ANALYSIS OF SCALLOP PATTERNS BY SIMULATION UNDER CONTROLLED CONDITIONS[1]

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ABSTRACT

Working with plaster of paris in an experimental flume, the authors have simulated the formation of scallop patterns, an intriguing feature of eroded limestone, under controlled conditions of velocity and viscosity. Analysis of the resulting scallop lengths as a frequency distribution has shown that certain of the statistical parameters are well correlated with the hydrodynamic conditions. Length is inversely related to velocity and directly to viscosity. These results are similar to those found by Curl (1966) in a theoretical dimensional analysis of the simpler flute problem. Work in limestone caverns has confirmed that these results apply to scallops generated on limestone. Certain lithologic effects have been noted, however, and these are believed to be correlated with the physical structure of the material.

INTRODUCTION: THE FORM OF CAVE SCALLOPS

The flow of a viscous fluid over a modifiable bed can produce a variety of small-scale relief patterns. Sediment ripples and dunes, which form on unconsolidated, non-cohesive beds, are perhaps the best known. Allen (1969) has studied "flute marks" in weakly cohesive clays. Features in many respects similar to these can result from the solution of limestone by flowing water. In the literature of limestone geomorphology, the terms "scallop" and "flute" have been used to refer to these small-scale, ripple-like features (fig. 1A) commonly developed on cavern walls. The features resemble a mosaic of inlaid scallop shells and are therefore treated as individuals defined by the ridges rather than the depressions of the surface.

As Bretz (1942, p. 731) noted many years ago, scallops (which he called "flutes") are generally steeper on one side, again in analogy to scallop shells. Further, the orientation of this steeper side is usually the same for all scallops on a cave wall. Scallops are in this respect similar to some sedimentary structures, such as current ripples in sand.

Observations of scallop patterns on limestone surfaces in the beds of streams have led many authors to the conclusion that they are the result of erosion by flowing water. The orientation of the scallop pattern is found to be such that scallops are always steeper on the upstream side. The pattern in figure 1A would therefore be interpreted as having been formed by a stream flowing in the direction indicated. Two definitive dimensions of a scallop are used: the width, the greatest extent perpendicular to the stream flow in the plane of the surface; and the length, the greatest extent between crests parallel to the flow.

Scallops are found in open channels, on the walls, ceilings, and floors of cave conduits, and on boulders or cobbles in stream courses. Although on any one surface there is little variation in scallop dimensions, these range widely between scattered locations. To quote extremes measured by the authors, scallop lengths of 2 mm are typical of steeply sloping conduits developed in Cambrian marbloid rocks in the Nakimu Caves, high in the Selkirk Mountains of British Columbia. They are commonly 2 m long in the great horizontal trunk aquifers of Mammoth Cave in the Central Kentucky Karst, developed in Mississippian limestones.

THEORIES OF SCALLOPS

Theories of speleogenesis recognize two processes of subterranean limestone removal, both dependent on flowing water. Lime_

stone may be removed by the action of dissolved carbon dioxide on calcium carbonate; alternatively, the erosion may be by abrasion by bedload or suspended sediment in the cave stream. Much has been written concerning the process of limestone removal responsible for the creation of a scalloped surface. Maxson (1940) observed scallop-like features on boulders in the Colorado River and believed their formation to be due to abrasion. Although most of the examples lower Reynolds Number and thus at lower velocities, *ceteris paribus*. Maxson considered that the qualitative nature of the relationship between scallop size and Reynolds Number was due to difficulty in the definition of the velocity parameter v. He further noted similarity between these features on river boulders and the faceting of desert pebbles by blown sand and proposed the same analysis, using appropriate values of density and viscosity.

Fig. 1A.—A scalloped boulder in the Nakimu Caves, British Columbia. Water flow direction indicated by arrow.

were on limestone boulders, he identified some on noncarbonate rocks. It is evident from the illustrations, however, that the features classified by Maxson cover a much wider range of form than those considered here.

