

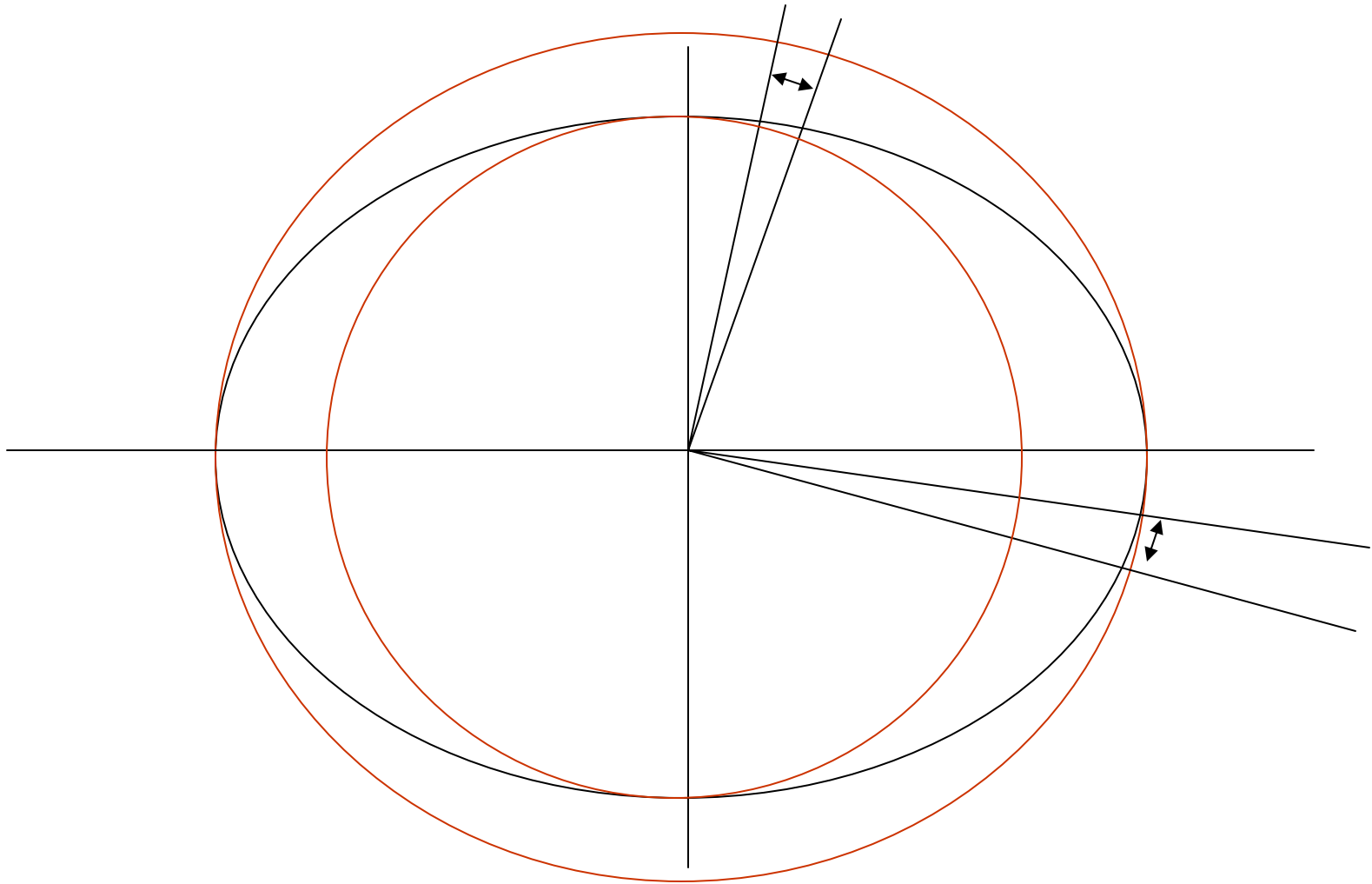
Is a degree of latitude, measured along a meridian, longer at the pole than the equator?

