Chapter 5

THE EVOLUTION OF TERRITORIAL ORGANIZATION: A CASE STUDY

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INTRODUCTION

Territorial patterns form a dominant feature in the spatial organization of societies, as has been noted in *The Science of Geography* (NAS./NRC., 1965: 31ff), and the analysis of these patterns has attracted the attention of several disciplines and approaches. It is the purpose of this paper to review some of the major work in this field and to apply specific hypotheses to a territorial pattern in Southern Ontario.

The paper is divided into six sections. Following the introduction, studies which are primarily concerned with the ownership and control of territory will be discussed. Next, models of territorial division which are based upon economic notions of cost and efficiency will be described. The fourth section will offer a summary of research which is primarily concerned with quantitative description of the shape of territories. This will be followed by a section which presents the most recent advances in the analysis of territorial patterns. These advances tend to overlap the three earlier sections and thus they will be treated separately. Specifically, they introduce the time dimension and the decision-making process. Section six will also offer an integrated approach, a set of postulated relationships, and a case study.

OWNERSHIP AND CONTROL OF TERRITORY

A twofold classification of human spatial patterns has been offered by Haggett (1965: 31-60) in his discussion of movement. He distinguishes between "fields" with undefined and indeterminate boundaries, and "territories" with specific boundaries (Haggett, 1965: 40-55). Territory can be defined in terms of control and occupance, whereas field is defined in terms of movement, without the caveat of ownership. Thus, operationally, the extent of a territory can be defined by the set of points within it, while field is more simply defined by an individual's location coupled to a distance function. Fields have been extensively examined in the geographical literature using various forms of gravity model (for a summary, see Olsson, 1965: 43-70). The study of territory has not attracted the same degree of attention by geographers; however, it has interested workers in anthropology, biology, psychology and sociology. Lorenz (1963), for example, attempts to explain territorial control in terms of equilibrium conditions between adjacent groups. Like Ardrey (1966) he considers that the behavior of man is akin to the behavior of animals, and they argue that a balance between aggression and respect for neighbors appears to dominate the process which explains animal territorial divisions. They suggest that this principle can be applied to human societies, but, clearly, making operational these ideas presents many problems of definition and data collection. Anthropologists have based their study of territory on analysis of the social characteristics of the occupants, their value system, kinship ties and laws of inheritance. Piddington (1952: 287-318) summarizes this approach. The relationship between the inherent qualities of space and the practices of the occupants have been debated by Hippocrates, Thucydides, Bodin, Montesquieu, Ratzel and Huntington; a critique of their views is presented by Firth (1961: 38-61). Though recognizing the falsehoods of pure determinism, it is necessary to retain the notion that land ownership and the right to occupy territory are a fundamental part of human social organization. Sommer (1966) has analyzed territorial arrangements in terms of patterns of social interactions, examining territorial defense and individual privacy. Hall (1966) offers an appraisal of the means by which different societies use space. Space perception and sanctions are influential, as well as the level of technology of the group. The perception of the environment has been studied by Gould (1969: 31-44), Lynch (1960), and Boulding (1959), among others, and it is implicit in their work that understanding the form of territory requires comprehension of the viewpoint of those who are responsible for its delimitation. Sets of mental maps of perceived territory may help in the quest to define and measure the parameters which influence an individual's impression of space.

The role of sanctions on land ownership has been discussed by Tönnies (1965: 223), who argues that "... positive law ... has its roots in family life and is based upon land ownership."

In summary, ownership of space is strongly linked to the structure of society and the studies reviewed thus far have stressed the fundamental need of man to own territory or control space. It is not clear if this is an individual or a group need. The desire to occupy and possess portions of space, according to Stea (1965), is as pervasive among men as among their animal forebears. The reasons for this need are still under investigation.

ECONOMIC MODELS OF TERRITORY

The delimitation of territories, so as to minimize the internal variation among selected parameters, and to maximize the between-territory variation, is a traditional problem in geography. The problem has been treated both qualitatively and quantitatively (1969: 196). The problem of spatial division is, in a mathematical sense, extremely complex. Socio-economic parameters are usually discretely distributed over space, and though the mathematical analysis of large discrete systems has improved in the last decade, the methods are still cumbersome. Iterative numerical methods involving large quantities of computational time are typical of this area of study. A review and bibliography of these methods is presented by Scott (1969) and specific applications have been offered by Weaver and Hess (1963: 228-303), Silva (1965: 20-27), and Mills (1967: 243-255) for defining compact, equal population voting districts; Yeates (1963: 7-10) and Koenigsberg (1968: 465-475) for delimiting school districts so as to minimize pupil's travel time and Goodchild and Massam (1969: 86-94) have constructed a set of least-cost models of administrative areas under differing sets of constraints. In all cases the solutions depend upon the initial conditions and definition of the space, and upon a set of constraint equations.

One recent attempt to program a sequential allocation problem is discussed by Scott (1969). His model is more amenable to the real-world situation where money for the construction of a set of destinations is available over a period of time. The destinations Scott examines are schools, though his ideas could readily be applied to other public services. Schultz (1969: 291-307), for example, considers the definition of refuse collection districts through time, as demand and supply conditions change. A review of dynamic programming is presented by McKinnon (1969). The constraint equations normally describe the capacities of the destinations, and as such, define the size of the areas to be delimited.

In most of the studies discussed above, only passing reference is made to the criteria used for determining the size of the areas, yet it is known that there is a theoretical relationship between size and efficiency. This is described by the long-run average cost curve and considers the influence of economies of scale. With respect to service areas, a special type of territory, Isard (1960: 528) argues that given a production function and a function which defines the utilization of the service, the optimum size of area can be defined. However, it appears very difficult to determine these functions empirically. Thompson (1967: 257-270) suggests that more research is needed to determine the nature of economies of scale and the supply of public services. He is supported by Teitz (1968: 35-51) who claims that this is one of the weakest aspects of location theory. The problem of determining the most efficient size of a system has been tackled by workers from the field of business management. However, they are not only concerned with cost factors but also those which relate to control and management of the organization. Terrein and Mills (1955: 11-13) have reviewed the speculations of social thinkers concerning the effect of the size of a group upon the internal relationships. Ross (1951: 148-154), Adler (1960: 80-84), and Arrow (1964: 397-408) are critical of the economist's approach which argues for increasing the size of organizations to take advantage of economies of scale without consideration of the problems of decision-making. More recently Morris (1968), an industrial engineer, has developed a series of systems models of organizations which can be used to study the delegation of decision-making to various levels in the organization. He offers a series of specific hypotheses concerning the relationships among parts of a decentralized system. Morris claims that it will be possible to make positive statements about the optimum size of organizations only after the hypotheses have been tested and the exact functional relationships are known. It is clear that good empirical data are needed before any significant advances can be made in this field, and it is disheartening to read planning reports which make specific recommendations for defining service and administrative boundaries without a discussion of the ways to evaluate alternative locations (United Nations, 1962). Costs and benefits of different boundary locations can be determined by mathematical solutions to transportation problems. However, it is questionable whether these models serve great utility in either the academic or the planning world unless realistic cost functions and constraint equations are available.