Maxson attempted an analysis by calculating a Reynolds Number, Re, based on an arbitrary dimension B of the scalloped boulder: $Re = (Bv\rho/\eta)$, where ρ is the fluid density, v the velocity, and η the viscosity. A quantitative relationship was found between Re and the size of the features, smaller features being formed at

Several speleologists have offered hypotheses to account for the wide variation in mean scallop length. Glennie (1963), following a suggestion of Ashwell (1962), proposed that a small mean length is the result of a fast, turbulent flow. A large mean length results from an extended period of steady flow. If allowed to remain undisturbed, such a steady flow would eventually produce a set of infinitely long scallops. Davies (1963) reported a conversation with Hsuan Yeh in which the latter suggested that the size variation is a matter of age, the length of a scallop increasing with time, apparently in-

definitely. However, if the scallops grow continuously, there must, on the same surface, be other features which continuously shrink. The nature of this competition between neighboring scallops for the available space must be considered.

From general observation, Ford (1964) proposed that large features are produced by turbulent, sediment-laden water corroding the limestone, whilst small features are produced by solution in slowly moving water. Eyre (1964) suggested the opposite relationship between stream turbulence and scallop length, noting that scallops which had formed on a bulge of the cave wall in Gaping Ghyll Caverns, Yorkshire, England, were smaller on the upstream side of the bulge than on the downstream side. This inverse velocity-size relationship was also the theme of a first suggestion by Curl (1959). Moore and Nicholas (1964, p. 11) published a graph of the relationship between length and velocity, showing an inverse power law of the form: $l = 56 \, v^{-0.75}$, where l is a measure of scallop length in centimeters, v is the stream velocity in centimeters per second. Moore and Nicholas give no source for their data. The first published experimental work is that of Rudnicki (1960), who produced scallops by immersing blocks of plaster of paris in natural streams and found that the number of scallops per unit area diminished in faster waters.

A hydrodynamic solution of the scallop problem is not feasible, whether on the basis of solution or abrasion. Curl (1966) reduced the problem to a two-dimensional one by eliminating all variation of form transverse to the flow. The features thus become similar to the swales of sand ripples. Curl called these features "flutes," a term used throughout this study. Approximate flute forms have been reported from several cave localities but are very much less common than scallops. Curl provided several illustrations of the form in nature and then proceeded to a dimensional analysis based on parameters likely to affect the flute pattern on a surface produced by solution alone.

There is much empirical evidence in sup-port of a wholly solutional origin of scallop patterns. Many eroded, scalloped cave walls have insoluble material standing out in sharp relief from the surface. The material is frequently fragile, responding to the slightest touch. In the Bonnechère Caves, Renfrew County, Ontario, shale beds protrude from the limestone walls of the passage and yet are readily crumbled. Oxidation may have severely reduced the mechanical stability of these materials after the passage was drained, but the presence of scallops on the roofs of other passages produced under very slow water flow adds weight to the solution hypothesis, as does the lack of contrast between scallops on roofs and those on the floors at the same sites. Finally, the authors have produced excellent patterns by eroding plaster of paris under controlled, sediment-free conditions in the laboratory. None of this evidence, however, completely rules out the possible influence of abrasion in the formation of such patterns.

Curl considered the following parameters to be those controlling the periodic length λ of a flute pattern: $\lambda = f(v, \, \rho, \, \eta, \, L, \, D)$, where L is a channel dimension, D is the combined diffusivity of the molecular species involved in the reaction. The reaction is controlled, in a simplified view, by the rate of diffusion of fresh CO_2 to the surface through the boundary layer and the rate of diffusion of bicarbonate back into the main stream.

Three dimensionless numbers may be constructed from these parameters, yielding $f \, [\lambda/L, \, \eta/D\rho, \, (v\lambda\rho)/\eta] = $ Constant. Curl suggested that the term λ/L could be ignored for flutes small compared with the channel dimensions and that the Schmidt Number, $\eta/D\rho$, could be ignored for extremely slow solution such as that of limestone. The equation becomes $(\lambda v\rho)/\eta = N_f$, a dimensionless "Flute Reynolds Number." From two brief field observations Curl derived an approximate value of 22,500 for N_f.