Length of a degree

<http://www.csgnetwork.com/degreelenllavcalc.html>

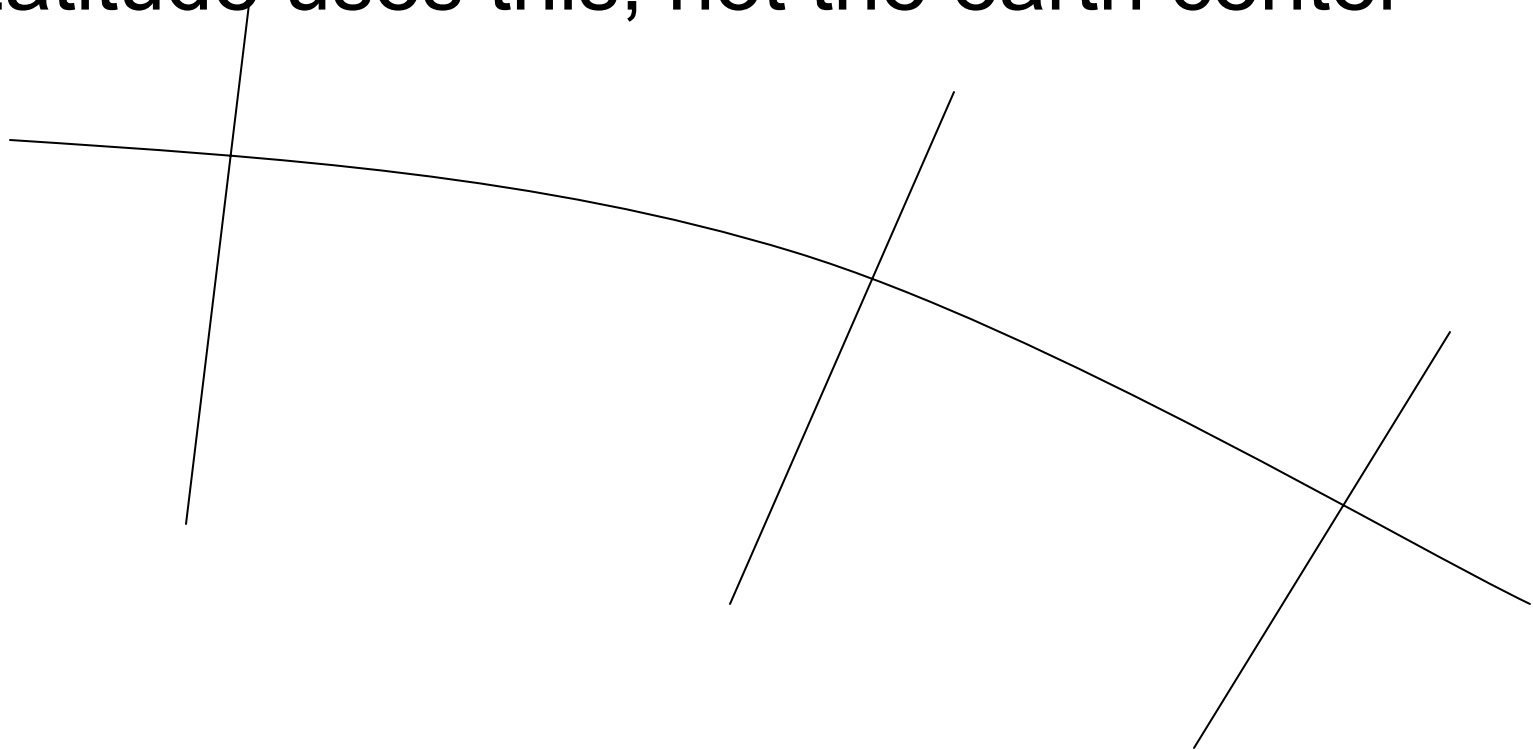
Latitude	Length of 1 Degree of Latitude
0	110574.27m
45	111131.75m
90	111693.92m

Counterintuitive?