QUANTITATIVE DESCRIPTIONS OF TERRITORIAL SHAPE

It has long been recognized that shape or pattern is related to process. Physical scientists have developed shape measures and observed the variation in values under different conditions and over varying lengths of time in an attempt to understand the forces which generate the patterns. Social scientists are trying to employ a similar strategy. This approach is partially dependent upon the objective measure of shape, and partially on the measurement of the forces operating. As shape changes through time it is argued that the functional efficiency will also change. Therefore, if it is possible to measure these attributes and determine the functional relationship, then it may be possible to construct predictive models of territorial divisions. Criticisms of this approach are offered by Massam (1970), who suggests that it is very difficult to infer why shapes change through time without knowledge of the decision-making process and an understanding of the rationale for dividing space into unique territories. Shape measures per se are useful to classify areas, but for little else unless they are related to the functional attributes of the area.

Haggett (1969: 70-73) has recently summarized several measures of shape used by geographers. He has used two of the indices, a Contact Number and a Shape Index, to evaluate the geometry of counties in Brazil. The study showed that, on the average, each county is in contact with about six others, suggesting to Haggett that "criticism of the hexagonal system proposed by Christaller and Losch as over theoretical, may have been too hasty." Other studies have compared patterns to shapes generated by random processes (Haggett, 1965: 308; Leopold and Langhein, 1962; Cole and King, 1968: 476). A comparison of indices for shapes generated under different processes, and the real-world situation, allows statements to be made about the relative level of organization of a system of territories. By examining the models through time, it is possible to see if the system approaches a greater degree of organization.

RECENT DEVELOPMENTS IN THE STUDY OF TERRITORY

Perhaps the most significant advances that have been made recently concern the addition of the time dimension and the decision-making process to the earlier models of territorial division, though there is still need for an integrated model of territorial evolution. It is hoped that some of the ideas presented in the latter part of this paper will help to rectify this situation. Some contemporary studies characterize relationships by expressing them in mathematical form, thus allowing easy manipulation. Solution sequences using different parameters can be readily obtained. Dacey (1969) and Casetti (n.d.) have proposed mathematical models of territorial evolution. The Dacey model depends upon probabilistic statements on the rise and demise of states, and Casetti has formulated sets of differential equations to express the rate of growth of revolutionary movements within and between states. Both models await empirical data before they can be tested, and suitable data are very difficult to obtain.

Territorial patterns evolve through time in response to decisions to change boundaries. These decisions are often based upon information on the functional efficiency of the territory. Of late a body of literature on learning theory has developed and some of this is closely linked to the field of information statistics and systems analysis. As the level of information in a system increases, it is suggested that the spatial pattern changes and it becomes less random. Conceptually, patterns achieve complete organization after an infinite time. However, it seems more appropriate to envisage the pattern as maintaining itself in the state of dynamic equilibrium as changes are slow, and there is a lag between needs, information and territorial reorganization. Boulding (1959) has drawn attention to the dynamic equilibrium condition at the state level. The level of order in a territorial pattern can be measured by comparing the actual shape or perceived shape to an optimum shape which represents the end-point on an organizational continuum (Massam, 1969). Through time the order changes first, in response to a changing end-point, and second, as the shape of the territory changes. The perceptual shape and the actual shape may vary over time. If the operational problems of pattern measurement can be overcome, then it is possible that some of the notions on organization, as envisaged by entropy, may provide a useful integrative framework within which shape, time, and information could be treated. Overviews of the utility of the concept of entropy are offered by Von Bertalanffy (1962: 1-20), Miller (1965), Olsson (1967: 13-45), and Wilson (1969: 225-233), and Medvedkov (1967: 165-168) has applied it to settlement pattern analysis, as have Semple and Golledge (1970: 157-160). Soja (1968: 39-57) has discussed it within the context of communications and territorial integration, and Berry and Schwind (1969: 5-14) have developed it to examine a diffusion process.

THE CASE STUDY

Introduction and Hypotheses

In this study, some of the ideas discussed in the earlier sections were applied to a particular spatial system. The Rural Operating Areas (ROA's) of the Ontario Hydro-Electric Power Commission were set up in 1948 to service electrical distribution in rural areas of

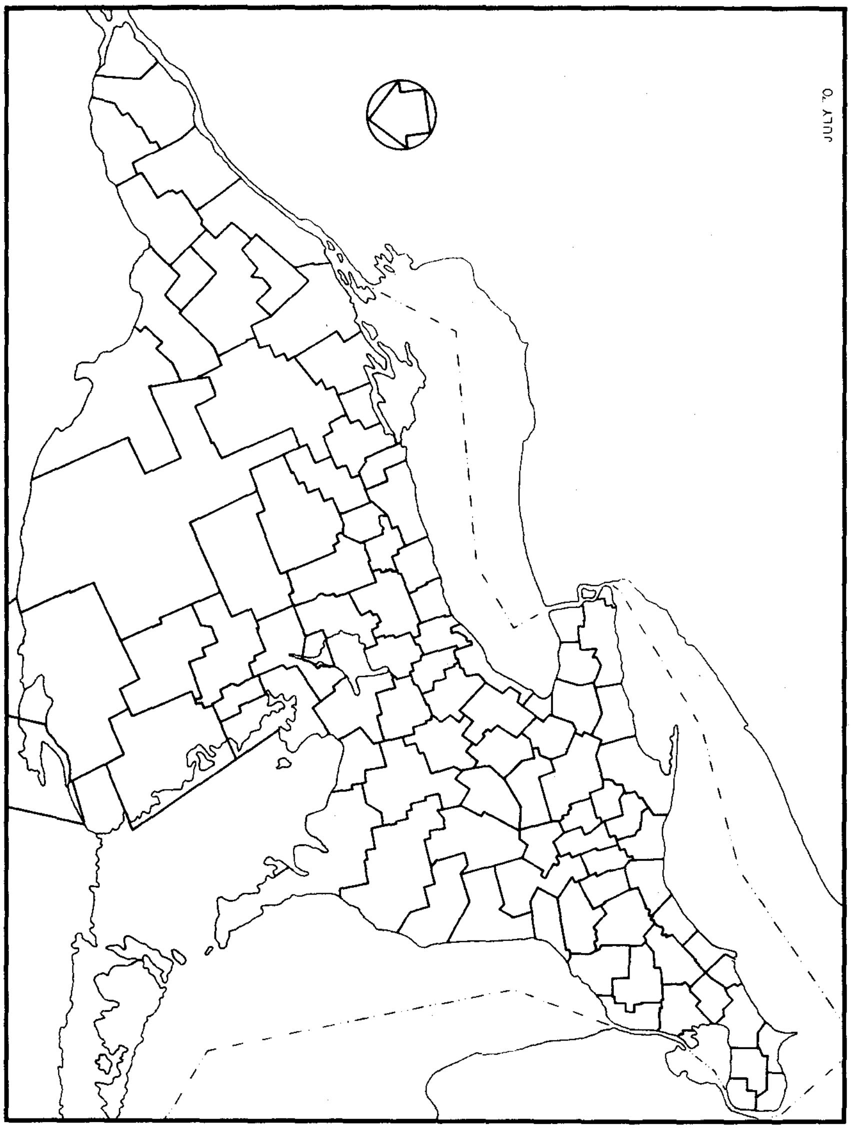

Figure 1. Ontario hydro rural operating areas, 1948.

the province. In the southern part of the province, the study area, there were initially 86 areas (Figure 1). In every year of operation at least one boundary change or amalgamation has taken place. In 1967 (Figure 2) there were 53 areas. Table 1 summarizes the changes that have taken place.

Each area is served from a service center located within the boundary. The service center is assumed to be a point location. It serves as the base for operations within the area and as an equipment storage depot.

