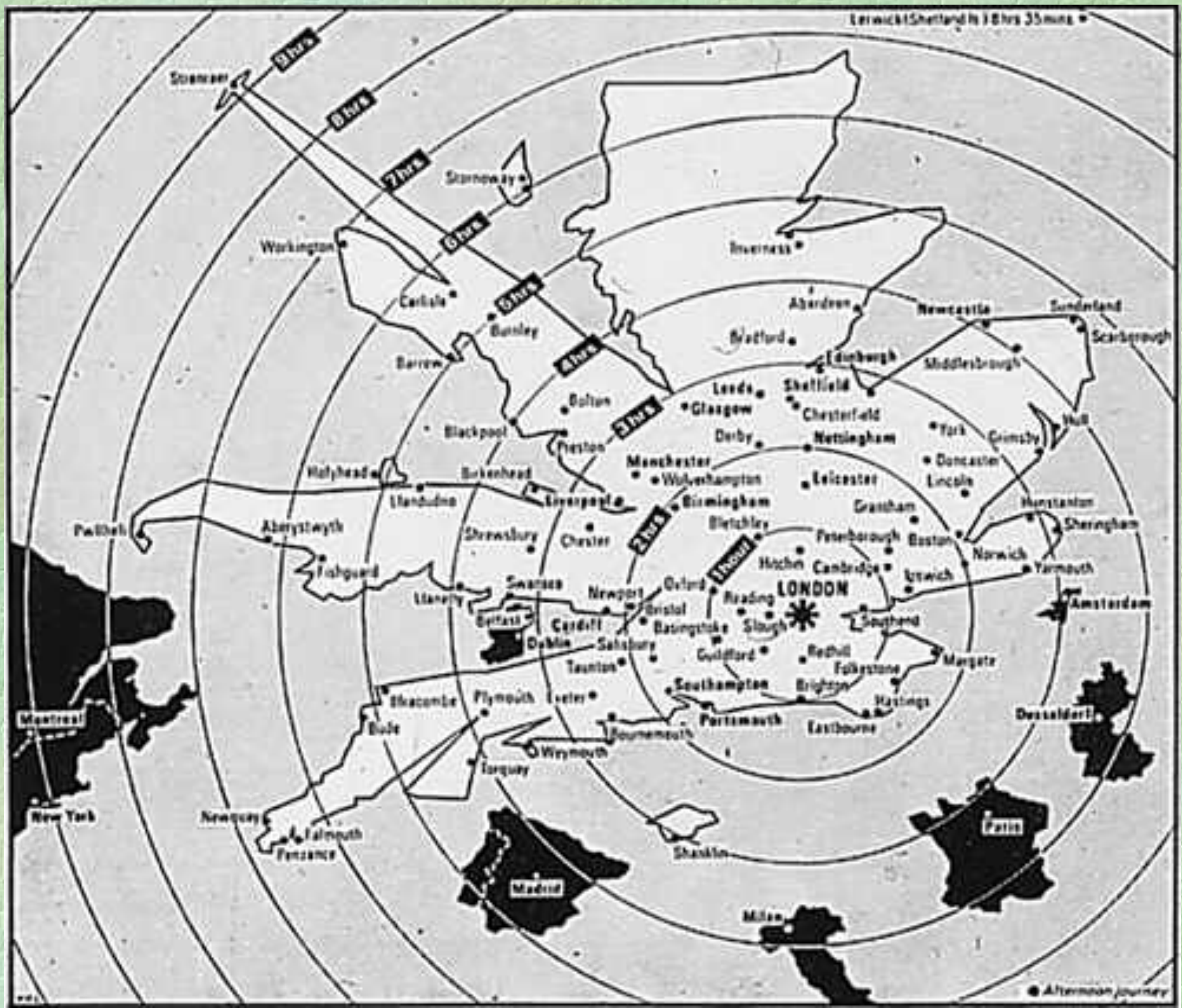
The assumption is here made that all of the land areas can be reached by flying, but of course only the airports can be so reached.

Schedules must also be taken into account.

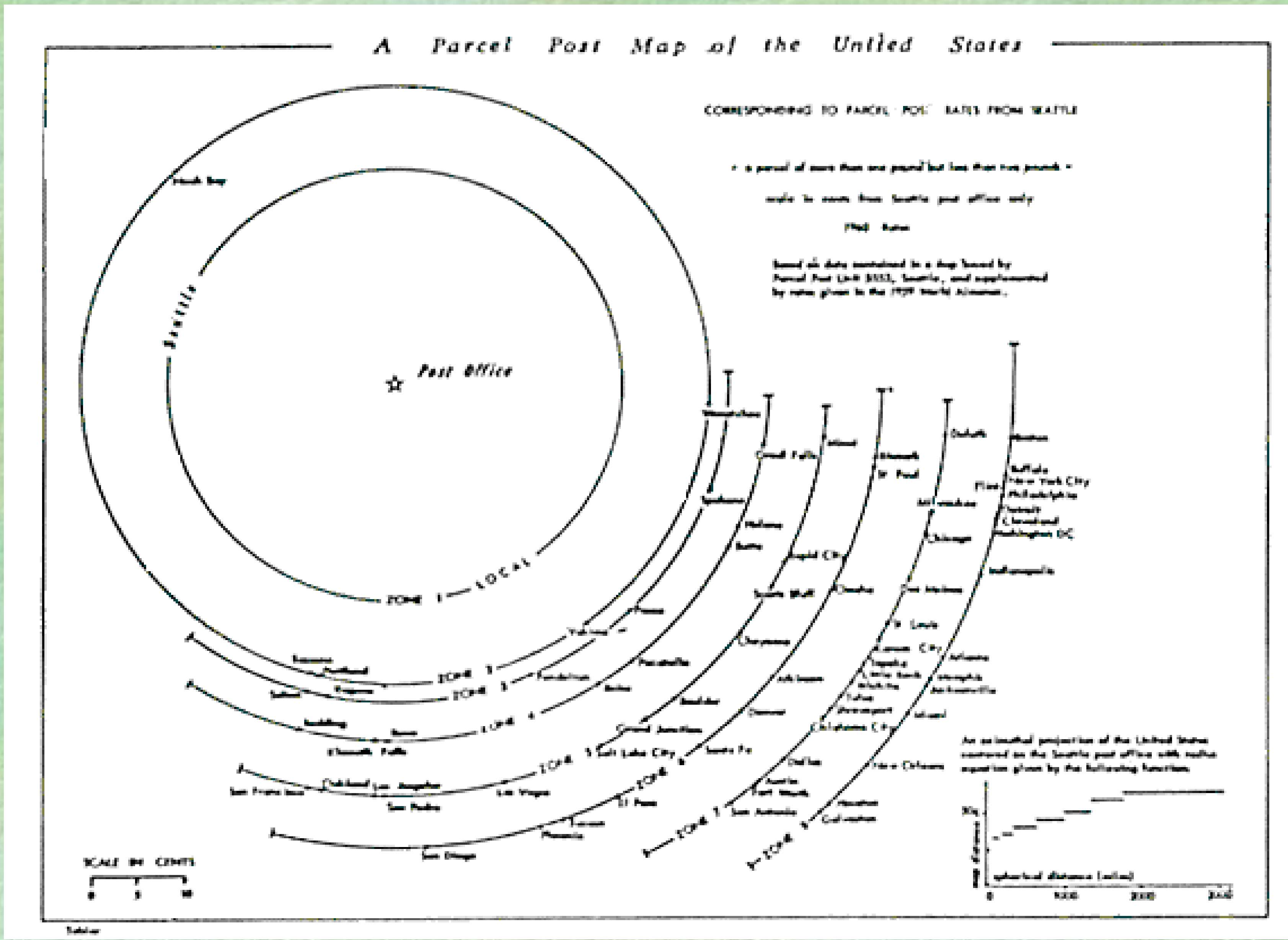
Still it is a useful graphic. Consider a service interested in a maximum number of clients reachable within one day.

Travel Time from London

In Hours



Parcel Post View From Seattle



Check an airline rate book and you will find that Santa Barbara is closer in cost space to New York City than it is to Arcata in Northern California.

Even worse, to fly from Santa Barbara to Columbus, Ohio, or to Milwaukee, Wisconsin, costs more than to fly to from Santa Barbara to New York City.

In cost-space Columbus and Milwaukee are further from Santa Barbara than is New York City.

Thus the United States must again be turned inside out, in a very complicated way.

Spend an hour or two with a rate table and you will find many such examples.

Try to construct an air travel cost map centered on College Station, placing US and foreign cities at their scaled proper cost distance. It will be most instructive.

There is not a monotonic relation between geographic distance in kilometers and geographic distance in monetary units.

Thus I contend that the world is shriveling, with many places becoming relatively more isolated, and only a few becoming more connected.

Next is another, carefully done, example.

It shows the one-hour vicinity of Liege, in Belgium.

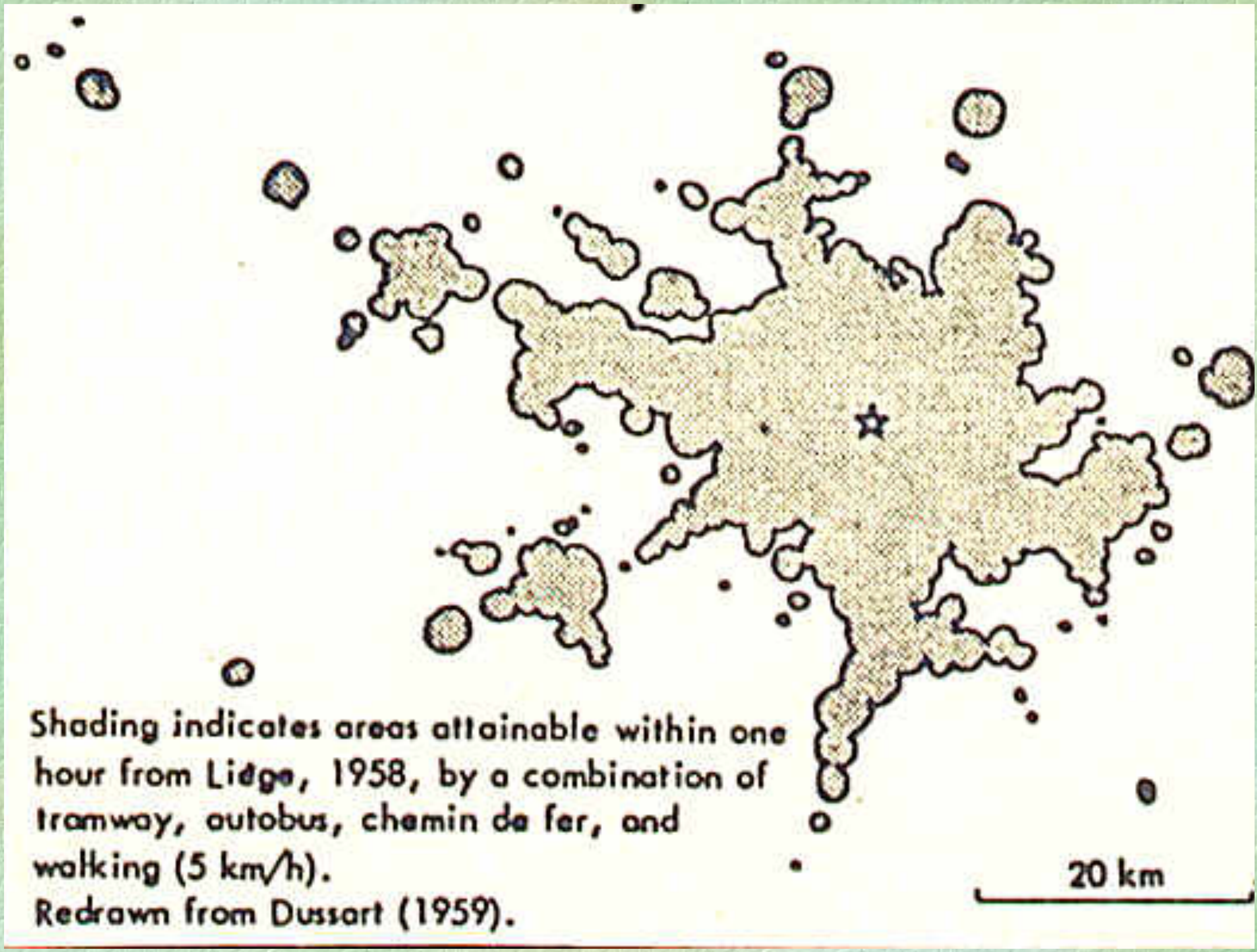
I say it is carefully done because the author recognizes several modes of transportation.

It is only a single time slice, although travel schedules and frequencies were taken into account.

An obvious question is "Can this type of diagram be made dynamic, in real time?"