It is essential to this theory of flute formation that the flute pattern be stable,

maintained on the limestone wall without change of form as the rock is eroded. There must therefore be a relationship at every point on the surface between the rate of rock removal, controlled by the diffusion equation and the Navier Stokes equations, and the slope of the surface at that point. The specific flute cross section results from this requirement.

The equations governing the diffusion of the molecular species involved in limestone solution differ only in their constants from those governing diffusion of any other molecular species, and are also identical in form with those governing the diffusion of heat. The theory of flute formation therefore predicts that similar patterns will occur on all surfaces eroded by solution in fluids obeying the hydrodynamic equations, and on surfaces eroded by heat transfer also. Similar patterns are well known on ablating snow surfaces. Snow scallops are generated when the dominant mode of removal is by warm moving air, rather than by direct solar heating. They are common on high alpine snowfields in late summer (where we have measured mean lengths ranging from 10 to 30 cm), and often occur during mild winter spells in temperate latitudes. Jahn and Klapa (1968) provide good illustrations of this phenomenon.

Curl suggested in his paper that the breakdown of the ideal flute pattern into the more common, transversely asymmetrical scallop pattern is due to natural fluctuation of the controlling parameters, notably the stream velocity. His flute analysis has had great success, both in predicting periodic lengths under known hydrodynamic conditions and in predicting the development of similar forms on snow and ice. The great majority of erosion patterns observed by speleologists have no such cross-flow symmetry, however. But information on ancient flow rates in dry, abandoned cave passages can be of great value in understanding both the underground and the related surface land forms. It is for this reason that the authors investigated the feasibility of extracting hydrodynamic data

from the record preserved by scallop patterns.

LABORATORY EXPERIMENTS: DESIGN

A direct analysis by solution of the relevant hydrodynamic and molecular diffusion equations is clearly infeasible. The presence of boundary layer separation in the flow pattern requires the inclusion of viscous terms in the equations, and there is no indication that the flow is steady.

Scallop patterns were therefore investigated by laboratory simulation under controlled conditions. Plaster of paris was selected as base material rather than limestone because the material could be more readily standardized, and because the erosion process would be speeded up by several orders of magnitude. The dimensions of the experimental flume cross section were constrained by the pumping capacity and yet had to be much greater than the size of scallops produced.

The flume is shown in figure 1B. Overall length is 6 m, width 2 m, and circuit length 12 m. The Plexiglas working section is 4 m in length. The cross section is a constant 32 cm square. Bends in the flume are semicircular and contain no baffles. Water is driven around the duct by a propellor directly in the flow. Since the processes relevant to scallop and flute formation occur very close to the surface, conditions in the main stream flow can be largely ignored. Therefore, smooth flow over the experimental area was not of major importance in the flume design.

The solubility of calcium sulphate in water is low; therefore it was necessary to continuously replace the flume water with a fresh supply, at a rate of 10 liters per minute. Temperature control, and hence viscosity control, was achieved by switching the incoming water between hot and cold by the use of solenoid-operated faucets. In this manner, temperature was maintained to $\pm 0.05°$ C over 100-hour periods.

Scallops were produced on upstream-facing, wedge-shaped blocks of plaster which were cast in the flume to ensure a tight seal.

Well-scalloped surfaces (fig. 2) developed in 12–60 hours, depending on flow velocity and temperature, after the removal of up to 1 cm of material from the surface. In all cases the initial surface was plane.

The flume was carefully kept clear of sediment during the experiments. Initial scallops were observed to originate about small-size imperfections in the base material, either insoluble fragments or voids. Such imperfections, on the surface of the eroding block, cause a boundary layer separation immediately downstream. In this area the erosion rate is increased, allowing a shallow pit to develop. If sediment is present as a bed load, such incipient scallops are observed to fill with material, the erosion rate is reduced, and the surface remains flat. Absence of sediment also ensured that ero-

sion was by solution alone. Early experiments, using commercial grade plaster of paris, failed because of a high silt content. A change to reagent grade led to immediate success.