The data base was particularly suitable for several reasons. Decisions regarding boundary and service center location are made by regional managers, who are assumed to have equal access to information and comparable levels of business acumen. In 1948 there were eight managers, but this number has since been reduced to five. Because the population served is purely rural, assumptions of isotropicity of population density and transportation are comparatively tenable.

Following the discussion in Section 5, the methodology used in the analysis isolated that part of the change through time which represented a trend toward an optimal arrangement. Although changes have taken place in the precise form of the optimal arrangement, due for example to modifications in the numbers of centers and distribution of customers, such changes are ignored by the form of analysis used. Two methods were used, first, a study of the trend in the average value of relevant indices evaluated for each year of the study period, and second, a study of the individual decisions made, combining them, by means of discriminant analysis, into a function representing the decision-maker's average criterion for elimination or amalgamation of an area.

The hypotheses regarding the optimal arrangement relate to the discussion of Section 3. Specifically, there are three. The first objective relates to the relative scale of each area. From a sophisticated point of view, it embodies the concept of economy of scale, which must result in areas of equal operational cost. From a point of view probably more relevant to the decision-maker, the areas all perform the same function, and so will operate best when of roughly equal scale. How scale is perceived will be determined empirically.

The other two objectives concern the geometry of an area once its scale has been determined. Minimization of operating cost requires

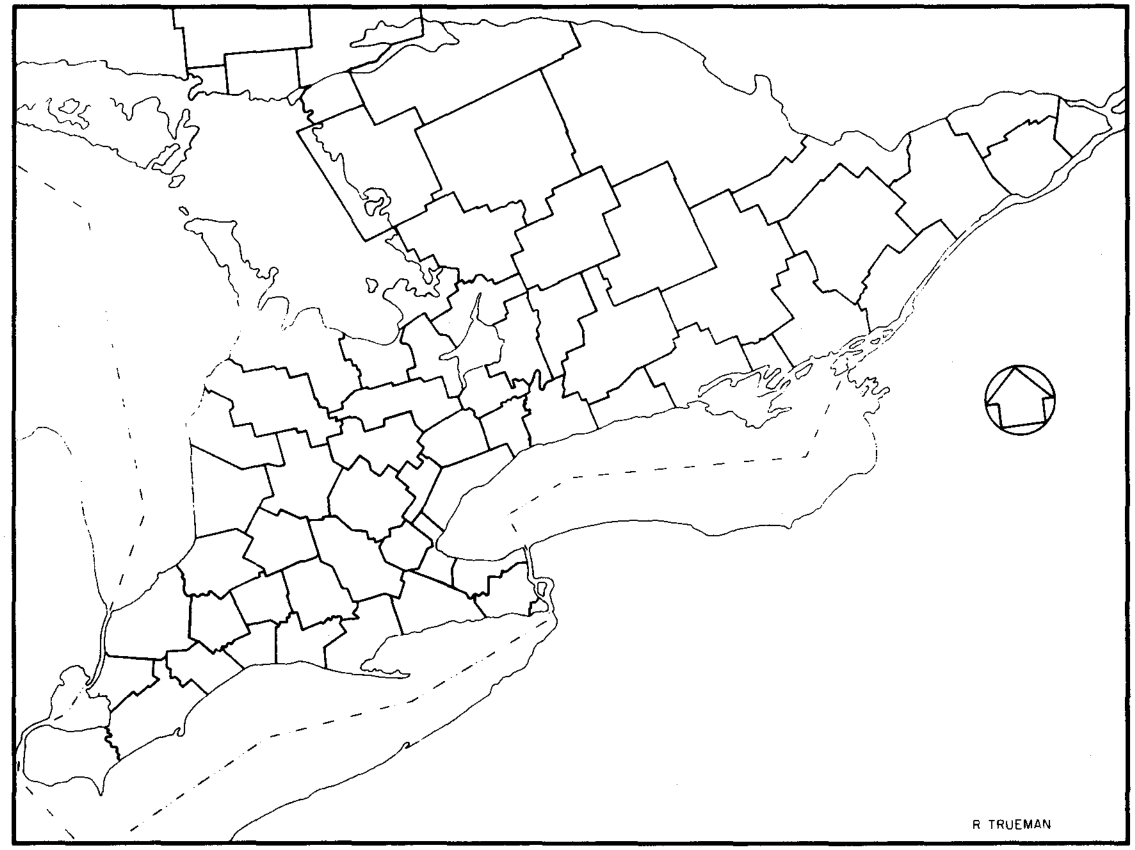

Figure 2. Ontario hydro rural operating areas, 1967.

TABLE 1
SUMMARY OF RURAL OPERATING AREAS[a]

I	II	III	IV	V	VI	VII	VIII
1948	86	3,122	6.197	317.1	1,048	1,452	0
1949	86	3,533	22.065	346.6	1,157	1,645	0
1950	89	3,467	19.674	351.1	1,195	1,809	0
1951	89	3,456	16.261	379.0	1,274	1,959	0
1952	91	3,442	15.974	391.1	1,315	2,093	7
1953	92	3,385	14.870	395.6	1,338	2,290	5
1954	89	3,454	14.425	416.8	1,405	2,524	0
1955	89	3,468	14.458	428.6	1,433	2,772	0
1956	89	3,447	14.107	439.2	1,439	2,850	1
1957	88	3,483	14.167	446.3	1,471	3,078	3
1958	85	3,607	14.351	465.7	1,498	3,288	2
1959	84	3,646	14.400	478.9	1,521	3,486	2
1960	82	3,732	14.405	498.8	1,570	3,733	4
1961	78	3,882	14.818	518.6	1,617	3,693	4
1962	78	3,871	14.851	523.0	1,607	3,861	5
1963	74	4,072	15.514	555.9	1,687	4,224	5
1964	71	4,213	15.698	579.7	1,748	4,382	10
1965	65	4,585	18.339	630.9	1,877	4,879	11
1966	58	5,008	20.117	693.3	2,011	5,551	10
1967	53	5,386	22.147	745.6	2,112	6,191	–

a. I = year of operation; II = number of rural operating areas; III = mean physical area (arbitrary units); IV = variance of physical area (arbitrary units); V = mean of miles of transmission line; VI = mean of number of farm customers; VII = mean of number of nonfarm customers; VIII = number of eliminations of rural operating areas.

that each area be as compact as possible, and that the service center be centrally located. Operationalization of these concepts is discussed below.

Maps are available for each year in the study period showing the boundaries of the ROA's and the locations of the service centers (Ontario Hydro-Electric Power Commission, 1948-1967). The numbers of miles of line, farm customers and nonfarm customers are also indicated for each area. The boundary of each ROA was coded as a series of straight line segments by defining two three-digit coordinates for each vertex. A Benson-Lehner digitizer was used, giving an accuracy of ±½ mile in the east-west direction and ±¼ mile in the north-south direction. All centers and boundaries were coded in this way on a standard grid. Throughout the study each area has been treated as a homogeneous two-dimensional shape by assuming that the demand for service is equally distributed throughout the area. All computing was carried out by teletype on the PDP-10/50 time-sharing facility at the University of Western Ontario.

Operational Considerations

The concept of Moment of Inertia, a measure which originates in the field of pure mechanics, forms the basis for the spatial measures used in the analysis. It measures the spatial relationship between a point and an area, and embodies the notion of the dispersion of the area about the point. The more distant the area, or parts of the area, from the point, the higher the Moment of Inertia. The Moment of Inertia of an area about a point was used to measure dispersion. For a continuous area, Moment of Inertia is defined as

$$I_x = \int_a r^2 da$$

that is, the sum over the area of each minute segment of the area multiplied by the square of the distance r separating it from the point x.