A One Hour Geographical Circle

Liege 1958



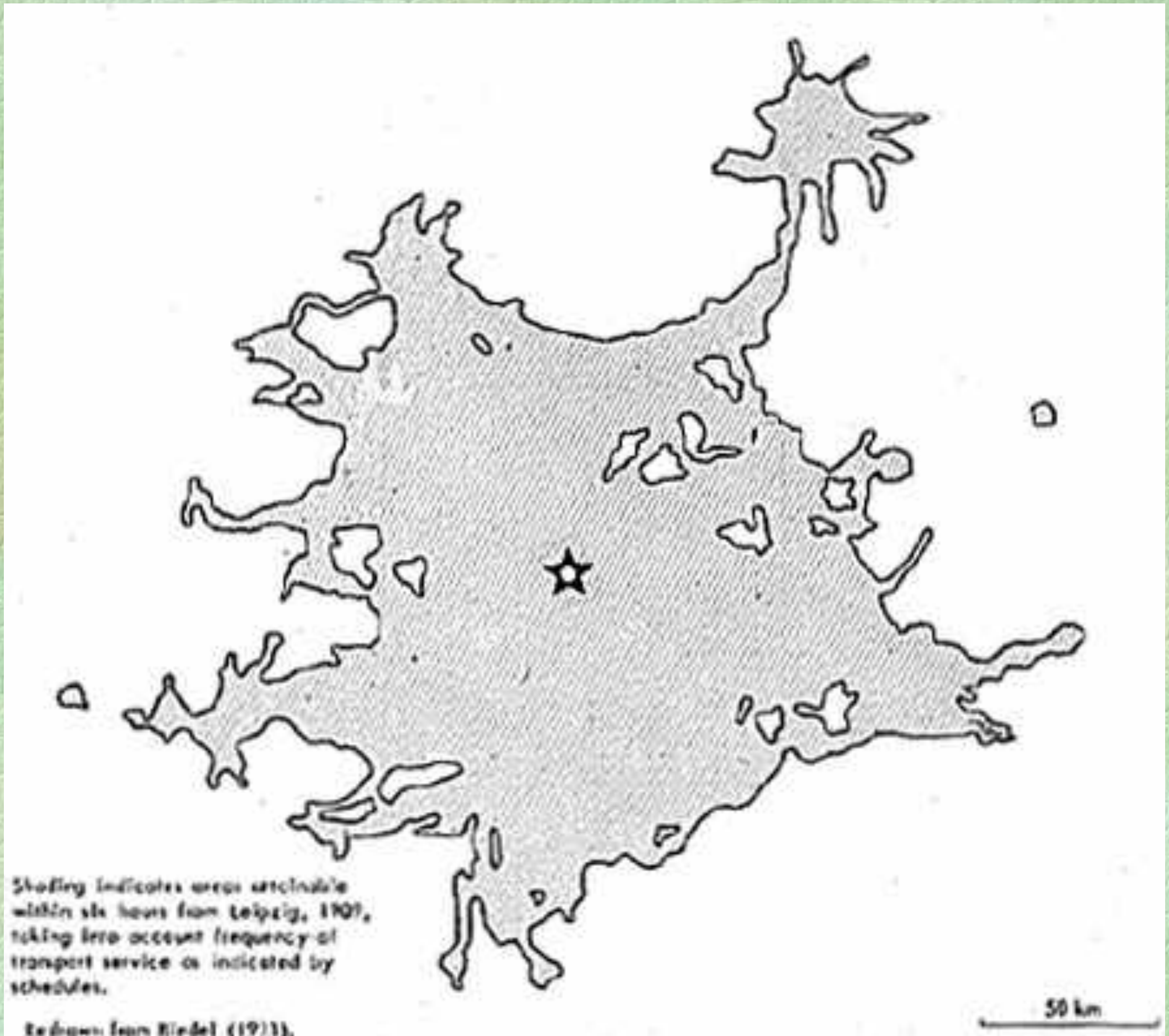
The next illustration is the six hour isochrone around Leipzig, circa 1911.

Looked at in another way, one can refer to this as a geographic circle, namely **the set of all places on the surface of the earth that can be reached in six hours from Leipzig**. The circle's radius is six hours.

Circles belong to the domain of geometry, but this is a strange type of circle. It is clearly different from the types of circles we are familiar with. It has holes and disjoint pieces. The length of the circumference is not $2 \pi r$, the area not πr^2.

This implies a geometry much, much more complicated than that of Einstein, which is relatively smooth and static by comparison.

Leipzig: The Six Hour Isochrone
1911



Now imagine what a one-hour circle around your home would look like on a map.

The person sitting next to you has one that is quite different.

Clearly the one-hour travel circle around the center of Paris will look different from the one-hour circle around the conference center here at Texas A & M. Include the two hour and three hour circles and then the orthogonal trajectories to get the polar geodesic coordinates.

All will fluctuate throughout the day.

Have you ever tried to draw a coordinate system on a potato, tomato, or cucumber taken from your kitchen?

Try it! Using polar coordinates is easiest.

You will find that a circle of a given radius will have a circumference that depends on where you center the circle.

Just like the travel time circles just considered!

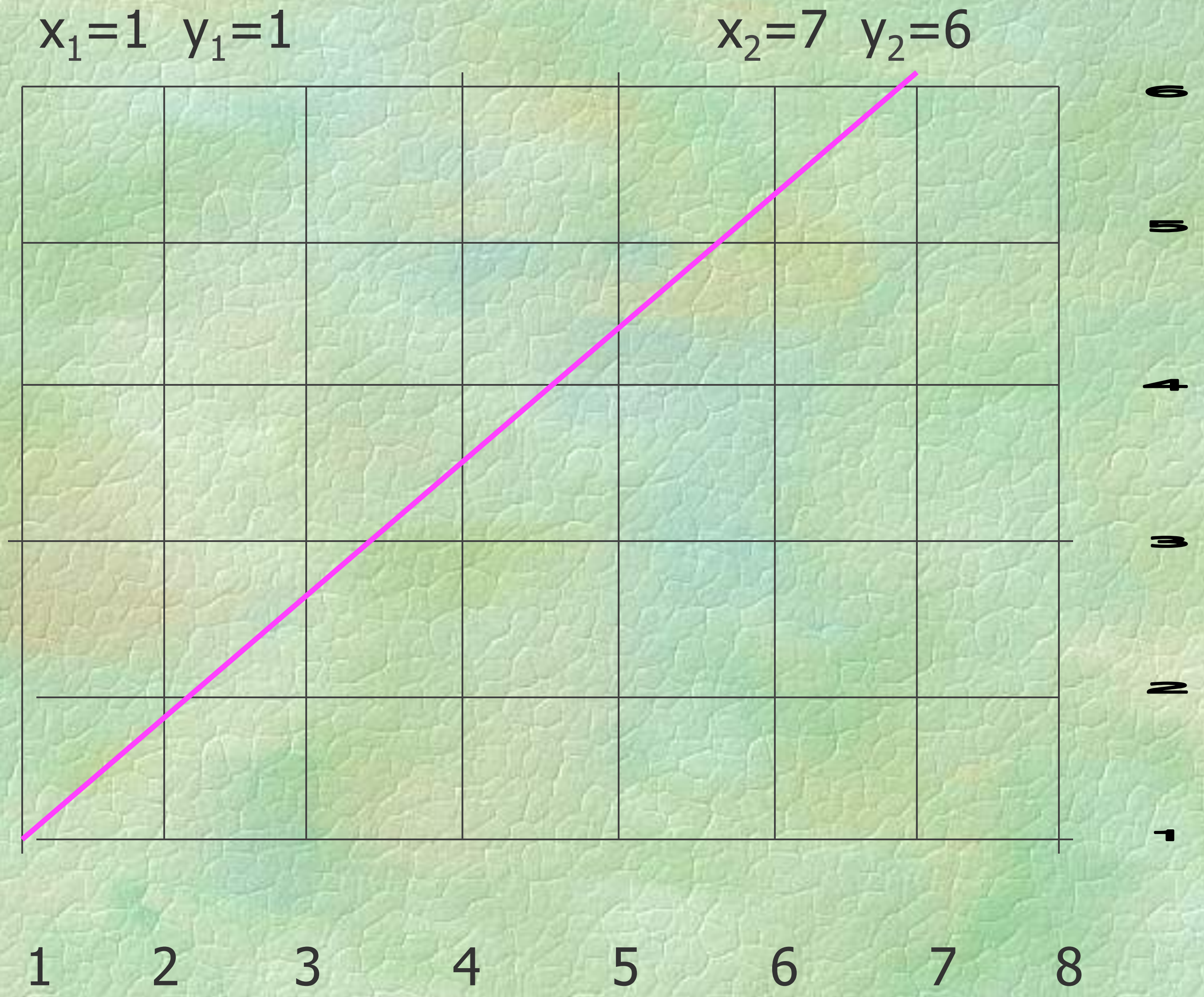
Mathematicians, particularly Gauss, worked out how to handle these types of geometry about 1820, when he invented the subject of differential geometry.

Now a little aside on geometry

Distances determine geometry

To calculate distances we use coordinates

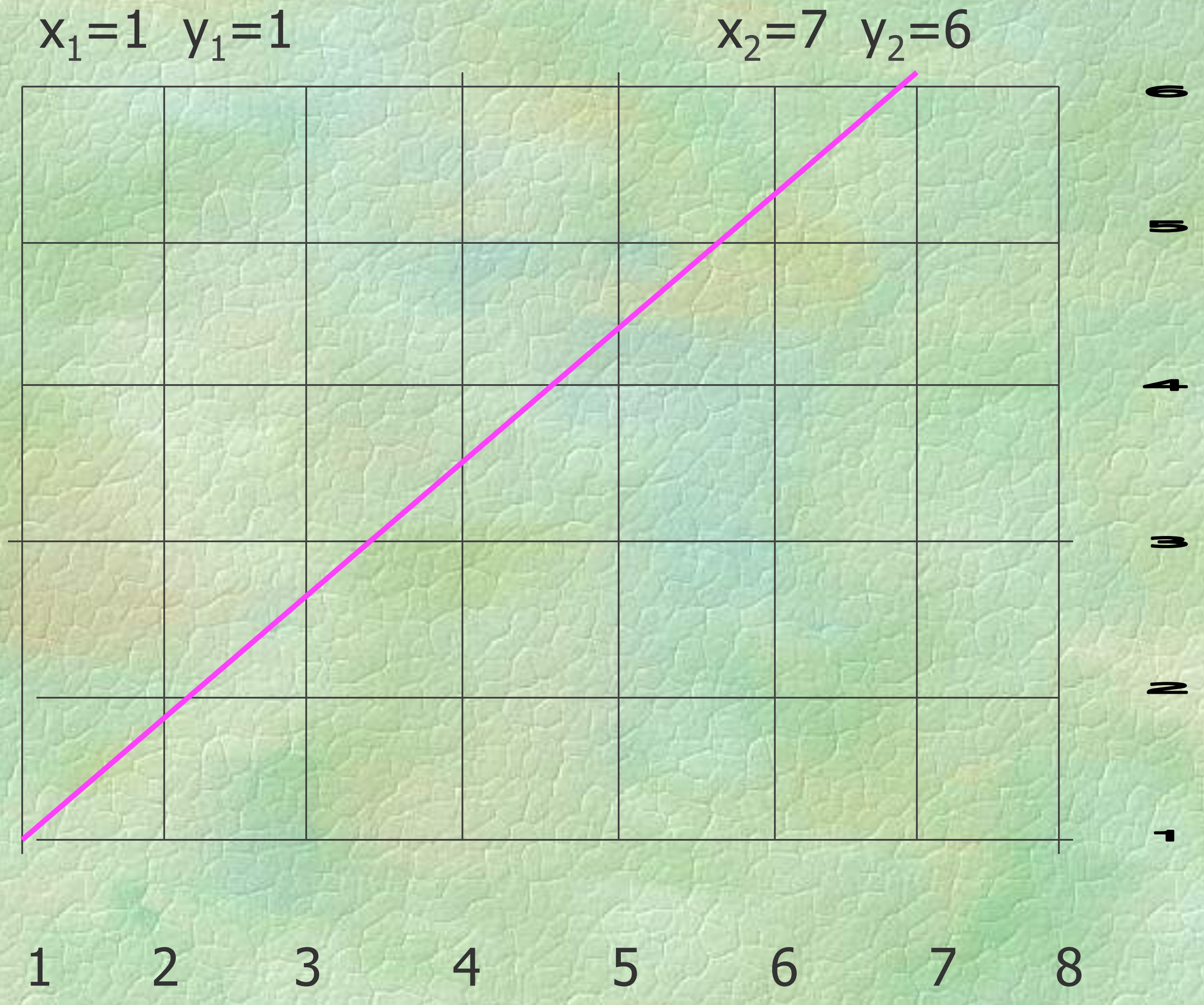
So here’s an example



$x_1=1$ $y_1=1$
$x_2=7$ $y_2=6$
1
2
3
4
5
6
1
2
3
4
5
6
7
8

Distance from (1, 1) to (7, 6)

$$D = [(1 - 7)^2 + (1 - 6)^2]^{1/2}$$
$$= [6^2 + 5^2]^{1/2}$$
$$= [36 + 25]^{1/2}$$
$$= 61^{1/2}$$
$$\cong 7.8$$



