

Research Article

Extending geographical representation to include fields of spatial objects

THOMAS J. COVA

Department of Geography, University of Utah, 260 S. Central Campus Dr.,
Rm. 270, Salt Lake City, UT 84112, USA; e-mail: covat@geog.utah.edu

and MICHAEL F. GOODCHILD

Department of Geography, University of California, Santa Barbara, CA 93106,
USA; e-mail: good@geog.ucsb.edu

(Received 11 November 2000; accepted 17 September 2001)

Abstract. This paper describes a means for linking field and object representations of geographical space. The approach is based on a series of mappings, where locations in a continuous field are mapped to discrete objects. An object in this context is a modeler's conceptualization, as in a viewshed, highway corridor or biological reserve. An object can be represented as a point, line, polygon, network, or other complex spatial type. The relationship between locations in a field and spatial objects may take the form of one-to-one, one-to-many, many-to-one, or many-to-many. We present a typology of object fields and discuss issues in their construction, storage, and analysis. Example applications are presented and directions for further research are offered.

1. Introduction

Field and object models have increasingly gained acceptance as two alternative approaches for conceptualizing and modelling geographical phenomena (Goodchild 1989, 1992, Coucleis 1992, Worboys 1995, Burrough and McDonnell 1998). In the context of a field perspective, each location in space is mapped to a value selected from an attribute domain. Elevation, temperature, and precipitation are three example spatial variables that are routinely modelled using a field perspective. In the context of an object perspective, space is perceived as a region populated with discrete entities, each with identity, spatial embedding, and attributes. Roads, buildings, and rivers are three classes of phenomena that are commonly modelled using an object perspective. Although most geographic phenomena are generally perceived using either a field or object perspective, the approach most suited in a given instance depends on the purpose and context of the modelling exercise. For this reason, the two model classes are best considered *conceptual perspectives* rather than inherent qualities of geographical phenomena (Peuquet *et al.* 1999). Any aspect of geographical reality can be conceptualized and modelled using either of these two approaches. The field-object dichotomy is valuable from both a theoretical and applied

perspective. In theoretical terms, the separation provides an elegant means for framing two opposing conceptualizations of geographical space (Coucleis 1992). This makes the classification valuable in GIScience research and teaching. From an applied perspective, GIS studies generally rely on one, or both, of these approaches in addressing a spatial problem. The dichotomy is reinforced in practice, as the decision to utilize one of these two modelling approaches can significantly influence the analysis functions available to an analyst and, thus, the results of a study. Operations like a mean-value focal operator (Tomlin 1990) are inherently field-based, while operations like a line buffer are object-based. It is the task of an analyst to select the most appropriate conceptualization, representation, and spatial operator set in a given modelling context.

Although the field and object class duality is valuable and well articulated, certain modelling approaches appear to combine aspects of these two perspectives. As Worboys (1995, p. 177) notes, 'There is some level at which fields and objects can coexist'. Worboys provides an example where a field is a form of object that can itself be ascribed identity, properties, and behaviour. This is akin to applying an object-based modelling approach to the task of representing fields. A second case that combines aspects of a field and object approach is the familiar concept of a density field. Density fields are field-based in nature despite the fact that the value at each location cannot be separated from the size and shape of the spatial object used to assign the value (Penquet *et al.* 1999). These examples demonstrate that object and field-based models of geographical reality can coexist.

The purpose of this paper is to describe a means for linking the field and object representations of geographical space. The approach is based on a series of mappings, where locations in a continuous field are mapped to discrete, geo-referenced objects. In this context, an object is a modeller's conceptualization, as in a viewshed, highway corridor, or biological reserve. Objects can be represented using points, lines, polygons, networks, or other complex spatial type. The paper begins with background on the field and object models of geographical phenomena. A conceptual framework is presented that includes a typology of object fields and a formal specification. We describe issues in constructing, storing, and analysing such fields. Three applications of object fields are presented, and the paper concludes with a discussion of areas for further research.

2. Background

2.1. Field models of geographical space

A field model is one of many conceptual models of geographical variation and a basis for much scientific and geographical modelling (Sachs 1973, Angel and Hyman 1976, Tobler 1978, 1991, Kemp 1997a,b, Heuvelink 1996, Goodchild 1997). In this model, every location in a spatial framework is associated with a set of attributes measured on a variety of scales. Fields are spatially continuous by definition, but continuous might also refer to the measurement scale (z value). A field can be viewed as a mapping between a locational reference frame and an attribute domain (Worboys 1995). For this reason, a field is commonly referred to as a single-valued function of space because it assigns a value to every location. Theoretically, the set of locations in a field is infinite, as we can always invent points at which to make measurements. The measurement scale for the attribute domain can be any common scale including binary, nominal, ordinal, interval, or ratio. The spatial frame of reference can be of one, two, or three dimensions, with an additional dimension

to represent time. $Z(x)$ is a common means of formalizing a field, where x is a locational vector. Time can be modelled in a field context as either discrete time slices or as a continuous field in time which leads to a 3-D volume (Goodchild 1992). It is possible to perceive geographical variation entirely as fields, and there are many terms to describe features in fields like peak, ridge, valley, plateau, and saddle.

The most common field types are scalar, vector, and tensor. In a scalar field, every location is assigned a scalar value from an attribute domain. This is the most common type of field in GIS modelling, and the term 'field' with no additional information generally refers to a scalar field. In a vector field, every location in space determines a set of values that describe the direction and magnitude of a vector at that point or its components in two or three dimensions. Vector fields can be used to represent land surface gradients like slope and aspect, or dynamic phenomena on the land surface like wind, water, and fire. Vector fields can also be represented as two scalar fields, one for direction and one for magnitude, or one for a vector's x -component and one for its y -component (Hunter and Goodchild 1995). Tensor fields are commonly used to represent strain or stress in multiple directions and are represented using a matrix at every location.

Representations of fields must always be approximate, as we cannot store an infinite number of locations. Spatial tessellations (regular, irregular, or hybrid) are the most common means for representing field-based models. A central issue in using a tessellation is the meaning of the value in a spatial unit. Is it the highest value in the unit, the median, or the mean? It is also possible to build representations by defining how the field varies within each spatial unit using a mathematical function, as in a Triangulated Irregular Network (TIN). Resolution is also a significant issue, as is the shape of each spatial unit. Regular and irregular point grids can also be used to represent a field. This is a common method for representing raw measurements about a field, as in the temperature at weather stations, elevation spot heights from GPS, or ozone concentrations from air quality monitoring stations. Digital contours are a means for representing fields using closed polylines. This is an incomplete representation because every location does not have a value. It also does not work for nominal data like land cover type. The accuracy of a contour model depends on the density of contours and sampling along the contours. Irregular polygons can also be used to represent a field, given that the polygons tessellate the area of interest. This representation is spatially complete, but each polygon is an average or dominant type as in a soil or vegetation map. Thus, the detailed variation within each polygon is lost. The accuracy and detail of this approach depends primarily on the size of the polygons. Common operations on fields include interpolation, classification, convolution, spatial overlay, statistical analysis, map algebra, spread functions, corridor analysis, terrain analysis, and many others.

2.2. Object-based models of space

There are many phenomena in geographical reality that are readily perceived as objects. Lakes, rivers, valleys, roads, land parcels, buildings, and islands are a few examples. In an object perspective, space is viewed as a container populated by these objects, each with identity, spatial embedding, and attributes. Natural language is much more suited to describing objects than fields. Goodchild (1997) notes that weather phenomena like temperature and pressure gradients, although modelled by experts using continuous fields, are translated into objects to communicate with the

public, as in, 'A high-pressure system is moving across Utah', or, 'A warm front is approaching the west coast'. Smith and Mark (1998) distinguish *flat boundaries* from *bona-fide boundaries* in defining objects. Flat boundaries exist only in virtue of demarcations induced by human cognition and action, as in a county, park, or sand dune. Bona fide boundaries refer to genuine discontinuities in geographical reality, as in the case of a lakeshore, river, or bridge. Bona fide objects that are cultural artifacts like buildings and roads are generally well defined in their spatial projection, while flat boundaries (and thus objects) can range from crisp to very ill defined, as in an administrative unit, mountain, or desert.