Moment of Inertia forms the basis for a convenient measure of the compactness of an area. The Parallel Axes theorem (Temperley, 1953: 30) states that:

$$I_x = I_G + Ah^2$$

where I_G is the Moment of Inertia about the center of gravity G, A is the total area of the shape, and h is the distance separating point x from the center of gravity. The center of gravity is that point about which the area would balance if suspended. Since the term Ah^2 must always be positive, it can be seen that the minimum value of I_x occurs when the point x is coincident with the center of gravity. Thus the Moment of Inertia of any shape about the center of gravity is minimum. Of all shapes, the circle has the minimum dispersion, and hence the minimum Moment of Inertia. The measure of shape, S, used in this study is derived from these considerations. S is defined as

$$S = \frac{A^2}{2\pi I_a}$$

This expresses the ratio of the Moment of Inertia of a circle of area A about its center, to the Moment of Inertia of the shape under examination, also of area A, about its own center of gravity. S varies from 0.0 for an infinitely dispersed area, for example a long thin strip, to 1.0 for a perfectly compact area. Most of the shapes encountered in this study have a shape index between 0.50 and 0.95.

As a measure of dispersion, the Moment of Inertia about a point also serves as a measure of the effort involved in distributing a service to all parts of the area from that point. But using the square of distance, excessive weighting is given to outlying points. Alternatively, the first moment $\int_a rda$ might be used. But this quantity has no simple minimum. The mathematical problems that would be involved in operationalizing the first moment are almost insurmountable at this time. Further, the ordinal methods of analysis used below render the Moment of Inertia a reasonable approximation to the first moment.

An index of spatial efficiency, E, is defined for each ROA by

$$E = \frac{I_G}{I_x}$$

where x is the actual service center of the ROA. This index measures the extent to which the service center has been located from the optimum point as regards provision of service at minimum cost. Alternatively, it is a crude measure of the minimum possible cost of service divided by the actual cost. Its minimum value is 0.0, for a service center located well away from the ROA, and 1.0 for a service center located at the center of gravity. In the study this index varied between 0.3 and 1.0.

To calculate these indices, each area was first divided into an exhaustive set of mutually exclusive triangles, using the vertices of the area as the vertices of the triangles. The area, location of center of gravity and Moment of Inertia about the center of gravity were calculated for each triangle as follows:

For a triangle with vertices (X_A, Y_A), (X_B, Y_B) and (X_C, Y_C) and sides of length a, b, and c.

$$\text{Area } \Delta = \sqrt{s(s-a)(s-b)(s-c)}$$

where s = (a + b + c)/2.

$$\text{Moment of Inertia} = \frac{\Delta(a^2 + b^2 + c^2)}{36} = I_\Delta$$

$$\text{Center of Gravity} = \left(\frac{X_A + X_B + X_C}{3}, \frac{Y_A + Y_B + Y_C}{3}\right) = (x_g, y_g)$$

Next, the entire area is found as $A = \Sigma\Delta$, and the center of gravity of the area as

$$X_G = \frac{\Sigma(x_g\Delta)}{\Sigma\Delta} \quad \text{and} \quad Y_G = \frac{\Sigma(y_g\Delta)}{\Sigma\Delta}$$

The Moment of Inertia of the entire area about its center of gravity is found by repeated application of the Parallel Axes theorem thus: $I_G = \Sigma\,(I_\Delta + \Delta d^2)$ where d is the distance between X_g, Y_g and X_G, Y_G. Finally, the Moment of Inertia of the entire area about any point X can be found by a further application of the Parallel Axes theorem

$$I_X = I_G + Ah^2$$

A, I_G, I_x, E and S were calculated for each ROA for each time period using these principles.

Analysis at the Aggregate Level

Both the shape index and the efficiency index are independent of scale. A large area can be just as inefficient to service as a small area, since efficiency is defined as a ratio between an optimum and the actual level. However, we might assume that a decision-maker has some idea of the importance of an area, based upon its size, or the number of customers it contains. Thus it is perhaps more important to rectify an inefficiency in a large area than in a small one, although the degree of inefficiency may be the same.

Accordingly, in studying the trends in the territorial organization through time, six different weights have been used for each ROA. They are:

(1) Equal weight (all areas the same)

(2) Miles of transmission line

(3) Number of farm customers

(4) Number of nonfarm customers

(5) Total number of customers (3 + 4)

(6) Physical area

These weights were applied to the measures of spatial organization, the efficiency and shape indices, to obtain twelve means for each year of the study period. The weightings represent an attempt to improve the simplistic geometrical measures of the ROA's by incorporating a series of variables which differentiate the areas on the basis of scale.

The trend in each mean was examined over the twenty-year period. A numerical evaluation of the consistency of trend was made by calculating the Spearman rank correlation coefficient. Thus only the consistency of the trend was measured, or the degree to which the means approached an unspecified monotonically increasing function of time. As there is no particular reason to expect a linear trend, the product-moment correlation coefficient could not be effectively interpreted. The rank correlations are given in Table 2. Their statistical significance was evaluated by reference to the null hypothesis that the twenty means were independently drawn from a

TABLE 2
SPEARMAN RANK CORRELATION COEFFICIENT OF WEIGHTED MEAN EFFICIENCY AND SHAPE INDICES AGAINST TIME, 1948-1967

Weighting	Efficiency Index	Shape Index
1. Equal (no weighting)	0.415[a]	0.224
2. Miles of transmission line	0.179	0.376
3. Number of farm customers	0.395[a]	0.012
4. Number of nonfarm customers	0.824[a]	–0.102
5. Total number of customers	0.705[a]	–0.048
6. Physical area	0.656[a]	0.483[a]
Physical area (1949-67)	0.902[a]	

a. Significant at 0.05 level.

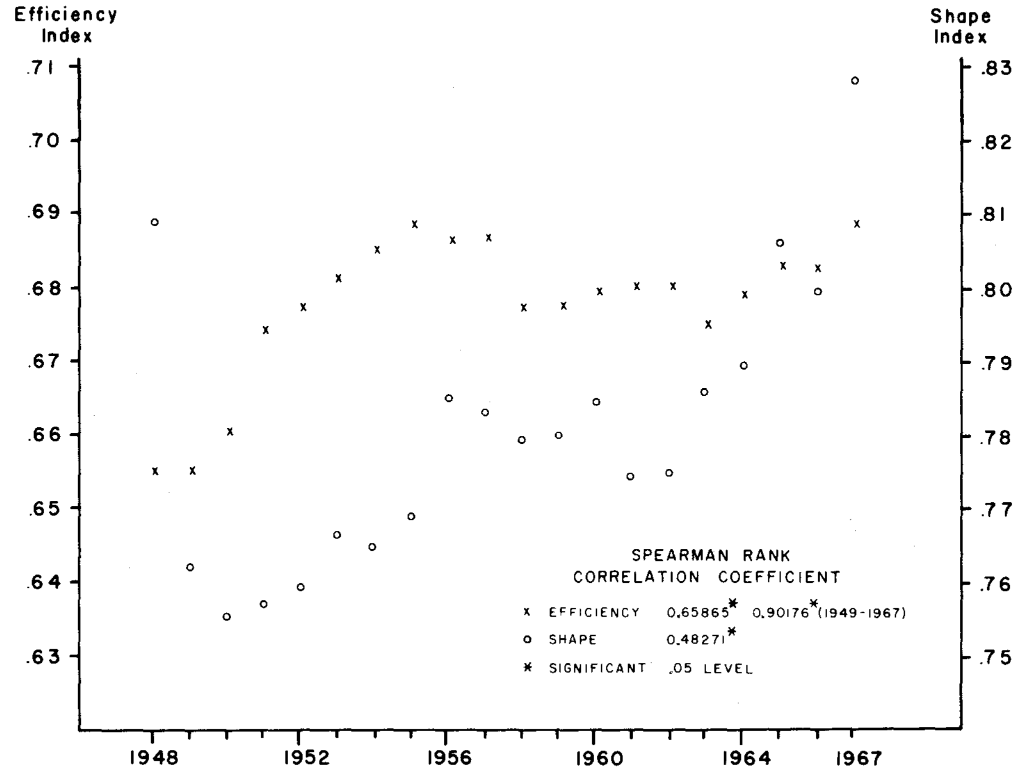

Figure 3. Efficiency and shape indices weighted by physical area, 1948-1967.

population of means showing no relationship in time. The test is nonparametric (Siegel, 1956: 284).