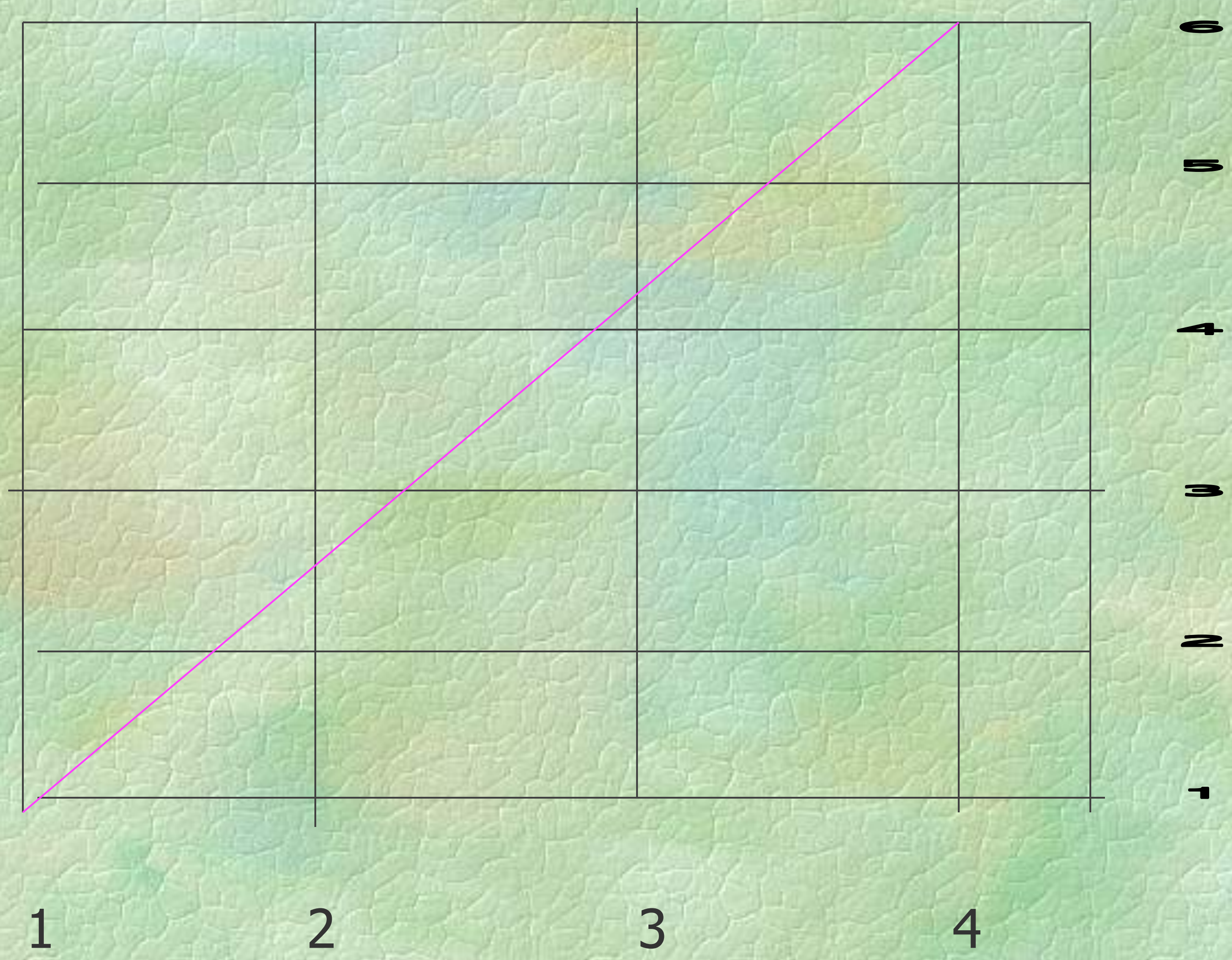
x1=1 y1=1
x2=7 y2=6
6
5
4
3
2
1
1 2 3 4 5 6 7 8

Changing the name (coordinates) of a point does not alter the distance between them.

But the rule whereby we calculate the distances does change.

The next slide shows the same line in a new, different naming scheme (coordinate system).

$x_1=1$ $y_1=1$ $x_2=4$ $y_2=6$



Distance from (1, 1) to (4,6)

$$D = [(1 - 4)^2 + (1 - 6)^2]^{1/2}$$

$$= [3^2 + 5^2]^{1/2} = 34^{1/2} \cong 5.8$$

which is wrong! But it can be fixed

$$D = [4(1-4)^2 + (1-6)^2]^{1/2}$$

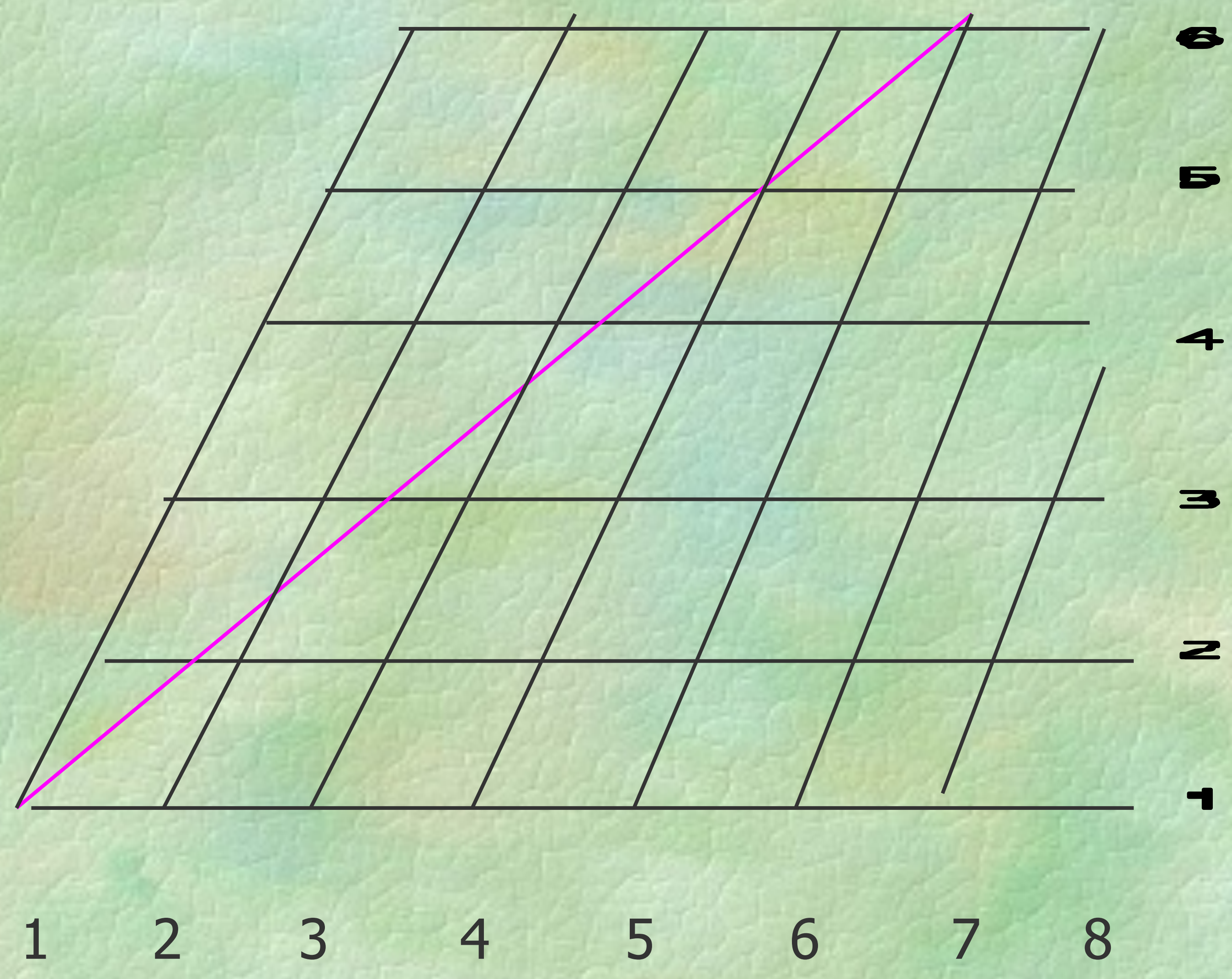
$$= [4(3^2) + 5^2]^{1/2}$$

$$= [36 + 25]^{1/2}$$

$$\cong 7.8$$

$$D^2 = W_x dx^2 + dy^2$$

$X_1=1 \quad y_1=1 \qquad x_2=5 \; y_2=6$



In these oblique coordinates

The name name of one point has again changed.

What is the new rule to compute the distance?

Try the following

$$D^2 = W_x dx^2 + W_{xy} dxdy + W_{yx} dydx + W_y dy^2$$

$$= W_x dx^2 + 2W_{xy} dxdy + W_y dy^2$$

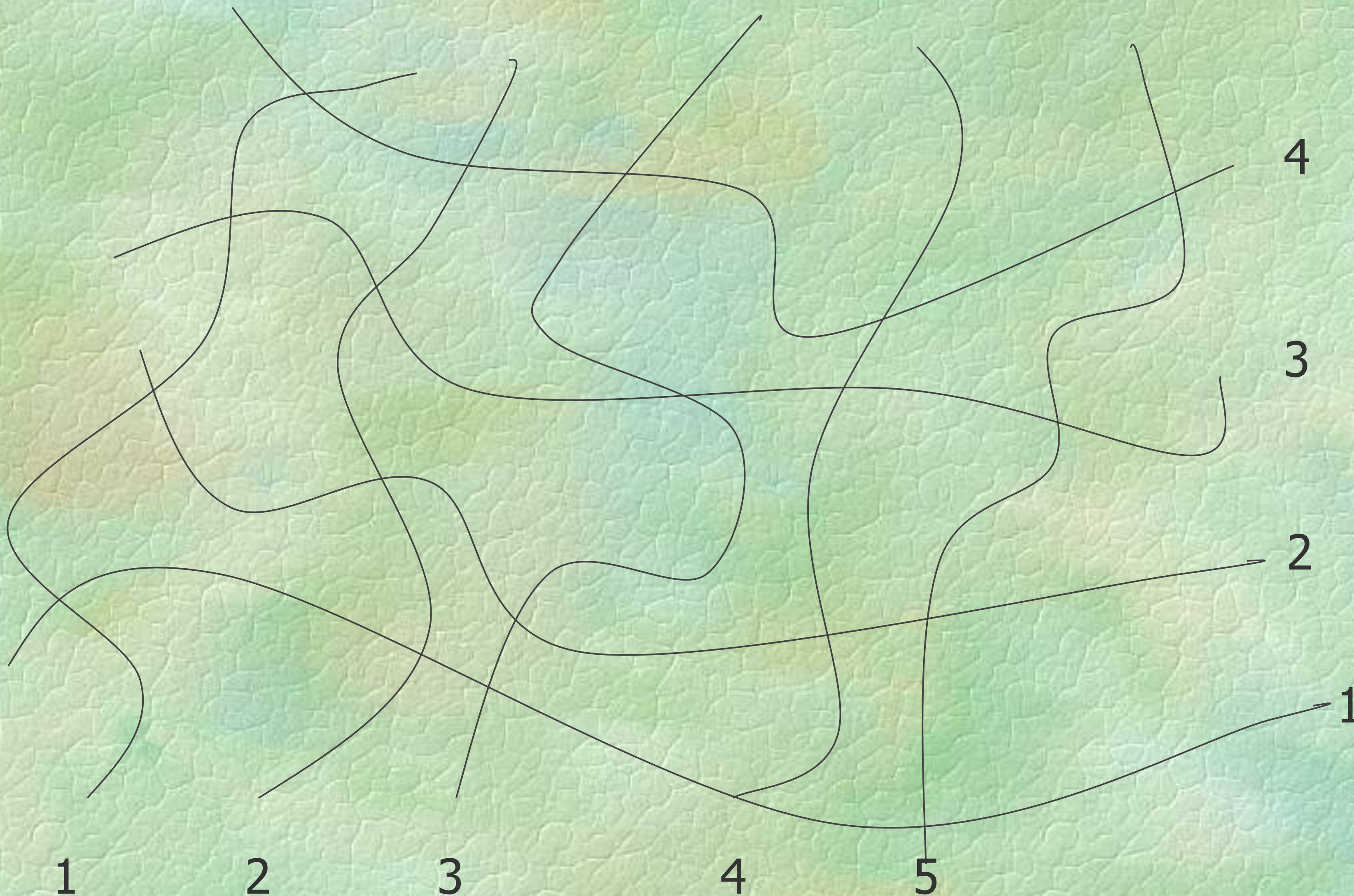
here $W_{xy} = W_{yx}$ is related to the cosine of the angle between the axes.

W_x and W_y are constants.

Work out the details for yourself.

A more complicated example

Curvilinear coordinates



With this new coordinate system the rule for the calculation of distances must again change.

Needed now is a rule to be applied to a curvilinear coordinate system.

Gauss invented such a rule in the early 1800's.

On surfaces that are not flat it is necessary to use such curvilinear coordinates and the Gaussian rule.

In the Gaussian metric formula

$$d_{12} = [(x_1 - x_2)^2 + (y_1 - y_2)^2]^{1/2}$$

becomes $d_{ij}^2 = Edx^2 + 2Fdxdy + Gdy^2$

in modern notation $ds^2 = g_{\alpha\beta}\, dx^{\alpha}\, dy^{\beta}$

Written out in full this is, in two dimensions,

$$ds^2 = g_{11}dx^2 + g_{12}dxdy + g_{21}dydx + g_{22}dy^2$$

The coefficients are no longer constants. They are different in each curvilinear quadrangle.

If distances are symmetric $g_{12} = g_{21}$

In polar coordinates:

$$d_{12} = [r_1^2 + r_2^2 - 2r_1r_2 \cos(\theta_1 - \theta_2)]^{1/2}$$

$$ds^2 = dr^2 + g_{22}d\theta^2$$

Some consequences

applying to non-flat surfaces

Circumference = $2\pi \sin(r/\sqrt{k}) = 2\pi r - (\pi k r^3/3) + \ldots$

Area = $2\pi (1 - \cos(r/\sqrt{k})) = \pi r^2 - (\pi k r^4/12) + \ldots$

($\sqrt{}$ = square root)

The Gaussian curvature k is given, in polar geodesic coordinates, by

$$k = -1/\partial(g_{22})^{1/2}/ (g_{22})^{1/2} dr^2$$

The relation between distance and curvature can be explained in more detail.