An object's spatial projection is commonly represented in a GIS environment with points, lines, and polygons. The object conceptual perspective in geographical data modelling is very compatible with object orientation in software engineering (Egenhofer and Frank 1987, Galagan and Roberts 1988, Oosterom and Vanderbos 1989, Worboys *et al.* 1990, Frank and Egenhofer 1992, Worboys 1994). Increasing attention is being paid to improving the representation of ill-defined objects (Burrough and Frank 1996). While crisp objects are more suited to being represented with points, lines, and polygons, ill-defined objects are more suited to techniques in fuzzy modelling, a field-based modelling approach to an object conceptual perspective. The most common GIS operations performed on objects are manipulation tasks like adding, deleting, updating, moving, and transforming. Common object-based analysis functions include spatial query, point pattern analysis, distance calculation, overlay analysis, buffer generation, network analysis, cluster analysis, spatial similarity analysis, shape analysis, and location modelling (Longley *et al.* 1999).

3. Object fields

3.1. Conceptual background

Coucleis (1992) and Worboys (1995) note that the field and object conceptual perspectives should not be considered mutually exclusive. Certainly the two perspectives are routinely combined in a variety of GIS studies. For example, a meteorological GIS might store a field representation of temperature and pressure surfaces in conjunction with an object representation of fronts, highs, and lows. In another case, a roads layer might be used to generate a distance-to-roads surface to identify areas in proximity to roads. Thus, the field and object perspectives can be used in conjunction as well as derived from one another. This section describes a system for relating the field and object perspectives through a series of mappings. The mappings serve as a conceptual bridge between the two perspectives.

An object field (OF) is defined as a continuous field in which locations are mapped to spatial objects. In other words, an object field relates locations in a field-space to objects in an object-space. Therefore, it shares qualities of both the field and object conceptual perspectives of geographical phenomena. The object type associated with a location may be a point, line, area, network, or other complex spatial type. Figure 1 depicts an object field as a series of mappings between the field and object perspectives. The field perspective is depicted on the left, where each location is mapped to an element of an attribute domain. As noted, a field location generally determines a scalar, vector, or tensor. The object perspective is depicted in the centre of the figure. In this case, objects of various types with identity, embedding, attributes, behaviour, and a representation populate a region. On the right is the multi-representational perspective (object field), where space is modelled as relationships between a field perspective and object perspective. This can be viewed as a

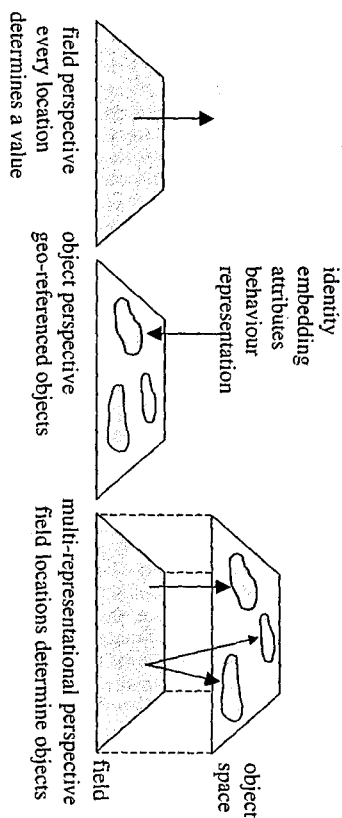


Figure 1. The field perspective, object perspective, and multi-representational perspective.

natural extension of the traditional field perspective where every location determines a scalar, vector, or tensor to one where every location determines a set of discrete, geo-referenced objects.

The utility of an object field can be elucidated with a few examples. Consider the case where a user would like to explore the terrain that can be viewed from various locations in a study area. A location's viewshed is the region that can be seen from that location. If a viewshed is identified for every location in an elevation field and associated with the location, this would yield an areal object field, as each location would be associated with an areal object (not necessarily singly bounded). Again, this is not possible in continuous space because there are an infinite number of locations in the field, but a discrete representation of the elevation field combined with strategies for reducing the number of locations in the field would allow for the construction of this unique type of field (O'Sullivan and Turner 2001).

A second example is a network-based field, where every location along a network is associated with a contiguous areal object. Assume that a decision maker would like to explore the potential effects of a toxic material spill along a highway transport route. In theory, every location along the route represents a potential spill site with varying consequences that depend on local meteorological conditions, topography (Hepler and Finco 1995), and the surrounding population distribution. Using a plume model, meteorological assumptions, a terrain model, and a discrete representation of the route, a plume 'object' could be simulated for every location along the route (Chakraborty and Armstrong 1996). This would yield an areal object field defined along a network (restricted field) that could be useful in assessing risk. An interesting aspect of this example is that the field is restricted to locations along a network but the spatial embedding of the associated objects is not.

A third example is a corridor object field. Consider the case where a right-of-way must be identified between two locations, an origin and destination. If the corridor must pass through a third location, a gateway location (Lombard and Church 1993), then a field of corridors can be defined. In theory, every gateway location in the field is associated with the 'best' corridor that passes through the location en route from the origin to the destination. Solving a corridor location problem instance for each location in a discrete representation of the field yields a corridor object field, as every location in the field is associated with a corridor

(Church *et al.* 1992). The corridor could be represented as a set of contiguous line segments or grid cells, depending on the problem context.

This represents a sample of the possible types and applications of object fields. Figure 2 graphically depicts a few more examples. An initial step is to develop an object-field typology. There are many types of field and object representations, each with unique characteristics. One approach is to distinguish OFs based on the method for conceptualizing and representing the underlying field combined with the method for conceptualization and representing the associated objects. Table 1 depicts a partial typology of object fields based on the above criteria. From left-to-right, the first major division is defined by the conceptualization of the underlying field. This might be a continuous field in n dimensions or a field restricted to locations along a network. If the field is conceptualized as continuous, then there are a variety of

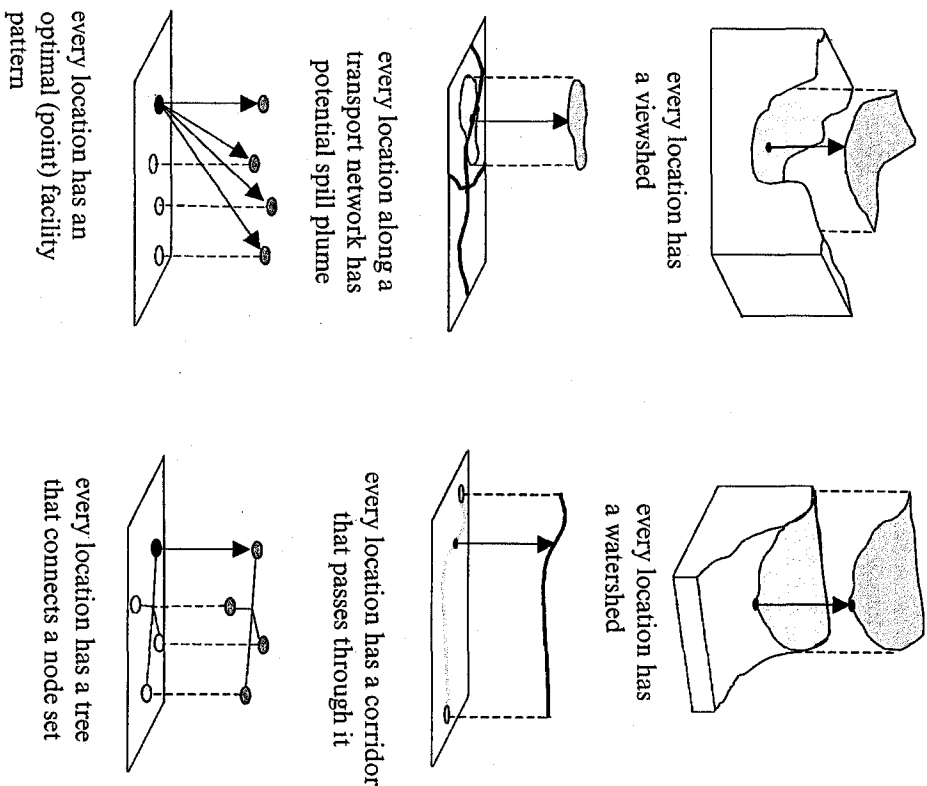


Figure 2. Example object field types.

Table 1. A partial object-field taxonomy.