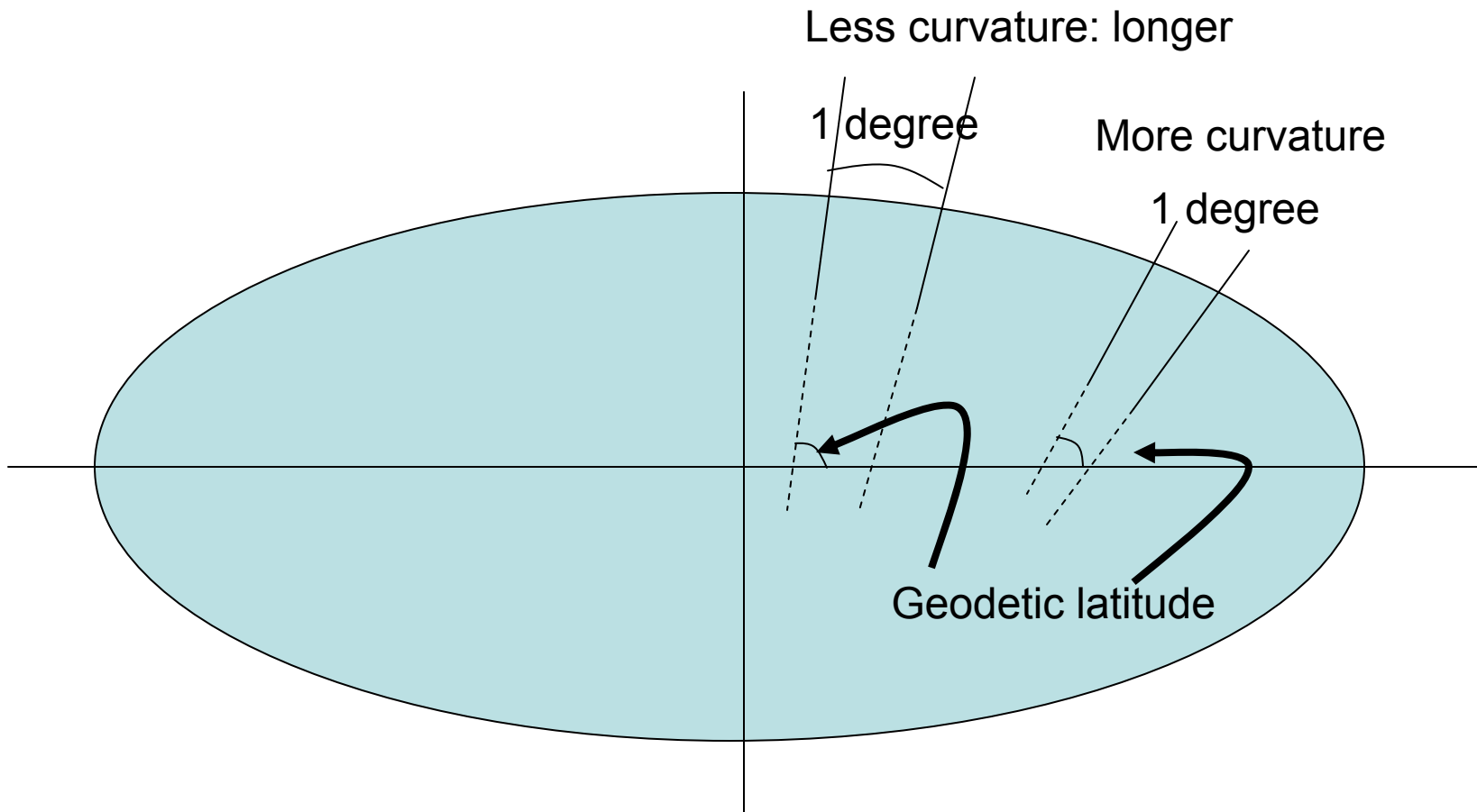


But...ellipsoidal “geodetic” latitude is not geocentric!