As an example use distances on the earth.

Construct a table of distances between all places on the earth.

Assume $2*10^7$ places.

The distance table contains n(n-1)/2 distances = $2*10^{14}$ distances.

Assume 200 distances per page for 10^{12} pages.

At 6 grams/page this is $6*10^6$ tons.

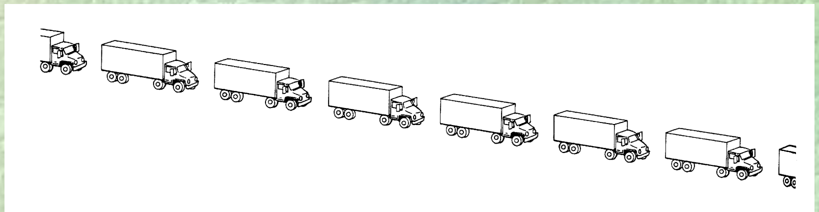
Assume 6 tons per truck for 10^6 truckloads.

Assume one truck every 5 seconds for $5*10^6$ seconds,

which is 2 months of day and night traffic.

C. Misner, K. Thorne, J. Wheeler, 1973, *Gravitation*, Freeman, 306-309.

Lots of trucks



But the quantity of information can be reduced.

Use only the distance to nearby locations

For each point record the distance to the nearest 100 points.

Now there are only $2*10^9$ distances.

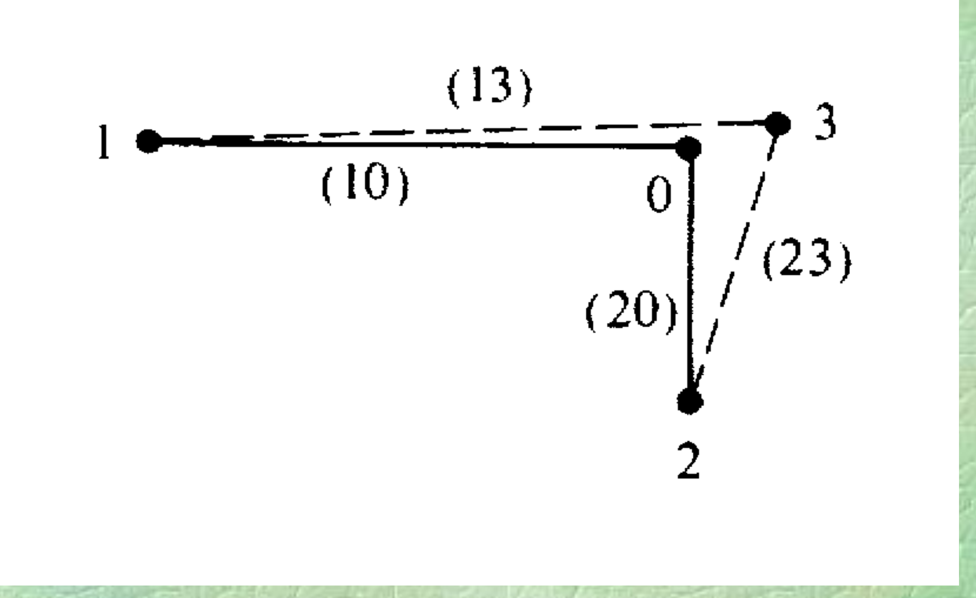
Or 10^7 pages of data, or 60 tons of paper.

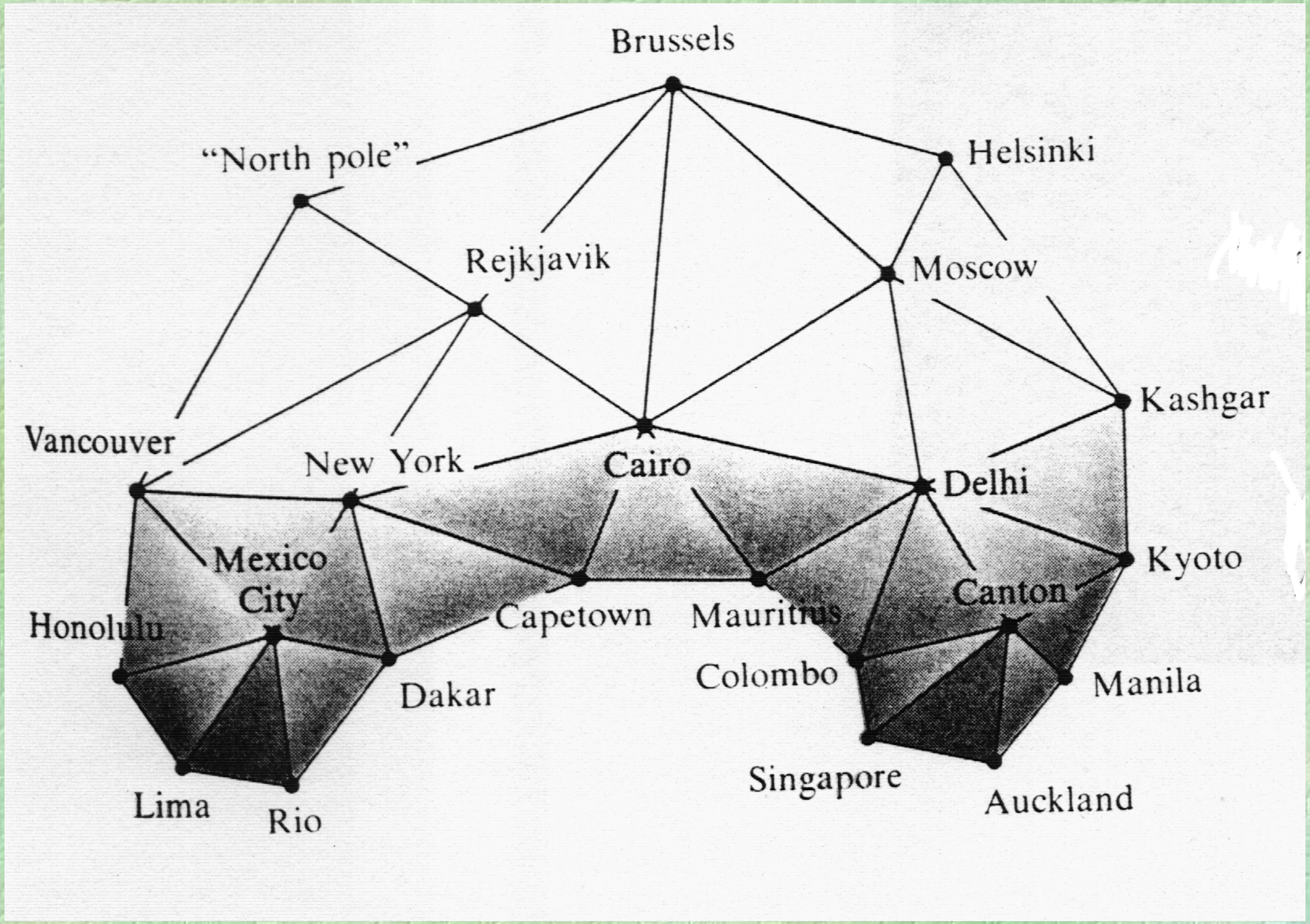
Needing only 10 truckloads, passing by in less than a minute.

Next assume that the surface is smooth and that the distance to points near another point can be approximated by the Euclidean distance formula.

A triangulation can thus be established and distances to points further away can be calculated using the triangles.

The approximation can be improved by taking more, and smaller, triangles.

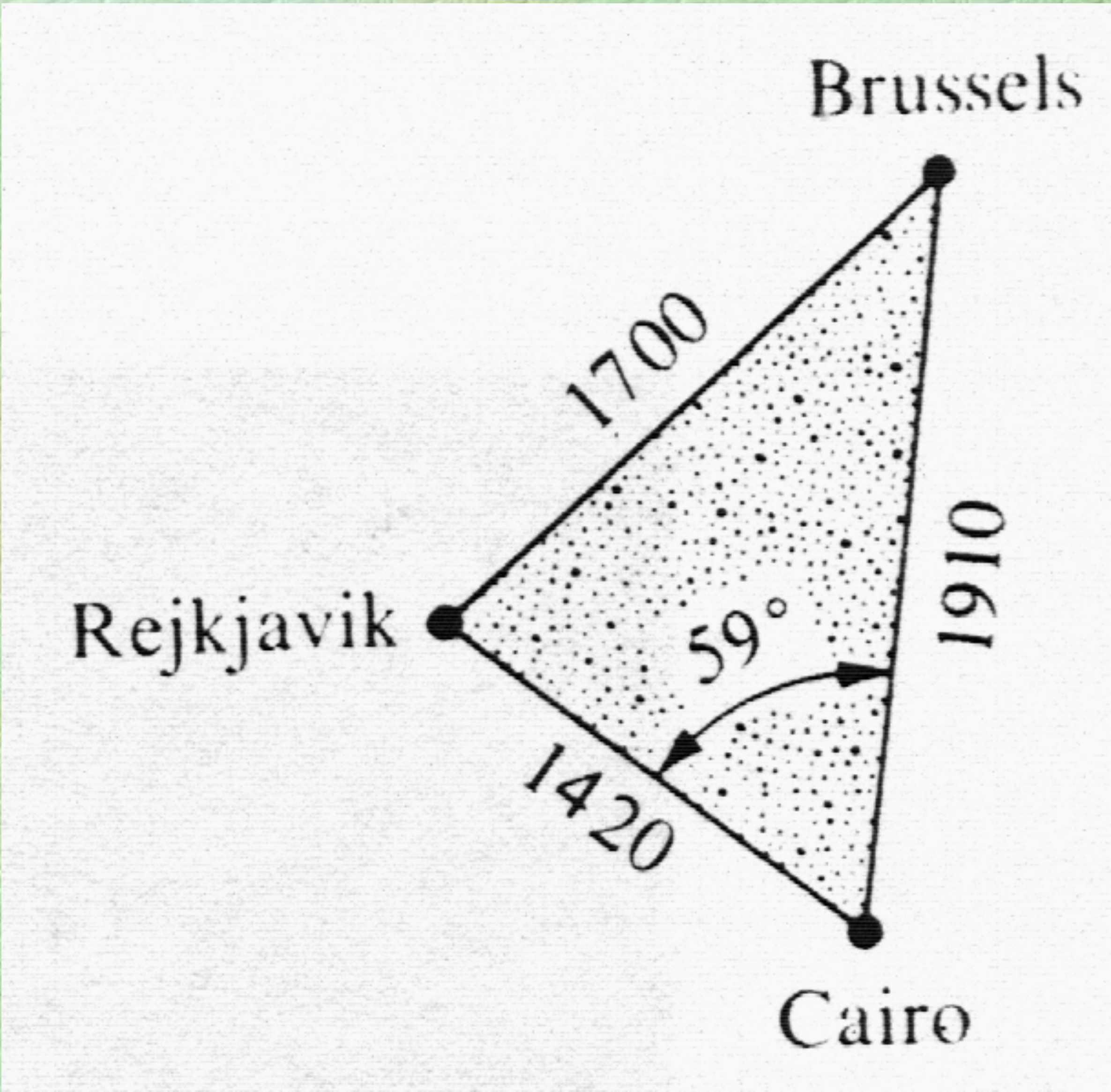

(13)
1
3
(10)
0
(23)
(20)
2



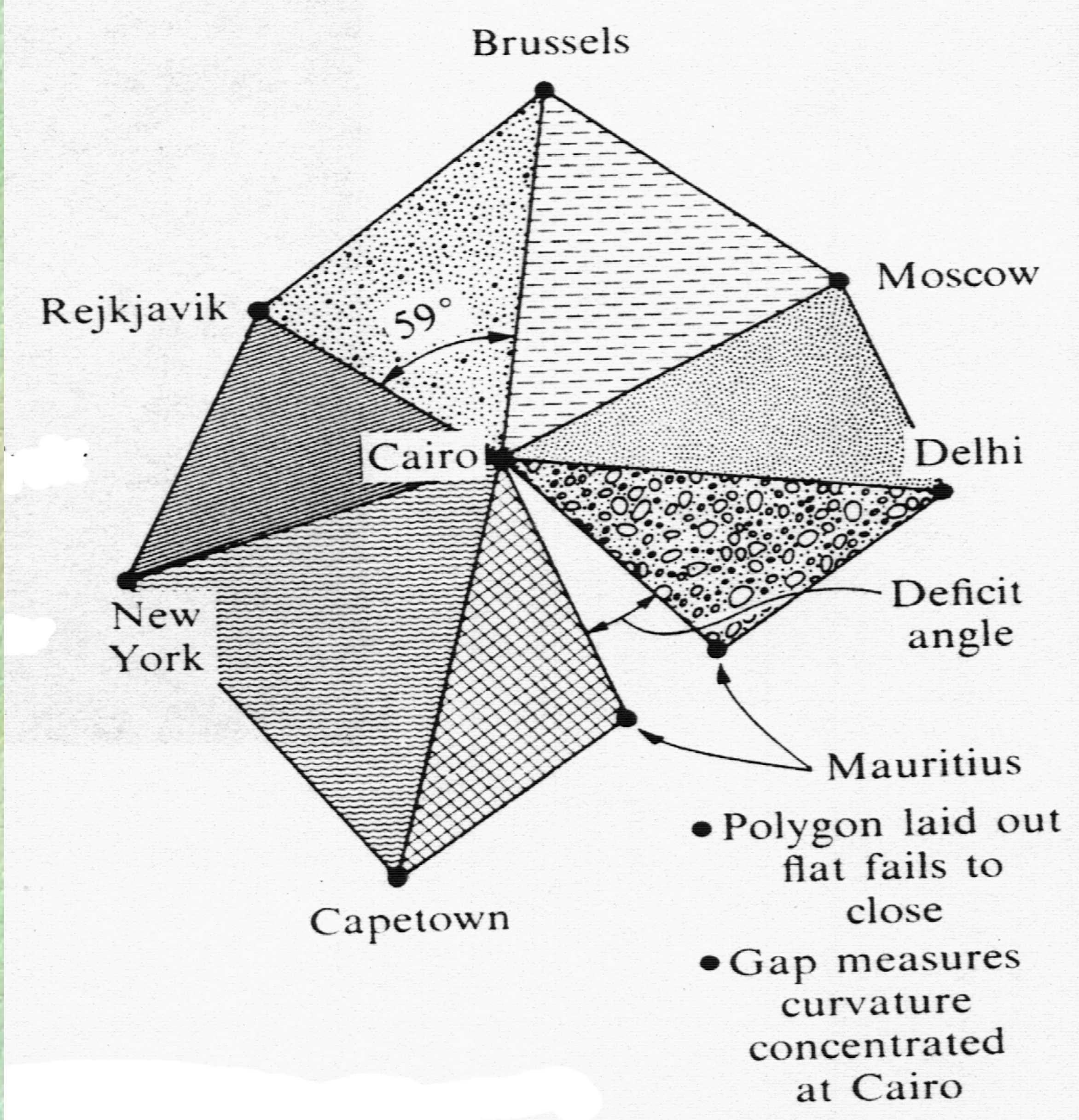
A sample triangulation

Compute the angles in a triangle

on the Euclidean assumption, using "small" triangles



Consider all triangles surrounding a vertex
and lay them flat



In this way the curvature can be approximated at all locations.