Underlying field		Objects	
Conceptualized	Represented	Conceptualized	Represented
Continuous field (1-d, 2-d, 3-d,...)	Raster (rectangular, triangular, or hexagonal)	Point set Corridor Area	Set of cells Set of points Set of cells Polyline Set of cells Closed polyline Nodes and arcs
	Point grid (lattice)	Network Point set Corridor Area Network Points	Set of points Set of points Set of points Set of points Nodes and arcs Set of spatial units Set of points Set of polygons Polyline Set of polygons Closed polylines Nodes and arcs Set of facets Set of points Set of facets Polyline Set of facets Closed polylines Nodes and arcs Set of nodes Node/arc subset Node/arc subset Nodes and arcs
	Irregular polygons	Corridor Area Network Point set Corridor	Set of points Set of polygons Set of polygons Closed polylines Nodes and arcs Set of facets Set of points Set of facets Polyline Set of facets Closed polylines Nodes and arcs Set of nodes Node/arc subset Node/arc subset Nodes and arcs
	Triangulated irregular network (TIN)	Corridor Area Network Point set Corridor	Set of points Set of polygons Set of polygons Closed polylines Nodes and arcs Set of facets Set of points Set of facets Polyline Set of facets Closed polylines Nodes and arcs Set of nodes Node/arc subset Node/arc subset Nodes and arcs
Network (restricted field)	Nodes and arcs	Network Point set Corridor Area Network	Set of points Set of points Set of points Set of points Nodes and arcs Set of spatial units Set of points Set of polygons Polyline Set of polygons Closed polylines Nodes and arcs Set of facets Set of points Set of facets Polyline Set of facets Closed polylines Nodes and arcs Set of nodes Node/arc subset Node/arc subset Nodes and arcs

methods for representing a field in a discrete domain. These include a raster (used here to refer to any regular tessellation of the plane), irregular polygons, or a triangulated irregular network. A network is generally represented using a node-arc structure, where dynamic segmentation can be used to model continuously varying network attributes (Nyerges 1990).

The next step is to examine the method for representing the object(s) associated with each location in the field. Four types of object conceptualizations are listed including point, corridor, area, and network. This set should not be considered exhaustive. The types are repeated for each conceptualization and representation of the underlying field. To the right of the object conceptualization are a few example representations. For example, one object field involves conceptualizing space as a continuous field in the plane, representing the field using a raster, associating a single corridor with each location (raster cell), and representing the corridor as a polyline.

There are a number of OF characteristics in addition to the primary ones used for the typology that might lead to further refinement. For example, are the objects associated with the field homogeneous, or is more than one type of object associated with the field? Is the spatial embedding of the objects crisp or fuzzy (Burrough and Frank 1996)? Are objects contained within objects in a hierarchical fashion, and, if

so, is the object hierarchy all one type? Can the objects that are associated with a location move or change with time? There are many more characteristics that could be used to further refine the typology.

A key quality that can be used to distinguish an object field from representing traditional spatial objects in the plane is the relationship between field locations and spatial objects. Object fields can be viewed as relating two spaces, one field-based and one object-based, that correspond in extent. Objects in object space can be points, lines, polygons, or other type and may overlap. As noted, there are four possible *relationships* between locations in field space and objects in object space: one-to-one, one-to-many, many-to-one, or many-to-many (figure 3). If the relationship is one-to-one, then each field location is associated with one object in object space. If the relationship is one-to-many, then a field location can be associated with many objects, but each object is only associated with one location in field space. If the relationship is many-to-one, then many locations in field space may be associated with one object, but each field location can only be associated with one object. Finally, if the relationship is many-to-many, then a location in field space can be associated with any number of objects, and an object can be associated with any number of field locations.

A second interesting quality of an object field arises from the separation between the field and object spaces. Because of this separation, field locations can be associated with objects that do not contain the location in their spatial embedding. For example, a location in one part of a study area can be associated with an object in another part of the study area. We refer to the case where a location is associated with an

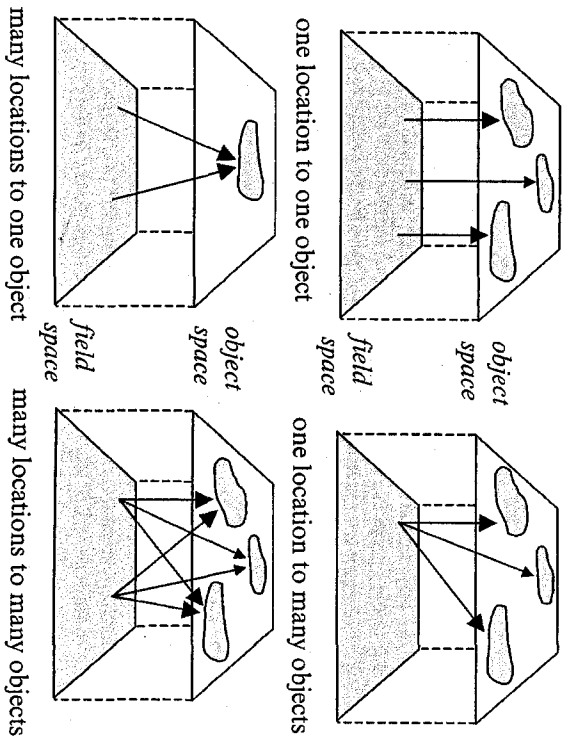


Figure 3. Relationship types between field locations and spatial objects.

object in object space that includes the corresponding location in its spatial embedding as *locationally inclusive*. Similarly, the case where a location in field space is associated with an object in object space that does not include the location is referred to as *locationally exclusive*.

An object field is similar to a meta-relational map in GeoAlgebra (Takeyama 1996, Takeyama and Couclelis 1997, Takeyama 1997, Couclelis 1997). A meta-relational map assigns a binary field, referred to as a relational map, to each location in a field to define the set of proximal locations that influence the given location (Takeyama 1997). For this reason, it is a mapping from each location in a field to its respective 'field of influence'. This is similar to relating each location to an object (or set of objects), as both involve relating a location to meta-information at other locations. Takeyama (1996) notes that the conceptual perspective in GeoAlgebra is field-based rather than object-based. In this way, one can consider meta-relational maps and object fields as analogous to the original field-object dichotomy. In other words, one is a field-of-fields and the other a field-of-objects.

3.2. Formalizing object fields

An OF can be formalized with existing approaches to formalizing fields and objects. For example, a scalar field can be defined as set of tuples that contain a location l and a value v selected from a set of locations L and a set of values V (Goodchild 1992). If geographical space is perceived as continuous, then L is an infinite set. Similarly, if the set of possible values at a location is perceived as continuous, then V is an infinite set. In computational practice, L and V are both finite, where any tuple selected from the Cartesian product is possible. Generally, there is a unique value at each location, so a scalar field is best considered a function f that maps a set of locations to a set of values $f: L \rightarrow V$. The arrangement of locations in a study area can be irregular or regular and vary in detail (resolution). The set of possible values V can be any of the common measurement scales binary, ordinal, interval, or ratio.

An object can be conceptualized in n -dimensional continuous space as possessing identity, spatial embedding, attributes, behaviours, and a representation (Worboys 1995). There are a variety of ways to represent an object's spatial embedding mathematically. One approach is to use an infinite binary field that defines whether the object is present or absent at each location in a field. An object's spatial embedding S can be defined as:

$$S = \{(x, f(x)) | x \in R^n, f(x) \in \{0, 1\}\} \quad (1)$$

where x is a location vector and f is a function that defines whether the object is present or absent at a given location. To represent fuzzy objects, the function f can alternatively be defined continuously on the interval 0 to 1. This allows an object to be present at a given location to a specified degree. For example, a lake with a dynamic boundary could be modelled as a fuzzy object where locations within the lake's changing boundaries are defined in terms of a degree-of-membership in the object 'lake'. Also, x may include a temporal dimension, which allows an object to have a spatio-temporal embedding that can be used to model movement.

In addition to a spatial embedding, objects have a set of attributes (properties) generally measured on a nominal, ordinal, interval, or ratio scale $A = \{a_1, a_2, \dots, a_i\}$, a set of behaviours $B = \{b_1, b_2, \dots, b_i\}$ represented by procedural functions (methods) that can be invoked on the object, and a pointer to a representation R (e.g. points,

lines, and polygons). Example object behaviours include a building object's ability to return the solar radiation that it would absorb and reflect given a sun angle and azimuth, or a watershed object's ability to calculate its output given a pattern and duration of precipitation. An object is thus a composite of its identity, embedding, attributes, behaviours, and representation:

$$O = \{i, S, A, B, R\} \quad (2)$$

An object field can then be formalized as a mathematical relation between field locations and geo-referenced objects:

$$R = \{(x, O) | x \in R^n, O \in U\} \quad (3)$$

where x is a location vector, R^n is n -dimensional real number space (spatial framework), O is an object, and U is the set of all objects of interest. In this way, each location in a spatial framework can be related to any number of objects, and an object may be related to any number of field locations, all selected from the Cartesian product of the location and object set $R^n \times U$. A key aspect of the formulation in (1)–(3) is the separation between field locations and an object's spatial embedding. This makes it possible to associate a location with an object that does not include the location in its spatial embedding (i.e. locational exclusion). The novelty of this formulation is that it contains elements of both the field and object perspectives of geographical space. The field perspective is represented in the relation between field locations and objects, and the object perspective is represented in that objects have identity, spatial embedding, attributes, behaviour, and a representation.