Five of the trends in the efficiency index are significant, but only one of the shape index trends. All of the significant trends are positive, suggesting that spatial reorganization has improved the level of spatial efficiency over time. Further, if the 1948 pattern is ignored, the most consistent trend is that of the indices weighted by area, as shown in Figure 3. This would suggest that area is the most common measure of importance used by the decision-makers, a most acceptable conclusion since area is the one quantity visually apparent from a map of the system. The fact that the only significant trend of shape is also in the area-weighted index is encouraging, and suggests that the decision-maker has incorporated shape into his criteria, albeit weakly. Analysis of trends in efficiency and shape has suggested a concern for these parameters on the part of the decision-maker. Amalgamations and boundary revisions have taken place in such a way that the system has become on the whole more compact and hence more efficient assuming that efficiency of servicing is directly related to the dispersion of the euclidean-space.

Both efficiency and shape are independent of scale, and so do not reflect any trend towards greater uniformity in the ROA's. But this also is an attribute of organization. So far, the level of organization within each ROA has been studied; attention is now turned to the relationship between ROA's.

Four measures of scale for each ROA will be used in this section:

(1) Physical area

(2) Miles of transmission line

(3) Number of farm customers

(4) Number of nonfarm customers

As the territorial pattern approaches greater uniformity, the variation between ROA's will decrease. The conventional measure of variation is the variance, defined as the mean square deviation of each item from the mean,

$$\sigma^2 = \sum \frac{(x - \bar{x})^2}{n}$$

TABLE 3
SPEARMAN RANK CORRELATIONS FOR INDEX OF VARIATION AGAINST TIME 1948-1967

Variable	Rank Correlation (R_s)
Physical area	−0.713[a]
Physical area (1949-1967)	−0.999[a]
Miles of transmission line	−0.690[a]
Number of farm customers	+0.571[a]
Number of nonfarm customers	−0.677[a]

a. Significant at 0.05 level.

To allow comparison between years with different numbers of ROA's of different average size, the variance was divided by the mean squared, that is

$$\frac{\sigma^2}{\bar{x}^2}$$

to give a scale-independent quantity, or index of variation. Trends in this index were then investigated for the four measures. Table 3 gives the rank correlations with time. This table shows that in all cases except one, there has been a trend towards greater uniformity between ROA's in those variables which relate to the magnitude of each operating area. Area again proves to be the most consistent, adding weight to earlier conclusions. Removal of the initial year, 1948, from the period of analysis changes R_s from −.712 to −.999. It should be noted that coverage of the southern part of the province was not complete until 1949; in this year several large areas were designated to cover the sparsely populated shield country. It is significant that the three variables, miles of line, farm and nonfarm customers, do not reveal any anomaly for 1948. Figures 4, 5, 6, and 7 show the trends of the index of variation for physical area, miles of transmission line, farm and nonfarm customers respectively.

Analysis at the Individual Level

The aggregate trends identified in the previous section are the result of a large number of changes and modifications to the system of rural operating areas through the study period. In this section the

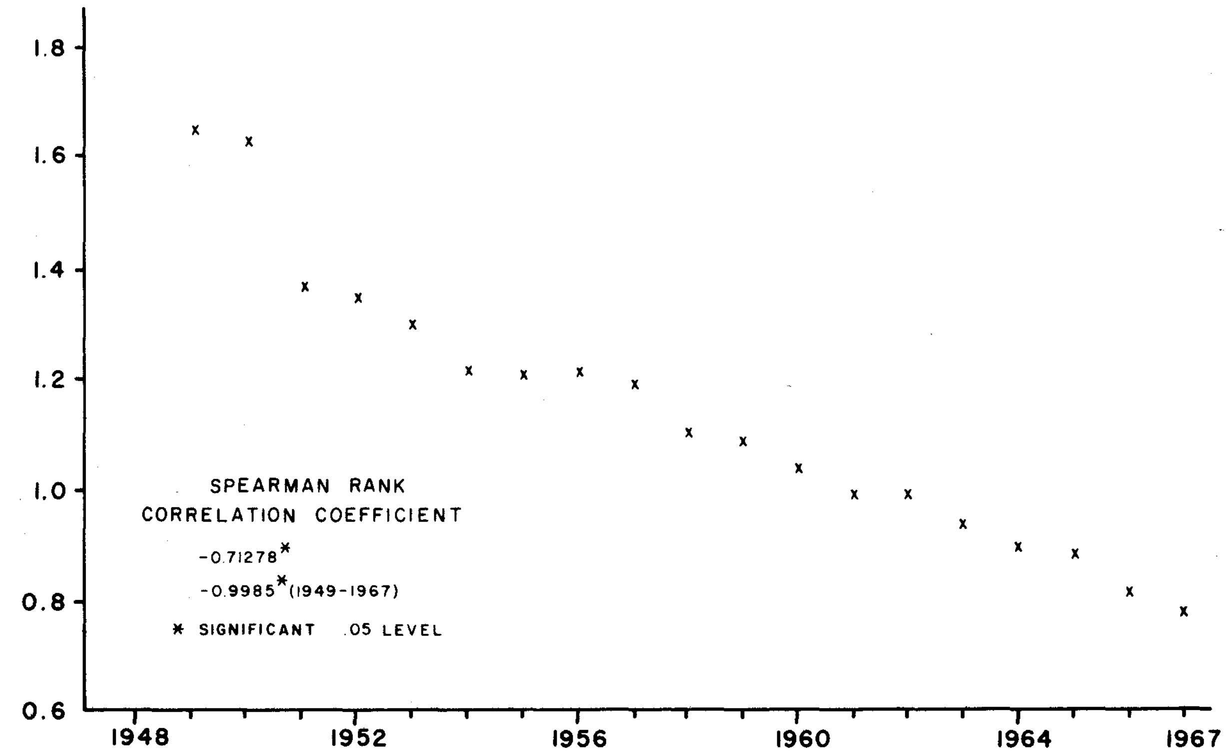

Figure 4. Trend of index of variation—physical area, 1949-1967.