The curvature can also be calculated from the Gaussian coefficients $g_{\alpha \beta}$

So, instead of using a large number of distances use the Gaussian formula

$$ds^2 = g_{\alpha \beta} \, du^{\alpha} \, du^{\beta}$$

To use the Gaussian formula we need the three metrical coefficients, g_{ij} $(i, j = 1, 2)$ at each of many geographical locations.

But we might give these as functions of latitude and longitude, in terms of a power series or an expansion in spherical harmonics, with a modest number say 100, of adjustable coefficients. Then the information about the geometry is caught up in these three hundred coefficients, a single page printout. Goodbye to any truck!

Considering the earth as a sphere the coefficients would all be constant, and therefore we need only one constant.

Now try using travel times, or costs, instead of spherical kilometers to calculate the curvature. The result could be quite interesting.

Some people have tried to fit mathematical functions to the geographic travel time or cost spaces.

They generally use either Minkowski or Manhattan metrics. Riemannian or Finsler geometry seems to me to offer more promise.

Geographers have long used travel time or cost maps.

Most travel time/cost maps are centered on one location.

We can go beyond this to consider all possible distances between places.

One technique is to adjust the map to correctly, in the least squares sense, represent distances.

That is, push the places apart until the scaled map distances are proportional to the given distances.

Here is an example computed by a student with distance data from a Rand McNally highway atlas.

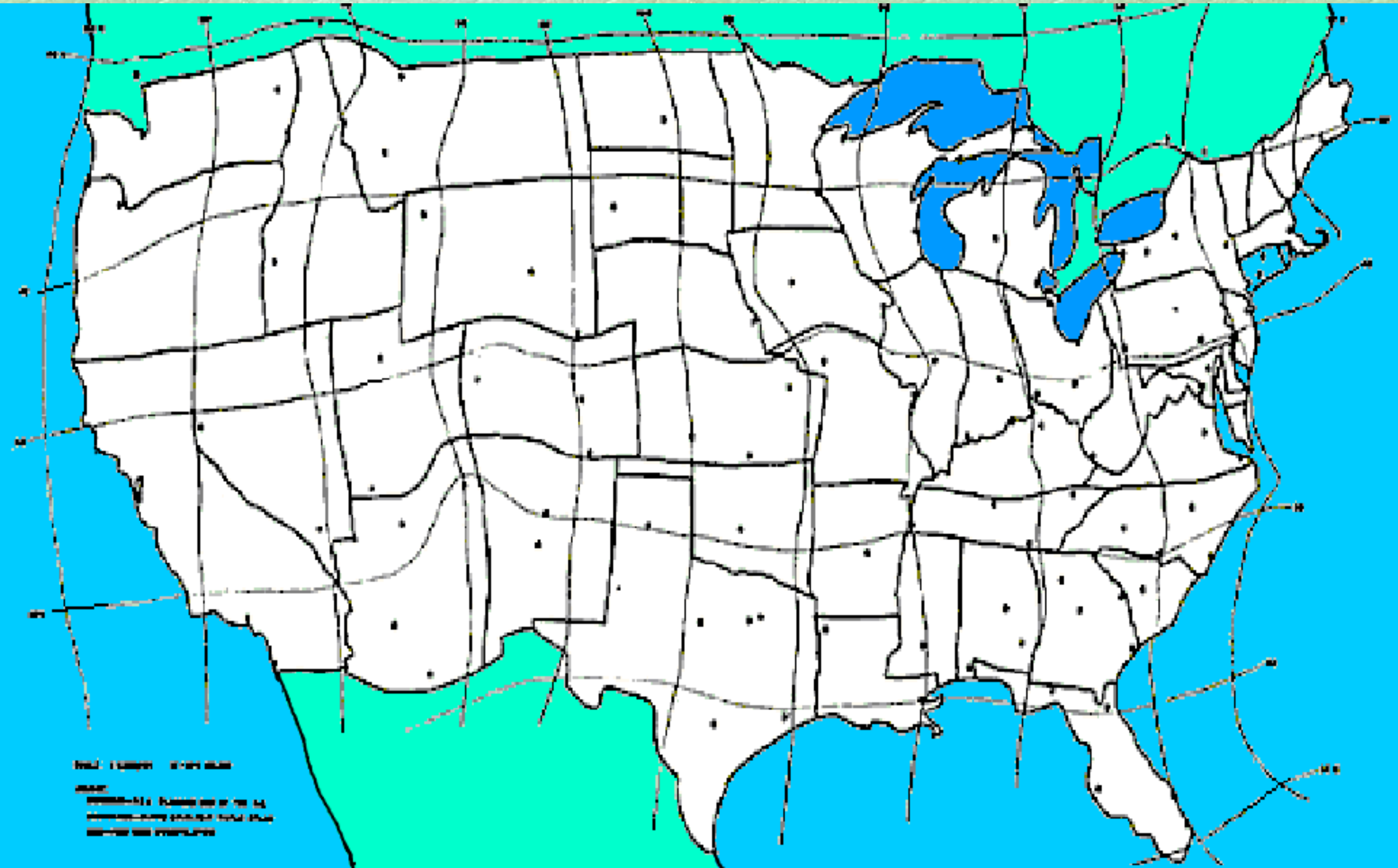
The towns are located at coordinate positions such that road distances are preserved as nearly as possible.

This is done using a trilateration or multidimensional algorithm.

Latitude and longitude lines and state boundaries are then interpolated to fit these locations, and the map drawn.

US Road Distances Map

Student drawing



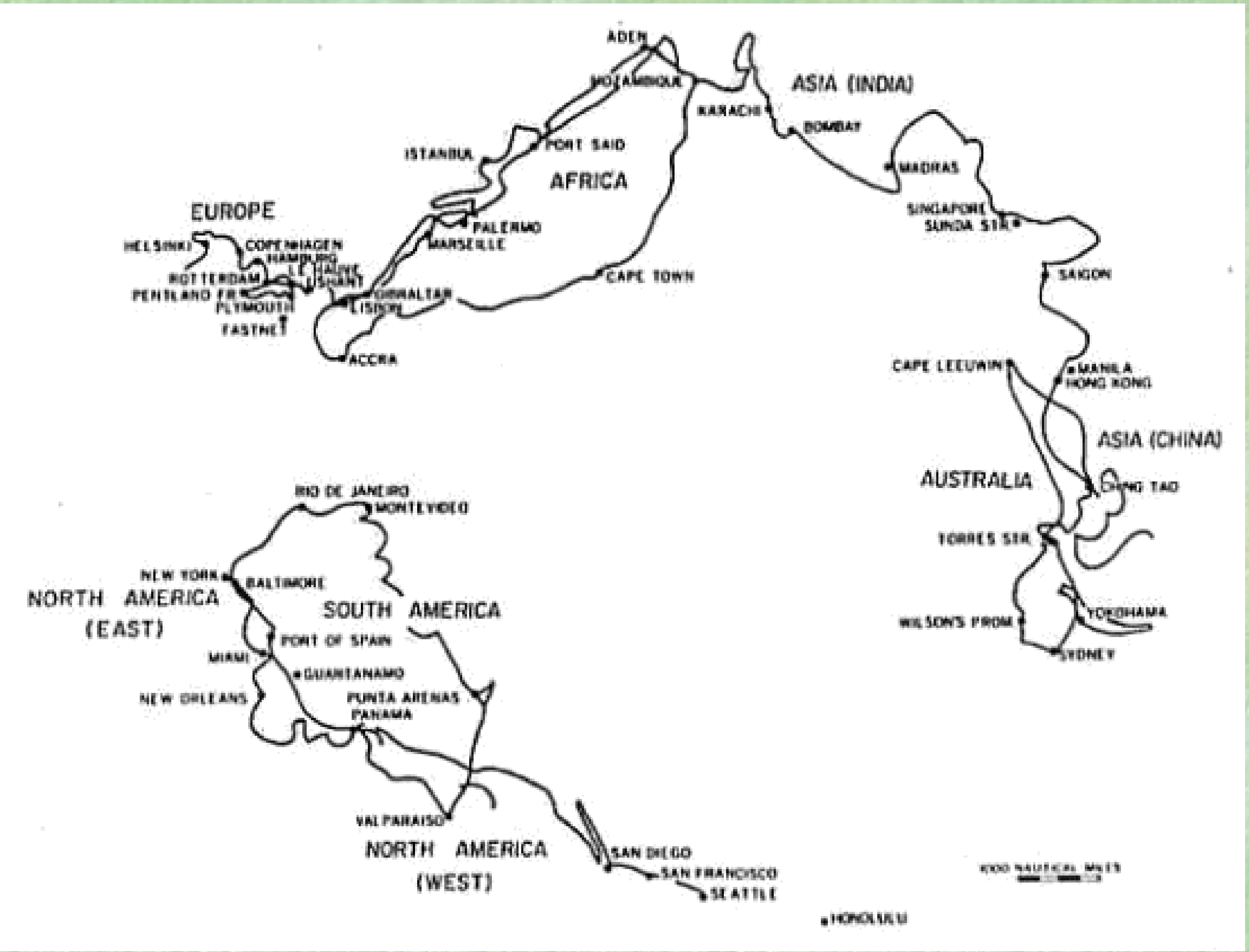
The road distance map is distorted relative to the normal map.

From the theory of cartography we know that map distortion can be measured using Tissot's strain tensor, and this can serve as a measure of the change introduced into the United States by the road system. In addition to distance changes there are also angular and area changes, and these can be computed. Perhaps these can be related to economic impacts.

The distorting effects of topography - think of the push to the West across the Rocky Mountains in the United States - can also be measured using Tissot's theorem.

World Ocean Distances Map

Based on Shipping Distances Between 42 Ports



The next example uses a 13 by 13 symmetric table of measures between thirteen places as input. Such a table might have come from a road atlas.

But road distances, or travel times or costs, are usually not the same as "as the crow flies" distances.

There are several ways of representing the time or cost metrics. For example, Insert resistor like symbols between the places, keeping their locations fixed.