Characteristic velocities used were those 3 cm above the surface of the wedge in midstream, measured by a Pitot probe. The flume was operated at three temperatures in the range 10°–30° C, and four velocities between 30 and 150 cm/second. Because there was an acceleration of fluid over the experimental surface, the influence of this variable was investigated by repeating the experiments with wedges of different slopes. The wedges were 65 cm long, rising to 6 cm or 10 cm at the downstream end. At the highest velocity (150 cm/second) and highest temperature (30° C) no scallops de-
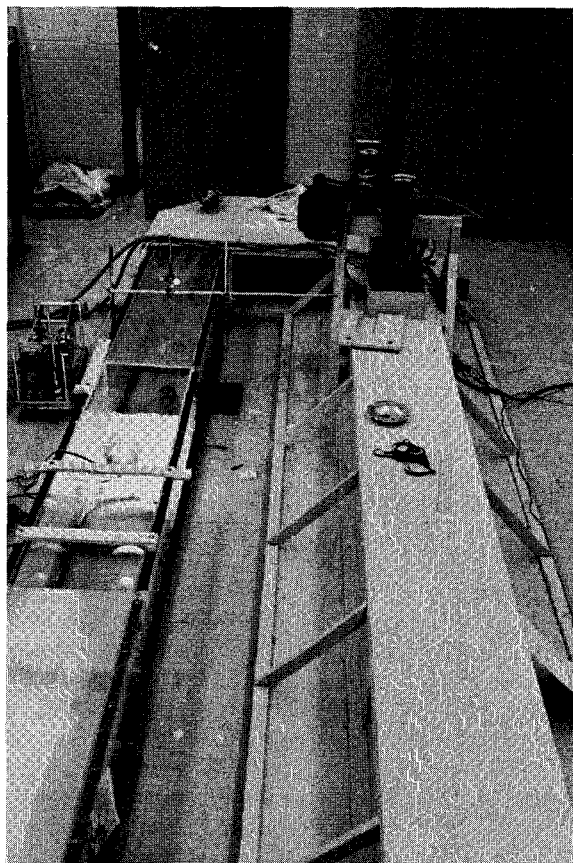

Fig. 1B.—The experimental flume

veloped on the steeply sloping block. With all other combinations of velocity, temperature, and slope, scallops were produced, amounting to a total of 23 successful experiments.

Because conditions varied over the wedge surface, due to the acceleration of the fluid, it was necessary to adjust measured scallop lengths according to their position on the wedge, before the frequency distribution of scallop lengths could be determined. Accordingly the distance of each scallop from

This procedure assumes only that the variation in scallop lengths over the wedge is a linear function of the distance from the downstream end. It does not imply the assumption of a linear relationship between size and velocity, nor does it preclude an absence of systematic variation over the wedge.

ANALYSIS OF EXPERIMENTAL RESULTS

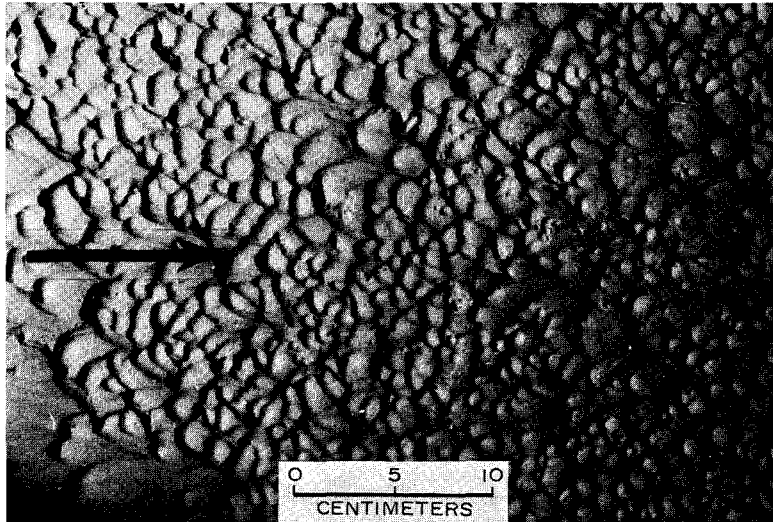Eleven length parameters were derived from each experiment: the mean length, and