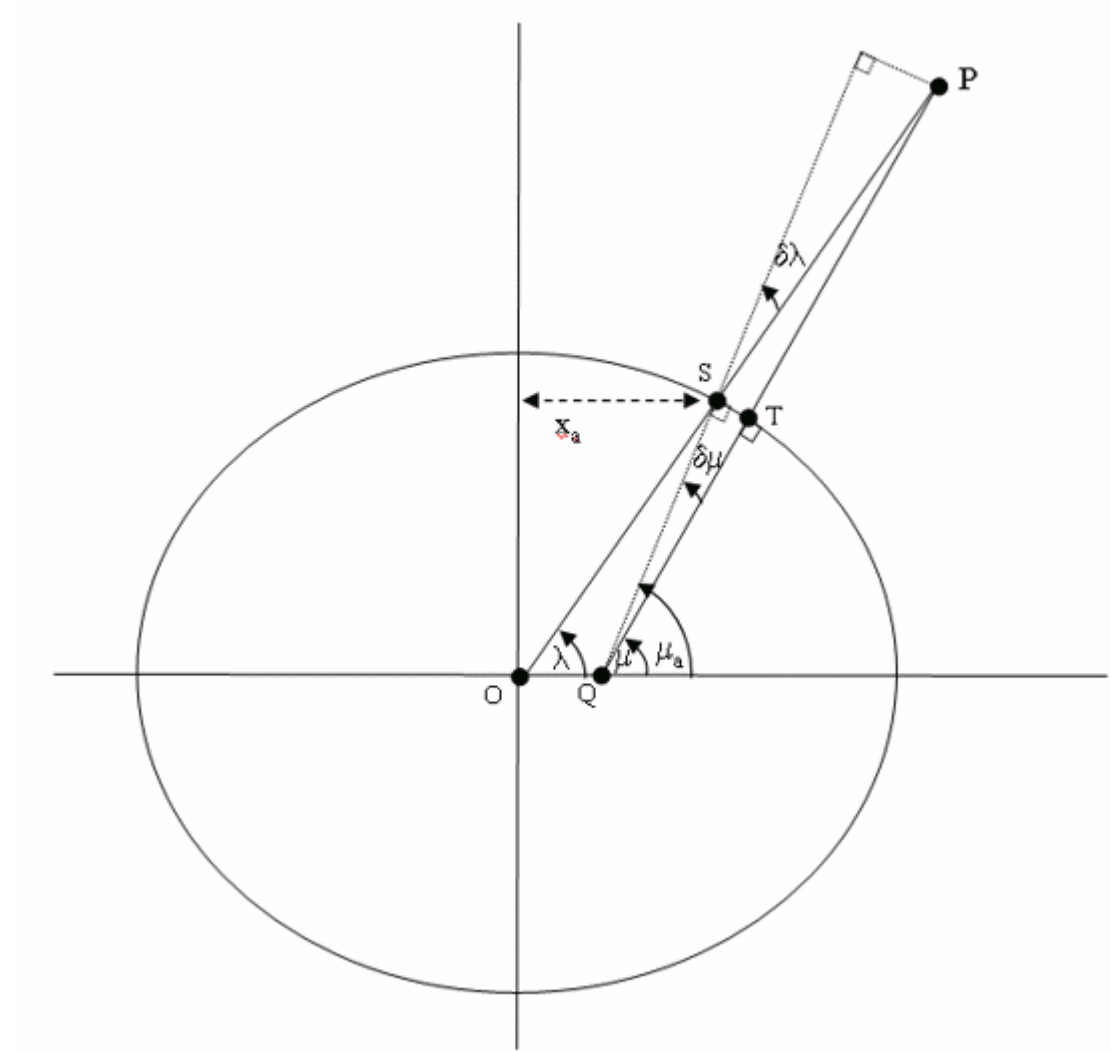
- Surface normal and the deflection of the vertical
- Latitude uses this, not the earth center