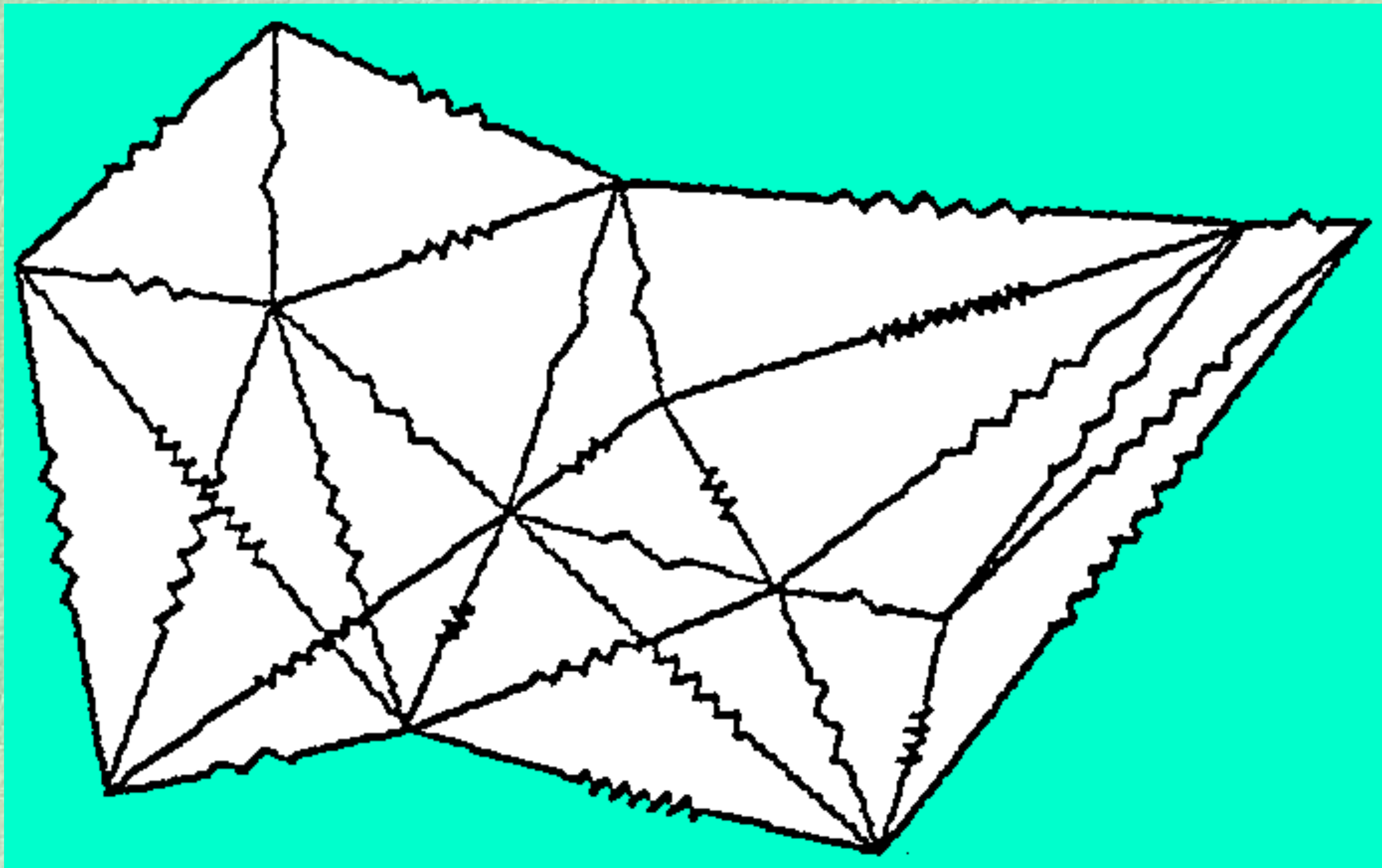
This should be done after parsing the table to get rid of the redundancies, using the triangular inequality.

Then the road distance, or travel time or cost, is represented by the total length of the line connecting the places.

One needs to measure along all of the wiggles between the places.

Such measurement is not really easy, but the graphic is effective.

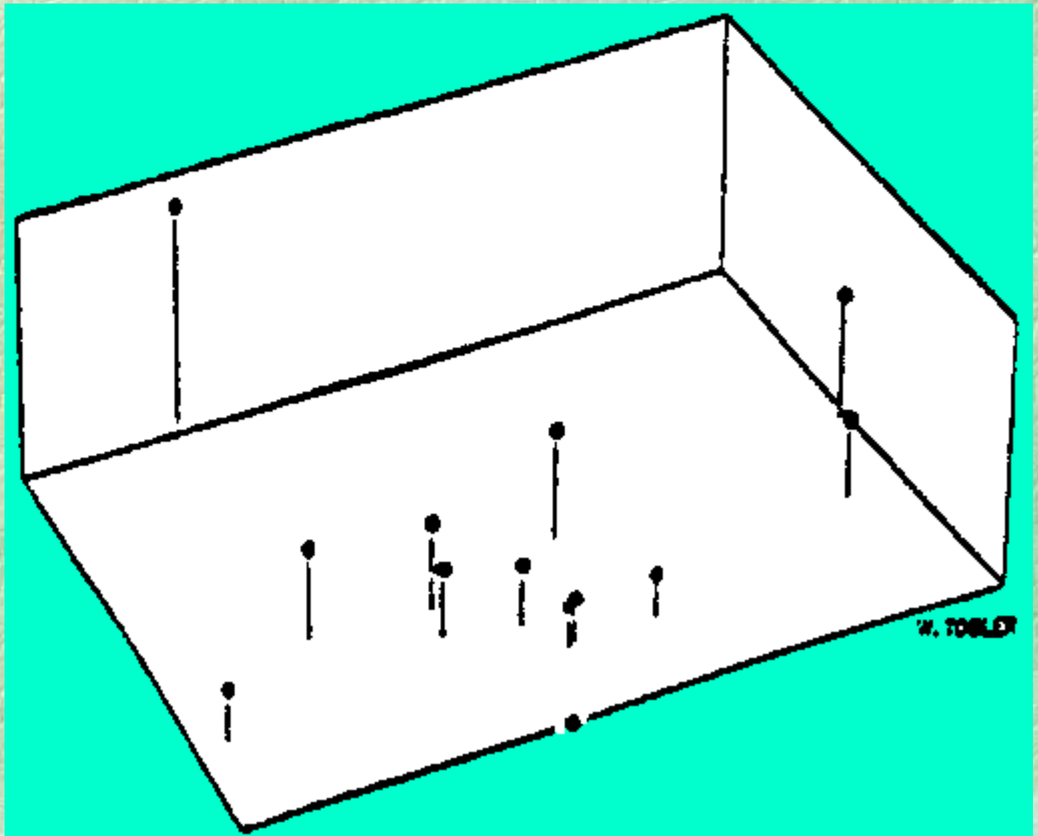
Measure Distances Along the “Road”



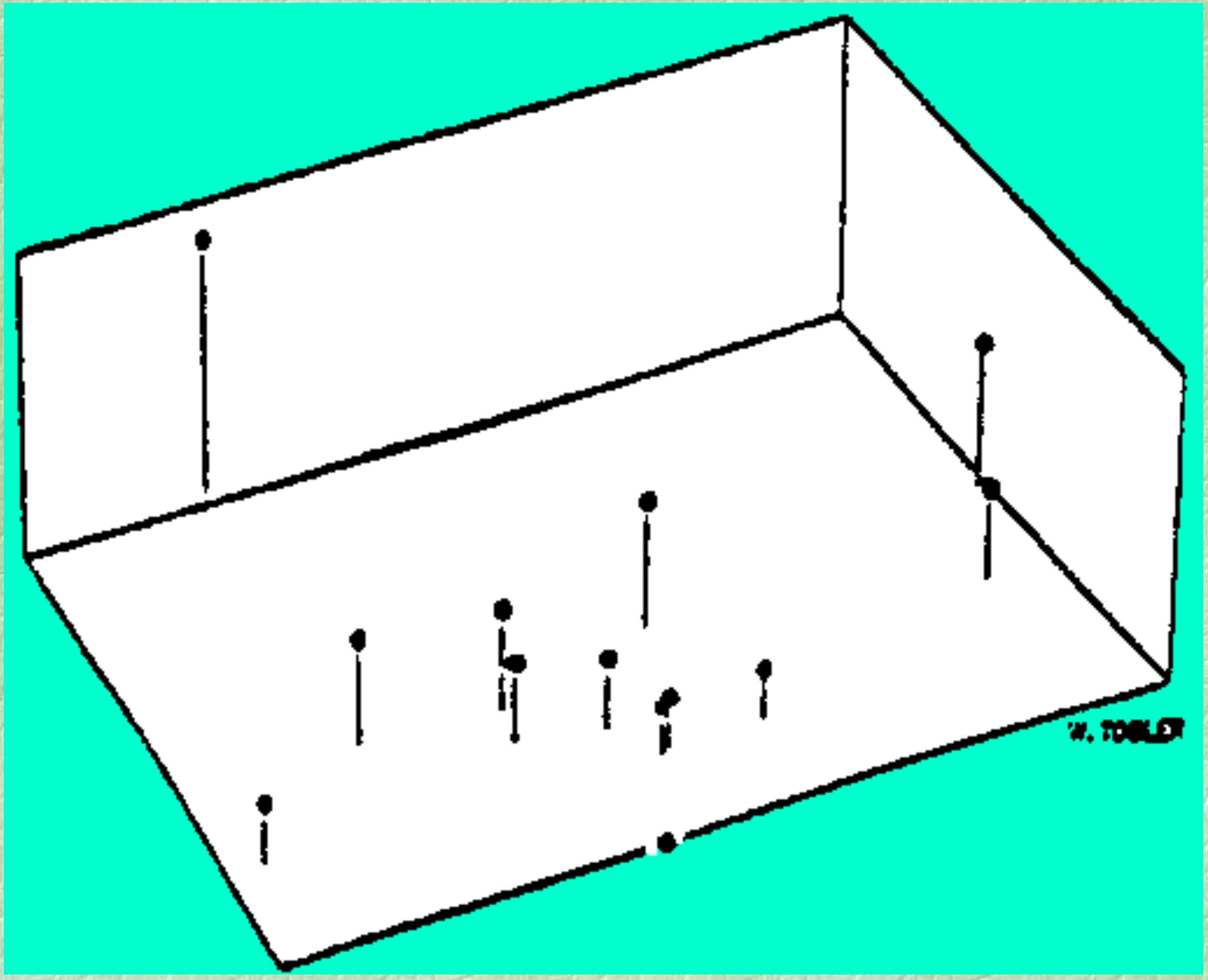
Alternately, raise the places into the air so that the Euclidean distance in three space gives the correct distance, in road distance space, in time distance space, or in cost distance space.

That is, use least squares to go from a representation in X, Y space go to a representation in X, Y, Z space.

Then, from the point representation in 3-D, interpolate to get a "transportation surface".