Figure 4 depicts four types of object fields that would be possible using this formulation. Figure 4(a) is a point-set object field where every location is associated

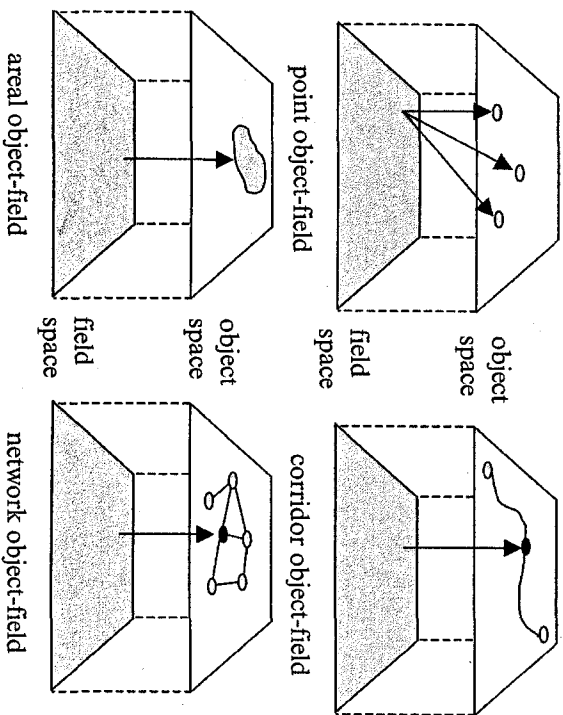


Figure 4. Four types of object fields: point, corridor, areal, and network.

with a set of points that may or may not include a point at the location. Figure 4(b) is a corridor object field where every location is associated with a corridor between two locations. Figure 4(c) is an areal object field where every location is associated with an area object (not necessarily singly bounded), and figure 4(d) is a network object field where every location is associated with a network of locations of which it may be a part. In this way, every location can be associated with any number and type of complex spatial objects located anywhere in the field.

4. Representing and constructing object fields

4.1. Representation

An object field is a multi-representational model of geographic variation that can be represented with any combination of existing data models for representing fields and spatial objects in a computational domain (Peuquet 1984, 1988, Goodchild 1992, Gahegan 1996, Burrough and McDonnell 1998, Longley *et al.* 1999). Table 1 depicts many of the combinations of field and object representations that can be used, but should not be considered exhaustive. There are three primary steps in constructing an object field to address a spatial problem: (1) define and represent the underlying field, (2) define and represent the objects, and (3) establish the relation between field locations and objects. In defining and representing the field, the central questions are the field's qualities like its spatial dimensions, extent, unit of discretization, and detail (resolution). The second step is defining and representing the object type, which involves defining the object embeddings, properties, behaviours, and associated representation. For example, the object associated with each location might be a potential habitat corridor between two existing biological reserves that is represented as a polyline and has a minimal attribute set of cost and width. The final step is establishing a relation between locations in the field and associated objects. This involves questions like the number of objects related with each location.

There are three approaches to generating an object field: manual, semi-automated, and automated. A manual approach involves digitizing the spatial form of each object, entering the object attributes, and relating each field location to a set of objects. A semi-automated procedure might generate the objects automatically but require manual assignment of field locations to objects, or the converse. Clearly, the most efficient method is a procedure that automates the complete construction of an object field. This procedure would generate both the objects and the location-object pairs in equation (1). This process might be analytical, as in the case of determining the visibility graph for every location (O'Sullivan and Turner 2001), process-based, as in simulating a toxic plume for every location along a network to derive a set of plume objects (Chakraborty and Armstrong 1996), or an exact or heuristic solution to a spatial optimisation problem (Lombard and Church 1993, Cova and Church 2000a, 2000b).

4.2. Data structure and database issues

The appropriate data structure for storing an object field depends on the representation of the field and associated objects. There is a rich history of spatial data structure development to draw from in GIScience (Peucker and Chrisman 1975, Chrisman 1978, Samet 1990, Burrough 1992, Frank 1992, Oosterom, 1994). In general, a random access data structure indexed by location will allow quick retrieval of the object(s) associated with a location in the field. This might be an array, list, or tree structure (e.g. quadtree) if region queries will be common. Furthermore,

contemporary extended relational database management systems (RDBMS) allow spatial objects to be stored directly in relational tables much like a text or numeric field (Worboys 1999). So, an object field can be stored in a database as a table where table rows correspond to field locations, and a column is defined that stores the object, or objects, associated with that location. Figure 5 depicts an example data structure for storing an object field in a computational environment.

The two most important variables in determining the space required to store an object field are the detail (resolution) of the discrete representation of the field and the average space required to store an object's spatial embedding, attributes, and other elements. The space required is thus mn , where n is the average number of bytes required for a spatial unit, and m is the average number of bytes necessary to store an object. This number can be very large in some cases, and developing strategies for storing object fields in an efficient manner is an important area for further research.

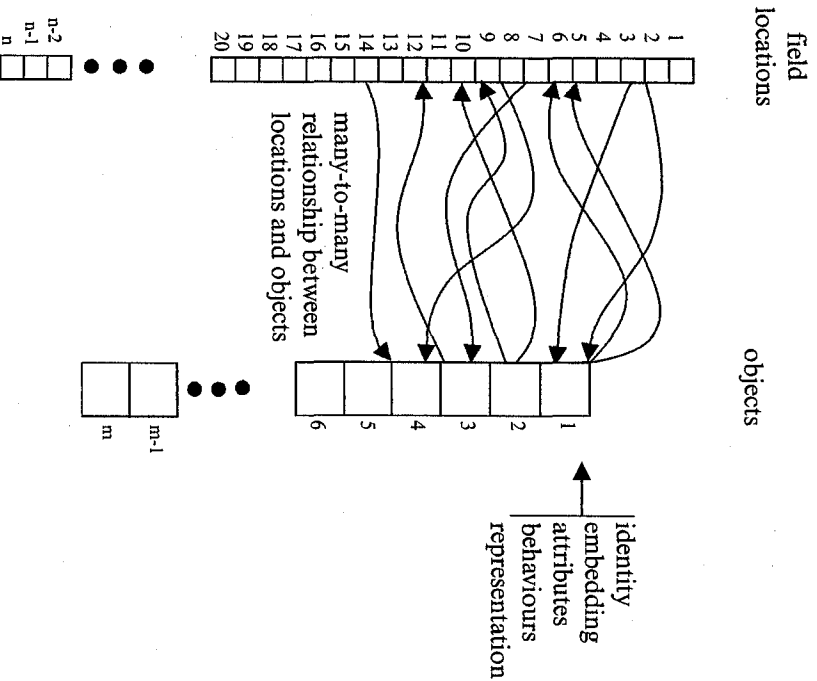


Figure 5. An example object field data structure.

5. Application examples

The central problem in constructing an object field is identifying a suitable representation of the field, objects, and relation between the two (equation 1). This section provides three examples of automatically constructing an object field to address problems in a spatial decision support context (SDSS). In an SDSS context, the challenge is to provide decision makers with tools to assist in locating facilities like fire stations, transmission lines, and biodiversity reserves (Densham, 1991, 1994, 1996, Church *et al.* 1992, Armstrong *et al.* 1993, Church 1999). The three examples include a point, corridor, and areal object field, respectively. The common theme in these examples is that an object field combines aspects of the field and object conceptual perspectives in a manner that is useful for exploring a population of competitive spatial alternatives.

5.1. Point-set field: multi-facility location problem

The problem in this example is a continuous p -median (facility location) problem. The objective is to locate p facilities in continuous space to serve n demand points, with varying demand, so as to minimize the total weighted travel distance from all demands to all facilities. This is a classic location problem and a complete formulation of the problem and common solution methods can be found in Ghosh and Rushton (1987). For our purposes, the important quality of this problem is that a facility can be located anywhere in the plane. A solution to this problem includes both a set of facility locations and a set of assignments from each demand to its closest facility. In a traditional location science context, the problem would be solved optimally for a given set of demands and the solution would be presented. In the context of this research, we can use this problem to generate a field of facility patterns, where every location is associated with the best pattern that can be identified, given that a facility must be placed at that location. Solving this problem in a computational domain requires a discrete representation of the field, demands, and facilities.