FIG. 2.—Scallops generated on plaster of paris in the experimental flume, at 35° C and 114 cm/second on the low gradient block. The arrow indicates flow direction.

the downstream end of the wedge was recorded in addition to its length. The effect of location was removed by fitting a least-squares regression line to the lengths and distances and removing that variation explained by the regression line. In this way all scallop lengths were adjusted to a value appropriate to a location at the downstream end of the wedge. In this way any systematic, linear variation in scallop lengths was eliminated. The velocity used in the analysis was the extrapolated velocity 3 cm above the downstream end of the wedge. Figure 3 shows an experimental length distribution, before and after adjustment.

10 percentiles of the length distribution, the tenth, twentieth, thirtieth, etc., through one hundredth. It was hypothesized that scallop patterns are the result of a breakdown of an ideal flute pattern due to fluctuation in the controlling parameters, and that the distribution of scallop lengths is related to the flute periodic length through one or more of its statistical parameters. The mean lengths are tabulated in table 1 for each set of operating conditions.

The particular model investigated was

$$ l = a_0 \, v^{a1} \, \eta^{a2} \, \alpha^{a3} , $$

where l is a length parameter, α is the acceleration term and the a's denote constants.

The acceleration term a was given a value of 1 for the steeply sloping wedge configuration, and 2 for the gentler slope.

This model can be linearized to the form $\log l = a_0' + a_1 \log v + a_2 \log \eta + a_3 \log a$, where $a_0' = \log a_0$. It is equivalent to the Curl flute model if $a_1 = -1.0$, $a_2 = +1.0$, and $a_3 = 0.0$.

The model was evaluated by a least-squares multiple regression for each length parameter. Results are given in table 2. In all cases the acceleration term was found to be insignificant at the 95 percent level (if the conventional assumption of a multivariate normal distribution may be made). This may be due to one of two factors. First,
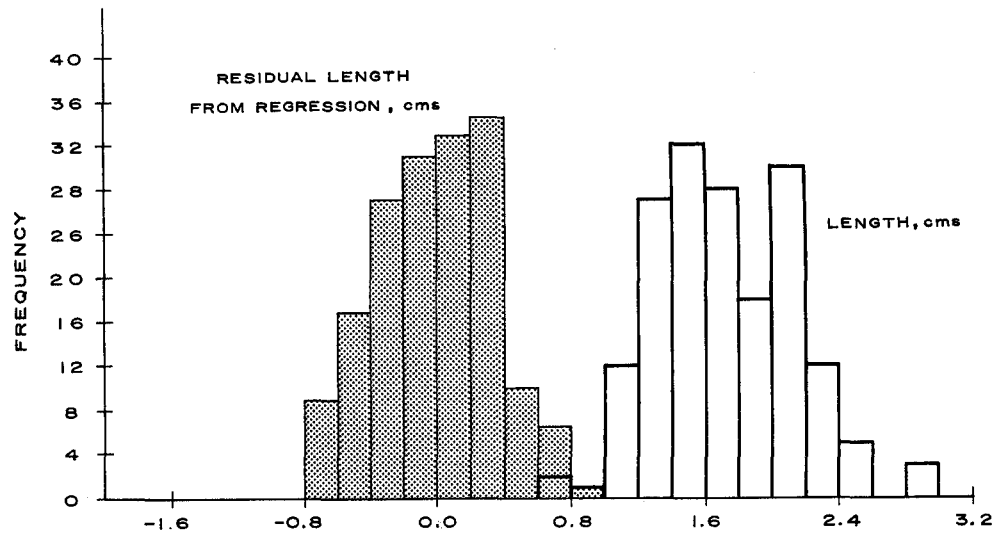

FIG. 3.—Distribution of scallop lengths from a typical experimental run, shown before and after correction for position on the plaster block.