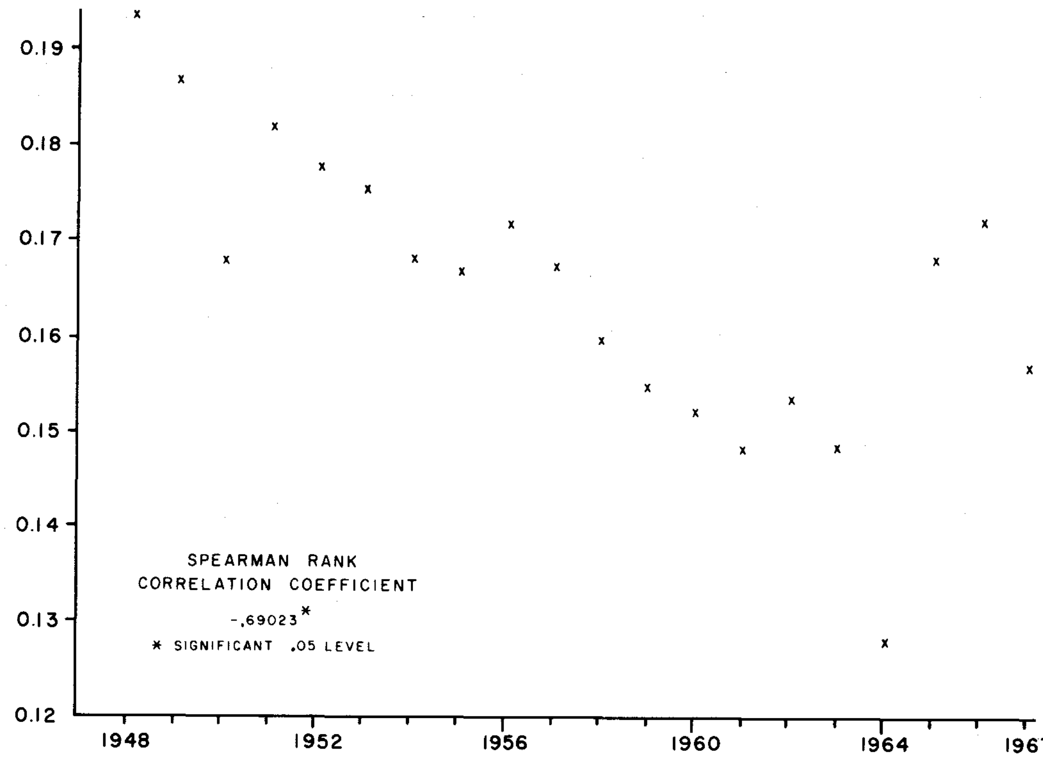

Figure 5. Trend of index of variation–miles of transmission line, 1949-1967.

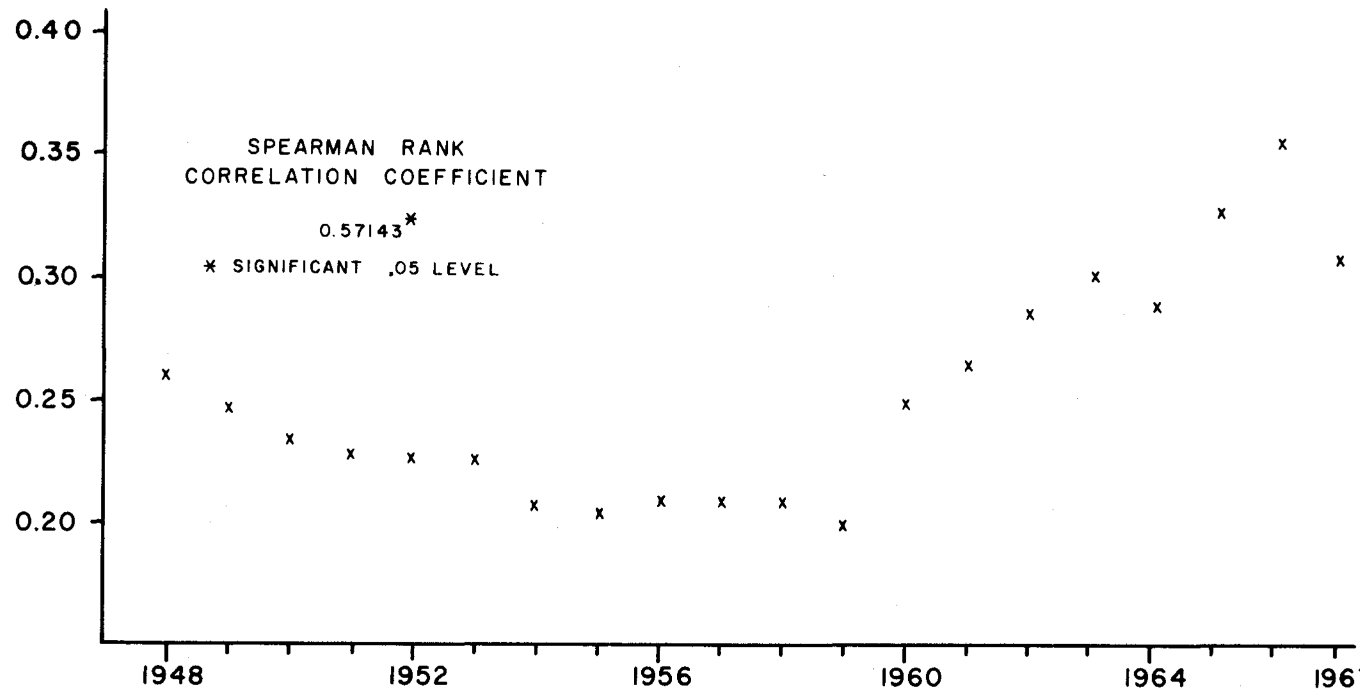

Figure 6. Trend of index of variation—number of farm customers, 1948-1967.

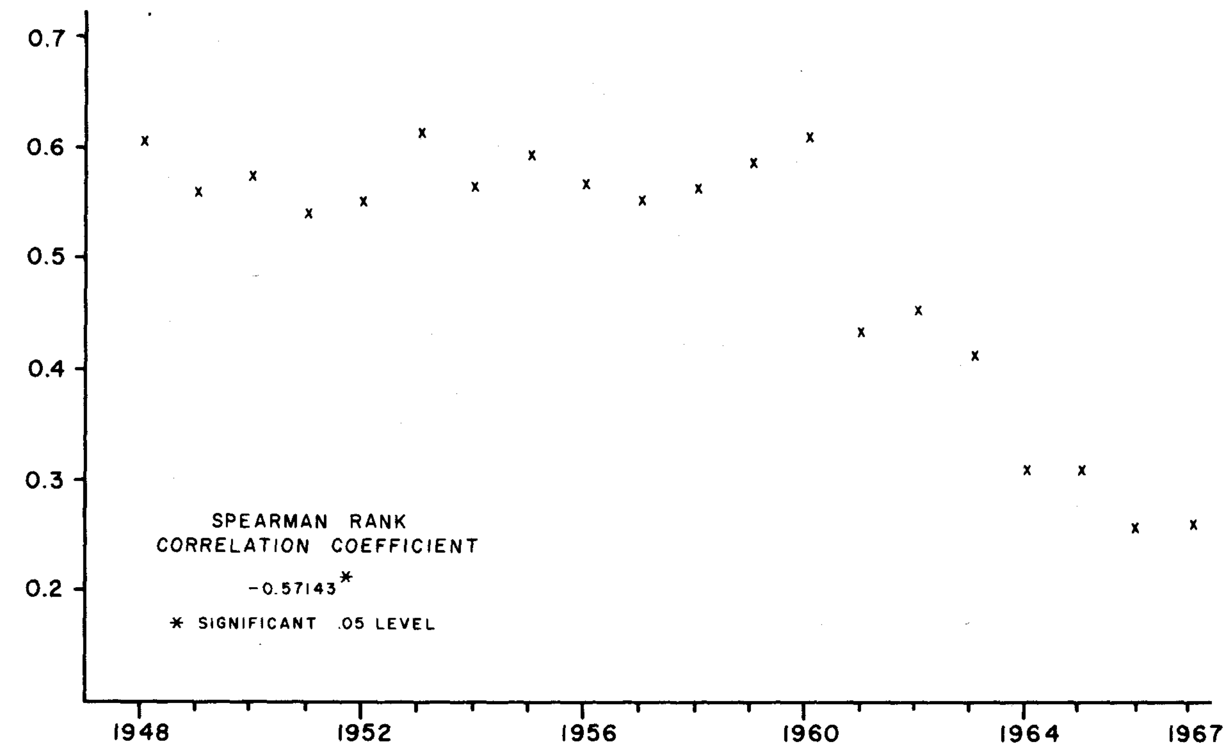

Figure 7. Trend of index of variation–number of non-farm customers, 1948-1967.

actual changes are classified according to the types of modification made, and analysed for the information they give about the decision-makers' criteria.