The discrete representation selected for the underlying field in this example is a raster, where a demand and facility are both represented as a single cell. To generate a point-set object field, a constraint is added that fixes one facility in a given location (cell). This constraint states that cell i must be selected for a facility. Incrementing i from 1 to n and iteratively solving the model above for every cell, yields a point-set object field. The results of each run are saved, and each cell is associated with its best facility configuration. Therefore, we would refer to the resulting object field as *locationally inclusive* because every field location is associated with a facility object that includes the location in its spatial embedding (as well as other facilities located elsewhere). To relate this back to the general formulation in equations (1)–(3), solving the problem for a particular cell returns the spatial embeddings (locations) of the point objects (S in equation 2), and associating the resulting solution with the given cell relates the location to the set of facility objects (x, O in equation 3).

Figure 6(a) depicts both the original demand map on the left and the resulting objective surface on the right. The objective surface represents the best objective value achieved for each cell (problem instance). The heuristic nature of the solution algorithm is evident in the speckled objective surface map. The arrow in each map indicates the cell that is currently selected. Superimposed on the demand map and the objective surface is the object field. The facility configuration and associated assignments displayed represent the best configuration, given that a facility must be located at the indicated cell. It is impossible to depict the entire object field in static

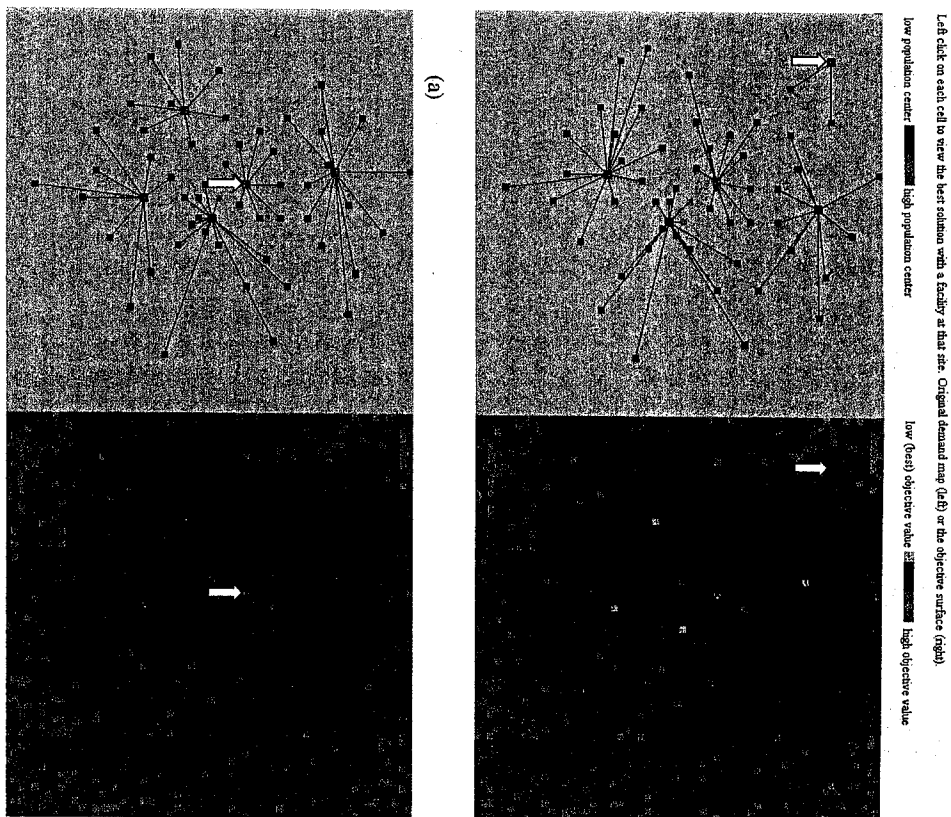


Figure 6. A point-set field that has a set of points (facilities) associated with each cell.

form, but figure 6(b) depicts the facility configuration and associated assignments for the best solution identified overall.

5.2. Corridor field: biological reserve corridor selection problem

The problem in this second example is to identify a suitable corridor between two existing habitat patches, a common spatial problem in biodiversity reserve design. There are a number of ways to formulate a corridor location problem in a GIS context. Ideally, the goal is a relatively suitable, direct corridor. This is a multi-objective spatial optimization problem (Cohon 1978). One approach for transforming

the problem into a single-objective context is to formulate the objective as a weighted trade-off between minimising corridor distance and maximising corridor suitability. Corridor suitability can be defined as the sum of the cell suitability values that comprise the corridor, and corridor length can be defined as the sum of the distances between cell centers that comprise the corridor.

To invert the suitability objective component, the suitability between two adjacent cells can be defined as one minus the sum of the cell suitability values divided by two times the maximum cell suitability value, which yields a value from 0 to 1. The resulting objective is then,

$$\text{minimize: } z = w_1 \sum_{i=1}^n \sum_{j=1}^n d_{ij} + w_2 \sum_{i=1}^n \sum_{j=1}^n (1 - (s_i + s_j)/2m) \quad (4)$$

where z is the objective value, w_1 and w_2 are adjustable weights that sum to 1; d_{ij} is the distance between adjacent cells i and j in the corridor; s_i is the suitability score for cell i , and m is the maximum suitability value for a cell. In this example, eight-neighbour adjacency was used to define the feasible paths between neighbouring cells.

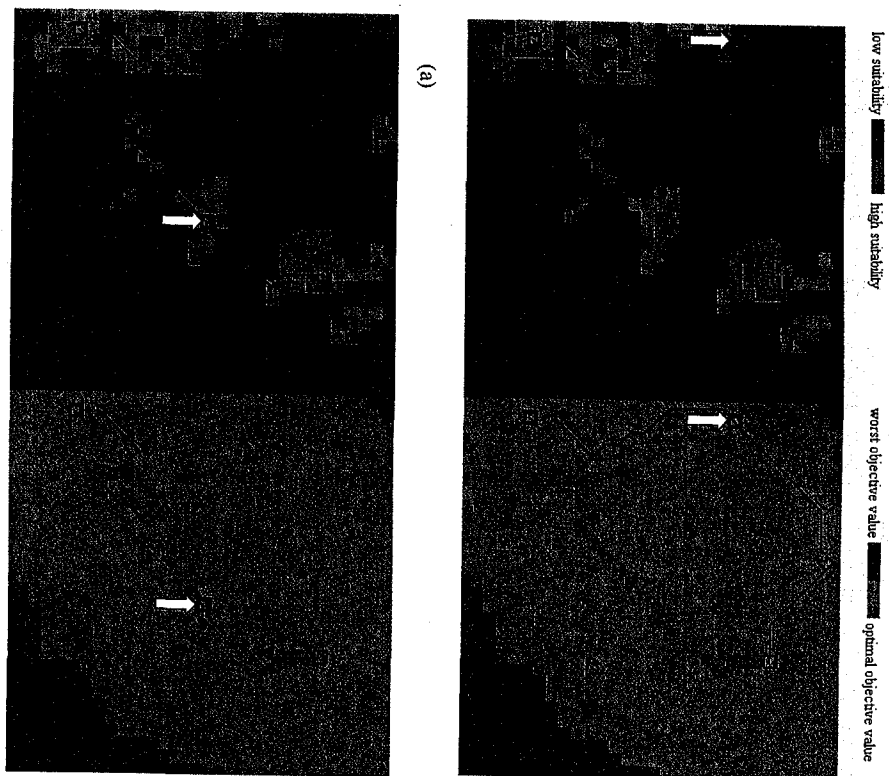
The problem of finding the best path between two cells in a raster can be solved optimally by converting the raster to a network and using Dijkstra's (1959) algorithm to solve for the least cost path. In this example, we used an eight-neighbour rule to transform the raster into a dense network. To generate a corridor object field, a gateway cell can be added to each problem instance that the path must travel through (Church 1992, Lombard and Church 1993). Iteratively solving the gateway problem for every potential cell in the suitability map results in a 'best' corridor for each gateway cell for a given model instance: a corridor object field. The corridor can be represented as either a set of contiguous cells or as a polyline (table 1). As in the last example, this is a locationally inclusive object-field because every cell is associated with a corridor that includes the cell in its spatial embedding. To relate this back to the general formulation, solving the problem for a particular cell returns the spatial embedding of the corridor object (S in equation 2), and associating the resulting corridor with that cell represents the relation from the location to the corridor object ((x, O) in equation 3). The corridor can either be represented as the set of contiguous cells that it passes through or as a polyline, the option selected here.

Figure 7(a) depicts the results of this process with the original suitability map on the left and the objective surface on the right. In this problem instance the distance was weighted lightly as 0.2 with the corridor suitability weighted as 0.8. The arrow in each figure represents the currently selected gateway cell. As Dijkstra's algorithm guarantees the optimal solution to a shortest path problem, the resulting objective surface on the right is optimal for the problem instance. Figure 7(b) shows the global optimal corridor for this problem instance, where selecting any cell along this route results in the same corridor.