TABLE 1

EXPERIMENTAL OPERATING CONDITIONS AND
RESULTING MEAN SCALLOP LENGTHS

TEMPERA-TURE (° C)	VISCOSITY (CGS UNITS)	VELOCITY (CM/SEC)		MEAN SCALLOP LENGTH (CM)	
		High Gradient	Low Gradient	High Gradient	Low Gradient
10.0.....	0.0131	129.3	112.1	1.305	1.633
10.0.....	0.0131	114.1	91.2	1.503	1.404
10.0.....	0.0131	99.2	80.2	1.786	1.755
10.0.....	0.0131	79.8	64.6	2.402	1.946
21.1.....	0.00988	129.3	112.1	0.905	0.984
21.1.....	0.00988	114.1	91.2	0.759	1.199
21.1.....	0.00988	99.2	80.2	1.169	1.374
21.1.....	0.00988	79.8	64.6	1.317	1.847
32.2.....	0.00761	....	112.1	....	0.767
32.2.....	0.00761	114.1	91.2	0.753	1.040
32.2.....	0.00761	99.2	80.2	0.711	1.008
32.2.....	0.00761	79.8	64.6	1.016	1.321

acceleration may play no part, independently, in the formation of scallops. Second, the actual value of the corresponding coefficient a_3 may be so low that differences in scallop measurements due to this factor are masked by sampling error and the probabilistic nature of the distributions themselves.

The values of a_1 and a_2 selected by the regression procedure are shown in table 2. A value of 1.0 (or -1.0) is expected in the case of linear dependence on the variable. The values determined differ from 1.0 (or -1.0) less in the case of velocity, which was directly measured, than viscosity, which was varied indirectly through the flume operat-

to this value, they are lower by as much as a factor of 2. These estimates are discussed later in a speleological context.

FIELD INVESTIGATION IN BONNECHÈRE CAVES, ONTARIO

The relationship between flow parameters and the scallop pattern cannot be investigated in the field in dry cave passages because the flow rates are never known. Relative flow rates may be deduced from geometrical considerations (for example, if it is known that a particular passage was full of water when the scallops were produced), and in this way the form of the relationship

TABLE 2

REGRESSION ANALYSIS OF FLUME DATA

Length Parameter	Pure Constant a_0	Velocity Coefficient a_1	Viscosity Coefficient a_2	Multiple Correlation Coefficient	Constant of Constrained Model $y = a x_2 / x_1$
Mean................	3.149	-0.9740	1.2997	.9353	11476
Percentile 10........	3.464	-1.1981	1.4824	.9270	8187
Percentile 20........	3.540	-1.1692	1.4967	.9343	9457
Percentile 30........	3.550	-1.1999	1.4753	.9203	10404
Percentile 40........	3.526	-1.1075	1.4713	.9275	11026
Percentile 50........	3.313	-1.0319	1.3701	.9331	11508
Percentile 60........	3.213	-0.9669	1.3233	.9199	12148
Percentile 70........	3.005	-0.8949	1.2250	.9157	12631
Percentile 80........	2.983	-0.8366	1.2008	.9154	13226
Percentile 90........	2.836	-0.8346	1.1264	.9004	14192
Percentile 100........	2.519	-0.4872	1.0074	.8673	16947

ing temperature. It should also be noted that four values of velocity were used, compared to three temperatures.

Although a full display of the five-dimensional data is not possible, figure 4 has been included to illustrate the intersection of the mean length regression plane with planes of constant temperature for the high gradient block configuration.

A second model of the form $l = a\eta/v$, where a is a constant, was also investigated by a similar least-squares technique. This model is less critical, but allows estimates of the constant a to be compared with Curl's Flute Reynolds Number, N_f. These estimates are given in table 2, and should be compared with the value 22,500 determined by Curl. Although the estimates converge

can be found. The laboratory results from plaster of paris were confirmed by comparison with limestone cave scallops at diverse sites in British Columbia, Ontario, California, Kentucky, and New York State. In one respect and in one instance, there was serious disagreement.