Five types of changes are identified as follows. Type A changes result when an area is abolished by being incorporated into a neighboring area. It is hypothesized that the criterion for this type of change is based upon the area's viability as an administrative unit. Type B changes are the converse in which an area engulfs a neighbor. It is hypothesized that these changes are prompted by the neighbor's properties and are unlikely to depend on the attributes of the area itself. Type C changes are those in which the location of the center is changed, while boundaries remain constant. Type D changes are the converse in which boundaries are revised without relocation of the center. Finally, type E changes result when previously unassigned areas are engulfed. This type occurred in 1948 when the northern part of the area was included in the system and has occurred on other occasions when boundaries have been redrawn to include urban areas (though the rural operating areas are not concerned with urban electrical distribution per se).

Table 4 shows the number of areas in existence in each year, together with the number of changes of various types made in that year, and the total number. Type C changes only occurred in 1952, but the other types are more evenly spread.

Correlations between the incidences of various types of changes in each year are shown in Table 5. Types A and B are highly correlated, as is to be expected since the presence of one implies the presence of the other. Types D and E are also correlated. There is, however, little correlation between the two groups. These regularities were formally identified by an R-mode principal component analysis of the data in Table 4. Two eigenvalues were greater than 1, and after varimax rotation the loadings were as given in Table 6. The factor scores enabled the years in the study period to be classified according to the strategy employed by the decision-makers. In the early period 1948-1956, changes were primarily by adjustment of boundaries and center locations; whereas, since 1957, changes have been made by abolition of certain areas and enlargement of others, with consequent attrition of the number of areas in existence.

To analyze these changes in a systematic fashion multiple discriminant analysis is used (King, 1970). Discriminant analysis is

TABLE 4
NUMBERS OF CHANGES

Year	Number of Areas	A	B	C	D	E	Total Change
1948	86	0	0	0	19	6	25
1949	86	0	0	0	4	0	4
1950	89	0	0	0	4	0	4
1951	89	0	0	0	1	0	1
1952	91	0	0	7	2	0	9
1953	92	4	3	0	11	1	19
1954	89	0	0	0	3	0	3
1955	89	0	0	0	4	0	4
1956	89	1	1	0	0	0	2
1957	88	2	3	0	0	0	5
1958	85	1	1	0	0	0	2
1959	84	4	1	0	5	0	10
1960	82	3	5	0	0	0	8
1961	78	0	0	0	0	1	1
1962	78	2	9	0	2	0	13
1963	74	3	5	0	1	1	10
1964	71	7	9	0	1	0	17
1965	65	7	6	0	2	0	15
1966	58	4	9	0	0	4	17
Total		38	52	7	59	13	169
Percentage		22.4	30.8	4.1	34.9	7.7	100.0

concerned with the problem of grouping, both in a predictive and in an explanatory sense. In the latter, it is concerned with finding a quality, or combination of qualities, of a set of objects that best distinguishes between the groups into which these objects have previously been placed; and in the former, it is concerned with the identification of criteria by which a new object may be classified into an existing set of groups.

In this paper, the technique is applied in a purely deductive sense, to answer the question: given a set of objects that have been placed

TABLE 5
CORRELATIONS BETWEEN INCIDENCES OF TYPES OF CHANGES

	A	B	C	D	E
A	1.000	0.7640	–0.2076	–0.1320	–0.0446
B	0.7640	1.000	–0.1951	–0.2772	0.0861
C	–0.2076	–0.1951	1.000	–0.0571	–0.1035
D	–0.1320	–0.2772	–0.0571	1.000	0.6339
E	–0.0446	0.0861	–0.1035	0.6339	1.000

TABLE 6
LOADINGS AFTER ROTATION

	Component 1	Component 2
1	0.8952	–0.1151
2	0.9149	–0.1245
3	–0.4535	–0.2349
4	–0.1588	0.8942
5	0.1251	0.8909

into groups, is it possible, by examination of the qualities of the objects, to discover the rules and criteria which led to the initial grouping?

Let the attributes of the objects be the variables $X_1, \ldots, X_n$, and let the number of groups be m. It is useful to distinguish two types of analysis, simple discriminant analysis for m = 2 and multiple for m > 2. The former problem is conventionally formulated in the following manner. Find a linear function of the variables $X_1 \ldots$, X_n, or $Y = \Sigma a_i X_i$, the discriminant function, such that the ratio of the difference between group means of this new variable to the standard error within the two groups is maximized (Fisher, 1936). The problem may be solved by analogy to the technique of multiple regression. If we consider Y to be the dependent, and $X_1, \ldots, X_n$ the independent variables, and assign Y a value of 1 for members of one group and 0 for members of the other, then the solution of the multiple regression problem is also a solution of the discriminant problem.

The method applied in this paper is due to the work of Cooley and Lohnes (1962), as implemented by Veldman (1967: 406). Conceptually multiple discriminant analysis is similar to principal component analysis, but whereas the latter extracts components, or eigenvectors, that explain a maximal amount of variance in the data, the former identifies vectors, or discriminant functions, that explain a maximal amount of between-group variance, or identifies directions along which the groups are as separated as possible. Both discriminant functions and principal components are orthogonal and are extracted in order of the amount of variance, or between-group variance, that they explain.

Six groups are identifiable in this study; the five change types A to E, and the group formed of those areas to which no change was made in each annual decision-making process. The latter group comprises some 1,394 instances.

Eight attribute variables were used, as follows:

X_1	Physical area	A
X_2	Moment of inertia about center of gravity	I_G
X_3	Moment of inertia about central depot location	I_X
X_4	Efficiency index	E
X_5	Shape index	S
X_6	Number of miles of transmission line in area	ML
X_7	Number of farm customers	F
X_8	Number of nonfarm customers	NF

The area served from Renfrew and later Cobden was excluded since its physical area, an order of magnitude greater than any other, would have made an overwhelming contribution to the various groups to which it has been allocated at different times.

The first two roots extracted the greater part of the between-group variance. Further roots were so small as to be dominated by round-off errors in the calculations. The overall group differentiation was tested by an F test of Wilk's Lambda (Cooley and Lohnes, 1962: 125) and found to be significant at the 99.9 percent level. The significance of the two roots individually can be tested by a χ^2 test (Rao, 1952); both roots were significant at the 99.9 percent level.

Correlations between the two roots and the eight attribute variables, analogous to factor loadings, are given in Table 7. Root 1 has a high positive correlation with area, second moment about the center of gravity, second moment about the central depot, number of miles of line, and perhaps number of farm customers. For these reasons, this root is identified as the scale dimension, measuring the size of an area's operation, its level of activity.