5.3. Areal field: wilderness study area search

The problem in this third example is to search for a viable wilderness study area (WSA) in south-eastern Utah within the boundaries of the USGS Huntington 1:100 000 quadrangle. The Wilderness Act of 1964 in the US requires that a wilderness area be contiguous public land of at least 5000 acres in size with no existing roads (US Congress, 1964). This can be approached as a site search problem with the goal of identifying a compact site as far as possible from any roads. As in the last example, this is a multi-objective spatial optimisation problem. The area and compactness

Left: click on each cell to view the best corridor that passes through that point. Suitability map (left) or the objective surface (right).



(b) Figure 7. A corridor field that has a corridor associated with each cell.

objectives can be formulated as constraints to transform the problem into a single-objective problem of maximising the site's distance to proximal roads (Cohon 1978).

To address the requirements that a wilderness area must be public land without a road, land screening (Dobson 1979) was initially performed to remove locations that are private land or contain a road. The objective of the search is to maximise the distance of the site from existing roads subject to constraints on area, compactness, and contiguity (Cova and Church 2000a, 2000b). The model includes the notion of a 'root' land unit that must be present in the site and a feasible neighbourhood around the root in which the best site must exist. The feasible neighbourhood

surrounding each cell is calculated using the area and compactness requirements of the site. Iteratively solving this model for every feasible cell in the suitability map (i.e. public and no road) yields a field of contiguous site areas (objects). As in the last two examples, the object field is locationally inclusive because the object (area) associated with each location includes the location in its spatial embedding. To relate this to the general formulation in equations (1)–(3), solving the problem for a particular cell returns the spatial embedding of the best contiguous site for that cell (S) in equation 2, and associating the resulting site with that cell relates each location with an areal object ((x, O) in equation 3). The site can be represented as the boundary surrounding the set of contiguous cells that comprise it.

In this problem instance, a site's compactness is defined in terms of a normalized area-to-perimeter squared ratio (Austin 1984, MacEachren 1985), with 1 representing a maximally compact site. A cell's suitability score is defined simply as its Euclidean distance to the nearest road. The study area was converted to a grid of 1 km resolution, and the proximity of each cell to its nearest road was calculated using the proximity operation in ArcView™. An additional land ownership map was acquired, and cells that contained a road or private land were screened. The site search problem must be solved for each remaining cell. In many cases, the cell cannot support a site with the required spatial characteristics. These cells are not related with an object (i.e. no site). The result is a field of areal objects, but many locations in the field have no associated object.


Figure 8(a) depicts the resulting areal object field and the site associated with a selected cell. Despite the fact that this is one of the least populated regions in the US, there are few cells capable of meeting the spatial requirements of the problem. For each viable cell, there is an associated compact, contiguous site of 5000 acres that is as far from proximal roads as possible. Figure 8(b) shows one of the best sites in this study area for a wilderness study area as it contains a significant buffer from any roads. While it is easy to identify the best areas for a wilderness study area without the use of the site object field, the added value of the site field is to answer the question for a given cell, 'Can this cell support a relatively compact wilderness study area?' It is therefore a search for all areas that could be a WSA rather than a search for areas that should be a WSA.

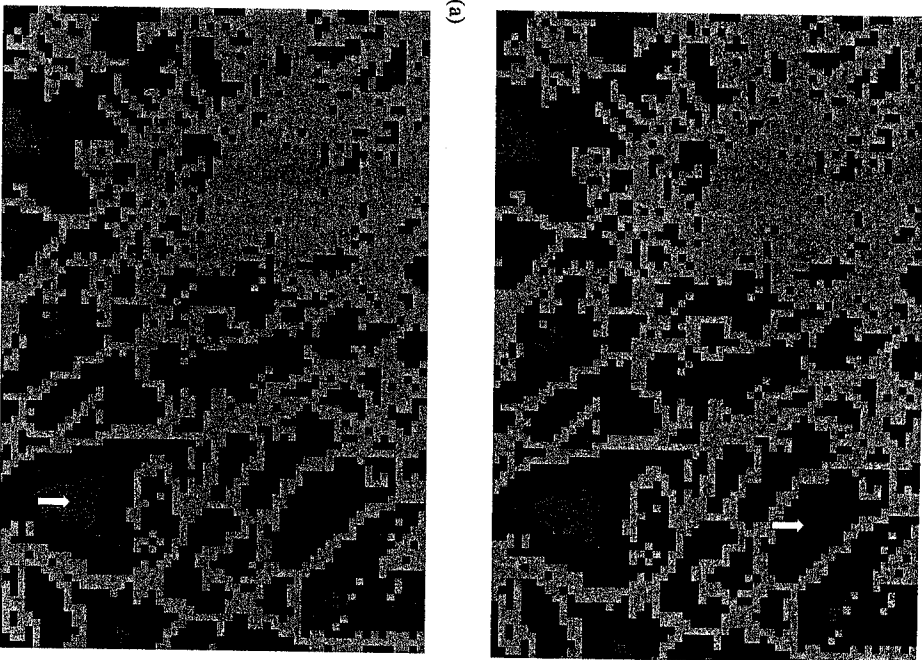
6. Discussion

The examples presented in the previous section demonstrate that object fields are relatively easy to construct, store, and analyse. This section addresses their potential value in spatial analysis and decision support. The primary benefit of an object field is that it reveals the spatial variation in an object, or object set, as a function of location. This information is not evident when a set of favoured locations is selected for a particular operation (e.g. toxic plume simulation). A secondary benefit is that object fields provide the ability to perform location-based spatial sensitivity analysis. We describe each of these concepts and argue that they can improve a user's understanding of geographical reality as well as that of a particular spatial model.

6.1. Analysis of model results as a function of location

The most central type of inquiry that can be pursued with an object field pertains to the spatial variation in a model's results as a function of location. In other words,

Left click on each cell to view its position in the stability map. Many cells will not contain a site road or private land.  maximum distance from road



(b)
Figure 8. An areal field that has a contiguous site associated with each cell.

how do the results of a spatial operation that returns an object vary by location? As noted, there are many types of spatial operations (e.g. analytical, simulation, or optimisation) that result in a geographical object (e.g. toxic plume, viewshed, corridor, best site) where a location is required as an input parameter, or the results of the operation would be affected if a location was constrained to being in the solution. These operations are typically performed for a small sample of locations in a study area, or globally, if an input seed location is not required. In the context of an object field, the operation is performed for all locations in a study area (or a much denser

sampling of locations). This comes with significant computational costs, and it is worth examining the benefits of this approach.

A central challenge in GIScience is to increase our understanding of geographical reality through spatial modelling. This raises the question of how object fields might serve to increase a user's understanding of geographical reality. When an operation is calculated or simulated for a sample of locations, as in a potential fire scar, the results are typically used to create a static map. A drawback to this approach is that it doesn't effectively communicate to the user how a fire scar varies continuously as a function of the ignition location. Discontinuities in the scar may occur over hilltops, it may contract and elongate in narrow canyons, expand in plains, and so forth. Simulating a fire scar for each location (or a dense sample of locations) would provide a user with a more intuitive sense of the spatial variation in the scar as a function of the ignition location. This same point can be made regarding a user's understanding of how a viewshed or watershed varies continuously across the landscape. Calculating, simulating, or solving for objects at all locations holds potential to improve a user's understanding of the phenomenon under study, and, therefore, aspects of geographical reality.

A second challenge in GIScience is to develop new approaches for improving a user's understanding of the similarity and differences between spatial operations to accomplish the same task (e.g. competing plume simulation models). This raises the question of how an object field might be used to meet this goal. A pitfall to avoid in pursuing this goal is revealing too much algorithm or implementation detail in comparing methods. This implies that there is a suitable level of detail to reveal to a user, where too little or too much is entirely possible. This goal can be stated as one of improving a user's understanding of a model's behaviour without introducing an overwhelming degree of technical detail. Object fields hold the potential to contribute to this end, as they increase a user's understanding of the behaviour of a spatial model (results) without revealing algorithmic detail. This understanding is garnered by improving the user's intuitive grasp of how a model's results vary across the study area as a function of location. In short, interactive exploration of an object field gives a user geographical control over a dimension of the problem that would not otherwise be possible if the operation was only performed for specific input locations and presented in a static map (Batty and Xie 1994, Bailey and Gatrell 1995).