One area of the Bonnechère Caves, Renfrew County, Ontario, has been formed by the simultaneous erosion of five distinct limestone beds of quite different characteristics. The geometrical configurations of the passages are such that they can only have been formed by a stream which filled them completely (Ford 1961). It is now abandoned and drained, except for standing pools of filtration water. Because the passages are rectangular and horizontal, and because the
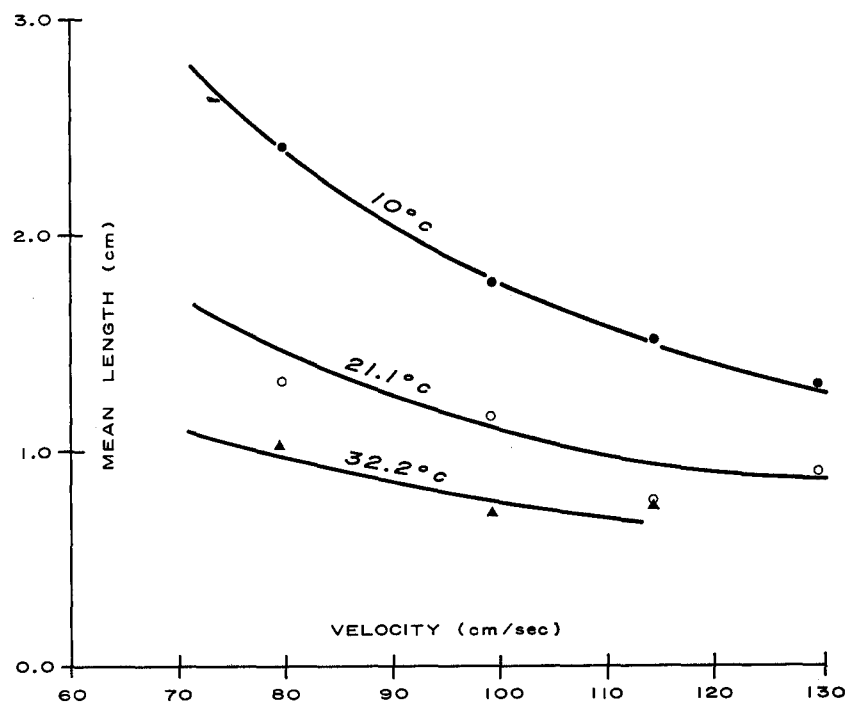
Fig. 4.—Experimental data for the high gradient blocks. Mean length is plotted against velocity for the three experimental temperatures.
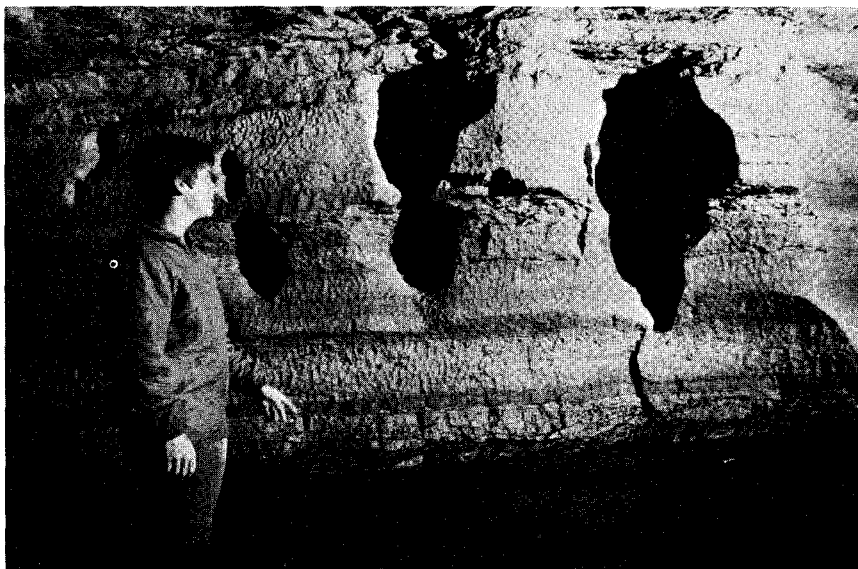