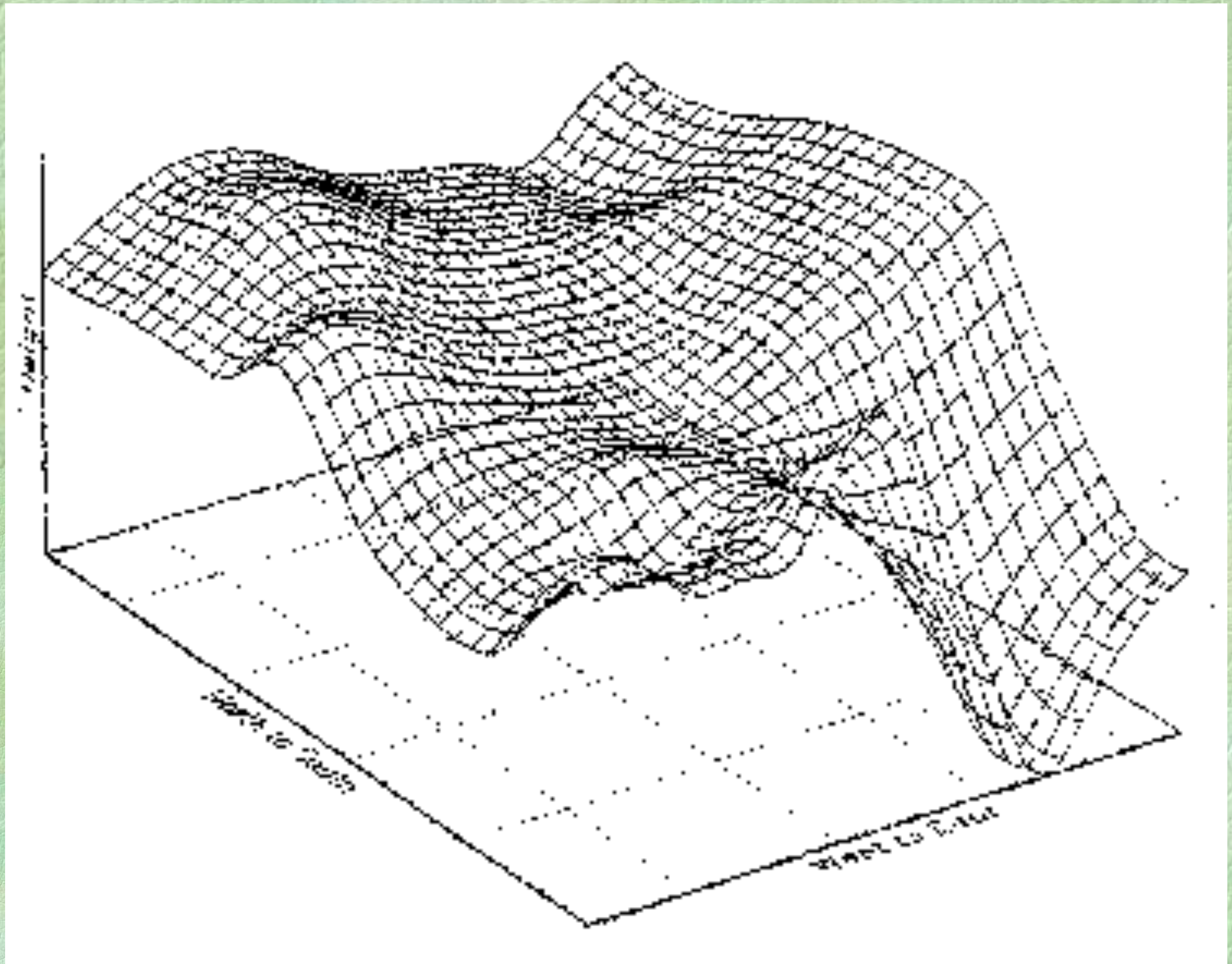


Measure distances through the air



Interpolated Transportation Surface

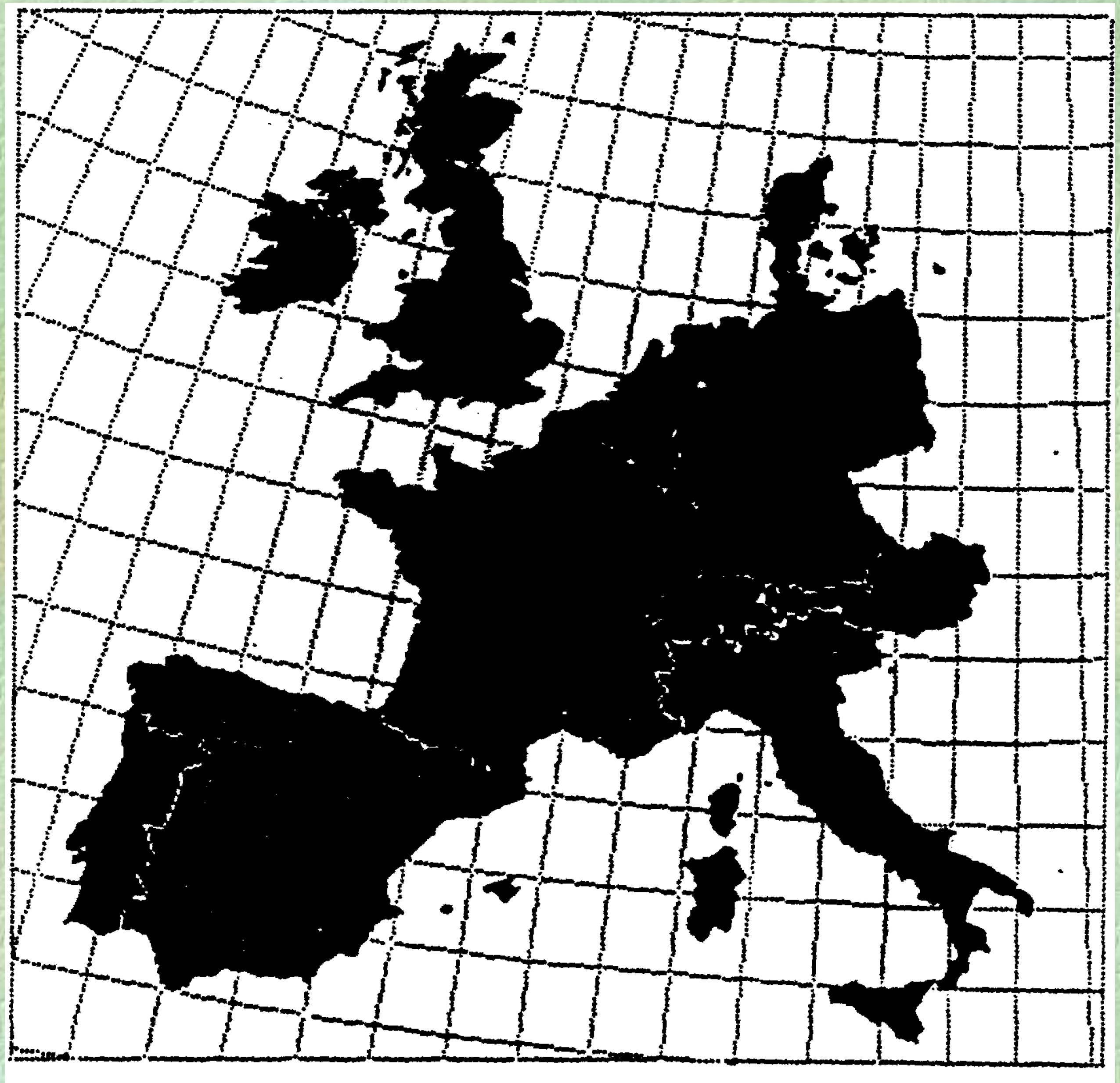
Measure distances as arc length on the surface



The next example, by Klaus Spiekermann and Michael Wegener, uses a similar technique to examine the effect of the European high speed rail connection, existing and contemplated.

The first diagram shows a conventional European map.

Europe Now



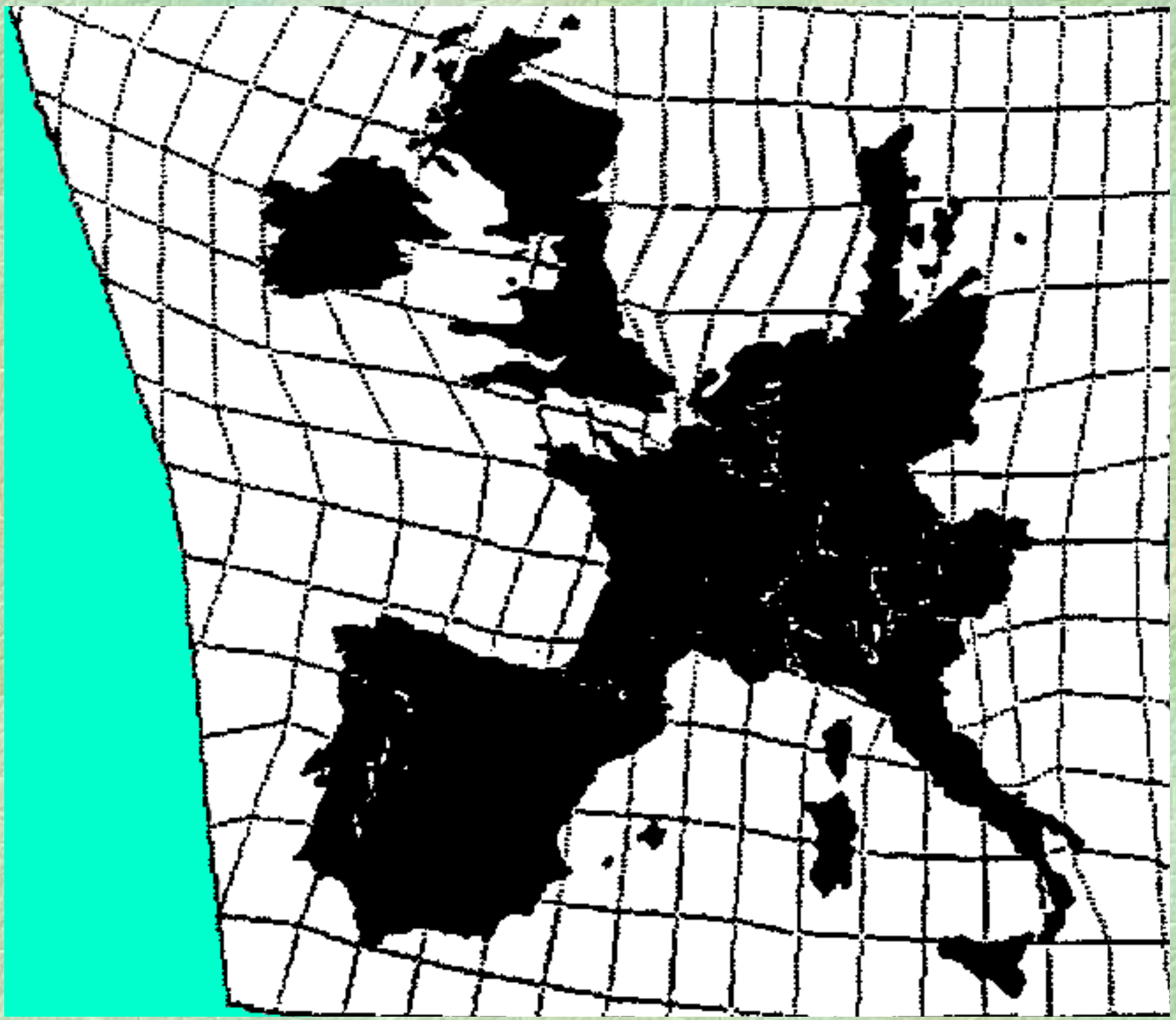
The next figure shows the warping that will be introduced by the high speed network, as measured in travel time space. The scale is given in hours, rather than kilometers and shows what Europe will look like when the high speed network is completed. Observe that France is furthest along, and therefore the most shrunken.

The assumption has been made that one can interpolate from the rail network to the entire continent.

Again, from cartographic theory, we know that not only are conventional distances distorted on this map but that areas and angles are also distorted, and we can compute by how much. Thus we can measure the distorting effect of transportation, using Tissot's theorem.

Europe After The Proposed TGV System

Spiekerman & Wegener



A profoundly more realistic example, by Alain L'Hostis, is from France.

Best viewed in color at http://www.inrets.fr/traces/equipe/lhostis/lhostis.htm

This is where I got the term shriveling. Maybe it should be wrinkling.

The map is in perspective, but shows three transportation systems simultaneously, using travel time as the metric.

First, on top and with the shortest connections, is the high speed rail system (TGV).

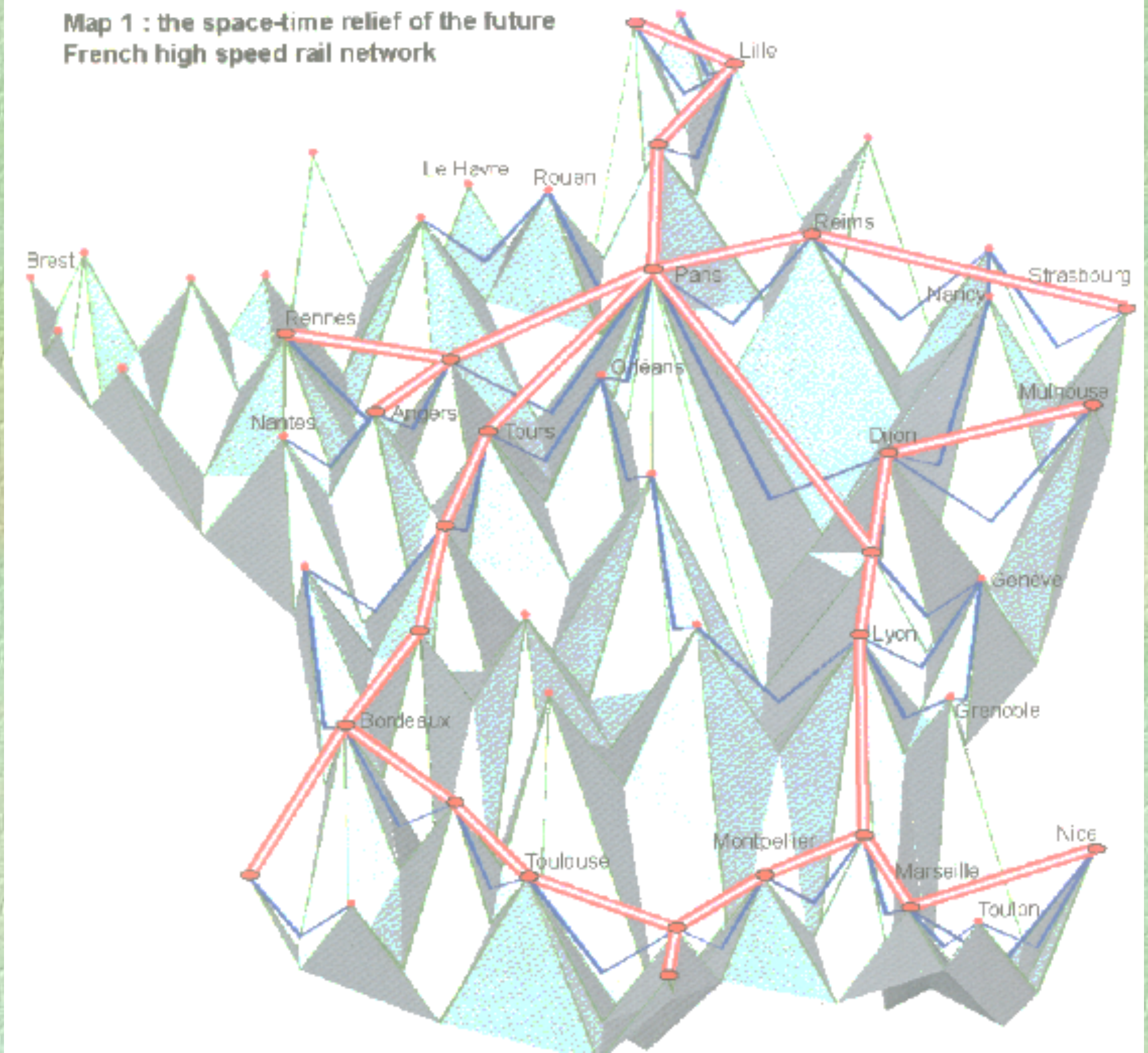
Then below this is the freeway system, in blue. Distances are to be measured along the blue lines, over hill and dale.

Below these two is the ordinary road network. Measure along the lighter lines.

This is a 3D version of the resistor diagram shown earlier.

The TGV Warps Space

By A. L' Hostis



One can get to any place in France using a combination of the three modes of transport.

From this diagram one can see how a new disease might diffuse from a rural location and quickly get transmitted to Paris, or how an idea could spread from Paris to other parts of the country before getting to remote “backwaters”.

Slightly unrealistic is the assumption that going from the TGV train to the freeway takes no time, air travel has been omitted, and that travel time is the same in both directions.

Admittedly measuring on this map would be difficult but this diagram is nevertheless a most marvelous invention, conceptually and graphically!

With GIS technology one might drape the conventional map of France on top of this transportation surface.
A dynamic version would pulsate.

Bunge suggested constrained balloons. Fix strings to well-connected places (NYC, LAX, etc) then inflate the balloon. The less well connected places will bulge out.