An improved understanding of geographical reality and the spatial model in use is likely to lead to improvements in the spatial model itself. This can be viewed as a beneficial feedback into model construction and refinement. If the user's mental model of the problem domain does not match the representation or implementation of the current model, this is likely to be revealed in the exploration of an object field. This occurs because the user is exploring a very large number of results in a very efficient manner. For example, a wildfire expert may discover that a fire simulation model is not behaving as a real fire would in certain terrain and wind conditions, or a conservation biologist might discover that a corridor location model is behaving erratically in areas where the suitability surface is rough. In other words, an object field reveals more to a user regarding the behaviour of a spatial model than select static calculations at specific locations. The corridor field example in the prior section is one example of where this is the case. Calculating the single best corridor in a study area, or a small set of competing corridors, falls short of revealing that there may be many near-optimal corridors proximal to these corridors (Church *et al.* 1992, Lombard and Church 1993). Furthermore, the degree to which these near-optimal

solutions are similar spatially to the model's defined optimal solution is also revealed. As no model can include all criteria, these near-optimal solutions are important and may result in a better final decision.

6.2. Location-based spatial sensitivity analysis

Another line of inquiry that can be pursued with an object field pertains to the spatial variation in the results of a spatial model as a function of the algorithm, data, and parameters in use. The objective here is to communicate the sensitivity of the results of a spatial model as a function of location. A fundamental tenet of geography is that everything varies spatially, and this holds for the spatial sensitivity of a model to changes in the algorithm, data, and parameters. A problem is identifying some means for communicating this sensitivity to a user. Object fields allow one to analyse how the results of a spatial model vary by location given changes in the algorithm, data, or parameters. This might be termed *location-based spatial sensitivity analysis*.

One example of this type of investigation is examining the spatial sensitivity of model results to changes in the input (seed) location. In any object field, there are areas where small changes in the seed location may result in very different spatial outcomes. For example, in the corridor location problem presented in the prior section, moving the gateway location one cell can result in an optimal path that shares very little with the neighbouring gateway location's optimal path. This means the decision maker is on a boundary between two very different spatial outcomes. In other areas, the resulting optimal corridor may be much less sensitive to changes in the gateway location. This would not be evident to the user if the corridor model had not been solved for all locations in the field. In another context, this might occur when a small move in the seed location across a ridge line results in an entirely different watershed, watershed, or fire scar. This understanding can be quickly garnered on the part of the user when the procedure used to calculate an object is performed at all locations in a study area and the user is allowed to interactively explore the results.

A second area of investigation that becomes possible in the context of an object field is analysis of the spatial sensitivity of a model's result to changes in the algorithm, data, and parameters in use as a function of location. In other words, a model can be altered in terms of the algorithm, data, and input parameters in use, and the resulting spatial change can be mapped to reveal the spatial variation in the sensitivity of the model to these changes. For example, consider two competing approaches for simulating a fire scar from an initial seed location. An interesting question would be the spatial variation in the differences between the two approaches. In some areas, the two may correspond very closely while in others they may vary substantially. This would also be true for changes in the data, where the results may remain very stable in one area to changes in the underlying data while in other areas, small changes in the data may result in entirely different spatial outcomes. Finally, changes in the parameters to a model or algorithm that generates an object would also vary spatially. Constructing an object field before and after the alteration, either in the algorithm, data, or parameters, would allow an analyst to generate a difference map that could be used to assess the spatial sensitivity to the alternation and communicate this to a user or decision maker.

7. Conclusion

This paper described a means for linking field and object models of spatial phenomena through a series of mappings. We presented a typology of object fields and a general formulation. The formulation includes unique qualities that can be used to distinguish object fields from traditional objects in the plane. Foremost, object fields allow a one-to-one, one-to-many, many-to-one, or many-to-many relationship between field locations and objects. Of lesser importance, but still unique in geographical data modelling, a location can be associated with an object that does not contain the location in its spatial embedding. The potential application of this quality remains open, as it was not used here. Three application examples were presented that share the common theme that object fields are useful for exploring a population of competing spatial alternatives in a spatial decision support context. Finally, the potential benefits of object fields in spatial analysis and spatial decision support were discussed. These include the ability to enhance a user's understanding of geographical reality and the particular spatial model in use as well as the ability to perform location-based spatial sensitivity analysis.

Object fields extend geographical representation in a direction that is recognized as an important research need (UCGIS 1998). A means for linking the field and object representations of geographical phenomena may lead to further innovation in using the two perspectives together. The approach to linking the field and object perspectives presented in this paper provides one example of a multi-representational framework that relies on the two perspectives. It should not be considered the only approach. The linkage between the two perspectives is achieved through mapping field locations to objects, but there are likely other means for using the two perspectives together. Innovation in this area may enhance existing concepts of geographical space in GIScience (Gatrell 1983, 1991).

Object fields are a potentially useful geographical data model, and there are a number of interesting GIS applications to pursue in environmental modelling, spatial decision support, and many other fields. Analysis of object fields is an interesting area in need of further research. This includes interactive exploration, as well as more rigorous techniques like quantifying the spatial dependence between objects, mapping the pattern of shared object embeddings, and exploratory analysis of object attributes in a field (Getis and Ord 1992, Anselin 1994, 1995). Topological relationships between objects in the field is another area of interest. For example, do the objects at neighbouring locations coincide, touch, or overlap, or are they disjoint (Egenhofer and Franzosa 1991)? Object-fields may have application in assessing spatial data uncertainty, particularly for procedures that generate objects as a function of location. Finally, there is a general need for GIS vendors and the Open GIS Consortium (OGC) to recognize that all data models are not grounded in either a field or object perspective and that some data models may include elements of both.

Acknowledgements

The authors would like to thank the anonymous reviewers for constructive, detailed comments. The interactive examples described in this paper can be found at: <http://www.geog.utah.edu/papers/cova-igis1.html>

References

- ANGEL, S. and HYMAN, G. M., 1976, *Urban fields: a geometry of movement for regional science* (London: Pion Limited).