Fig. 5.—Scallops developed on different beds under similar hydrodynamic conditions in the Bonnechère Caves, Renfrew County, Ontario.

beds are also almost horizontal, the five limestones must have been subjected to similar hydrodynamic conditions. Comparison of scallop length distributions on the separate beds (fig. 5) shows a wide, systematic variation, however. Beds were numbered 0 through 4 from the uppermost downward, and samples of 25 scallops were measured on each bed at eight different sites. The mean lengths are given in table 3 as a ratio relative to the mean length in Bed 2. Although the mean lengths in each bed vary widely between sites, the ratios deviate by less than 15 percent from the mean values shown in table 3.

Several possible explanations for this

Samples of the beds were analyzed by buffered EDTA titration to determine the $CaCO_3$, $MgCO_3$, and insoluble contents. These chemical parameters showed no clear correlation with scallop sizes. Apparently, variations in the mean lengths cannot be explained simply by a variation in the rate of solution.

Thin polished slices were cut from samples of each limestone, at random orientations to the geological structure. The slices showed widely varying physical characteristics. Some beds showed greater irregularity, which could have led to a greater likelihood of scallop initiation and hence greater competition between scallops, but such a con-

TABLE 3

MEAN SCALLOP LENGTHS AND CHEMICAL ANALYSES OF VARIOUS
LIMESTONES IN BONNECHÈRE CAVES, ONTARIO

Bed	Thickness (Feet)	Mean Ratio	$CaCO_3$ (%)	$MgCO_3$ (%)	Insoluble (%)
0.........	2.0	1.30	95.6	1.3	3.1
1.........	0.4	1.95	95.1	0.1	4.8
2.........	1.1	1.00	93.4	1.7	4.9
3.........	2.0	1.53	91.9	5.0	3.1
4.........	3.4	1.60	93.9	2.5	3.6

lithologic control of the scallop patterns in Bonnechère Caves were considered. The lithology may affect the solution kinetics of the system or its hydrodynamics. Under given conditions of temperature, pH and CO_2 concentration, limestone solution depends primarily on the mineralogical composition of the carbonate rock. For example, the solution of dolomite proceeds at a much slower rate than the solution of calcite. Alternatively, the physical structure of the rock, the presence of insoluble fragments and voids, may produce local variation in solution rates, altering wall friction, the boundary layer thicknesses, and hence the mass transfer rates. Again, irregularities in the material may cause a larger number of initial vortices, leading to development of a larger number of scallops, with consequent increase in the competition between scallops for the available space.

cept would be difficult to quantify and test empirically.

The textures of the eroded surfaces of these limestones showed great variation. The exposed surface of Bed 1 had the smoothest appearance, while Bed 2 was roughest. Again, the texture of the eroded surface is responsible for scallop initiation, so that the greatest competition would occur on the roughest surface, leading to smaller scallops. But again any empirical test of this notion would be difficult.

CONCLUSIONS

On a standardized material such as a single, uniform bed of limestone or a block of plaster of paris, certain statistical parameters of scallop length distributions are well correlated with the fluid dynamic parameters in a manner similar to the model developed by Curl for the special flute form.

Of the 11 parameters tested in this study, the mean length appears to be most highly correlated with the fluid parameters, with a multiple correlation coefficient of .9353.

The lower percentiles of the length distribution show a greater correlation with the fluid parameters than the higher percentiles, implying that other, unknown factors play a larger part in the generation of the latter.

The best-fit values of the constant analogous to Curl's Flute Reynolds Number differ considerably from 22,500. The value corresponding to mean length was found to be 11,476, while the tenth percentile gave a value of 8,147 from flume observations. Measurements made on limestone showed, however, that the value of this constant depends on the nature of the eroded material, in particular upon some indefinite quality of its physical structure.

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