Root 2 has large negative correlations with the second moment measures, I_G and I_X, and positive correlation with the shape index S. I_G and I_X by themselves measure the dispersion of customers around the center of gravity and central depot, respectively. Dispersion can be identified, to some extent, with the transport component in the cost of operation of the area. A high score on this dimension can be

TABLE 7
CORRELATIONS BETWEEN DISCRIMINANT ROOTS AND ATTRIBUTE VARIABLES

	Root 1	Root 2
Area	0.8208	–0.2051
Moment of inertia of center of gravity	0.7997	–0.7917
Moment of inertia, service center	0.7519	–0.8336
Efficiency index	–0.4506	–0.2091
Shape index	–0.4034	0.6965
Miles of line	0.8386	0.0361
Farm customers	0.5948	0.1072
Nonfarm customers	0.3179	–0.0727

identified, then, with a low cost of transportation and shape index close to 1.0.

The roots have been selected by the analysis so that the six groups are as separated as possible in the directions that the dimensions represent in eight-dimensional attribute space. The location of each group can be represented by its centroid, or the arithmetic means of the attribute variables for each group. In Figure 8, the six group centroids are plotted in terms of the two discriminant roots. This, then, is the plane on which the groups are maximally separated.

In order to identify the criteria which led to the selection of specific areas for specific types of change, the centroid of that group is compared with the centroid of the "no change," or 0 group. The most distant, possessing therefore the most distinct and consistent criterion, is the type A group, areas abolished and engulfed by neighbors. The location of this centroid indicates that the criterion was one of small scale, poor shape, and high dispersion, as hypothesized.

The centroid of group B lies high on the scale dimension, indicating that the criterion for this type of change was one of large scale. The position indicates that a piece of territory previously belonging to an abolished area was likely to be assigned to a larger neighbor. This result is unexpected; it was hypothesized that the decision to engulf a neighboring area would depend upon the attributes of the neighbor only. The B centroid would, according to this hypothesis, lie close to the 0 centroid.

Group C has only seven observations, all decisions being made in the same year, so it is most unlikely that this type of change was the

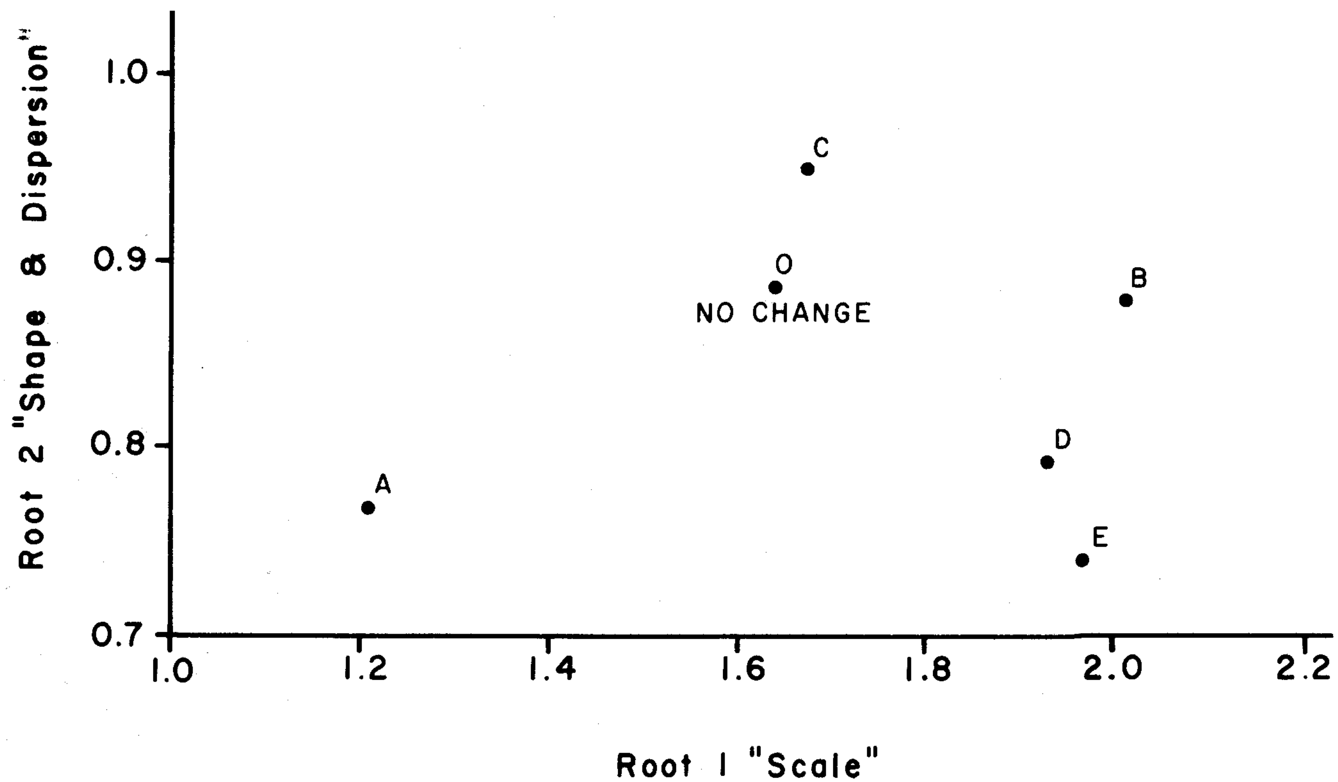

Figure 8. Six group centroids plotted in terms of the two discriminant roots.

result of the application of any consistent criterion to the attribute variables. The closeness of the centroid to the 0 centroid bears this out.

The D centroid is identified by high scale and low shape. Thus decisions to make type D changes, to revise boundaries, were based upon the shape index of each area, modifications being made to areas of poor shape, and in those cases where the scale of operation was large. This criterion reflects a feeling that it is more important to correct poor shape in large-scale administrative areas than small. The type E centroid lies close to D. Thus, if an unassigned piece of territory was to be allocated to a neighboring area, the allocation was likely to be made in such a way as to correct poor shape in a large area.

CONCLUSIONS

This study has attempted to make operational some methods for evaluating the evolution of the spatial pattern of service areas. With reference to rural operating areas in Ontario, three trends have been identified: a tendency for the shape of each area to approach optimum compactness; and a trend toward uniformity in the scale of each area. The analysis suggests that decisions to amalgamate areas are based primarily upon a visual impression of the map of ROA's and the objective of making the set of ROA's as uniform and efficient as possible. However, using discriminant analysis, several conclusions can be made regarding the decisions behind the evolution of this spatial system. First, two major groups of change types can be identified, the A, B group and the D, E group. The latter were predominant during the period 1948-1956, and reflect a policy of improvement of efficiency without creation or abolition of operating areas. Beginning in 1956, however, A and B types predominate, as a result of a policy decision to reduce the number of areas. The analysis reveals that this has been achieved by elimination of small, inefficient areas, thus at the same time revising the general efficiency of the system.

Type C changes, relocation of service centers, were only made in one year. These changes are costly since the capital investment in a service center is considerable, and must also be politically less desirable than boundary relocation, or perhaps even outright abolition.

The administrators involved in these decisions show a considerable sensitivity to derived measures of spatial efficiency, in addition to their concern for the direct scale quantities, as is indicated by the size of the loadings of the two discriminant roots on these variables.

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