- ANSELIN, L., 1994, Exploratory spatial data analysis and geographic information systems. In *New Tools for Spatial Analysis*, edited by M. Painho (Luxembourg: Eurostat), pp. 45-54.
- ANSELIN, L., 1995, Local indicators of spatial association (LISA). *Geographical Analysis*, 27, 93-115.
- ARMSTRONG, M. P., 1993, Perspectives on the development of group decision support systems for local problem-solving. *Geographical Systems*, 1, 69-81.
- ARMSTRONG, M. P., DENSHAM, P. J., LOLOMS, P., and RUSHTON, G., 1992, Cartographic displays to support local decision making. *Cartography and Geographic Information Systems*, 19, 154-164.
- AUSTIN, R. F., 1984, Measuring and comparing two-dimensional shapes. In *Spatial Statistics and Models*, edited by G. L. Gaile and C. J. Willmott (Boston: D. Reidel Publishing Company), pp. 293-312.
- BAILEY, T. C., and GATRELL, T., 1995, *Interactive spatial data analysis* (Harlow Essex: Longman Scientific & Technical).
- BATTY, M., and XIE, Y., 1994, Modelling inside GIS: part 1. Model structures, exploratory spatial data analysis and aggregation. *International Journal of Geographical Information Systems*, 8, 291-307.
- BURROUGH, P. A., 1992, Are GIS data structures too simple minded? *Computers & Geosciences*, 18, 395-400.
- BURROUGH, P. A., and FRANK, A., 1996, *Geographic objects with indeterminate boundaries* (London: Taylor & Francis).
- BURROUGH, P. A., and McDONNELL, R. A., 1998, *Principles of Geographical Information Systems* (Oxford: Oxford University Press).
- CHAKRABORTY, J., and ARMSTRONG, M. P., 1996, Using geographic plume analysis to assess community vulnerability to hazardous accidents. *Computers, Environment, and Urban Systems*, 19, 341-356.
- CHRISMAN, N., 1978, Concepts of space as a guide to cartographic data structures. In *Proceedings of the First International Advanced Study Symposium on Topological Data Structures for Geographic Information Systems*, edited by G. Dutton (Cambridge: Harvard Laboratory for Computer Graphics and Spatial Analysis), pp. 1-19.
- CHURCH, R. L., 1999, Location modelling and GIS. In *Geographical information systems: principles, techniques, applications, and management*, Volume 1, edited by P. A. Longley, M. F. Goodchild, D. J. Maguire and D. W. Rhind (New York: John Wiley & Sons), pp. 293-303.
- CHURCH, R. L., LOBAN, S. R., and LOMBARD, K., 1992, An interface for exploring spatial alternatives for a corridor location problem. *Computers & Geosciences*, 18, 1095-1105.
- COHON, J. C., 1978, *Multiobjective programming and planning* (New York: Academic Press).
- COUCLEIS, H., 1992, People manipulate objects (but cultivate fields): beyond the raster-vector debate in GIS. In *Theories and Methods of Spatio-Temporal Reasoning in Geographic Space*, Lecture Notes in Computer Science 639, edited by A. U. Frank and I. Campari (Berlin: Springer-Verlag), pp. 65-77.
- COUCLEIS, H., 1997, From cellular automata to urban models: new principles for model development and implementation. *Environment and Planning B: Planning and Design*, 24, 165-174.
- COVA, T. J., and CHURCH, R. L., 2000a, Exploratory spatial optimization and site search: a neighborhood operator approach. *Computers, Environment, & Urban Systems*, 21, 401-419.
- COVA, T. J., and CHURCH, R. L., 2000b, Contiguity constraints for single-region site search problems. *Geographical Analysis*, 32, 306-329.
- DENSHAM, P. J., 1991, Spatial decision support systems. In *Geographical Information Systems: Principles and Applications*, Volume 1, edited by D. J. Maguire, M. F. Goodchild and D. Rhind (New York: John Wiley & Sons).
- DENSHAM, P. J., 1994, Integrating GIS and spatial modelling: visual interactive location selection. *Geographical Systems*, 1, 203-219.
- DENSHAM, P. J., 1996, Visual interactive location analysis. In *Spatial Analysis: Modelling in a GIS Environment*, edited by P. Longley and M. Baty (Cambridge: GeoInformation International), pp. 185-205.
- DIKSTRA, E. W., 1959, A note on two problems in connexion with graphs. *Numerische Mathematik*, 1, 269-171.
- DOBSON, J. E., 1979, The regional screening procedure for land use suitability analysis. *The Geographical Review*, 62, 224-234.
- EGENHOFFER, M. J., and FRANZOSA, R. D., 1991, Point-set topological spatial relations. *International Journal of Geographical Information Systems*, 5, 161-174.
- EGENHOFFER, M. J., and FRANK, A. U., 1987, Object-oriented databases: database requirements for GIS. In *Proceedings of the International GIS Symposium: The Research Agenda, Volume 1* (Washington, DC: US Government Printing Office), pp. 189-211.
- FRANK, A. U., 1992, Spatial concepts, geometric data models, and geometric data structures. *Computer & Geosciences*, 18, 409-417.
- FRANK, A. U., and EGENHOFFER, M., 1992, Computer cartography for GIS: an object-oriented view on the display transformation. *Computers & Geosciences*, 18, 975-987.
- GAHEGAN, M., 1996, Specifying the transformations within and between geographic data models. *Transactions in GIS*, 1, 137-152.
- GAHEGAN, M., and ROBERTS, S. A., 1988, An intelligent object-oriented geographical information system. *International Journal of Geographical Information Systems*, 2, 101-110.
- GATRELL, A. C., 1983, *Distance and space: a geographical perspective* (Oxford: Clarendon Press).
- GATRELL, A. C., 1991, Concepts of space and geographic data. In *Geographical Information Systems*, edited by D. J. Maguire, M. F. Goodchild and D. W. Rhind (Harlow: Longmans), pp. 119-134.
- GETIS, A., and ORD, J. K., 1992, The analysis of spatial association by the use of distance statistics. *Geographical Analysis*, 24, 189-206.
- GHOSE, A., and RUSHTON, G., 1987, *Spatial analysis and location-allocation models* (New York: Van Nostrand Reinhold).
- GOODCHILD, M. F., 1989, Modelling error in objects and fields. In *Accuracy of spatial databases*, edited by M. F. Goodchild, and S. Gopal (London: Taylor & Francis).
- GOODCHILD, M. F., 1992, Geographic data modelling. *Computers & Geosciences*, 18, 401-408.
- GOODCHILD, M. F., 1997, Representing fields. Core Curriculum in GIScience, National Center for Geographic Information and Analysis (NCGIA), Santa Barbara.
- HEPNER, G., and FINCO, M., 1995, Modelling dense gas contaminant pathways over complex terrain using a geographic information system. *Journal of Hazardous Materials*, 42, 187-199.
- HEUVELINK, G. B. M., 1996, Identification of field attribute error under different models of spatial variation. *International Journal of Geographical Information Systems*, 10, 921-935.
- HUNTER, G. J., and GOODCHILD, M. F., 1995, A new model for handling vector data uncertainty in geographic information systems. *Journal of the Urban and Regional Information Systems Association*, 8, 51-57.
- KEAR, K. K., 1997a, Fields as a framework for integrating GIS and environmental process models. Part 1: representing spatial continuity. *Transactions in GIS*, 1, 219-234.
- KEAR, K. K., 1997b, Fields as a framework for integrating GIS and environmental process models. Part 2: specifying field variables. *Transactions in GIS*, 1, 235-246.
- LOMBARD, K., and CHURCH, R. L., 1993, The gateway shortest path problem: generating alternative routes for a corridor location problem. *Geographical Systems*, 1, 25-45.
- LONGLEY, P., GOODCHILD, M. F., MAGUIRE, D. J., and RHIND, D. W., 1999, *Geographical information systems: principles, techniques, applications, and management* (New York: John Wiley & Sons).
- MACEachREN, A. M., 1985, Compactness of geographic shape: comparison and evaluation of measures. *Geografiska Annaler*, 67B, 53-67.
- NYERGES, T. L., 1990, Localized referencing and highway segmentation in a geographic information systems. *ITTE Journal*, 60, 27-31.
- OOSTEROM, P. V., 1994, *Reactive data structures for geographic information systems* (Oxford: Oxford University Press).
- OOSTEROM, P. V., and VANDERBOS, J., 1989, An object-oriented approach to the design of geographic information systems. *Computers & Graphics*, 13, 409-418.
- O'SULLIVAN, D., and TURNER, A., 2001, Visibility graphs and landscape visibility analysis. *International Journal of Geographical Information Science*, 15, 221-237.

- PEUCKER, T. K., and CHRISMAN, N. 1975, Cartographic data structures. *American Cartographer*, **2**, 55-69.
- PEUQUET, D. 1984, A conceptual framework and comparison of spatial data models. *Cartographica*, **21**, 66-113.
- PEUQUET, D. 1988, Representation of geographic space: toward a conceptual synthesis. *Annals of the Association of American Geographers*, **78**, 375-394.
- PEUQUET, D., SMITH, B., and BROGAARD, B. 1999, *The ontology of fields: report of a specialist meeting held under the auspices of the Varenhous Project* (Santa Barbara: National Center for Geographic Information and Analysis).
- SACHS, M. 1973, *The field concept in contemporary science* (Springfield: Charles C. Thomas).
- SAMET, H. 1990, *The design and analysis of spatial data structures* (Reading: Addison-Wesley).
- SMITH, B., and MARK, D. 1998, Ontology and geographic kinds. In *Proceedings of International Symposium on Spatial Data Handling (SDH '98)*. Vancouver, B.C., pp. 308-318.
- TAKEYAMA, M. 1996, Geo-algebra: a mathematical approach to integrating spatial modelling and GIS [Unpublished doctoral dissertation thesis]. Santa Barbara, University of California.
- TAKEYAMA, M. 1997, Building spatial models within GIS through geo-algebra. *Transactions in GIS*, **2**, 245-256.
- TAKEYAMA, M., and COUCLERIS, H. 1997, Map dynamics: integrating cellular automata and GIS through geo-algebra. *International Journal of Geographical Information Science*, **11**, 73-91.
- TÖBLER, W. R. 1978, Migration fields. In Clark, W., and Moore, E., eds., *Population Mobility and Residential Change*, Studies in Geography No. 25 (Evanston: Department of Geography), pp. 215-232.
- TÖBLER, W. R. 1991, Visual evidence for urban potential fields. *Mappemonde*, **4**, 46-47.
- TOMLIN, C. D. 1990, *Geographic information systems and cartographic modelling* (Englewood Cliffs: Prentice Hall).
- UCCGIS. 1998, Extension to geographic representation, GIScience Research Agenda, <http://www.uccgis.org/research98.html>.
- US CONGRESS. 1964, Wilderness act. P.L. 88-577. 78 Stat. 890.
- WORBOYS, M. F. 1994, Object-oriented approaches to georeferenced information. *International Journal of Geographical Information Systems*, **3**, 385-399.
- WORBOYS, M. F. 1995, *GIS: a computing perspective* (London: Taylor & Francis).
- WORBOYS, M. F. 1999, Relational databases and beyond. In *Geographic Information Systems: Principles, Applications, and Management*, edited by P. Longley, M. F. Goodchild, D. Maguire, and D. Rhind (New York: John Wiley & Sons) 373-384.
- WORBOYS, M. F., HEARNshaw, H. H., and MACaURE, D. J. 1990, Object-oriented data and query modelling for spatial databases. *International Journal of Geographical Information Systems*, **4**, 369-383.