(Bunge, personal communication, ca, 1960)

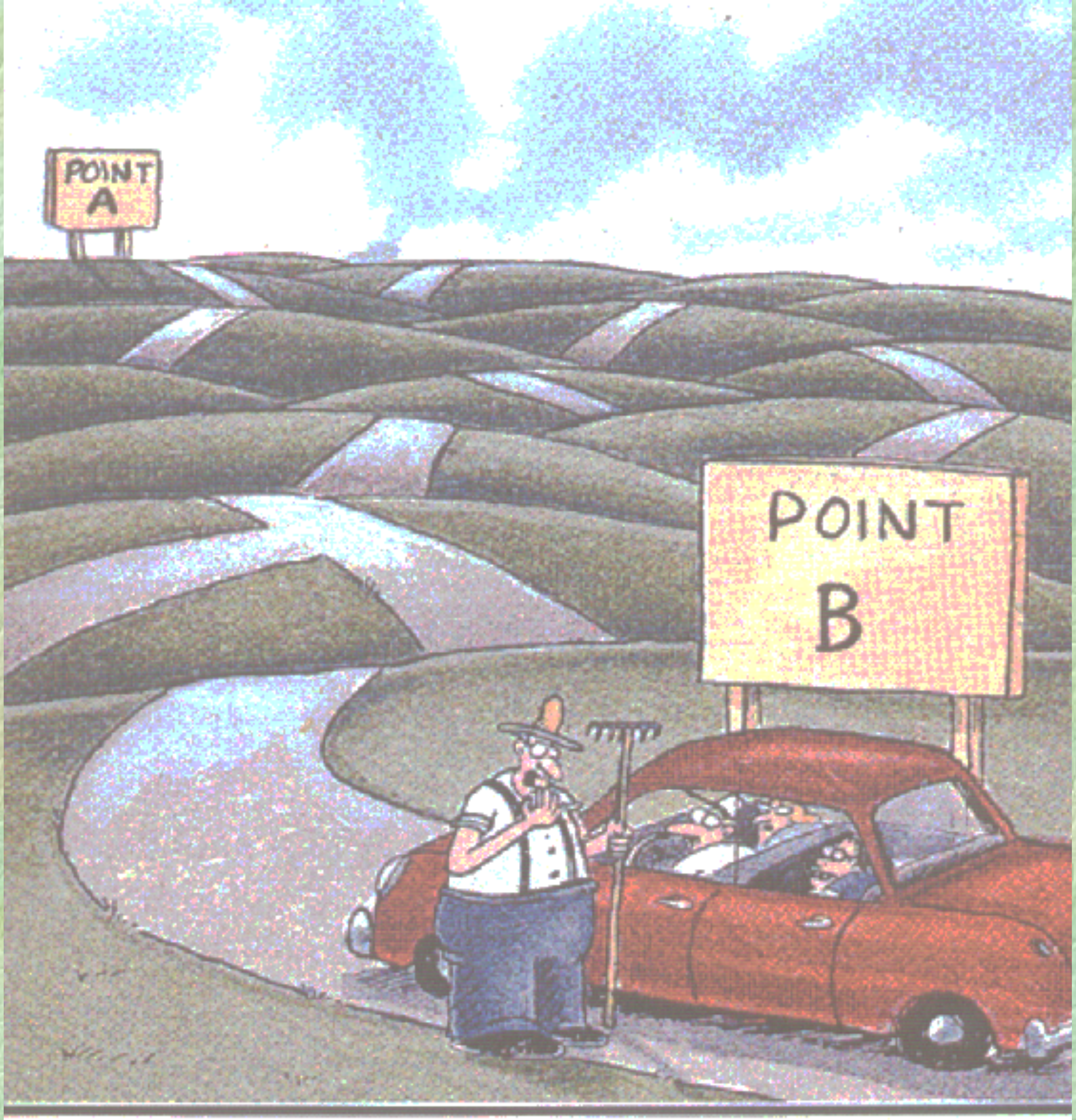
We are not finished

An important property of geographic distance measures has been overlooked

But first consider another aspect of the geometry of geography, as is represented by a cartoon from Gary Larson's "Far Side."

In this diagram a traveler in a vehicle is sitting at location **B** questions a farmer. Location **A** can be seen in the distance. In response to the query "How to get from **B** to **A**" the farmer at **B** opines that he is not sure, since most people want to go in the opposite direction. Does the farmer know that from **B** to **A** is not the same as from **A** to **B**?

Travel times, routes, or travel costs are usually not symmetric, and this is a complication.



"Well, lemme think. ... You've stumped me, son.
Most folks only wanna know how
to go the other way."

An example of an asymmetric geographical table.

Polynesian Communication Charges ($)

To From	CI	Fiji	F P	Ki	N C	PNG	SI	To	Tu	Va	WS	Aust	Fr	Ja	NZ	UK	USA
Cook I		5.29	5.29	5.29	5.29	5.29	5.29	5.29	5.29	5.29	5.29	5.29	9.35	9.35	3.70	9.35	9.35
Fiji	3.29		3.50	3.29	3.50	3.29	3.29	3.29	na	3.50	3.29	3.29	5.55	5.55	3.29	5.55	5.55
F Polynesia	14.85	14.85		na	7.95	14.85	14.85	14.85	na	7.95	14.85	12.73	15.91	24.39	12.73	21.21	24.39
Kiribati	6.50	6.50	6.50		6.50	6.50	6.50	6.50	6.50	6.50	6.50	6.50	9.28	9.28	6.50	9.28	9.28
N Caledonia	13.85	13.85	7.79	na		11.70	11.70	13.85	na	7.79	11.70	11.70	15.61	17.53	11.70	19.91	23.36
Papua N G	5.67	5.67	5.67	5.67	5.67		5.67	5.67	5.67	5.67	5.67	5.67	11.33	11.33	5.67	7.55	11.33
Solomon I	5.30	4.12	5.30	5.30	5.30	4.12		5.30	5.30	5.30	5.30	3.82	8.53	8.53	4.12	8.53	8.53
Tonga	3.47	3.47	3.47	3.47	3.47	3.47	3.47		3.47	3.47	3.47	3.47	6.95	6.95	3.47	6.95	6.95
Tuvalu	5.80	4.64	5.80	5.80	5.80	5.80	5.80	5.80		5.80	5.80	3.48	9.28	9.28	9.28	9.28	9.28
Vanuatu	7.96	5.24	7.96	7.96	5.24	7.96	7.96	7.96	7.96		7.96	5.24	10.29	10.29	5.24	10.29	13.02
W Samoa	3.86	3.86	3.86	3.86	3.86	3.86	3.86	3.86	3.86	3.86		3.86	5.14	5.14	3.86	5.14	5.14
Australia	3.71	3.71	3.71	3.71	3.71	3.02	3.02	4.87	na	3.71	3.71		3.71	4.87	3.02	3.71	3.71
France	12.80	12.80	7.53	na	7.53	12.80	12.80	12.80	na	12.80	12.80	11.17		11.17	11.17	2.63	5.47
Japan	6.27	6.27	6.27	6.27	6.27	6.27	6.27	6.27	na	6.27	6.27	6.27	7.88		6.27	7.88	5.20
N Zealand	2.79	2.79	2.79	2.79	2.79	2.79	2.79	2.79	2.79	2.79	2.79	2.79	5.40	5.40		5.40	5.40
UK	8.21	8.21	8.21	8.21	8.21	8.21	8.21	8.21	8.21	8.21	8.21	5.84	2.66	8.21	5.84		4.11
USA	5.77	5.77	5.49	5.77	5.77	5.77	5.77	5.72	5.77	5.77	5.72	4.66	5.27	4.14	5.53	4.08	

R.G. Ward, 1995, "The Shape of the Tele-Cost Worlds", A. Cliff, et al, eds., *Diffusing Geography*, p. 228.

Here is another very small example of a geographic table. It shows mail delivery times, in days, between a few cities in the United States.

The cities are New York, Chicago, Los Angeles, Washington D.C., St. Louis, and Houston.

There are many examples of such asymmetric tables, especially when considering costs. Any such table can be decomposed into symmetric and skew symmetric parts, each of which contributes to the total variance.

Table of Mail Delivery Times

Transit time for US mail, in days (1973)

From: \ To:	NYC	CHI	LAX	WDC	STL	HOU
NYC	0.9	1.8	2.5	2.0	2.3	2.3
CHI	2.6	0.8	3.1	2.2	1.9	2.3
LAX	2.5	2.2	1.1	2.2	2.3	2.6
WDC	1.8	2.3	2.6	1.3	2.4	2.5
STL	2.4	2.1	3.1	2.4	0.9	2.5
HOU	2.3	1.9	2.8	2.2	2.2	1.1

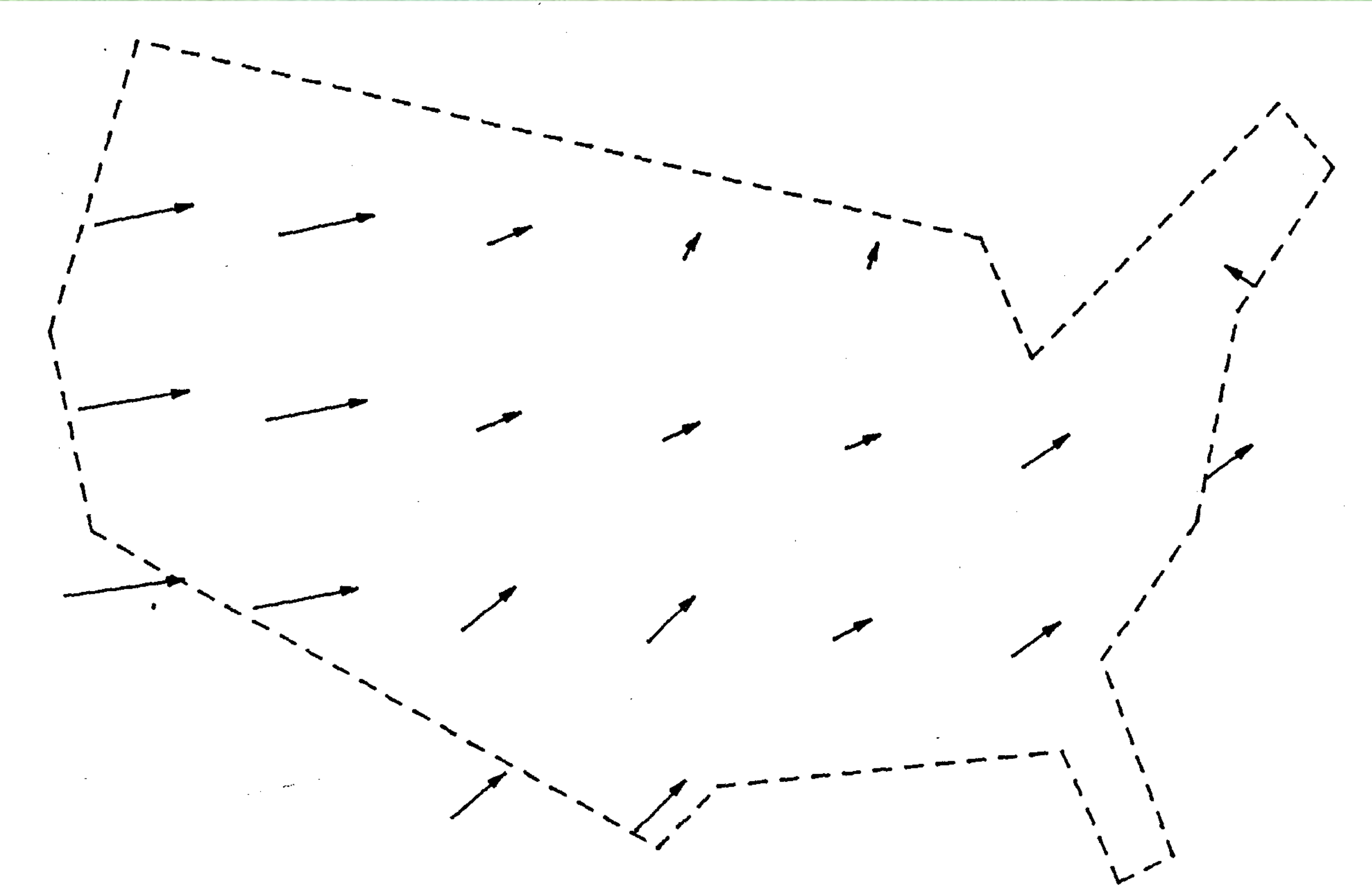
The asymmetry can be exploited, as can be seen in the next diagram.

Having produced a vector diagram, this can often also be converted into a potential field, by inverting the gradient operation.

That is, given the vector field, find the "topography".

My main application has been to asymmetric migration tables.

A Map of Wind Computed from Mail Delivery Times



From Wind to Pressure Field

- An interesting property of vector fields, as on the foregoing map, is that they may be inverted.
- If you think of a vector field as having been derived from the topography of some surface this assertion is that the topography can be calculated when only the slope is known.
- At least up to a constant of integration (the absolute elevation) and if the data are curl free.
- In the particular instance here, this says that the barometric pressure could be estimated from the mail delivery times.

What I am asserting is

that one approach to the asymmetry problem is to add a vector field to the distances.

This makes movement in particular directions easier or more difficult.

Such a vector field might be applied to simulations of the spread of ideas, and so on.

There are several possible mathematical implementations to this idea.

See: W. Tobler, "Spatial Interaction Patterns". *Journal of Environmental Systems*, 6, 4 (1976-77): 271-301.

Finally, in contemplating relations between places on the earth I hope that I have convinced you that it is often not the geodetic distance that is most important but rather the time or cost which must be overcome. Some places are now closer but others are relatively further away.

This is why I assert that

The earth is shriveling as it shrinks.

Thank You For Your Attention

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