So, on the ellipsoid



Geodetic to geocentric



If geocentric: Calculate Length of an elliptical arc

- For a circle, arc length = $2 \pi R A/360$
- But for an ellipse, lengths are more complex
- Involves either approximation (Simpson's Rule or Prismoidal formula) or elliptic integrals
- More generic Vincenty formulation with code: (<http://www.movable-type.co.uk/scripts/latlong-vincenty.html>)

Distance along any geodesic line on the ellipsoid

a, b = major & minor semiaxes of the ellipsoid

f = flattening $(a-b)/a$

WGS-84 $a = 6\,378\,137$ m (± 2 m) $b = 6\,356\,752.3142$ m $f = 1 / 298.257223563$

ϕ_1, ϕ_2 = geodetic latitude

L = difference in longitude

$U_1 = \text{atan}((1-f) \cdot \tan \phi_1)$ (U is 'reduced latitude')

$U_2 = \text{atan}((1-f) \cdot \tan \phi_2)$

$\lambda = L, \lambda' = 2\pi$ while $\text{abs}(\lambda - \lambda') > 10^{-12}$ { (i.e. 0.06mm)

$\sin \sigma = \sqrt{(\cos U_2 \cdot \sin \lambda)^2 + (\cos U_1 \cdot \sin U_2 - \sin U_1 \cdot \cos U_2 \cdot \cos \lambda)^2}$ (14)

$\cos \sigma = \sin U_1 \cdot \sin U_2 + \cos U_1 \cdot \cos U_2 \cdot \cos \lambda$ (15)

$\sigma = \text{atan2}(\sin \sigma, \cos \sigma)$ (16)

$\sin \alpha = \cos U_1 \cdot \cos U_2 \cdot \sin \lambda / \sin \sigma$ (17)

$\cos^2 \alpha = 1 - \sin^2 \alpha$ (trig identity; §6)

$\cos 2\sigma_m = \cos \sigma - 2 \cdot \sin U_1 \cdot \sin U_2 / \cos^2 \alpha$ (18)

$C = f/16 \cdot \cos^2 \alpha \cdot [4 + f \cdot (4 - 3 \cdot \cos^2 \alpha)]$ (10)

$\lambda' = \lambda \quad \lambda = L + (1-C) \cdot f \cdot \sin \alpha \cdot \{\sigma + C \cdot \sin \sigma \cdot [\cos 2\sigma_m + C \cdot \cos \sigma \cdot (-1 + 2 \cdot \cos^2 2\sigma_m)]\}$ (11)

$u^2 = \cos^2 \alpha \cdot (a^2 - b^2) / b^2$

$A = 1 + u^2/16384 \cdot \{4096 + u^2 \cdot [-768 + u^2 \cdot (320 - 175 \cdot u^2)]\}$ (3)

$B = u^2/1024 \cdot \{256 + u^2 \cdot [-128 + u^2 \cdot (74 - 47 \cdot u^2)]\}$ (4)

$\Delta \sigma = B \cdot \sin \sigma \cdot \{\cos 2\sigma_m + B/4 \cdot [\cos \sigma \cdot (-1 + 2 \cdot \cos^2 2\sigma_m) - B/6 \cdot \cos 2\sigma_m \cdot (-3 + 4 \cdot \sin^2 \sigma) \cdot (-3 + 4 \cdot \cos^2 2\sigma_m)]\}$ (6)

$s = b \cdot A \cdot (\sigma - \Delta \sigma)$ (19) $\alpha_1 = \text{atan2}(\cos U_2 \cdot \sin \lambda, \cos U_1 \cdot \sin U_2 - \sin U_1 \cdot \cos U_2 \cdot \cos \lambda)$ (20)

$\alpha_2 = \text{atan2}(\cos U_1 \cdot \sin \lambda, -\sin U_1 \cdot \cos U_2 + \cos U_1 \cdot \sin U_2 \cdot \cos \lambda)$ (21)

Where:

s is the distance (in the same units as a & b) α_1 is the initial bearing, or forward azimuth

α_2 is the final bearing (in direction $p_1 \rightarrow p_2$)

Solved first by measurement!

